EXPONENTIAL STABILITY FOR THE NONLINEAR SCHRÖDINGER EQUATION WITH LOCALLY DISTRIBUTED DAMPING

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This work is concerned with the stabilization of defocusing nonlinear Schrödinger equations (dNLS)

$$\begin{cases} i\,\partial_t y + \Delta y - |y|^p\, y + i\, a(x)\, y = 0 \quad \text{in } \Omega \times (0,T), \\ y(0) = y_0 \quad \text{in } \Omega, \end{cases} \tag{1}$$

where Ω is a general domain, and a is a nonnegative function that may vanish on some parts of the domain. We first study (dNLS) on a bounded domain Ω in $\mathbb{R}^N$ with boundary Γ of class C^2. In this case we assume $y = 0$ on Γ. Then, we extend the theory to unbounded domains in the particular cases $\Omega = \mathbb{R}^N$ and Ω being an exterior domain.

Assumptions. The power index p can be taken as any positive number. The nonnegative real valued function $a(\cdot) \in W^{1,\infty}(\Omega)$ represents a localized dissipative effect.

If Ω is a bounded domain we will assume that a satisfies the geometric condition $a(x) \geq a_0 > 0$ (for some fixed $a_0 \in \mathbb{R}_+$) for a.e. x on a subregion $\omega \subset \Omega$ that contains $\overline{\Gamma(x^0)}$, where $\Gamma(x^0) = \{x \in \Gamma : m(x) \cdot \nu(x) > 0\}$. Here, $m(x) := x - x^0$ ($x_0 \in \mathbb{R}^N$ is some fixed point), and $\nu(x)$ represents the unit outward normal vector at the point $x \in \Gamma$. On the other hand, if Ω is the whole space, we assume $a(x) \geq a_0 > 0$ in $\mathbb{R}^N \backslash B_{R'}$, where $B_{R'}$ represents a ball of radius $R' > 0$. We assume the same if Ω is an exterior domain: $\Omega := \mathbb{R}^N \setminus \mathcal{O}$, where $\mathcal{O} \subset\subset B_{R'}$ being $\mathcal{O}$ a compact star-shaped obstacle, namely, the following condition is verified: $m(x) \cdot \nu(x) \leq 0$ on Γ_0, where Γ_0 is the boundary of the obstacle $\mathcal{O}$ which is smooth and associated with Dirichlet boundary condition as in Lasiecka et al. [3]. In this case, the observer x_0 must be taken in the interior of the obstacle $\mathcal{O}$. Regarding to the localized dissipative effect, we consider $a(x) \geq a_0 > 0$ in $\Omega \backslash B_{R'}$. Moreover, in all cases, we assume that the damping coefficient $a(\cdot)$ satisfies:

$$|\nabla\, a(x)|^2 \lesssim a(x),\ \forall\, x \in \Omega. \tag{2}$$

The main goal of the present paper is to achieve stabilization with the (natural) weaker dissipative effect $ia(x)y$ instead of relying on a strong dissipation such as $ia(x)(-\Delta)^{1/2}a(x)y$. It will turn out that the assumption (2) enables us to avoid using such strong dissipation. We want to achieve stabilization in all dimensions $N \geq 1$ and for all power indices $p > 0$. For this purpose, we first construct approximate solutions to problem by using the theory of monotone operators. We show that these approximate solutions decay exponentially fast in the L^2-sense by using the multiplier technique and a unique continuation property. Then, we prove the global existence as well as the L^2-decay of solutions for the original model by passing to the limit and using a weak lower semicontinuity argument, respectively.

In addition, we implement a precise and efficient algorithm for studying the exponential decay established in the first part of the paper numerically. Our simulations illustrate the efficacy of the proposed control design.

References

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