A PERIOD DETECTOR FOR PSEUDO-PERIODIC SIGNALS

Bart F. Rice

Lockheed Martin Technical Operations
Sunnyvale, California
Tel: +44 1423 565 947; Fax: +44 772 646
e-mail: rice@brice.demon.co.uk

ABSTRACT

The spreading codes of direct sequence spread spectrum signals are often periodic. From a segment of signal containing multiple periods, this periodicity can be detected and the period estimated. The approach taken is to bandlimit the signal (to introduce variation into the envelope), compute the envelope of the filtered signal (to remove the information), and compute the autocorrelation of the envelope. The autocorrelation function contains a peak at an offset equal to the period. The peak-to-noise ratio can be used to obtain an estimate of the input signal-to-noise ratio. Narrowband interference can corrupt the process, so it can be important to precede application of the technique by a pre-whitening filter. The technique applies even for "featureless" signals, in which measures have been taken to suppress rate lines in the outputs of quadratic or higher-order chip rate detectors.

I. INTRODUCTION

Radiometers, quadratic chip rate detectors [1-5], or higher-order detectors [6-10] have been applied for detection of direct sequence spread spectrum signals (DS4s) with low signal-to-noise ratios (SNRs). Despite the fact that the spreading codes of direct sequence spread spectrum signals are often periodic, relatively little effort has been devoted to exploiting this periodicity for detection [11]. However, periodicity of the spreading code is a highly exploitable characteristic of DS4s, even when steps have been taken by the communications systems designers to suppress other signal cyclostationarities, such as the chip rate.

To create the spread signal, the periodic spreading sequence is mixed with a narrowband, constant-envelope information signal, so the resulting signal is not periodic but "pseudo-periodic," because of the "underlying" periodic spreading waveform. Certain non-linear transformations, such as envelope detection, remove the effect of the information signal, with the result that the output of the transformation is periodic rather than pseudo-periodic.

The period of the transformed signal may not be the same as that of the spreading code. For example, if the signal consists of a square wave of *±1*s, then every sample of the envelope of the signal has the value *+1*, so that the envelope of such a sampled signal has period equal to *1/(sample rate)*. To ensure that the period of the transformed signal is equal to that of the spreading waveform, the signal is filtered prior to envelope detection. The bandwidth of the filter must be such that the minimum periodicity of the envelope of the filtered signal is the same as the period of the spreading waveform, and therefore it must be greater than *1/period*. It must also be greater than the symbol rate of the information signal, so that envelope detection removes the effect of the information signal without introducing distortions to the envelope.

Period detection is explained in section 2. The principal implementation is simply the autocorrelation of the envelope of the filtered signal. A second method, essentially a cepstrum, applies when the period of the spreading code is short compared to the data window corresponding to the size of the FFT employed in the analysis. The first version is more generally useful and is the one emphasized. Performance of each technique is described for a number of test cases.

If the signal environment contains narrowband interferers, it is wise to "pre-whiten" the environment prior to period detection. Section 3 describes the effects of interference and the benefits of pre-whitening.

The output of the autocorrelation of the envelope of the filtered signal contains peaks at offsets equal to multiples of the period of the spreading code. In section 4, it is explained how the estimate input SNR from the peak-to-signal ratio.

Period detection techniques also apply when the signal is otherwise "featureless," a term that has mainly been used to describe waveforms with the property that spectra of outputs of non-linear transformations of the signal (such as a conventional delay-and-conjugate-multiply chip rate detector) exhibit no keying rate component. Results of applying period detection to several types of featureless waveforms are given in section 5.

II. PERIOD DETECTION

The main principle behind period detection is that the output of a non-linear transformation applied to a periodic signal is still periodic. To detect the periodicity of the spreading sequence, the carrier and information must be removed. This is accomplished via a non-linear process, such as envelope detection. Now, one must be careful in applying this technique lest the periodicity of the underlying spreading waveform be "modified." To prevent this, the signal must be appropriately filtered prior to computing the envelope. Fig.1 depicts the operation of two versions of the period detector.

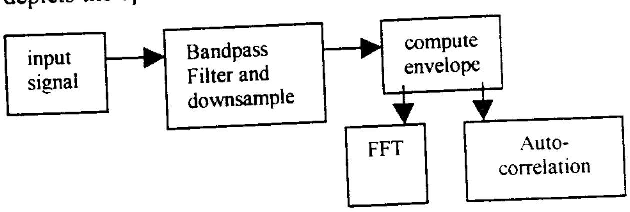

Figure 1. Block diagram of period detector.

Fig. 1 allows for two period techniques. In one, the autocorrelation function of the filtered envelope is computed. The envelope of the filtered signal is padded with *0*'s, an FFT taken, and the squared magnitudes of the FFT coefficients are inverse transformed. The second method applies if the period is short compared to the time span corresponding to the FFT size. The envelope of the filtered signal has a strong periodic component at a frequency equal to *1/(period of the spreading code)* and harmonics thereof. Since the period is short, there are many harmonics less than the half-sample rate, so that the spectrum of the filtered envelope itself has a periodic component equal to the period. Consequently, the period appears as a spike in the magnitude of an FFT of sufficient length performed on the magnitude spectrum of the envelope of the filtered signal (similar to a cepstrum).

The squared envelope of the output of the filter can be represented as a function of the Fourier coefficients of the periodic spreading code. Let $x(t)i(t)$ denote the (analytic) signal, where $x(t)$ is periodic with period T and the "information signal" $i(t)$ is assumed to satisfy $i(t)i^*(t) = 1$. ('*' denotes complex conjugate). $x(t)$ can be represented by a Fourier Series [12, p. 78],

$$x(t) = \frac{1}{\sqrt{T}} \sum_{k=-\infty}^{\infty} a_k e^{j\frac{2\pi kt}{T}}, \quad a_k = \frac{1}{\sqrt{T}} \int_0^T x(t) e^{-j\frac{2\pi kt}{T}} dt,$$

$k = 0, \pm 1, \pm 2, \cdots$. Likewise, the squared envelope $y(t) = x(t)i(t)x^*(t)i^*(t) = x(t)x^*(t)$ is periodic and has a Fourier Series representation,

$$y(t) = \frac{1}{\sqrt{T}} \sum_{k=-\infty}^{\infty} b_k e^{j\frac{2\pi kt}{T}}, \quad b_k = \frac{1}{\sqrt{T}} \int_0^T x(t)x^*(t) e^{-j\frac{2\pi kt}{T}} dt,$$

$k = 0, \pm 1, \pm$ $\cdots$. It follows easily that

$$b_m = \sum_{k=-\infty}^{\infty} a_k a^*_{k+m}, \quad m = 0, \pm 1, \pm 2, \cdots.$$

Note that if $x(t)$ has a constant envelope ($x(t)x^*(t) = 1$, say), then $b_m = 0$ when $m \neq 0$ (therefore, the Fourier Series coefficients of a constant-envelope signal are uncorrelated). Hence, bandlimiting is critical if any information is to be gleaned from $y(t)$.

Band-limiting $x(t)$ to the frequency band $[0,B]$ produces a signal $u(t)$ with a Fourier Series representation $u(t) = \frac{1}{\sqrt{T}} \sum_{k=-M}^{M} a_k e^{j\frac{2\pi kt}{T}}$, where $B \approx M/T$. Thus,

$$b_m = \sum_{k=-M}^{M-m} a_k a^*_{k+m}, \quad m = 0, \pm 1, \pm 2, \cdots.$$

To maximize the component at frequency $1/T$, the bandwidth B of the filter should be chosen to maximize $|b_1|$. To do so would require knowledge of $\{a_k\}$. In section 4, a formula that estimates the ratio of the ratio of the autocorrelation peaks to the root-mean-squared (rms) autocorrelation values is derived. This estimate provides a more tractable method for determining the optimal filter bandwidth.

Fig. 2 shows the result of an application of the cepstral version of the period detector. The signal was a degree-*5* linear recursive sequence (period = *31* chips) with *8* samples per chip. The sample rate was 1, so $T = 1/248$. No noise was added. The data snapshot consists of *24000* samples, or *3000* chips, or between *96* and *97* periods. The signal was filtered to *0.25* × chip rate. Only one FFT was computed, over the snapshot length. The multiple spikes are separated by $1/T$.

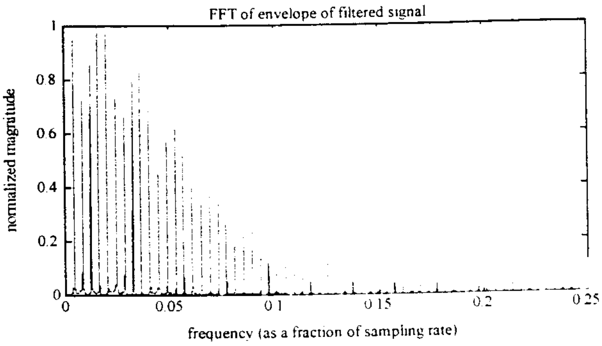

Figure 2. The (linear) spectrum of the envelope of a filtered (to 0.25 × chip rate) spread spectrum signal with 96 periods of the spreading code.

Fig. 3 shows the result of taking an FFT of the waveform in Fig. 2, only with noise added, so that the SNR was *−10* dB. Since, in effect, the sampling rate was halved, the separation of the peaks in Fig. 3 is *124* samples (*31 × 4*) rather than the actual number of samples per period, *248*.

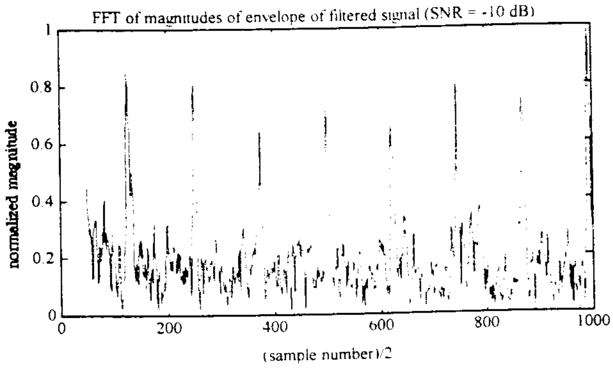

Figure 3. A portion of the FFT of the data shown in Fig. 2, except that SNR = −10 dB.

Results of applying the autocorrelation method for period detection are depicted in Fig. 5 (no noise added). Further examples are given in section 4. The peaks are shown in dB in Fig. 4, whereas the previous plots employ a linear scale.

In section 4, a method for estimating SNR from the heights of these peaks above the noise floor is given.

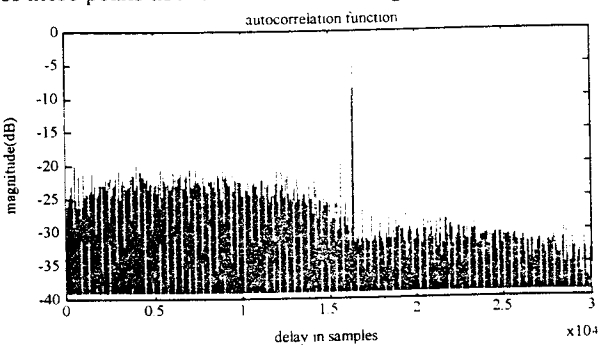

Figure 4. Autocorrelation function of envelope of filtered signal with a period of 2047 chips (8 samples per chip). The peak appears at an offset of 8 × 2047 = 16376 samples.

Note that period detection also provides an estimate of the chip rate, if the number of chips per period is known. For example, if it is supposed that the number of chips per period is N (often $N = 2^n - 1$ for some integer n), then the duration of a chip is the period divided by N. If the uncertainty in (or standard deviation of) the period estimate is Δt, then that of the chip duration is $\Delta t/N$.

III. THE IMPORTANCE OF PRE-WHITENING

Because of the relatively narrowband filter used to introduce variation into the signal envelope in Fig. 1, narrowband interference is often rejected, so that the period detector is relatively interference tolerant. However, when the filter is placed where there is significant interference, the performance of the period detector can be severely degraded. Therefore, it is important that the interference be mitigated.

Fig. 5 contains three plots. The middle plot shows a spectral estimate of a typical UHF environment. The top plot shows the amplitude response of a *1024*-point "whitening filter" designed to produce an output that is spectrally flat. The bottom plot shows the response of the filtered input.

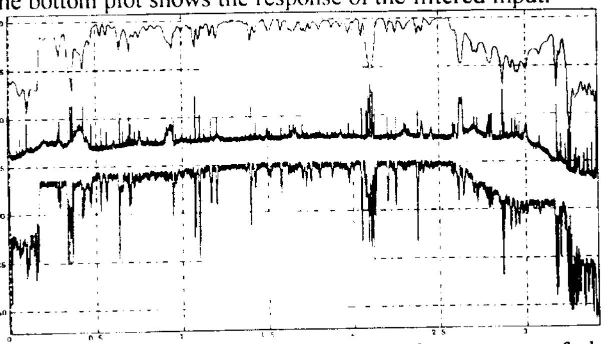

Figure 5. The top trace is the amplitude response of the whitening filter. The center trace is the spectrum of the input environment. The botton trace is the spectrum of the output of the whitening filter. This output has a relatively flat output spectrum, so that non-linear transformations of the data do not contain significant cross products of narrowband interferers.

A DS4 was added to the environment shown as the middle trace of Fig. 5, at a level about *6-to-7* dB below the noise floor. The signal had BPSK modulation and a chip rate of *2* MHz. It was centered at the quarter sample rate. The spreading code was an M-sequence (maximal length binary linear recursive sequence) of period *32767* chips. Figs. 6 and 7 show the outputs of the autocorrelation period detector (using a filter bandwidth of *100* kHz, centered on the interferer at *2.1* MHz, to introduce variation into the signal envelope), with and without pre-whitening. In Fig. 6, no correlation peaks are discernible.

IV. ESTIMATING THE INPUT SNR

Assuming that the background noise is white, the SNR of the input signal can be estimated from the heights of the correlation peaks above the local noise floor, using the following rationale. If the filter is centered near the center frequency of the signal, and if the filter bandwidth is small compared to the chip rate (which can be determined by applying a chip-rate detector to the output of the whitening filter), then the spectrum of the filtered signal is relatively flat in the filter passband. Thus the distribution of the filtered signal-plus-noise can be approximated as Gaussian.

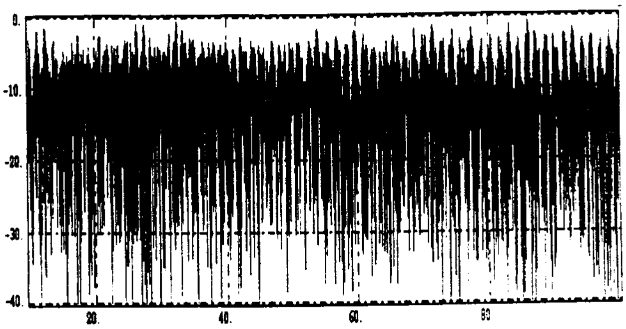

Figure 6. Without interference mitigation, the period detector is vulnerable to interference in the passband of the filter used to introduce variation into the envelope.

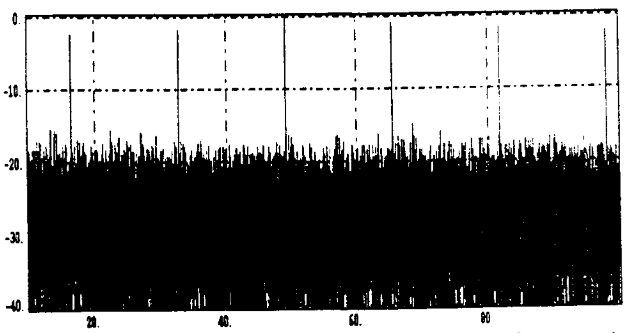

Figure 7. After pre-whitening, the period detector is relatively unaffected by the interferer.

Suppose that the output of the filter is $\{t_k\} = \{s_k\} + \{n_k\}$, where the filtered signal $s_k = N(0,\sigma^2)$ (i. e., s_k is Gaussian with mean 0 and variance σ^2) and the filtered noise $n_k = N(0,\lambda^2)$. Thus, $t_k = N(0,\sigma^2 + \lambda^2)$.

In the example of Figs. 11 and 12, the sequence $\{t_k\}$ was the output of a *3*-pole Bessel filter. $\{t_k\}$ was converted to a complex signal (using an IIR phase splitter), from which its envelope $\{u_k\}$ was computed. For ease of analysis here, $u_k = t_k^2$. Thus, $u_k = (\sigma^2+\lambda^2)\chi^2$, so that $E(u_k) = \underline{u} = (\sigma^2 + \lambda^2)$ and $var(u_k) = (\sigma^2 + \lambda^2)^2$ ('E' denotes "expected value").

The autocorrelation of $\{u_k\}$ at offset p is computed as

$$c = \sum_{k=1}^{BT} (u_k - \underline{u}) \cdot (u_{k+p} - \underline{u}) = \sum_{k=1}^{BT} u_k \cdot u_{k+p} - BT \cdot \underline{u},$$

where BT = bandwidth-time product (which is the number of independent samples in the filter output in time T).

<u>Case</u>: $p \neq period$: Then, $E(c) = 0$ and $var(c)$ is:

$$E(c^2) = E(\sum_{k=1}^{BT} (u_k - \underline{u})^2 \cdot (u_{k+p} - \underline{u})^2 = \sum_{k=1}^{BT} var(u_k)^2$$

$$= 4BT(\sigma^2 + \lambda^2)^4.$$

<u>Case</u>: $p = period$: Here, $u_k = s_k^2 + 2s_k n_k + n_k^2$, $u_{k-p} = s_k^2 + 2s_k n_{k-p} + n_{k-p}^2$. It follows that $E(u_k u_{k-p}) = 3\sigma^4 + 2\sigma^2\lambda^2 + \lambda^4$ and that $E(c) = 2\sigma^4 BT$.

From the preceding, the ratio of $E(c)$ when $p = period$ to the rms estimate of c over all values of p is

$$R = \frac{E(c_{p=period})}{\sqrt{E(c^2_{p\neq period})}} = \frac{\sqrt{BT}(\sigma^2/\lambda^2)^2}{(1+\sigma^2/\lambda^2)^2}.$$

It remains to relate the ratio σ^2/λ^2, the SNR at the output of the bandpass filter, to the input SNR. Assume that the input signal spectrum is proportional to $sinc^2(\pi f/f_b)$, where f_b is the symbol rate. Let f_s denote the sampling rate, so that the signal energy is confined to the frequency interval $[0, f_s/2)$. The result is:

$$SNR = \frac{\sigma^2}{\lambda^2}\cdot\frac{2}{f_s}\cdot\frac{\int_0^{f_s/2} sinc^2(\pi f/f_b)df \cdot \int_0^{f_s/2}|H(f)|^2 df}{\int_0^{f_s/2} sinc^2(\pi f/f_b)\cdot|H(f)|^2 df}.$$

This formula easily converts into a formula for σ^2/λ^2 as a function of *SNR*, so that *R* also can be expressed as a function of *SNR*. This approximation is valid only if the bandwidth *B* of the filter is small with respect to f_b.

The period detector was applied to signal-plus-environment using a filter bandwidth of first *100* kHz and then *25* kHz. The integration time was *0.164* seconds. In the first (second) case, the measured value of *R* was about *28* (*24.5*) dB. From the formulas above, the estimated input SNR was about *−7.5* (*-7*) dB, close to the previously estimated value of (*−6*)-to-(*-7*) dB.

V. FEATURELESS WAVEFORMS

Much effort has been devoted to the analysis of the performance of the well-known delay-and-conjugate multiply (second order) chip rate detector [2, 3] and to the development of techniques to defeat it [13, 14]. In contrast to the chip rate detector, the period detector applies also when the spreading sequence is a periodic featureless waveform. In Fig. 8, two periods of a repetitive Gaussian waveform are displayed, once with no noise added and once with an SNR of *- 5 dB*. Fig. 9 shows the results of applying the period detector using *16* periods at *-5 dB SNR*.

Other chip-rate suppression techniques have also been tested, with similar results. The results are noticeably degraded when the chips are "jittered" to broaden the chip rate line at the output of a chi-rate detector, is applied.

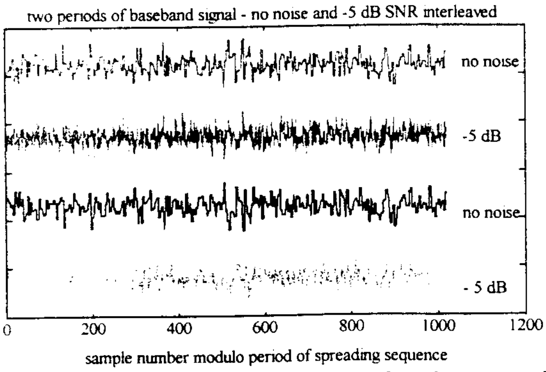

Figure 8. Two periods of a signal spread with a periodic Gaussian waveform, with and without noise.

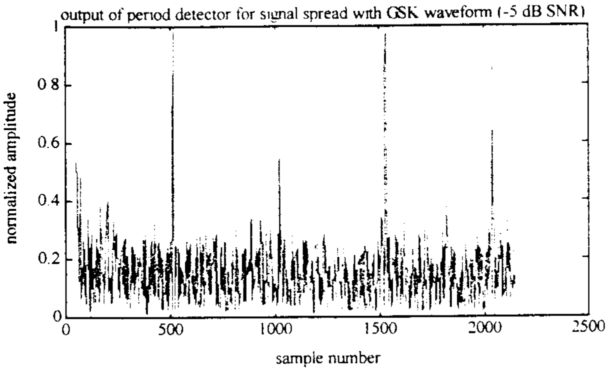

Figure 9. The period detector produces peaks at multiples of the period, even for a featureless waveform at -5 dB SNR (16 periods processed).

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