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Influence of deterministic fluctuations on the 8-state Potts model

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Abstract

We study a layered 8-state Potts model with an aperiodic modulation of the exchange couplings. Depending on its geometric properties, the aperiodic sequence may induce a 2nd order phase transition.

In the light of the Imry–Wortis criterion [1], fluctuations may cause a 1st order phase transition to change into a 2nd order one. It is the case for the random-bond 8-state Potts model as first shown by Chen et al. [2]. The role played by aperiodic fluctuations at 1st order phase transition is not currently understood.

The system considered is a layered square 8-state Potts model defined by the following Hamiltonian

$$-\beta\mathcal{H} = \sum_{i,j} K_i\left[\delta_{\sigma_{i,j},\sigma_{i,j+1}} + \delta_{\sigma_{i+1,j},\sigma_{i,j}}\right],$$
$$\sigma_{i,j} = 0\ldots q-1, \tag{1}$$

where the exchange couplings K_i in each layer i are given by an aperiodic sequence $\{f_k \in \{0;1\}\}_{k=0\ldots L-1}$: $K_i = Kr^{f_i}$.

It can be shown that duality arguments apply for sequences which are identical when read from the left or the right apart from the exchange $0 \leftrightarrow 1$, i.e. when $\{f_k\}_{k=0\ldots L-1} = \{1 - f_{L+1k}\}_{k=0\ldots L-1}$. The four sequences under investigation: periodic (known to lead to a 1st order phase transition and denoted PS), Thue–Morse (TM), Paper–Folding (PF) and Three–Folding (TF) satisfy this property [3,4].

The aperiodic sequences are generated by iterating substitution rules. The Thue–Morse sequence is for example generated with the rules:

$$\begin{cases} 0 \to S(0) = 01 \\ 1 \to S(1) = 10 \end{cases} \tag{2}$$

so that the first iterations are $0 \to 01 \to 0110 \to 01101001 \to \cdots$. The geometric properties of a sequence are given by its substitution matrix M defined as $M_{ij} = n_i(S_j)$ where $n_i(S_j)$ is the number of digits i in the sequence (S_j). The length L_n of the sequence after n iterations of the substitution rules and the fluctuations of the density $\Delta\rho$ with respect to its asymptotic value ρ_∞ behave as

$$L_n \sim \lambda_0^n,$$
$$\Delta\rho = \frac{1}{L_n}\sum_{i=0}^{L_n-1}(f_i - \rho_\infty) \sim \left(\frac{\lambda_1}{\lambda_0}\right)^n \sim L_n^{\omega-1}, \tag{3}$$

where λ_i is the ith eigenvalue of M and $\omega = \ln|\lambda_1|/\ln\lambda_0$ is the wandering exponent. This exponent is equal to $\omega = -\infty$ for PS and TM (bounded fluctuations) and $\omega = 0$ for PF and TF (logarithmically divergent fluctuations).

Large-scale Monte Carlo simulations using the Swendsen–Wang algorithm show that PS and TM do

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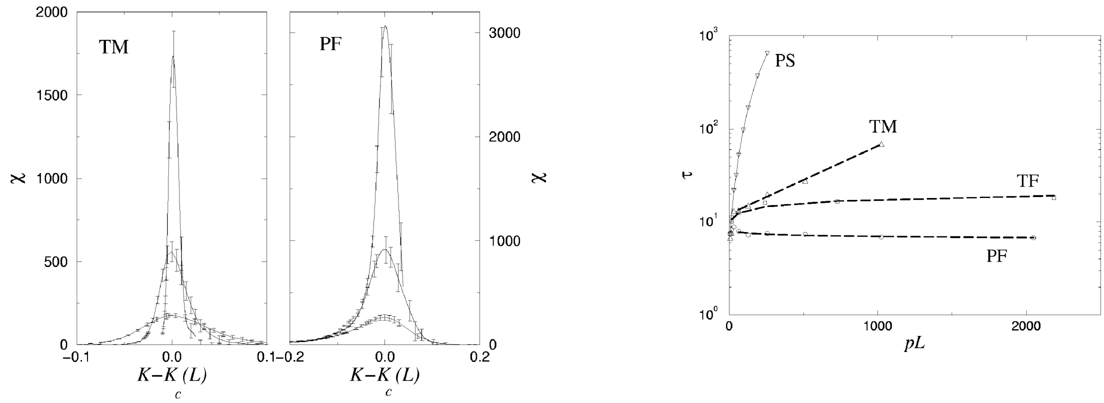

Fig. 1. Magnetic susceptibility χ for TM (δ-like asymptotic behaviour) and PF (power-law) (left) and size-dependence of the energy auto-correlation time which is expected to be exponential for a 1st phase transition and a power law for a 2nd order one (right).

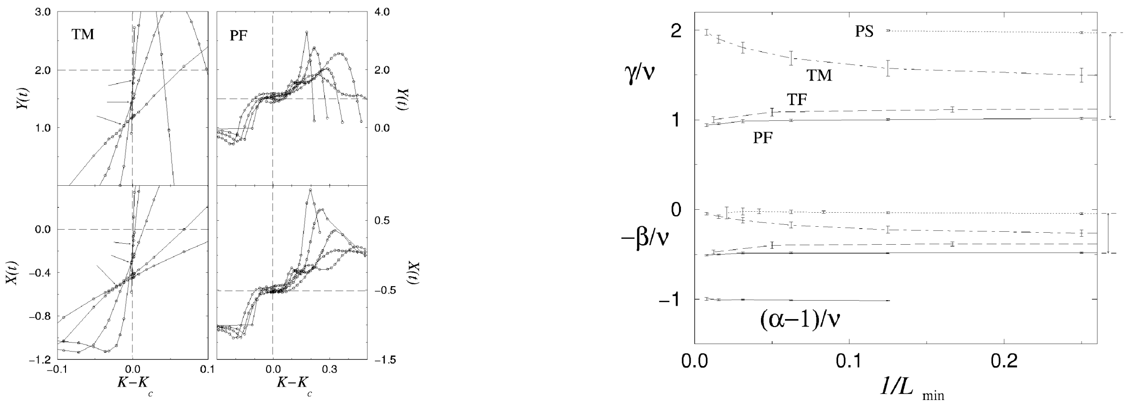

Fig. 2. Effective exponent as expected by phenomenological renormalization of the total magnetization and the magnetic susceptibility for TM and PF at $r = 5$ (left) and effective size-dependent critical exponents obtained by finite-size scaling at the critical point for $r = 5$ (right).

not modify the order of the phase transition undergone by the 8-state Potts model (they nevertheless significantly smooth it). On the other hand, the development of singularities in the magnetic susceptibility and the behaviour of the energy auto-correlation time are strong evidences of a 2nd order phase transition for PF and TF.

Finite-size scaling and phenomenological renormalization allow the calculation of the critical exponents for PF and TF. It appears that both PF and TF belong to the same universality class defined by $y_t \simeq 1.00$ and $y_h \simeq 1.50$. Moreover, these critical exponents do not depend (within error bars) on the perturbation amplitude r.

References

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