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NUMERICAL REPRESENTATIONS FOR SOLVING A CLASS OF THE SYSTEM OF SECOND ORDER TWO-POINT NONLINEAR BOUNDARY VALUE PROBLEMS

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ABSTRACT:
The aim of the present research is to solve a nonlinear system of second order two-point boundary value problem by using a rapid Adams technique as well as well–known explicit Runge-Kutta numerical scheme. The designed methodology of both the numerical schemes is also presented. Fourdifferent examples of the system of nonlinear two-point boundary value problems have been discussed and the numerical results obtained from the above mentioned two techniques have been compared with the exact solutions that depict the correctness of the model as well as the numerical schemes. The achieved result details of the system of two-point boundary value problems is presented in the form of tables as well as numerical configurations.

Keywords: System of two-point boundary value problems, explicit Runge-Kutta method, Adams technique, nonlinear, numerical solutions

RESUMEN

El objetivo de la presente investigación es resolver un sistema no lineal de problemas de valor límite de dos puntos de segundo orden mediante el uso de una técnica rápida de Adams, así como un conocido esquema numérico explícito de Runge- Kutta. Tambien se presenta la metodología diseñada de ambos esquemas numéricos. Se ha discutido cuatro ejemplos diferentes del sistema de problemas de valor límite de dos puntos no lineales y se han comprobado los resultados numéricos obtenidos de las dos técnicas mencionadas anteriormente con las soluciones exactas que representan la corrección del modelo, así como los esquemas numéricos. Los detalles de los resultados obtenidos del sistema de problemas de valor de límite de dos puntos se presentan en forma de tablas y configuraciones numéricas

PALABRAS CLAVES: Sistema de problemas valor límite de dos puntos, método explícito de Runge - Kutta, técnica de Adams, soluciones numéricas no lineales

1 INTRODUCTION

The study of the differential equations is very important for the researchers all the time because of its vast uses and huge implementations in many areas of the science, technology and engineering. History is full of the literature of the differential equations, which have many types e.g., delay differential equations, pantograph differential equations, perturbed differential equations, singular differential equations, fuzzy differential equations, fractional order differential equations, fractional differential equations and many more. The implementations of the differential models have been seen in many biological models e.g., heat conduction model in human heat, HIV infection model, SIR infection model, prey-predator biological model, corneal shape model of eye surgery, human physiological models, etc. All the above models are famous due to the

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significance and importance andhave been solved in different times using different numerical, analytical and deterministic techniques. have their own importance and perfections.

This study is related to the system of the two-point boundary value problems (STP-BVPs), that has unique significance and impotence for the researchers due to a verity of applications in the field of technology, engineering and science [1-2].To see the significance of this important system, no one can deny the implementations of this model. These important STP-BVPs is used in various physical type of models, for example, chemical reaction models, transfer of heat, fluid dynamics specially in fluid viscosity, defection of beams and in the solutions of the optimal control problems, reaction diffusion models, etc. By keeping the view on the uses of these famous STP-BVPs in many areas, author is interested to solve and produce the numerical results by using the two techniques named as Adams and explicit Runge-Kutta techniques. For solving the single TP-BVP, many analytical and numerical techniques have been used. Some of them are Adomian decomposition and extended form of the Adomian decomposition technique [3–8], the transformed form named as differential transformation method[9], a famous variational iteration method [10], a technique based on perturbation methods [11–13], homotopy asymptotic scheme [14], and many more [15–20].

The system of differential equations is also much important and has been presented in many applications. The general form of the STP-BVPs is written as:

$$\frac{d^2u}{dt^2} f_1(t, u, v), \qquad a \quad t \quad b$$
$$\frac{d^2v}{dt^2} f_2(t, u, v). \tag{1}$$

The corresponding boundary conditions are

$$u(a) \; A_1, \quad u(b) \; B_1,$$

$$v(a) \; A_2, \quad v(b) \; B_2.$$

The aim of the present model is to present the numerical results by using the Adams and explicit Runge-Kutta techniques. This model has never been solved before using these two techniquetogether. The remaining part of the paper is described as follows: Section 2 narrates the numerical technique, the numerical outcomes are provided in Section 3 and conclusions along with future research directions are listed in the final Section.

2 NUMERICAL METHODS

To solve the system of two-points boundary value problems, predictor-corrector Adams numerical method along with explicit Runge-Kutta technique.

2.1 Predictor-corrector numericalAdams scheme

To solve and find the numerical measures of the system of two points boundary value problems,The formulation of corrector- predictor numerical scheme is used and complete further in two steps.

Step 1: The approximate valuesof the prediction analysis is accomplished as:

Step 2: To calculate the numerical values of the correction is capable with the equal number of prediction as:

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$$\frac{du}{dt} f_1\ t, u, v, \qquad u\, t_0 \quad u_0 \quad .$$
$$\frac{dv}{dt} f_2\ t, u, v, \qquad v\, t_0 \quad v_0 \qquad (2)$$

For the generalized Adams-Bashforth numerical scheme by using the predictor-corrector is shown as:

$$D_{n\,1}\ u_n \quad \frac{3}{2} f_1 h\, t_n\ , u_n \frac{1}{2} f_1\, h\, t_{n\,1}, u_{n\,1} \quad .$$
$$_{n\,1} \quad v_n \quad \frac{3}{2} f_2 h\, t_n\ , v_n \frac{1}{2} f_2\, h\, t_{n\,1}, v_{n\,1} \qquad (3)$$

Two-steps Adams-Moulton corrector is given as:

$$u u_{n\,1} \quad _n \quad \frac{1}{2} \quad f_{1} h\, t_{n\,1}, D_{n\,1} \quad _n\ h\, t_n, u_n$$
$$v_{n\,1}\ v_n \quad \frac{1}{2} f_2\, h\, t_{n\,1}, D_{n\,1} \quad h \quad t_n, v_n \qquad (4)$$

Thepredictorcorrector 4-stepsmethodis as follows:

$$D_{n\,1}\ u_n \quad \frac{1}{24} 55h\, t_n, u_n \quad 59 f_1\ t_{n\,1}, u_{n\,1} \quad 37 f_1\ t_{n\ 2}, u_{n\ 2} \quad 9 f_1\ t_{n\,3}, u_{n\ 3}$$
$$_{n\,1} \quad v_n \quad \frac{f_2}{24} 55h\, t_n, v_n \quad 59 f_2\ t_{n\,1}\ , v_{n\,1} \quad 37 f_2\ t_{n\ 2}, v_{n\ 2} \quad 9 f_2\ t_{n\,3}, v_{n\ 3}\ . \qquad (5)$$

The Adams-Bashforth Moulton 4-step method is given as:

$$u_{n\,1} u_n \quad \frac{1}{24} 9\, h\, t_{n\,1}, D_{n\,1} \quad 19 f_1\ t_n, u_n \quad 5 f_1\ t_{n\,1}, u_{n\,1} \quad f_1\ tx_{n\ 2}, u_{n\ 2}$$
$$_{n\,1} \quad v_n \quad \frac{2}{24} 9\, h\, t_{n\,1}, D_{n\,1} \quad 19 f_2\ t_n\ , v_n \quad 5 f_2\ t_{n\,1}, v_{n\,1} \quad f_2\ tx_{n\ 2}, v_{n\ 2} \qquad (6)$$

2.2 Explicit Runge-Kutta numerical technique

To solve the two-point boundary value problems, the explicit Runge-Kutta method is performed. The general form of explicit Runge-Kutta method is presented as:

$$u_{n\,1} \quad u_{n\,1} \quad \sum_{j\,1}^{s} b_j k_j, k_{21} \quad h\, t_n, u_n\ ,$$
$$v_{n\,1} \quad v_n \quad f_2 \sum_{j\,1}^{s} b_j k_j\ , k_{12}\, h\, t_n, v_n,$$
$$k_{21} h \quad x \quad c f_{2\,1}, u_n\ f\ a_{1\,21\,11} k$$
$$k_{22} \quad h\, x_n \quad c_2 f_2, u_n \quad f_1\ a_{22} k_{21}\ , \qquad (7)$$

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$$k_{31} \quad h\, x_n \quad c_3\, f_1, u_n \quad f_1 \;\; a_{31} k_{11} \quad a_{32}\, k_{21}$$

$$k_{32} \quad h\, x_n \quad c_3\, f_2\,,\, v_n \quad f_2 \;\; a_{31}\, k_{11} \quad a_{32}\, k_{21} \quad \ldots.$$

$$k_{s1} \quad h\, x_n\, c_s\, f_1,\, u_n\, f_1\, a_{s\,1} k_{11}\, a_{s\,2}\, k_{21} \ldots a_{ss\,1}\, k_{s1\,1}\, h$$

$$k_{s\,2} \quad x_n\, c_s\, f_2\,,\, u_n\, f_1\, a_{s\,2}\, k_{21}\, a_{s\,2}\, k_{22} \ldots a_{ss\,1} k_{s\,2\,1}\,.$$

3 RESULTS AND DISCUSSIONS

In this section, the numerical investigations for nonlinear STW-BVPs with stiff/non-stiff limitations is presented. Four problems of the designed model have been numerically investigated and the numerical outcomes are drawn in Tables for each problem.

Problem1: Consider the nonlinear system of two-point boundary value problem is:

$$3\frac{d^2u}{dt^2}\,(u^2 \quad v^2) \quad 2 \quad 36t^2 \; 2t^8, \quad 0 \quad t \quad 1$$

$$3\frac{d^2v}{dt^2}\,(u^2 \quad v^2)\; 2 \quad 36t^2 \; 2t^8. \qquad (8)$$

The boundary conditions are written as:

$$u\,(0) \quad 1,\; u(1) \;\; 2$$

The exact solutions of the above system are $[1 \quad t^4, 1 \;\; t^4]$.

Table 1: Comparison of Exact and numerical solutions for u(t) based on Example 1

t	Exact Solution	Adams Method	Explicit RK	Absolute Error
0	1.0000000	1.0000000	1.0000000	0
0.05	1.0000063	1.0000063	1.0000063	8.44E-15
0.1	1.0001000	1.0001000	1.0001000	1.67E-14
0.15	1.0005063	1.0005062	1.0005063	2.51E-14
0.2	1.0016000	1.0016000	1.0016000	3.33E-14
0.25	1.0039063	1.0039062	1.0039063	4.17E-14
0.3	1.0081000	1.0081000	1.0081000	5.02E-14
0.35	1.0150063	1.0150062	1.0150063	5.91E-14
0.4	1.0256000	1.0256000	1.0256000	6.86E-14
0.45	1.0410063	1.0410062	1.0410063	7.75E-14
0.5	1.0625000	1.0625000	1.0625000	8.68E-14
0.55	1.0915063	1.0915062	1.0915063	1.06E-13
0.6	1.1296000	1.1296000	1.1296000	1.30E-13

0.65	1.1785063	1.1785062	1.1785063	1.44E-13
0.7	1.2401000	1.2401000	1.2401000	1.54E-13
0.75	1.3164063	1.3164062	1.3164063	1.77E-13
0.8	1.4096000	1.4096000	1.4096000	1.99E-13
0.85	1.5220063	1.5220062	1.5220063	2.09E-13
0.9	1.6561000	1.6561000	1.6561000	2.63E-13
0.95	1.8145063	1.8145062	1.8145063	3.46E-13
1	2.0000000	2.0000000	2.0000000	2.56E-13

Table 2: Comparison of Exact and numerical solutions for v(t) based on Example 1

	tExact	Method	Explicit RK	Absolute Error
0	1.0000000	1.0000000	1.0000000	0
0.05	0.9999938	0.9999937	0.9999937	2.03E-09
0.1	0.9999000	0.9999000	0.9999000	1.62E-09
0.15	0.9994938	0.9994938	0.9994937	1.72E-09
0.2	0.9984000	0.9984000	0.9984000	1.82E-09
0.25	0.9960938	0.9960938	0.9960937	1.92E-09
0.3	0.9919000	0.9919000	0.9919000	2.02E-09
0.35	0.9849938	0.9849938	0.9849937	2.12E-09
0.4	0.9744000	0.9744000	0.9744000	2.22E-09
0.45	0.9589938	0.9589938	0.9589937	2.33E-09
0.5	0.9375000	0.9375000	0.9375000	2.43E-09
0.55	0.9084938	0.9084938	0.9084937	2.53E-09
0.6	0.8704000	0.8704000	0.8704000	2.63E-09
0.65	0.8214938	0.8214938	0.8214937	2.74E-09
0.7	0.7599000	0.7599000	0.7599000	2.84E-09
0.75	0.6835938	0.6835938	0.6835937	2.95E-09
0.8	0.5904000	0.5904000	0.5904000	3.06E-09
0.85	0.4779938	0.4779938	0.4779937	3.18E-09
0.9	0.3439000	0.3439000	0.3439000	3.30E-09
0.95	0.1854938	0.1854938	0.1854937	3.43E-09
1	0.0000000	0.0000000	0.0000000	3.57E-09

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Problem2: Consider the nonlinear system of two-point boundary value problem involving trigonometric functions:

$$3\frac{d^2u}{dt^2}(u^2 \quad v^2) \quad 3\cos t \; 2\cos^2 t, \; 0 \quad t \quad 1$$

$$3\frac{d^2v}{dt^2}(u^2 \quad v^2) \; 3\cos t \; 2\cos^2 t. \qquad (9)$$

The boundary conditions are written as:

$u(0) \;\; 1,$

$v(0) \qquad 1, \; v(0) \;\; 0.$

The exact solutions of the above system are $[\cos t, \;\; \cos t]$.

Table 3: Comparison of Exact and numerical solutions for u(t) based on Example 2

t	Exact Solution	Adams Method	Explicit RK	Absolute Error
0	1.0000000	1.0000000	1.0000000	0
0.05	0.9987503	0.9987502	0.9987503	2.21E-08
0.1	0.9950042	0.9950041	0.9950042	2.16E-08
0.15	0.9887711	0.9887711	0.9887711	2.14E-08
0.2	0.9800666	0.9800666	0.9800665	2.14E-08
0.25	0.9689124	0.9689124	0.9689123	2.14E-08
0.3	0.9553365	0.9553365	0.9553363	2.14E-08
0.35	0.9393727	0.9393727	0.9393725	2.14E-08
0.4	0.9210610	0.9210610	0.9210608	2.22E-08
0.45	0.9004471	0.9004471	0.9004470	2.90E-08
0.5	0.8775826	0.8775825	0.8775825	2.92E-08
0.55	0.8525245	0.8525245	0.8525245	2.32E-08
0.6	0.8253356	0.8253356	0.8253356	2.18E-08
0.65	0.7960838	0.7960838	0.7960838	1.82E-08
0.7	0.7648422	0.7648422	0.7648422	1.76E-08
0.75	0.7316889	0.7316889	0.7316889	1.74E-08
0.8	0.6967067	0.6967067	0.6967067	1.73E-08
0.85	0.6599831	0.6599831	0.6599831	1.65E-08
0.9	0.6216100	0.6216100	0.6216100	1.63E-08
0.95	0.5816831	0.5816831	0.5816831	1.57E-08
1	0.5403023	0.5403023	0.5403023	1.56E-08

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Table 4: Comparison of Exact and numerical solutions for v(t) based on Example 2

	tExact	Method	Explicit RK	Absolute Error
0	-1.0000000	-1.0000000	1.0000000	0
0.05	-0.9987503	-0.9987502	-0.9987503	2.21E-08
0.1	-0.9950042	-0.9950041	-0.9950042	2.16E-08
0.15	-0.9887711	-0.9887711	-0.9887711	2.14E-08
0.2	-0.9800666	-0.9800666	-0.9800665	2.14E-08
0.25	-0.9689124	-0.9689124	-0.9689123	2.14E-08
0.3	-0.9553365	-0.9553365	-0.9553363	2.14E-08
0.35	-0.9393727	-0.9393727	-0.9393725	2.14E-08
0.4	-0.9210610	-0.9210610	-0.9210608	2.22E-08
0.45	-0.9004471	-0.9004471	-0.9004470	2.90E-08
0.5	-0.8775826	-0.8775825	-0.8775825	2.92E-08
0.55	-0.8525245	-0.8525245	-0.8525245	2.32E-08
0.6	-0.8253356	-0.8253356	-0.8253356	2.18E-08
0.65	-0.7960838	-0.7960838	-0.7960838	1.82E-08
0.7	-0.7648422	-0.7648422	-0.7648422	1.76E-08
0.75	-0.7316889	-0.7316889	-0.7316889	1.74E-08
0.8	-0.6967067	-0.6967067	-0.6967067	1.73E-08
0.85	-0.6599831	-0.6599831	-0.6599831	1.65E-08
0.9	-0.6216100	-0.6216100	-0.6216100	1.63E-08
0.95	-0.5816831	-0.5816831	-0.5816831	1.57E-08
1	-0.5403023	-0.5403023	-0.5403023	1.56E-08

Problem3: Consider the nonlinear system of two-point boundary value problem involving exponential functions:

$$3\frac{d^2u}{dt^2} (u^2 \quad v^2) \quad e^{2t^4} e^{2t^4} \qquad 36t^2 e^{t^4} \; 48t^6 e^{t^4}, \; 0 \quad t \quad 1$$

$$3\frac{d^2v}{dt^2} (u^2 \quad v^2) \; e^{2t^4} e^{2t^4} \qquad 36t^2 e^{t^4} \; 48t^6 e^{t^4}. \tag{10}$$

The boundary conditions are written as:

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$u(0)\ \ 1, \quad u(1)\ \ e$

$v(0)\ \ 1, \quad v(1)\ \ \frac{1}{e}.$

The exact solutions of the above system are $[\,e^{t^4}, e^{t_4}\,]$.

Table 5: Comparison of Exact and numerical solutions for u(t) based on Example 3

t	Exact Solution	Adams Method	Explicit RK	Absolute Error
0	1.0000000	1.0000000	1.0000000	0
0.05	1.0000063	1.0000062	1.0000063	4.15E-08
0.1	1.0001000	1.0000999	1.0001000	8.83E-08
0.15	1.0005064	1.0005062	1.0005064	1.30E-07
0.2	1.0016013	1.0016011	1.0016013	1.70E-07
0.25	1.0039139	1.0039137	1.0039139	2.07E-07
0.3	1.0081329	1.0081327	1.0081329	2.43E-07
0.35	1.0151194	1.0151191	1.0151194	2.81E-07
0.4	1.0259305	1.0259302	1.0259306	3.18E-07
0.45	1.0418586	1.0418583	1.0418586	3.55E-07
0.5	1.0644945	1.0644941	1.0644947	3.93E-07
0.55	1.0958236	1.0958232	1.0958237	4.30E-07
0.6	1.1383729	1.1383725	1.1383733	4.66E-07
0.65	1.1954304	1.1954299	1.1954310	4.97E-07
0.7	1.2713763	1.2713758	1.2713764	5.21E-07
0.75	1.3721876	1.3721870	1.3721906	5.44E-07
0.8	1.5062152	1.5062146	1.5062153	5.62E-07
0.85	1.6854056	1.6854050	1.6854076	5.68E-07
0.9	1.9272613	1.9272608	1.9272614	5.60E-07
0.95	2.2580605	2.2580599	2.2580674	5.41E-07
1	2.7182818	2.7182813	2.7182818	5.09E-07

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Table 6: Comparison of Exact and numerical solutions for v(t) based on Example 3

	tExact	Method	Explicit RK	Absolute Error
0	1.0000000	1.0000000	1.0000000	0
0.05	0.9999938	0.9999937	0.9999938	1.79E-08
0.1	0.9999000	0.9999000	0.9999000	2.97E-08
0.15	0.9994939	0.9994938	0.9994939	4.29E-08
0.2	0.9984013	0.9984012	0.9984013	5.39E-08
0.25	0.9961014	0.9961013	0.9961014	6.16E-08
0.3	0.9919327	0.9919326	0.9919327	6.69E-08
0.35	0.9851058	0.9851057	0.9851058	7.37E-08
0.4	0.9747249	0.9747248	0.9747249	7.89E-08
0.45	0.9598231	0.9598230	0.9598231	8.45E-08
0.5	0.9394131	0.9394130	0.9394131	9.02E-08
0.55	0.9125556	0.9125555	0.9125556	9.53E-08
0.6	0.8784467	0.8784466	0.8784467	1.00E-07
0.65	0.8365188	0.8365187	0.8365188	1.05E-07
0.7	0.7865492	0.7865491	0.7865492	1.10E-07
0.75	0.7287633	0.7287632	0.7287631	1.13E-07
0.8	0.6639158	0.6639156	0.6639158	1.14E-07
0.85	0.5933290	0.5933289	0.5933289	1.13E-07
0.9	0.5188710	0.5188709	0.5188710	1.10E-07
0.95	0.4428579	0.4428578	0.4428579	1.04E-07
1	0.3678794	0.3678793	0.3678794	9.58E-08

Problem4: Consider the nonlinear system of two-point boundary value problem involving stiff exponential functions:

$$3\frac{d^2u}{dt^2}\ (u^2\ \ v^2)\ 2\ 2e^{2t^4}\ \ 36t^2e^{t^4}\ \ 48t^6e^{t^4}, \qquad 0\ t\ 1 \tag{11}$$

$$3\frac{d^2v}{}\ (u^2\ \ v^2)\ 2\ 2e^{2t^4}\ \ 36t^2e^{t^4}\ \ 48t^6e^{t^4}.$$

The boundary conditions are written as:

$$u(0)\ 2, \qquad u(1)\ 1\ e$$

$$v(0)\ 0, \qquad v(1)\ \ 1\ e.$$

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The exact solutions of the above system are [1 e^{t^4} ,1 e^{t_4}] .

Table 7: Comparison of Exact and numerical solutions for u(t) based on Example 4

t	Exact Solution	Adams Method	Explicit RK	Absolute Error
0	2.0000000	2.0000000	2.0000000	0
0.05	2.0000063	2.0000062	2.0000063	3.31E-08
0.1	2.0001000	2.0000999	2.0001000	6.64E-08
0.15	2.0005064	2.0005063	2.0005064	9.75E-08
0.2	2.0016013	2.0016012	2.0016013	1.27E-07
0.25	2.0039139	2.0039137	2.0039139	1.54E-07
0.3	2.0081329	2.0081327	2.0081329	1.79E-07
0.35	2.0151194	2.0151192	2.0151194	2.03E-07
0.4	2.0259305	2.0259303	2.0259306	2.31E-07
0.45	2.0418586	2.0418584	2.0418586	2.56E-07
0.5	2.0644945	2.0644942	2.0644947	2.85E-07
0.55	2.0958236	2.0958233	2.0958237	3.13E-07
0.6	2.1383729	2.1383726	2.1383733	3.40E-07
0.65	2.1954304	2.1954300	2.1954310	3.65E-07
0.7	2.2713763	2.2713759	2.2713763	3.86E-07
0.75	2.3721876	2.3721872	2.3721902	4.06E-07
0.8	2.5062152	2.5062148	2.5062152	4.24E-07
0.85	2.6854056	2.6854052	2.6854078	4.33E-07
0.9	2.9272613	2.9272609	2.9272614	4.29E-07
0.95	3.2580605	3.2580601	3.2580679	4.25E-07
1	3.7182818	3.7182814	3.7182818	4.07E-07

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Table 8: Comparison of Exact and numerical solutions for v(t) based on Example 4

t	Exact	Method	Explicit RK	Absolute Error
0	0.0000000	0.0000000	0.0000000	0
0.05	0.0000000	0.0000000	0.0000000	6.25E-06
0.1	-0.0000999	-0.0001000	-0.0001000	6.64E-08
0.15	-0.0005063	-0.0005064	-0.0005064	9.75E-08
0.2	-0.0016012	-0.0016013	-0.0016013	1.27E-07
0.25	-0.0039137	-0.0039139	-0.0039137	1.54E-07
0.3	-0.0081327	-0.0081329	-0.0081328	1.79E-07
0.35	-0.0151192	-0.0151194	-0.0151192	2.03E-07
0.4	-0.0259303	-0.0259305	-0.0259303	2.31E-07
0.45	-0.0418584	-0.0418586	-0.0418583	2.56E-07
0.5	-0.0644942	-0.0644945	-0.0644943	2.85E-07
0.55	-0.0958233	-0.0958236	-0.0958234	3.13E-07
0.6	-0.1383726	-0.1383729	-0.1383727	3.40E-07
0.65	-0.1954300	-0.1954304	-0.1954300	3.65E-07
0.7	-0.2713759	-0.2713763	-0.2713762	3.86E-07
0.75	-0.3721872	-0.3721876	-0.3721873	4.06E-07
0.8	-0.5062148	-0.5062152	-0.5062148	4.24E-07
0.85	-0.6854052	-0.6854056	-0.6854052	4.33E-07
0.9	-0.9272609	-0.9272613	-0.9272610	4.29E-07
0.95	-1.2580601	-1.2580605	-1.2580602	4.25E-07
1	-1.7182814	-1.7182818	-1.7182815	4.07E-07

The numerical formulation based on Adams and Explicit Runge-Kutta of Example 1 to Exampole 4 for u(t) and v(t) is presented in Table 1 to Table 8. The graph of absolute error are also shown in the tables. The results of AE are calculated on the basis of exact solutions and Adams results. One can see that the results of Explicit Runge-Kutta are better than Adams results. The step size is used 0.05 for calculating the results in the interval 0 and 1. The better absolute error results show the higher accuracy and perfection of the results.

4 CONCLUSION

The present study is investigated to find the numerical results based on the two techniques Adams and Explicit Runge-Kutta for finding the numerical solution of the two-point nonlinear system of equations. Four different numerical examples have been studied for solving this nonlinear model based equations, which have different nature stiff/ non-stiff. The comparison of the results of each example is also presented by using the exact and Adams results. The results of Explicit Runge-Kutta are found better as compared to Adam numerical results. The absolute error is found very good

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up to higher level of accuracy. The software used for solving the nonlinear second order nonlinear system of two-point boundary value problem is Mathematica 10.4 and MATLAB R2016a.

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