A FAST NEAR-FIELD BEARING ESTIMATION VIA SPATIAL EXPONENTIAL KERNEL DISTRIBUTION

S.-H. Chang[†], *W.-W. Lin*[†], *and W.-H. Fang*[*]

[†] Department of Electrical Engineering, National Taiwan Ocean University,
Keelung, Taiwan, R.O.C. `E-mail:b0091@ntou66.ntou.edu.tw`

[*] Department of Electronics Engineering, National Taiwan University of Science and Technology,
Taipei, Taiwan, R.O.C. `E-mail:whf@et.ntust.edu.tw`

ABSTRACT

With its wide applications in various contexts of signal processing, the Direction of Arrival (DOA) of signal sources emerges to be a cascading issue. The majority of research, however, emphasizes far-field estimation instead of near-field one. In addition, almost all of the high-resolution DOA algorithms are based on the eigenstructure decomposition which calls for lots of computations. In this paper, we present a new algorithm by using the spatial Exponential Kernel Distribution (EKD) function in conjunction with a source parameter estimation procedure to evaluate the involved instantaneous parameters. Since the EKD function can suppress cross terms effectively, the new algorithm, as justified by simulations, can render precise estimates for near field, multi-component signal sources. Furthermore, it avoids the computationally intensive eigendecomposition and scanning scheme by using the simpler time-frequency distribution function. To further speed up the processing, the kernel can be initially evaluated and stored in a matrix form.

1. INTRODUCTION

The Direction of Arrival (DOA) of signal sources has found applications in various contexts of signal processing, including sonar, radar, seismology and the acoustics field [1, 2]. However, most of the research focuses on the far-field estimation problems. As a consequence, these algorithms in general can not offer accurate estimates when the underlying assumptions are invalid. To remedy this difficulty, various algorithms such as the Far-Field Approximation (FFA) [3], the 2-D MUltiple SIgnal Classification (MUSIC) [4], the Maximum Likelihood (ML) [4], and the time-frequency distribution-based [5, 6] algorithms have been addressed, which pinpoint the near-field signal estimation problems.

This work was supported by National Science Council of the R.O.C. under Grant 87-2213-E-019-006 and 87-2213-E-011-014.

The 2-D MUSIC and the ML algorithms are more versatile than the FFA algorithm due to the fact that they can estimate both of the bearing and range parameters of signal sources. However, all of these algorithms are based on the eigendecomposition and therefore suffer the same drawback of enormous amount of computations. On the other hand, Breed and Posch [5] exploited the spatial Wigner distribution to estimate the bearing and range of near-field sources. Swindlehurst and Kailath [6] explored the spatial Wigner distribution in conjunction with the eigenspace technique to estimate the parameters involved. The spatial Wigner distribution holds its advantage of a rapid and accurate estimation of near-field sources. All the benefits, however, would slip away if it were applied to the multi-component signal environment because of the cross-term interference.

In this paper, we propose a new algorithm by using the spatial Exponential Kernel Distribution (EKD) along with a simple estimation procedure to evaluate the parameters involved. Since the EKD can suppress cross terms effectively, this new algorithm, as justified by furnished simulations, can offer precise estimates even for near-field, multi-component signal sources. Furthermore, the computational load is less demanding because it is free of eigendecomposition and scanning schemes. Besides, to further speed up the processing, the kernel used can be initially evaluated and stored in the matrix form as that of [7].

Using the definition of the Cohen distribution function [8], the EKD, also known as the Choi-Williams distribution [9], can be expressed as:

$$\begin{aligned} C(t,\omega) &= \frac{1}{2\pi}\int\int\int_{-\infty}^{\infty} e^{j(\theta u-\tau\omega-\theta t)} \\ &\quad \phi(\theta,\tau)y(u+\frac{\tau}{2})y^*(u-\frac{\tau}{2})dud\tau d\theta \\ &= \frac{1}{2\pi}\int\int\int_{-\infty}^{\infty} e^{j(\theta u-\tau\omega-\theta t)} \end{aligned}$$

$$e^{\frac{-\theta^2\tau^2}{\sigma^2}} y(u+\frac{\tau}{2})y^*(u-\frac{\tau}{2})dud\tau d\theta \quad (1)$$

where $y(t)$ is the time signal and we have used the fact that the kernel function $\phi(\theta,\tau)=e^{\frac{-\theta^2\tau^2}{\sigma^2}}$ for the EKD in which the quantity σ stands for a smoothing factor that decides the decaying speed of the kernel function. After some manipulations, (1) can be alternatively expressed as

$$\begin{aligned} E_y(t,\omega) &= \int_\tau \left[\int_u \frac{1}{\sqrt{4\pi\tau^2/\sigma}} e^{-\frac{(u-t)^2}{4\tau^2/\sigma}} y(u+\frac{\tau}{2}) \right. \\ &\quad \left. y^*(u-\frac{\tau}{2})du\right] e^{-j\omega\tau}d\tau \ . \qquad (2) \end{aligned}$$

The EKD function agrees with many desired properties of the time-frequency distribution functions[8]. Also, its kernel function $\phi(\theta,\tau)$ is a second-order low-pass filter in the ambiguity domain. This explains why it can effectively suppress the induced cross terms.

2. PROBLEM FORMULATION

Consider a uniform linear Array (ULA) composed by M omnidirectional sensors. The distance between any two sensors is D, and the total length of array stretches to $l=(M-1)D$. For the signal source, it consists of N uncorrelated narrow-band signals with white Gaussian noise as shown in Fig. 1. Since the distance between the signal source and the array system has become finite, the incident wavefront should be regarded as spherical wave instead of planar wave. As a consequence, the actual distance between each of the sensors and the signal source should be taken into account.

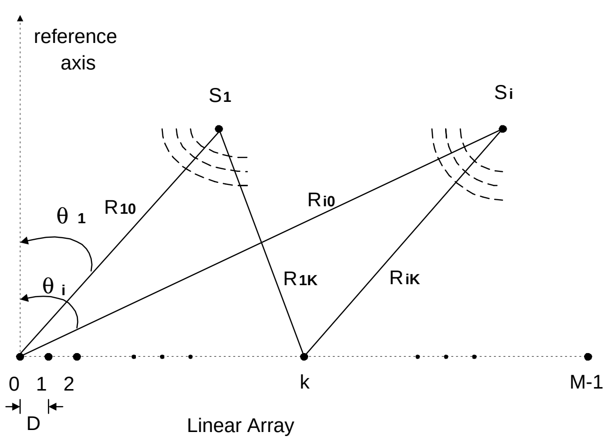

Figure 1: The diagram of a near-field signal source.

If we set the first sensor as the reference node, then the signal received by k-th sensor $y_k(t)$ can be written as:

$$y_k(t) \cong \sum_{i=1}^{N} s_i(t)e^{j\omega_i t}e^{-j\frac{2\pi}{\lambda}(\alpha_i\cdot k-\beta_i\cdot k^2/2)} + n_k(t) \quad (3)$$

where

$$\alpha_i = D\sin\theta_i \text{ and } \beta_i = \frac{(D\cos\theta_i)^2}{R_i},\ i=1,\cdots,N, \quad (4)$$

and the parameters α_i and β_i represent the incident bearing and the source range of the near-field signal source, respectively.

3. ANALYSIS OF THE SPATIAL EKD FUNCTION FOR NEAR-FIELD SOURCE

Let us consider the near-field spatial EKD Function given by

$$\begin{aligned} K_y(k,\tau) &= \int_{-\infty}^{\infty} \frac{1}{\sqrt{4\pi\tau^2/\sigma}} e^{-\frac{(u-k)^2}{4\tau^2/\sigma}} \\ &\quad \cdot y_{u+\frac{\tau}{2}}(t)y^*_{u-\frac{\tau}{2}}(t)\ du + \sigma^2\delta_\tau, \qquad (5) \end{aligned}$$

where k is the center index of the sliding window, σ^2 is the noise power, and δ_τ is Kronecker delta function. Using (3), we can simplify (5) as

$$\begin{aligned} K_y(k,\tau) &= \int_{-\infty}^{\infty} \frac{1}{\sqrt{4\pi\tau^2/\sigma}} e^{-\frac{(u-k).^2}{4\tau^2/\sigma}} \sum_{i=1}^{N} s_i(t) \\ &\quad \cdot s_i^*(t)e^{j(\frac{2\pi}{\lambda})(u\beta_i-\alpha_i)\tau}du + \sigma^2\delta_\tau, \qquad (6) \end{aligned}$$

where we have used the fact that since the EKD function possesses the feature of suppressing the cross terms $s_is_j^*$, only auto terms $s_is_i^*$ are preserved. Extending from the kernel function to the spatial EKD function, we arrive at

$$\begin{aligned} ED(k,\omega) &= 2\sum_{\tau=-\infty}^{\infty} K_y(k,\tau)e^{-j2\omega\tau} \\ &= 2\sum_{i=1}^{N} s_i(t)s_i^*(t) \sum_{\tau=-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{\sqrt{4\pi\tau^2/\sigma}} \\ &\quad e^{(-\frac{(u-k)^2}{4\tau/\sigma})}e^{-j2\omega\tau}\cdot e^{j(\frac{2\pi}{\lambda})(u\beta_i-\alpha_i)\tau}du \\ &= A_1\sum_{i=1}^{N} s_i(t)s_i^*(t) \\ &\quad \cdot\delta\left[2\omega-(\frac{2\pi}{\lambda})(k\beta_i-\alpha_i)\right]. \qquad (7) \end{aligned}$$

where A_1 is a proper constant. Consequently, from the plotted spatial $ED(k,\omega)$, the decline of the N peak values of the near-field source can be observed. The slope $b_i=\frac{2\pi}{\lambda}\beta_i$ and the intercept $a_i=-\frac{2\pi}{\lambda}\alpha_i$ of the i-th line then signify the parameters β_i and α_i, respectively.

Therefore, the DOA bearing and the range of the i-th near-field signal source can be determined by

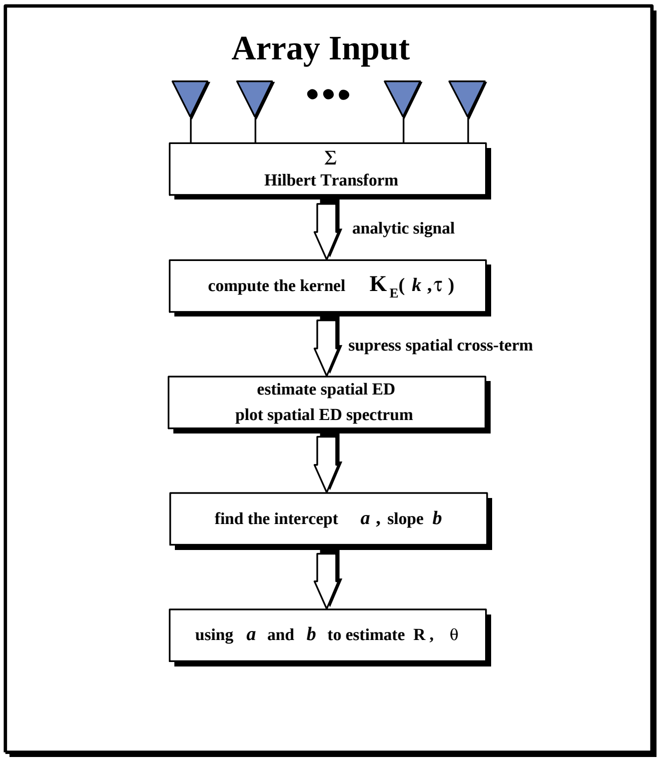

Figure 2: The flow chart of the application of the spatial EKD function to near-field source parameter estimation.

1. DOA bearing

$$\theta_i = \sin^{-1}(-\frac{\lambda a_i}{2\pi D}),\ i = 1, \cdots, N. \tag{8}$$

2. Range

$$R_i = \frac{2\pi(D\cos(\sin^{-1}(-\frac{\lambda a_i}{2\pi D})))^2}{\lambda b_i},\ i = 1, \cdots, N. \tag{9}$$

The spatial EKD function in conjuction with the source parameter estimation procedure stated above can be summarized by the flow chart as shown in Fig. 2.

4. COMPUTER SIMULATIONS AND DISCUSSION

In this section, some simulations are carried out to demonstrate the performance of the proposed approach for the near-field multi-component bearing and the range estimation problems. Comparison has also been made with the previous work which is based on the spatial Wigner distribution.

Consider two stationary signals $s_1(t) = exp(j2\pi f_1 t)$, and $s_2(t) = exp(j2\pi f_2 t)$ with $SNR = 20\ dB$ that locate in the near field. The ULA employed consists of 26 sensors. The DOA bearings are $s_1(t) = 10°$ and

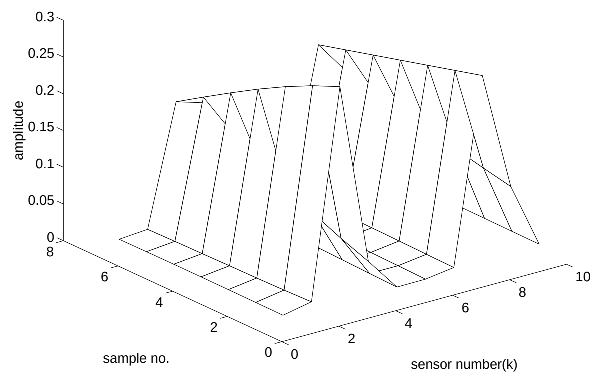

Figure 3: The near-field spatial EKD function of multi-component signal ($SNR = 20\ dB$, $\theta_1 = 10°$, $\theta_2 = 45°$, $R_1 = 100$, $R_2 = 100$).

$s_2(t) = 45°$, and the range of $s_1(t) = 100\lambda$ (set the wavelength of the incident bearing as the fundamental unit) and $s_2(t) = 100\lambda$. The sampling points for both the EKD and the Wigner function are equal to 200.

We can observe from Fig. 3 that the spatial EKD function contains two peaks which denote that there are two signals inside the near-field signal source. From Fig. 3, we can obtain two sets of slopes and intercepts to determine the bearing and the range of the signal source via (8) and (9), respectively, as $\hat{\theta_1} = 9.6980$, $\hat{R_1} = 94.1919$, $\hat{\theta_2} = 45.0162$, $\hat{R_2} = 97.9033$. These estimates are close to the true values. Figs. 4 and 6 describe the decline of the peak values for the first and the last samples by using the EKD and the Wigner distribution functions, respectively. As an illutation, the spatial Wigner distribution for several consecutive samples is depicted in Fig. 5. It is obvious from Fig. 5 that the spatial Wigner distribution function can be seriously jeopardized when two signals are close to each other because of the occurrence of annoying cross terms.

5. CONCLUSION

This paper presents a new algorithm for the estimation of the DOA bearing and the source range for near-field, multi-component signal sources. This algorithm extends the previous one which is based on the spatial Wigner distribution to the EKD functions. The new approach together with a parameter estimation procedure can accurately evaluate the parameters since the annoying cross-term interference has been effectively suppressed. In addition, it is computationally efficient because it does not require computationally involved

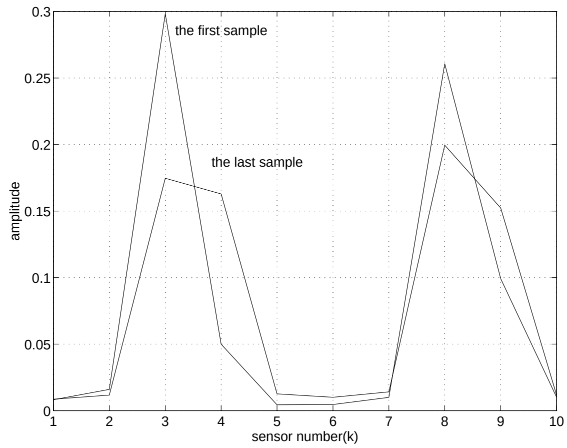

Figure 4: The peak values decline diagram of the first and the last samples inside the near-field spatial EKD function of multi-component signal ($SNR = 20\ dB$, $\theta_1 = 10°$, $\theta_2 = 45°$, $R_1 = 100$, $R_2 = 100$).

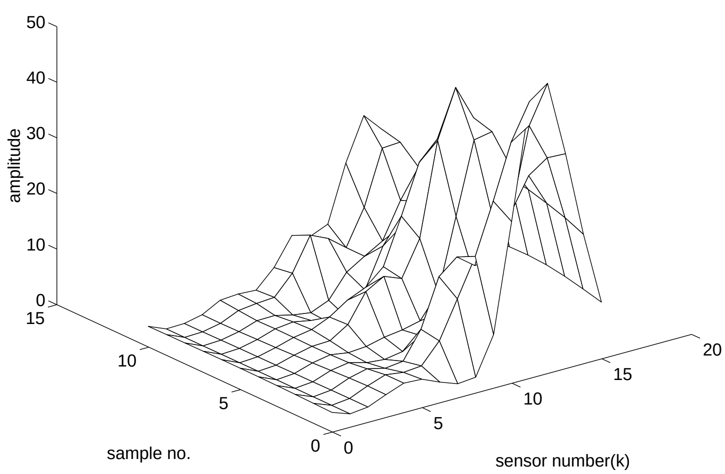

Figure 5: The near-field spatial Wigner distribution function of multi-component signal ($SNR = 20\ dB$, $\theta_1 = 10°$, $\theta_2 = 45°$, $R_1 = 100$, $R_2 = 100$).

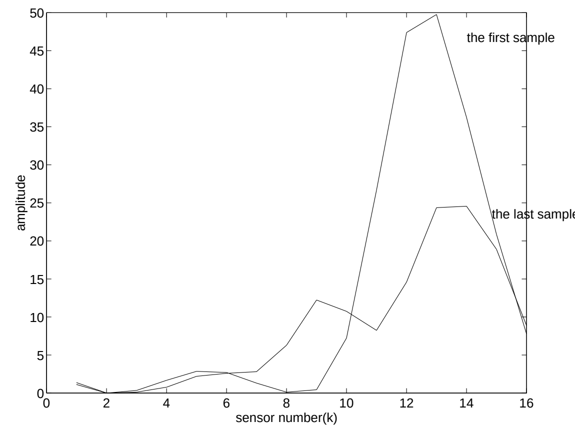

Figure 6: The peak values decline diagram of the first and the last samples inside the near-field spatial Wigner distribution function of multi-component signal ($SNR = 20\ dB$, $\theta_1 = 10°$, $\theta_2 = 45°$, $R_1 = 100$, $R_2 = 100$).

eigendecomposition. As a result, it offers an appealing alternative to such problems.

References

[1] R. O. Schmidt,"Multiple emitter location and signal parameter estimation, in *Proc. RADC Spectrum Estimation Workshop*, Rome, NY, 1979.

[2] R. Roy and T. Kailath, "ESPRIT-estimation of signal parameters via rotational invariance techniques," *IEEE Trans. on Acoustic., Speech, Signal Processing*, pp. 984-995, vol. 37, 1989.

[3] Y.-M. Chen, *Bearing Estimation under the Mismatches in Signal Characteristics and Array Configurations,* Ph. D. Thesis, National Taiwan University, Taipei, Taiwan, 1991.

[4] Y.-D. Huang and M. Barkad, "Near-field multiple source location by passive sensor array," *IEEE Trans. Antennas and Propagation.*,vol. 39, no. 7. pp. 968-974. July. 1991.

[5] B. R. Breed and T. E. Posch, "A Range and azimuth estimator based on forming the spatial Wigner distribution," *IEEE ICASSP-84,* vol. 3, pp. 41B.9.1-2, 1984.

[6] A. L. Swindlehurst and T. Kailath, "Near-field source parameter estimation using a spatial Wigner distribution approach," *Proc. SPIE*, vol. 975, pp. 86-92, 1989.

[7] D. T. Barry, "Fast calculation of the Choi-Williams time-frequency distribution," *IEEE Trans. on Signal Processing,* vol. 40, no. 2, Feb. 1992.

[8] L. Cohen, *Time-Frequency Analysis*, Prentice Hall., 1995.

[9] H. Choi and W. J. Williams, "Improved time-frequency representation of multicomponent signal using exponential kernels," *IEEE Trans. on Acoust., Speech, Signal Processing,* vol. 37, no. 6, pp. 862-871, June 1989.