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Monte Carlo investigation of the influence of randomness at first-order phase transitions

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Abstract

We report on an extensive Monte Carlo and Transfer Matrix study of disordered 2D 8-state Potts model using conformal invariance techniques.

Critical properties of disordered spin systems attracted a lot of attention, especially since Harris proposed a relevance criterion for second-order phase transitions. The effect of quenched randomness at first-order phase transitions was studied later, after the pioneering work of Imry and Wortis who argued that a rounding of the transition occurs, possibly leading to continuous transitions [1], as it is the case in 2D. These problems have been extensively studied numerically recently in 2D in the case of the self-dual 8-state random-bond Potts model (RBPM) with a bimodal coupling distribution [2]. Both Monte Carlo (MC) simulations [2–5] and transfer matrix (TM) calculations [6] were performed, leading to partially conflicting results (Table 1), which eventually found an explanation in terms of a crossover behaviour in a recent work of Picco [4]. A disorder amplitude r in the range 8–20 appears to be adapted to a numerical analysis and gives a good estimate of disordered fixed point exponents.

The PM is well known to exhibit a first-order phase transition when the number of states q is larger than 4, the larger the value of q, the sharper the transition. The choice of a value $q = 8$ is motivated by the value of the correlation length in the pure case ($\xi = 23.87$). MC simulations can thus be performed easily with systems of larger sizes which enable to discriminate between a first-order regime and a second-order transition.

Table 1
Results obtained by different authors on the eight-state RBPM

Authors	r	β/ν	Technique
Chen et al. [2]	2	0.118(2)	MC
Cardy and Jacobsen [6]	2	0.142(4)	TM
Chatelain and Berche [3]	10	0.153(3)	MC
Picco [4]	10	0.153(1)	MC

The question of the application of conformal invariance techniques in these systems is of great interest, since these techniques are known to be very powerful. When one knows the critical behaviour of a two-dimensional system in the infinite or semi-infinite geometry, conformal invariance tells us how the correlations behave in restricted geometries, such as strips or squares. After the replica average [...], one can expect that the symmetry properties required by conformal invariance do hold in the disordered system. The method which is proposed here is to apply the trans-

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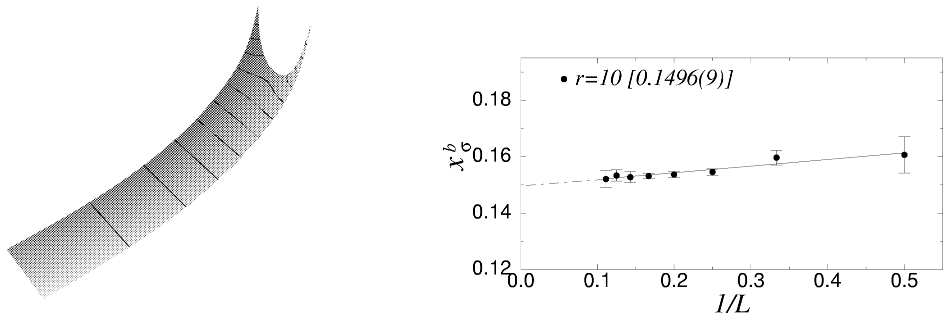

Fig. 1. Order parameter correlation function in the strip geometry and effective scaling dimension extrapolated to $L \to \infty$.

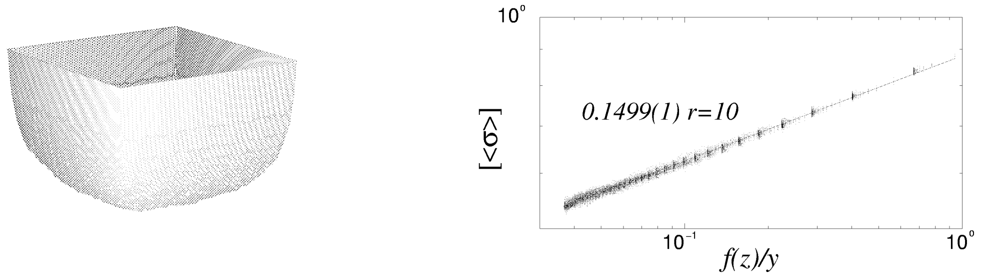

Fig. 2. Order parameter profile in the square geometry and magnetic scaling dimension.

formation of correlation functions under adapted conformal mappings $w(z)$

$$G_\sigma(w_1, w_2) = \left|w'(z_1)\right|^{-x_\sigma^{\mathrm{b}}} \left|w'(z_2)\right|^{-x_\sigma^{\mathrm{b}}} G_\sigma(z_1, z_2) \quad (1)$$

to deduce the magnetic scaling dimension $x_\sigma^{\mathrm{b}} = \beta/\nu$ in the disordered fixed point regime $r = 10$. The comparison with standard Finite-Size Scaling (FSS) results $x_\sigma^{\mathrm{b}} = 0.150$–$0.155$ will constitute the crucial test.

The first restricted geometry considered in the following is a strip, obtained from the infinite complex z-plane via the logarithmic mapping

$$w(z) = \frac{L}{2\pi} \ln z.$$

The exponential decay of the correlations along the strip (Fig. 1) then follows

$$\left[\left\langle G_\sigma^{\mathrm{strip}}(u_2 - u_1)\right\rangle\right] = \mathrm{Cst}(v) \exp\left[-\frac{2\pi}{L} x_\sigma^{\mathrm{b}} (u_2 - u_1)\right] \quad (2)$$

Table 2
Results obtained by conformal mappings compared to FSS results.

FSS	Conformal mappings		
$[\langle M_b\rangle]$	$[\langle G_\sigma^{\mathrm{strip}}(u)\rangle]$	$[\langle G_\sigma^{\mathrm{sq.}}(w)\rangle]$	$[\langle \sigma^{\mathrm{sq.}}(w)\rangle]$
0.153(1)	0.1496(9)	0.152(3)	0.1499(1)

and allows the determination of the scaling dimension (Table 2) for different strip sizes in the range $L = 2$ to 9 (average over 40×10^3 disorder realizations).

In the square geometry (of size N^2), the Schwarz–Christoffel mapping $z = \mathrm{sn}(2Kw)/N$ leads to the magnetization profile (Fig. 2), deduced from the algebraic decay of the corresponding profile from the distance to a surface with fixed boundary spins in the semi-infinite geometry. The resulting expression is of the form

$$\left[\left\langle \sigma^{\mathrm{sq.}}(w)\right\rangle\right] = \mathrm{Cst}\big(f(z)/y\big)^{x_\sigma^{\mathrm{b}}}$$

and, again, the magnetic scaling dimension can be extracted (Table 2, 3000 disorder realizations on a square of size 101^2). One should mention that the shape of the correlation function in the square geometry gives a third determination (Table 2).

In conclusion, the results deduced from the application of conformal covariance of correlations do agree with previous standard FSS results. This is clearly in favor of the validity of conformal invariance at randomness-induced second-order phase transitions, and thus provides a very accurate technique for the investigation of their critical properties. This work was supported by the CNUSC under project No C981009.

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