ANALYSIS OF A NONLINEAR EQUALIZER BASED ON HIGH-ORDER STATISTICS

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ABSTRACT: In this paper, we analyse the equalizer structure when the transmitted signal is iid. Conditions for recovering an iid signal are established, which points out limitations of the classical linear equalizer and suggests the application of nonlinear structures. A simple nonlinear equalizer, previously proposed in [1], is then compared to the classical CMA equalizer.

1. INTRODUCTION

Equalization of a baseband received signal is generally carried out by means of a linear adaptive FIR filter, according to figure 1, which also depicts the classical mathematical model of a communications system. In this figure, {h(n)} and {c(n)} are the impulse responses of the channel and of the equalizer, with length N and L, respectively. {v(n)} is the impulse response of the global linear system comprising the channel and the equalizer, and it is defined as the convolution of {h(n)} and {c(n)}: $\{v(n)\} = \{h(n)\} * \{c(n)\}$ (1)

where "*" stands for the convolution operator.

In figure 1, we suppose that the equalizer also includes the automatic gain control and y(n) is the equalized signal.

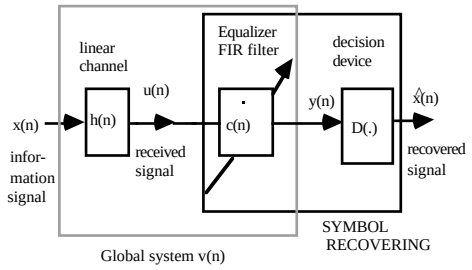

Figure 1 - Communications system model and traditional equalizer

In this context, equalization is generally analysed as a deconvolution process, so that coefficients of the FIR filter are set in order to achieve the well-known "zero-forcing" condition [2]:

$\{v(n)\} = \pm\{\delta(n-d)\}$ (2)
$y(n) = \pm x(n-d)$ (3)

Where $\delta(n)$ is the Dirac function, d is the equalization delay and "±" represents the signal ambiguity.

Although the previous theoretical framework for analysing equalization has been used for a long time, it presents some drawbacks. Condition (2) is very stringent, since it requires that the intersymbol interference (ISI) be zero, so that the equalized signal y(n) be exactly equal to the transmitted one x(n). According to [3], when the signal x(n) is iid and the channel model is nonminimum-phase, condition (2) cannot be achieved unless the equalizer length L is infinite. As a consequence, condition (2) cannot be verified for a practical situation where L is finite.

On the other hand, one could propose another equalization condition by considering that x(n) is an iid signal. In this context, equalization is considered perfect when the equalized signal y(n) is iid. Such a condition will be called "iid condition".

In fact, there are some applications where generation of an iid signal y(n) is very important: for example, in the case of time-synchronization signals. These signals are used as a standard to assure accurate transmission rates and clock syncrhonism all over a multi-user communications system. This is specially important in the case of mobile communications. Such signals are generally binary random sequences, as for the Global Positioning System (GPS) [4]. In this case, the more a GPS signal approaches an iid sequence, more robust it is with respect to random interferences associated to other signals, which assures high reliability [4]. As a consequence, equalization of a time-synchronization signal is constrained to recover the best iid signal as

possible, in order to assure the system synchronism requirements.

In this paper, we analyse the equalizer structure from the perspective of the iid condition. We are mainly interested in providing new insights in order to propose better equalizers for mobile communications systems. In the next section, we present the theoretical framework of the paper and we recall some important definitions. Then, in section 3, we apply some recent results of [5] in order to establish conditions for recovering an iid signal y(n). Motivated by these conclusions, we recall in section 4 the nonlinear equalizer proposed in [1], which is experimentally compared to the classical CMA equalizer in section 5.

It should be noticed that [6] studied deconvolution as a kind of entropy maximization, in the context of geophysical prospecting. Besides, [7] already analysed deconvolution as a kind of "redundancy reduction", in the context of audio signal deconvolution. Both articles are based on information theory. In this way, although these articles are related to ours, the present paper relies on a different theoretical framework (HOS) and is developed in the context of communications systems.

2. THEORETICAL FRAMEWORK AND DEFINITIONS

From now on, we suppose that the following hypothesis hold.

(H1) x(n) is iid, zero-mean, real (PAM modulation) and strictly stationary. In this case, the power of x(n) ($E\{x^2(n)\}$) is finite.
(H2) The density function $f_X(x)$ is nongaussian and symmetric. $f_X(x)$ is also continuous, so that it admits a characteristic function $\phi_X(\omega)$.
(H3) The channel {h(n)} is linear, causal, invariant, and doesn't present any zero exactly on the unit circle. Besides, {h(n)} is nonminimum-phase.
(H4) The signal-to-noise ratio is high, so that the additive noise may be neglected.
(H5) The global system {v(n)} is time-invariant.
(H6) The automatic gain control is perfect, so that the power of y(n) is the same as the power of x(n).
(H7) The equalizer {c(n)} is a FIR filter of infinite length ($L \to \infty$)

Notice that such hypothesis, which are unrrealistic and stringent, are frequently supposed in the classical equalization litterature (for example, see [3]).

We now recall the concept of an iid signal in the context of HOS. According to [5], a "pure iid signal" x(n) is defined by the following equations:

$$C_{mX}(\tau_1,\tau_2,...,\tau_{m-1}) = R_{mX}\,\delta(\tau_1,\tau_2,...,\tau_{m-1});$$
$$\forall\, m \in \mathbb{N}^*;\ m \geq 2 \qquad (4)$$

$$S_{mX}(\omega_1,\omega_2,...,\omega_{m-1}) = R_{mX};$$
$$\forall\, \omega_i;\ \forall\, m \in \mathbb{N}^*;\ m \geq 2 \qquad (5)$$

Where:

$C_{mX}(\tau_1,\tau_2,...,\tau_{m-1})$: m-th order cumulant of x(n); $\tau_i \in \mathbb{N};\ \forall\, i = 1, ..., m-1$.

$\mathbb{N}$: Set of natural numbers.

$\mathbb{N}^*$: Set of nonzero natural numbers.

R_{mX}: Real constant.

$\delta(\tau_1,\tau_2,...,\tau_{m-1})$: Generalized Kronecker symbol

$$\delta(\tau_1, \tau_2, \ldots, \tau_{m-1}) \stackrel{\Delta}{=} \begin{cases} 1; & \text{if } \tau_1 = \tau_2 = \ldots = \tau_{m-1} = 0 \\ 0; & \text{otherwise} \end{cases} \qquad (6)$$

$S_{mX}(\omega_1,\omega_2,...,\omega_{m-1})$: m-th order polyspectrum of x(n).

If equations (4)-(5) are valid only for (q-1) different values of m, we then define a "qth-order iid signal" x(n) by the following equations.

$$C_{mX}(\tau_1,\tau_2,...,\tau_{m-1}) = R_{mX}\,.\delta(\tau_1,\tau_2,...,\tau_{m-1});$$
$$\forall\, m \in \mathbb{N}^*;\ 2 \leq m \leq q \qquad (7)$$

$$S_{mX}(\omega_1,\omega_2,...,\omega_{m-1}) = R_{mX};$$
$$\forall\, \omega_i;\ \forall\, m \in \mathbb{N}^*;\ 2 \leq m \leq q \qquad (8)$$

When the equalization is perfect in the zero-forcing (ZF) sense, then equation (3) is valid and the cumulants of y(n) are given by (4). In this way, y(n) is a pure iid signal. So, the ZF equalization condition leads to the iid condition. Notice, however, that the ZF condition requires an infinite-length linear equalizer. As a consequence, for a practical situation, where L is finite, one should ask whether a linear equalizer is capable of generating a pure iid signal y(n). In fact, by supposing a particular nongaussian density $f_X(x)$, [5] demonstrated that the linear equalizer cannot recover a q-th order iid signal y(n), even in the case of a minimum-phase channel. The same limitation has been demonstrated in the case of nonlinear channels [5].

3. ANALYSIS OF A NONLINEAR EQUALIZER STRUCTURE

The following theorem provides conditions for generating a pure iid signal y(n). This result was originally obtained for a general filtering

application [5]; here we provide its interpretation in the context of equalization.

Theorem 1 : "Suppose that (H1)-(H4) and (H6) hold. Suppose also that the global system comprising the channel and the equalizer is nonlinear, so that the equalized signal y(n) is related to the transmitted signal x(n) by the following equation:

$$y(n) = a_0 + \sum_{i=1}^{L_V} a_i . x^i(n) \qquad (9)$$

Where a_i is a real constant, bounded, for all i = 0, 1, ..., L_V; $L_V \geq 2$.

Let $y^c(n)$ be the zero-mean version of the signal y(n):

$$y^c(n) = y(n) - E\{y(n)\} \qquad (10)$$

Then,

If x(n) is q-th order iid, with $q = l.L_V$ =>

$$y^c(n) \text{ is l-th order iid} \qquad (11a)$$

If x(n) is a pure iid signal ($q \to \infty$) =>

$$y^c(n) \text{ is pure iid} \qquad (11b)$$

$$q \in \mathcal{N}^*;\ q \geq 6;\ l \in \mathcal{N}^* \qquad (12a)$$

$$L_V \in \mathcal{N}^*;\ 2 \leq L_V \leq (q/3) \qquad (12b)"$$

Notice that the only constraint imposed on L_V is (12b). In fact, inequalities (12a)-(12b) are supposed in order to assure a l-th order iid signal $y^c(n)$ so that l is greater than 2. In this way, the case $L_V = 1$ is not considered.

When the equalizer is linear, then the global system (channel+equalizer) is the linear system {v(n)} (fig. 1). In this way, one must have $L_V = 1$ in (9). However, such a value of L_V doesn't assure constraint (12b).

Theorem 1 and the discussions of the last section motivate us to replace the linear equalizer of fig. 1 by a nonlinear structure. This is also desirable to enhance the system performance for other situations where the linear equalizer is inefficient, e.g., spectral-null channels, nonlinear channels and low signal-to-noise ratios [2].

4. A SIMPLE NONLINEAR EQUALIZER

Figure 2 depicts the equalizer proposed in [1].

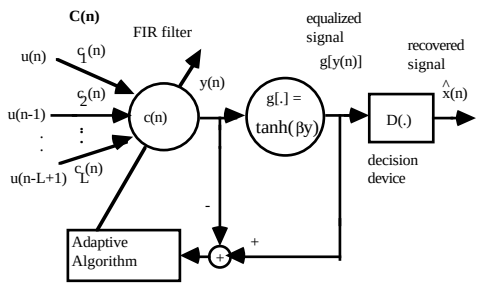

Figure 2 - Nonlinear equalizer [1]

This equalizer is composed of a linear FIR filter {c(n)}, followed by a nonlinearity g[.]. Notice that, for the classical linear equalizer the equalized signal is y(n), the output of the linear filter. In our solution, the equalized signal results from the nonlinear filtering by g[.]. We choose g[.] as the hyperbolic tangent function, in order to minimize the following cost function:

$$J = \frac{1}{2} E\left\{ \left(g[y(n)] - y(n) \right)^2 \right\} \qquad (13)$$

$$g[y] \overset{\Delta}{=} \tanh(\beta y) = \left(1 - e^{-2\beta y} \right) \Big/ \left(1 + e^{-2\beta y} \right) \qquad (14)$$

where β is a positive real constant.

The associated gradient algorithm which minimizes the cost function (13) is given by:

$$\mathbf{C}(n+1) = \mathbf{C}(n) + \\ + \{\mu . f(y(n)\}. (g[y(n)] - y(n). \mathbf{U(n)} \qquad (15)$$

$$f(y) \overset{\Delta}{=} 1 - \frac{dg[y]}{dy} \qquad (16)$$

where:

C(n) equalizer weight vector, at time n;

$$\mathbf{C}(n) = \left[c_1(n) \ldots c_L(n) \right]^T$$

$c_i(n)$ i-th equalizer coefficient at time n;

L equalizer length;

μ step-size of the algorithm;

U(n) vector of L sample measurements received at the channel output.

$$\mathbf{U(n)} = \left[u(n) \ldots u(n - L + 1) \right]^T$$

It has been demonstrated in [8] that, supposing (H1)-(H7), an uniform density $f_X(x)$ and that $0.75 \leq \beta \leq 1.0$, all the local minima of cost function (13)-(14) lead to the perfect ZF equalization condition (2)-(3). This justifies the choice of g[y] in (14) and also attests that the learning of this blind equalizer is efficient, or at least more efficient than the training of other more

complex nonlinear structures, such as neural networks.

5. SIMULATIONS

We now present some simulation results to illustrate the behaviour of the proposed nonlinear equalizer and to compare its performance to that of the classical CMA Bussgang algorithm [9], which employs a linear filter. Such comparison is carried out in terms of residual bit-error-rate (BER) in steady-state. This means that all equalizers parameters are set in order to achieve the same convergence time. A receiver is considered to provide good equalization as the steady-state BER is under 5%. Monte-Carlo type simulations were carried out with 30 input data sequences x(n), which are 2PAM signals. For the proposed nonlinear structure and for CMA, we considered a 30-tap linear filter. In both cases, filter coefficients were initialized according to the "center-spike" procedure [9]. We considered two channel models.

The first channel is a nonminimum-phase MA system with a pole exactly on the unity circle, given by $H(z) = 0.5 - 1.5z^{-1} + z^{-2}$, and a signal-to-noise ratio of 30 dB. Notice that, in principle, a linear structure is not able to equalize such channel due to the spectral null. Figure 3 depicts the results. Graphics 1 and 2 present respectively the CMA and the nonlinear equalizer performance.

Figure 3- Comparison of CMA (plot 1) and the nonlinear equalizer of [1] (plot 2) for a spectral-null channel $H(z) = 0.5 - 1.5z^{-1} + z^{-2}$, SNR = 30 dB.

The second channel is a simple nonlinear model . The signal $u_1(n)$, at the input of the equalizer, is given by $u_1(n) = u(n) + 0.2u^2(n) + b(n)$, where b(n) is the additive noise and u(n) is the output of the linear channel given by $H(z) = 0.35 + 0.87z^{-1} + 0.35z^{-2}$. The signal-to-noise ratio was fixed to 10 dB. Notice that it is a low value for communications applications. Figure 4 depicts the results.

Figure 4- Comparison of CMA (plot 1) and the nonlinear equalizer of [1] (plot 2) for a simple nonlinear channel $u_1(n) = u(n) + 0.2u^2(n) + b(n)$, SNR = 10 dB.

These simulation results point out that the nonlinear structure proposed in [1] is more effective than the classical linear solution in the case of spectral-null channels, simple nonlinear channels and for low signal-to-noise ratio situations.

6. CONCLUSION

In this paper, equalization has been analysed in the context of recovering an iid transmitted signal. Our results are independent of equalizer learning. Simulation results point out that a simple nonlinear equalizer is more effective than the classical CMA solution.

7. REFERENCES

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