# Novel quantitative push gravity theory poised for verification

Gerasimos D Danilatos
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ESEM Research Laboratory
28 Wallis Parade
North Bondi, NSW 2026
Australia
gerry@danilatos.com

**Abstract**

The working of this version (v2) is typed in blue font to facilitate reading by those who have already read the first version: New work provides compelling evidence for a genuine re-appraisal of an old way to explain gravity, which has been sidelined in the periphery of science for a long time. A novel quantitative push gravity theory has been advanced on the basis of a set of primary principles (postulates), from which the derivation of classical acceleration and force by stationary massive bodies in the steady state is possible. In contrast to prior conceptions, it is shown that the absorption of gravity particles by matter need not be extremely weak and linear, in order to derive and explain the observed classical laws of gravity. Any value of the absorption coefficient by a uniform spherical mass produces a gravitational field obeying the inverse square of distance law. The gravitational constant (big G), is itself a function of the ratio of the absorption coefficient over the density of matter. The latter ratio (mass attenuation coefficient) now becomes the new universal gravitational constant of the cosmos, whilst G can vary in different locations of the universe. The measured mass of planets and stars is only an effective or apparent mass actually smaller than the real mass due to a self-shadowing or shielding effect of the absorption of gravitational particles. Any given mass appears quantitatively different depending on its spatial distribution. We now find that Newton's gravitational law uses only the apparent (or effective) masses with a potentially variable G, but the inverse square distance relationship is locally preserved in the cosmos. The radiant flux of energetic particles being uniform over a region of space creates a maximum acceleration of gravity for all material bodies in that region, so that any further mass accretion over a certain upper limit does not create additional acceleration; this limit is reached when practically all gravitational particles are absorbed (saturation state) by the massive body above a saturation mass. The latter limit should be measurable, for which some tentative situations and experiments are proposed for prospective experiments and tests. The internal field of a spherical mass and the external field of a two layered sphere have been derived. The superposition principle of gravity fields has been reformulated and the Allais effect explained and measured. The equivalence principle can now be properly understood and explained in a way that the principle per se becomes redundant. For moving bodies, the established relationships from special and general relativity may continue to operate within the gravitational fields created by push particles, but may need to be adapted and re-aligned within the greater framework of push gravity principles operating at any distance. These advances constitute the main (or first) part of this report purported to become a valid mathematical formulation for a basic physical interpretation or embodiment of gravity poised for verification. In the second part of the report, an attempt is made to overcome the main remaining objection of presumed catastrophic thermal accretion of absorbed particles. A further attempt is made also for the push-gravity principles to explain the vastly higher intensity gravitational fields of white dwarfs, neutron stars and black holes. It is proposed that the field of white dwarf stars is created also by push particles but of a different kind, namely, by those responsible for mediating the electric field. In the same way, the field of neutron stars is created by yet a third kind of push particles, namely, those responsible for mediating the nuclear field. The effective mass attenuation coefficient is variable around those massive bodies. In general, push particles may exist with different energy (or mass) having different mean free paths as they traverse different concentrations of masses like black holes, neutron stars, dwarfs, stars, planets, ordinary masses, atoms, nuclei, protons and all the known or unknown sub-nuclear particles. The invariable principle of momentum transfer (push) by particles directly relating to their absorption rate by the various concentrations (density) of masses could be the basis and the starting principle for a prospective unification theory of everything. The veracity or not of these attempts in the second part of the report may not negate the general theory of the first part, but they follow as speculative but logical proposals, or conclusions of the observed phenomena seen from the perspective of push gravity.

# Introduction

Scientific theories are built from concepts and discoveries that have come before, and are constantly evolving and being refined. When it comes to the theory of gravity, there should be no difference. While maintaining established laws, e.g. in relativity, re-assessing sidelined theories of push gravity may help fill-in gaps of our understanding. This paper humbly seeks to re-assess what this author views as compelling evidence for push gravity and its only goal is to have related theories re-evaluated and ultimately incorporated into more extensive scientific writings.

The push (or shadow) gravity theory (PG) is not generally widely known, by and large, despite of it dating back since Newton's time. It remains outside the mainstream of established theories of gravity and is not generally accepted or introduced, even in a negative way, to primary, secondary or tertiary schooling systems. Whilst fully acknowledging its shortcomings and reasons for which it continues to receive little attention, the present paper reports new advances of this theory, which should provoke a renewed consideration beyond prior attempts to break out of the hitherto impasses in science. Push gravity is now developed on a new basis thought to overcome at least most, if not all, of the prevailing objections.

It would be helpful and productive, if we initially avoid the existing stalemate imposed by the existing criticisms and objections against the push gravity theory by patiently examining the mathematical or quantitative relationships newly derived in this work. The results and conclusions produced are important in their own right. We have opted to work out (or rework) a number of significant derivations of fundamental importance first and then follow with appropriate discussion here and elsewhere. Afterwards, we can re-visit all known arguments and objections to PG, most of which may be shown to be invalid, redundant, or not necessarily valid. Triggering a protracted series of arguments and counter-arguments from the outset could be counter-productive for the ordinary reader to proceed and appreciate the main findings and purpose of this report.

In the above sense, we initially assume the validity of a set of PG principles or postulates, which allow a novel derivation of the laws of gravity in the steady state of initially stationary (or slow moving) material bodies. In particular and based on these principles, Newton's law of gravity can be easily derived but with a new understanding of the universal constant of gravity. This is followed by the general case, where the same principles are used to derive some novel relationships beyond Newtonian mechanics, from whereby is is shown that Newton is the limiting case for very weak absorption of the push particles. A universal law for the acceleration of gravity is produced, which reveals the most fundamental physical quantities involved. Both the gravitational field around a material sphere and the force between two material spheres is established. An attempt is then made to use and propose tests, observations and experiments to verify the new physics. By this methodology, we will provide completely independent means of describing some fundamental physics and phenomena providing better explanations than hitherto existing theories.

This approach then, overall, will further assist towards invalidating most of the objections and at least neutralizing the others, or placing them on a rational speculation for an interim period. This will allow experts in the areas of particle physics, theoretical physics, astrophysics and mathematics to find new and fruitful ground for further progress to both use and advance the presented theory to its logical conclusions. Ultimately, work can include general relativity, a generalized theory of fields and a unified theory of everything.

In setting out such an ambitious goal, it should be clarified from the outset that the presented PG theory is thought to by no means be in conflict with the theories of relativity, by and large. The mathematical tools of the general relativity may still be applicable and useful to PG in the case of moving bodies close to the speed of light, but this is left for later works, i.e. when the time dependence phenomena are considered under PG. For the present work, we start only with the steady state condition of push particle flows around stationary bodies, which is sufficient to reveal some important misses of existing theories in general and, perhaps, of the general relativity in particular.

Unavoidably, we include a certain amount of speculation and heuristic findings, which should be separated out from the fundamental derivations of PG. For this purpose, we divide this report in two parts, the first of which concentrates on the definite new mathematical derivations of PG, whilst the second part expands the first to include possible applications under certain conditions and speculations.

For the above purposes, there is no extensive literature relating to the novel developments of PG in the first part. However, by attempting in the second part to apply the new findings to existing data and theories, the task overwhelms the expertise of this author who takes a great risk in possible misapplication of what otherwise can be a valid PG theory. Therefore, this report does not contain an exhaustive search of literature on all aspects touched upon, but only a limited reference to prior works as needed or known to the present author, who also resorts and refers to Wikipedia to indirectly provide a more extensive list of references. These misses may be excused, whilst they could also be rectified by others in the spirit of further progress along the path ushered by the following work.

# Part One (1)

## 1  Early push gravity theory

Nicolas Fatio de Duillier is considered to be the first who proposed an explanation of the phenomenon of gravity. That subject was one among many and various interests that he worked on around the same time that Newton developed his own laws of gravity. Fatio's works are not readily available in present journals for direct accessibility and reference, but can be found in a Wikipedia article (Wikipedia contributors, 2018b) containing numerous references (de Duillier, 1929; Gagnebin, 1949) and further reading on push gravity. His theory is "*based on minute particles streaming through space and pushing upon gross bodies...*" via collisions between ordinary matter and ethereal corpuscles, which was thought to be his greatest work. This theory was later reworked and presented also by Le Sage (Wikipedia contributors, 2018a). However, Fatio's mechanical theory of gravity soon fell into oblivion, chiefly because no drag by the ether on the motion of the planets could be detected in celestial motions; it was finally abandoned on account of a number of serious objections by renowned scientists around the beginning of the 20th century. As we know, Special Relativity (SR) and General Relativity (GR) have become the prevailing or established theories for over a century to date.

A few works have appeared from time to time attempting to revive PG. However, the latter still remains outside the mainstream physics.

## 2  Push Gravity (PG) principles

The fundamental principle or assumption of push gravity (PG) as understood or proposed in this report is that the forces we experience by an assumed gravitational field attached to material bodies is actually generated by the flow and absorption of a radiation energy in the form of elementary particles or waves, or both, traveling randomly but homogeneously in all directions in the interstellar/interplanetary space, or at least in regions of the order of magnitude of solar systems. This is a form of radiant flux, the nature of which need not be specified at the outset, but which, for convenience, we can initially assume that it consists of elementary particles to be called **gravions** (gravity + ion (from "ιόν" meaning "going") and are characterized by the following postulated properties:

1. They rarely interact (or collide) between themselves resulting in relatively very long mean free paths as compared with planetary size orbits.

2. They interact with material objects at any point at a rate in direct proportion to the density of the matter they traverse.

3. During their interaction with matter, they become partially or totally absorbed but re-emitted in a different form of particles (energy) with much shorter mean free path so as not to pertain (mediate) further to gravitational force, but likely to pertain to other types of forces or reactions.

4. Conservation of momentum: During their interaction with matter, they transfer momentum to the material body, a process that appears as a force acted upon the material body.

We further use two complementary provisional assumptions to connect the theory with existing theories, namely:

5  The gravions are relativistic.

6  The speed of gravions is the speed of light (photons).

The third principle (#3) is analyzed and discussed in considerable detail in Part 2.

The sum total of all gravion absorption by a material body results in a depletion zone around the said body, a process that appears as a gravitational field acting on any other material body inside the said field.

The nature of gravions and the nature of their interaction with matter remains to be found, so that "particle" and "matter" are as yet undefined entities, as they may pertain to energy or mass in particle or waveform according to established ideas and principles in physics.

Dibrov (2011) called the particles fations, or other names may be found, but we opted to use a fresh term for good reasons, such as to dissociate, not critically, the presented theory here from previous ones on this subject. The gravions may be identical to the known gravitons from elsewhere, but a new term purports to avoid possible conflicts or transferring properties from existing theories not necessarily needed or assumed by the PG as presented here. After all, gravions and gravitons might be the same thing, except that we attempt
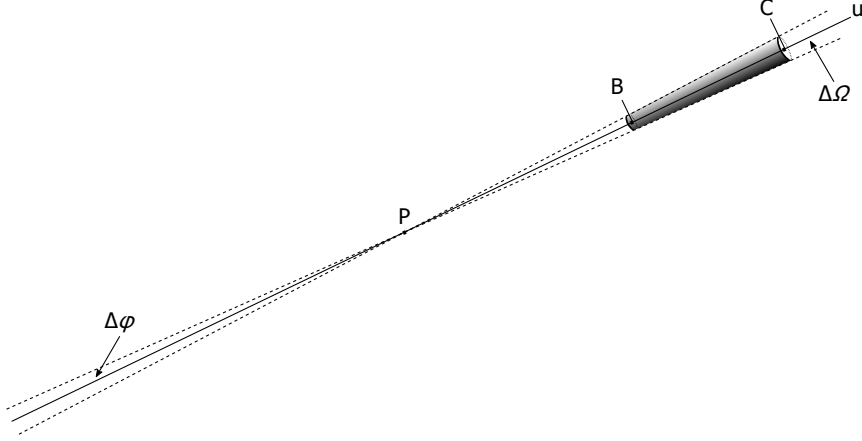
Figure 1: *Push Gravity principle.*

to start afresh (i.e. be independent) in this work. The term gravion has been coined by incorporating the root word "ion" meaning "going" or "traveling" from ancient Greek (hence, by denoting particles or energy that is flowing).

## 2.1 Formulation of principle

The preceding principles can be formulated as follows:

Let us denote the radiant flux of gravions (energy) by $\Phi_0$, which is the radiant energy emitted, transmitted or received anywhere per unit time (in Watts), i.e. the rate of flow of particles/energy by gravions. The radiant flux received by a surface, per unit solid angle $\Omega$, per unit area $S$ in a particular direction defines the radiance $L_0$ by:

$$L_0 = \frac{\partial^2 \Phi}{\partial \Omega \partial S} \tag{1}$$

At any point in space, we will need to find and use the flux density $J_0$ (also called intensity), namely, the flux per unit area received within a solid angle $\Delta\Omega$

$$J_{0\Delta\Omega} = L_0 \Delta\Omega \tag{2}$$

If within this solid angle there is a finite material body, the received flux will be diminished due to absorption. Referring to Fig. 1, the radiance and the flux density at any point in free space is initially the same from all directions resulting in zero force, except when at a point P the flux density is affected by the presence of matter in the direction $u$ within a cone with small semi-angle $\Delta\varphi$ subtending a small solid angle $\Delta\Omega$. Due to the absorption of gravions by matter contained in the distance BC, there is a deficiency from that direction and hence an excess flux from the opposite direction within the same angle.

We can treat the problem as we use the general case of any radiation absorption by matter and write the well known equations of absorption. In the general case, the flux density (intensity) $J(u)$ at any point $u$ along the line $u$ diminishes in proportion to product $J(u)du$

$$dJ_{\Delta\Omega} = -kJ_{\Delta\Omega}(u)\mathrm{d}u \tag{3}$$

where the constant of proportionality $k$ is the coefficient of absorption for gravions (or attenuation coefficient in the Beer-Lambert law). Upon integration, we obtain the classical exponential transmission equation

$$J_{\Delta\Omega} = J_{0\Delta\Omega} \exp(-ku) \tag{4}$$

where $J_{0\Delta\Omega}$ is the incident (initial) intensity per above. The absorbed intensity $J_{a\Delta\Omega}$ is simply the difference

$$J_{a\Delta\Omega} = J_{0\Delta\Omega} - J_{\Delta\Omega} \tag{5}$$

and the corresponding absorption fraction $f_{a\Delta\Omega}$ in the small solid angle $\Delta\Omega$ is:

$$f_{a\Delta\Omega} = \frac{J_{0\Delta\Omega} - J_{\Delta\Omega}}{J_{0\Delta\Omega}} = 1 - \exp(-ku) \tag{6}$$

For the case in Fig. 1, by setting $AB = \ell$ we simply write

$$f_{a\Delta\Omega} = 1 - \exp(-k\ell) \tag{7}$$

where $k$ is constant if the density is uniform.
We note that for very small values of $k \ll 1$, Eq. 4 reduces to

$$J_{\Delta\Omega} = J_{0\Delta\Omega}(1 - k\ell) \tag{8}$$

and

$$J_{a\Delta\Omega} = J_{0\Delta\Omega} - J_{\Delta\Omega} = J_{0\Delta\Omega}k\ell \tag{9}$$

and

$$f_{a\Delta\Omega} = k\ell \tag{10}$$

The above equation is the basic assumption of Fatio's theory and all subsequent theories of push gravity, i.e. the absorption of gravions by a planet is very weak and linear, because only then could they reproduce Newton's equation of gravity.

In the above and subsequent notation, we use the subscript "a" to denote the presence of absorption so that $f_a$ is a shorthand notation for the absorption fraction of gravions per unit area inside a finite solid angle:

$$f_a = \int f_{a\mathrm{d}\Omega}\mathrm{d}\Omega \tag{11}$$

This fraction will be used later for finding the total energy absorbed by a sphere.

# 3   Newton's gravity law

Based on the given PG principle, we can derive Newton's equation of gravity in a simple way as follows:

Referring to Fig. 2, let us consider a point O at distance $r$ from the center of a sphere at point P with radius $R$. We draw a straight line $u$ from point O traversing the sphere along the cord AB, the length $\ell(\varphi)$ of which is given by:

$$AB = 2(AM) = 2\sqrt{R^2 - r^2 \sin^2 \varphi} = 2r\sqrt{a^2 - \sin^2 \varphi} \equiv \ell(\varphi) \tag{12}$$

since

$$OM = r \sin \varphi \tag{13}$$

and

$$a = \frac{R}{r} = \sin \varphi_0 \tag{14}$$

while we want these quantities expressed as a function of the angle $\varphi$ in the range

$$0 \leq \varphi \leq \varphi_0 \tag{15}$$

We also need the initial $u_1 = OA$ and final $u_2 = OB$ lengths on the line OAB along $u$ corresponding to points A and B

$$u_1(\varphi) = r \cos \varphi - \sqrt{R^2 - r^2 \sin^2 \varphi} = r\left(\cos \varphi - \sqrt{a^2 - \sin^2 \varphi}\right) \tag{16}$$
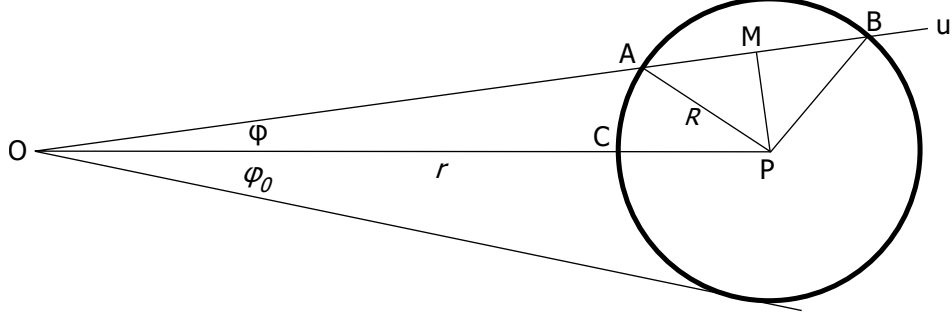
Figure 2: *Derivation of push gravity around a sphere.*

and

$$u_2(\varphi) = r\cos\varphi + \sqrt{R^2 - r^2\sin^2\varphi} = r\left(\cos\varphi + \sqrt{a^2 - \sin^2\varphi}\right) \tag{17}$$

We note that the above equations describe the given circle in polar coordinates, when the origin O lies away from the circle, which then it is simplified to just the cord length, when the origin lies on the surface ($r = R$) by the well known cosine equation:

$$\ell(\varphi) = 2r\cos\varphi \tag{18}$$

The elementary annular solid angle $d\Omega$ at angle $\varphi$ around the axis OP is

$$d\Omega = 2\pi\sin\varphi d\varphi \tag{19}$$

Gravions arrive at point P from all directions uniformly in the absence of any mass around. However, if the sphere contains a uniform mass we can initially assume that some gravions are absorbed by the mass in direct proportion to the elementary solid angle and the length of the cord AB at angle $\varphi$. This creates a depletion of gravions from that direction, from which the total depletion (fractional absorption) of gravions is found by the double integral

$$f_a = \int\limits_0^{\varphi_0}\int\limits_{u_1}^{u_2} 2\pi\sin\varphi d\varphi k du \tag{20}$$

where we use the previously defined absorption coefficient being $k \ll 1$ along the length $\ell(\varphi)$. Integrating with $u$ along the $\ell(\varphi)$, we get

$$f_a = \int\limits_0^{\varphi_0} 2\pi\sin\varphi d\varphi\cdot k(u_2 - u_1) = 2\pi\int\limits_0^{\varphi_0}\sin\varphi k\ell(\varphi)d\varphi \tag{21}$$

or

$$f_a = 4\pi kr\int\limits_0^{\varphi_0}\sin\varphi\sqrt{a^2 - \sin^2\varphi}d\varphi \tag{22}$$

Now, since the flux of gravions is a directional quantity (vector) at the test point O, the components normal to direction OP cancel out and only the components along OP add to a total directional flux for the generation of an acceleration of gravity $g$. The latter components are integrated by multiplying the above integrand by $\cos\varphi$:

$$f_g = 4\pi kr\int\limits_0^{\varphi_0}\sin\varphi\cos\varphi\sqrt{a^2 - \sin^2\varphi}d\varphi \tag{23}$$

to find the total component of accelerating fraction $f_g$ below:

$$f_g = \left[-\frac{4\pi kr}{3}\left(a^2 - \sin^2\varphi\right)^{3/2}\right]_0^{\varphi_0} \tag{24}$$

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By substituting the integration limits on account of the above relationships, we finally get:

$$f_g = \frac{4\pi k r}{3} a^3 = \frac{4\pi k R^3}{3r^2} \tag{25}$$

By introducing an average density $\rho$ of the spherical mass, the last result becomes :

$$f_g = \frac{k}{\rho} \frac{4\pi \rho R^3}{3r^2} = \frac{k}{\rho} \frac{M}{r^2} \tag{26}$$

where $M$ is the total mass of the sphere. This is essentially Newton's law of gravity subject to a proportionality constant to yield the force of the gravions on a test unitary mass, which is the acceleration at point O.

In the above and subsequent notation, we use the subscript "g" to mean the component of absorption responsible for the generation of acceleration $g$.

It should be noted that the ratio $\Lambda = \dfrac{k}{\rho}$ is the mass attenuation coefficient of the Beer-Lambert law in any absorption situation written in alternative form as a function of the area density (or mass thickness) $\lambda = \rho \ell$, that applies also in flux density attenuation in PG, i.e.

$$J = J_0 \exp \left( -\frac{k}{\rho} \rho \ell \right) \equiv J_0 \exp \left( -\Lambda \lambda \right) \tag{27}$$

The fraction $f_g$ as initially derived above is a pure (dimensionless) parameter involving only geometrical parameters (Euclidean geometry) that appears to be a fundamental property of nature. The inverse square of distance law appears from the outset together with the sphere diameter and the absorption co-efficient $k$, which implies an absorbing entity like the mass, or density of the mass to appear in the next step.

This fraction was obtained by integrating over all absorption possible around the axis of symmetry defined by points O and P and yielding the simplest solution for a sphere. However, for any other shape, we should integrate around three normal independent axes (x,y,z) and add the corresponding acceleration fractions vectorially, as can also be shown during the derivation of force between two spheres later.

# 4   Beyond Newton

Next, we obtain the expected acceleration from the previous derivation, as a consequence of the push gravity principle.

## 4.1   Universal gravitational constant in weak absorption regime

The simple derivations above can already lead to a better understanding of the universal constant $G$ (or bigG).

From the absorption and acceleration fractions $f_a$ and $f_g$ introduced in the previous section, we convert to the corresponding fractions of absorption and acceleration for the flux density $J_a$ and $J_g$ below:

$$J_a = J_0 f_a \tag{28}$$

$$J_g = J_0 f_g \tag{29}$$

where $J_a$ is the flux density absorbed by the presence of a mass (here spherical uniform mass) and $J_g$ is the component of $J_a$ in the direction of the axis of symmetry responsible for the generation of acceleration.

We now proceed to find the constant of proportionality to reproduce Newton's gravitational law from Eq. 26 by

$$J_g = J_0 \frac{k}{\rho} \frac{M}{r^2} = J_0 \Lambda \frac{M}{r^2} \tag{30}$$

using the newly introduced constant $\Lambda$.

The physical meaning of this constant is the number of absorption events per unit density of matter in units of inverse mass-thickness ($m^2/kg$). In other words, it is the number of absorption events per kilogram per square meter. The inverse ($1/\Lambda$) is the mass-thickness (or area density) per absorption event. This is a new cosmic constant the magnitude of which remains to be found.

It is generally known in flow problems that the product of pressure times the velocity of the flow yields the flux intensity. Thus, if we divide $J_g$ by the velocity $c$ of the radiant flux (gravions), we obtain the pressure $p_g$ exerted by the gravions at O:

$$p_g = \frac{J_g}{c} \tag{31}$$

An elementary test mass $dm$ is located at point O with a surface area $dS$ and thickness $\Delta x$ having a density $\rho'$ with corresponding absorption coefficient $k'$. The force $dF$ on this test mass is then given by:

$$dF = p_g dS \cdot k'x = \frac{J_g}{c} dS \cdot k'x \tag{32}$$

where we multiply by $k'x$ to allow only for the fraction of gravions absorbed by the test mass, considering that $k'$, in general, is the number of absorption events per unit length. The force per unit mass, i.e. the acceleration $g$ is then

$$g = \frac{\mathrm{d}F}{\mathrm{d}m} = \frac{\frac{J_g}{c} dS \cdot k'x}{\rho' dS \cdot x} = \frac{J_0}{c} \frac{k}{\rho} \frac{M}{r^2} \frac{k'}{\rho'} = \frac{J_0}{c} \Lambda^2 \frac{M}{r^2} \tag{33}$$

The above equation is exactly Newton's law, where the factors of proportionality between $g$ and $M/r^2$ must correspond to the universal constant $G$:

$$G = \frac{J_0}{c} \Lambda^2 \tag{34}$$

The above is already an important derivation for the universal gravitational constant in terms of other constants, namely, the gravion speed and intensity of the neighboring universe, and the mass attenuation coefficient (new universal constant). Eqs. 33 and 34 are thought to be new fundamental derivations beyond Newton even within the realm of Newtonian mechanics for weak absorption.

## 4.2   General gravitation law in any absorption regime

Having considered the case of weak absorption, we now proceed to investigate what happens if absorption is strong, or to any arbitrary degree, i.e. the absorption coefficient can take any value. This actually means that we allow gravitational shielding inside a material body and between bodies. We may also refer to this condition as self-shadowing within the bulk of a massive body. In other words, we allow "gravitational shielding" as a core condition of a general push gravity theory, as opposed to considering it a case for rejecting PG, as has been done by the hitherto critics. This ushers a novel approach to push gravity.

In the general case, where self-shadowing (shielding) is caused by a significant $k$, we follow the same initial procedure as previously with reference to Fig. 2: The force is proportional again to the elementary annular solid angle $2\pi \sin\varphi$, but now multiplied by the absorption fraction of the flux intensity along the length AB (Fig. 2) provided by Eq. 6; we also multiply by $\cos\varphi$ to allow, as previously, only for the component of force along the direction OP, so that we only need to integrate with respect to angle as follows:

$$f_g = \int\limits_0^{\varphi_0} 2\pi \sin\varphi \cos\varphi \mathrm{d}\varphi \cdot [1 - \exp(-k\ell(\varphi))] \tag{35}$$

and

$$f_g = 2\pi \int\limits_0^{\varphi_0} \sin\varphi \cos\varphi \left[ 1 - \exp\left( -2kr\sqrt{a^2 - \sin^2\varphi} \right) \right] \mathrm{d}\varphi \tag{36}$$

The final integration of the above expression in the given subtended angle $\varphi_0$ by the sphere is fortunately an analytical expression of the form:

$$f_g = 2\pi \left[ \frac{\sin^2\varphi}{2} - \frac{\exp\left( -2kr\sqrt{a^2 - \sin^2\varphi} \right)\left( 2kr\sqrt{a^2 - \sin^2\varphi} + 1 \right)}{4k^2r^2} \right]_0^{\varphi_0} \tag{37}$$

and with the given values of integration from 0 to $\varphi_0$, we finally obtain
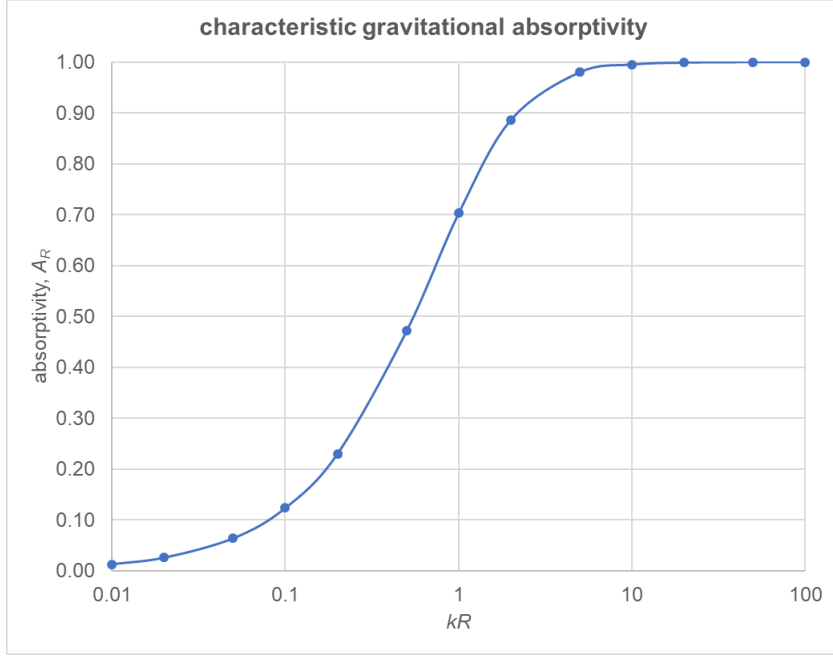
Figure 3: *Dependence of characteristic gravitational absorptivity $A_R$ on $kR$.*

$$f_g = \pi \left[ R^2 - \frac{1}{2k^2} + \frac{\exp(-2kR)(2kR+1)}{2k^2} \right] \frac{1}{r^2} \equiv \frac{\pi A}{r^2} \tag{38}$$

where we have now a new parameter $A$, which is a function of $k$ and $R$ only (i.e. independent of $r$):

$$A = \left[ R^2 - \frac{1}{2k^2} + \frac{\exp(-2kR)(2kR+1)}{2k^2} \right] \tag{39}$$

Like in Eq. 25, we find that the fraction $f_g$ derived in the general case of strong absorption is again a dimensionless parameter (quantity) that appears to be a fundamental property of nature and that the inverse square of distance law is preserved. This law is a consequence of the geometry alone (Euclidean) by any uniform flux propagated and absorbed in space. It is the law in the steady state around any absorbing medium (mass), whilst the time dependence remains to be introduced at a later stage of PG development.

Now, we follow the same procedure, as previously, to obtain the acceleration: For the test mass acted upon by a pressure $p_g$, Eq. 29 now becomes

$$J_g = J_0 \frac{\pi A}{r^2} \tag{40}$$

In view of above, Eq. 33 is modified to become:

$$g = \frac{\mathrm{d}F}{\mathrm{d}m} = \frac{\frac{J_g}{c}\mathrm{d}S \cdot k'x}{\rho'\mathrm{d}S \cdot x} = \frac{J_0}{c}\frac{k'}{\rho'}\frac{\pi A}{r^2} = \frac{J_0}{c}\Lambda\frac{\pi A}{r^2} \tag{41}$$

The above provides the equation of acceleration in PG, which again preserves the inverse square of distance law. However, the factor(s) of proportionality between $g$ and $M/r^2$ is different from the corresponding PG derivation in Newton's equation, the significance of which will be described later. To understand the difference, we need to first investigate the properties of the newly derived parameter $A$.

# 5 Investigation, consequences and new physics with parameter $A$

## 5.1 PG versus Newton

From from Eqs. 25 and 38, we see that the corresponding $f_g$ (or $g$, or force) is always proportional to $1/r^2$ regardless of the values of $k$ and $R$. The general assumption by previous proponents of PG that the gravion absorption should be very weak (in order to produce Newton's Law), is now found to be redundant together
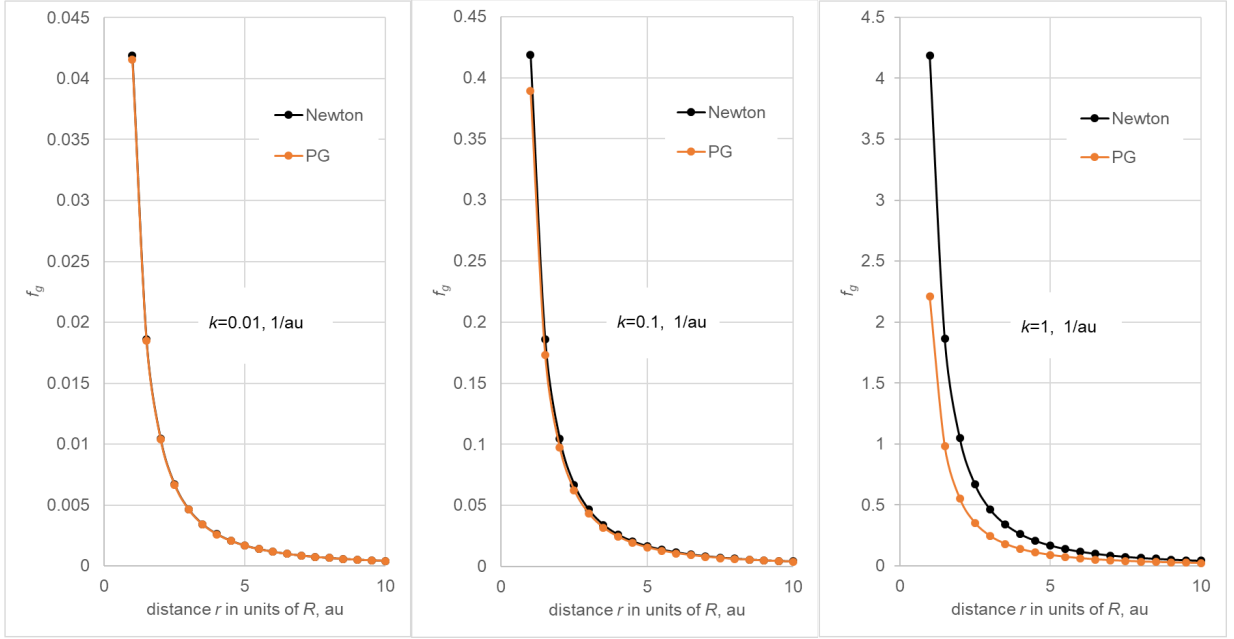
Figure 4: *Gravity acceleration factor $f_g$ vs. distance $r$ in units of radius $R$ for three different absorption coefficients $k$ using linear (Newton) and exponential (PG) absorption.*

with the allegation that the "gravitational shielding" is a reason to reject PG. On the contrary, this is now found to be a fundamental underlying mechanism of PG. This is already an important finding.

It is helpful and instructive to normalize the distance $r$ over the radius of the sphere $R$:

$$n_R = \frac{r}{R} \tag{42}$$

whereby we re-write the newly found expression as

$$\frac{f_g}{\pi} = \left[ 1^2 - \frac{1}{2k^2 R^2} + \frac{\exp(-2kR)(2kR+1)}{2k^2 R^2} \right] \frac{1}{n_R^2} = \frac{A_R}{n_R^2} \tag{43}$$

by introducing the characteristic parameter $A_R$:

$$A_R = 1 - \frac{1}{2k^2 R^2} + \frac{\exp(-2kR) \cdot (2kR+1)}{2k^2 R^2} \tag{44}$$

The latter parameter depends only on the product $kR$ and it is plotted in Fig. 3. This shows that $A_R$ is monotonically (absolutely) increasing, as it should, but it reaches a saturation limit at very high values of $kR$. For simplicity, we may also set $R = 1$ and plot against $k$, or set $k = 1$ and plot against $R$, in either case reproducing the same curve. The important observation is that most of the change in value of $A_R$ takes place over the span of about four orders of magnitude of $k$, or $R$, or $kR$. Overall, the magnitude varies from near zero to unity. This means that increasing the radius of a sphere, the shadowing parameter $f_g$ (and acceleration) at the surface will reach a saturation value as opposed to infinity predicted by Newton. Likewise, by increasing the absorption capacity (density, or $k$) of a constant radius sphere, the shadowing parameter $f_g$ (and acceleration) at the surface will again reach a saturation value, not the infinity provided by Newton. This new parameter $A_R$ characterizes the absorbing ability of a spherical mass or planet and may be referred to as *characteristic gravitational absorptivity*, or absorptivity, for short.

For a direct comparison, we plot simultaneously $f_g$ against normalized distance $r_R$ for $r_R > 1$, i.e. by setting $R = 1$ au (arbitrary unit) in Eqs. 25 and 38 as shown in Fig. 4 for three fixed values of $k$ in a range spanning three orders of magnitude. Initially, we may avoid the involvement of mass $M$ and density $\rho$ by investigating only the quantity $f_g$. For very low values of $k$, the pair of curves are indistinguishable. We note that as we increase $k$, the shadowing derived from PG is increased **absolutely** (see actual values), as it should, because more absorption by the gravitating mass means more **net** push by gravions. However, the curve lies below the corresponding expected Newtonian force, as it should. This is to be expected from the general absorption Eqs. 6 and 10, whereby the second equation is a straight line tangent to the first near (or at) the origin (at very short distance, or very low $k$), always yielding a higher value above the downward concave line of PG absorption. The latter is a consequence of the self-shadowing (gravitational shielding)
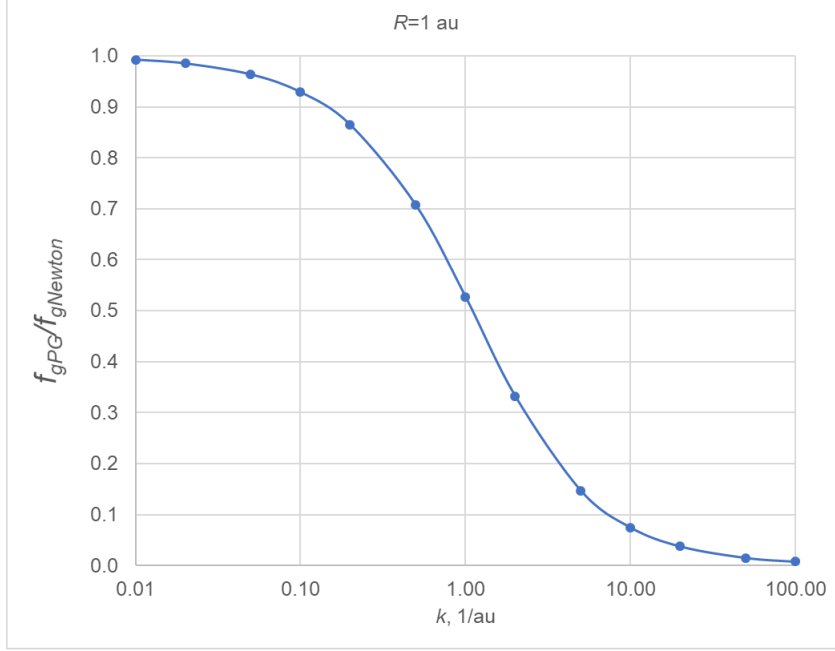
10

Figure 5: *Dependence of ratio $f_{gPG}/f_{gNewton}$ on $k$*

effectively creating a hidden mass, which, if it could exert an "attractive" force (per Newton), it would be greater than the corresponding PG force found.

The above analysis is also consistent with a comparison between Newton and PG as provided in Fig. 5 by plotting the ratio of $f_{gPG}/f_{gNewton}$ from (Eq. 38)/(Eq. 25) vs. $k$ for a constant sphere radius $R = 1$ au. The absorption ratios by PG/Newton approaches unity for very small values of $k$ (k<0.01), as it should, but vanishes for very large values of $k$, which means that $f_g$ becomes infinity in Newton, whilst it reaches a saturation value in PG. This is reasonable and helpful in understanding the mechanism of shielding. Noted that the horizontal axis is logarithmic tending to uplift (concave up) the initial straight line (Newtonian) but eventually tending to reach a saturation value asymptotically (concave down). PG is the overriding physics in all cases, whilst Newtonian physics is an approximation in the limiting case of very low values of $k$. The above ratios are given by:

$$q = \frac{f_{gPG}}{f_{gNewton}} = \frac{g_{PG}}{g_{Newton}} = \frac{3A_R}{4kR} \tag{45}$$

The same ratio can be obtained from the integrands of Eqs. 23 and 36

$$q = \frac{f_{gPG}}{f_{gNewton}} = \frac{1 - \exp\left(-2kr\sqrt{a^2 - \sin^2\varphi}\right)}{2kr\sqrt{\alpha^2 - \sin^2\varphi}} \tag{46}$$

This is the ratio of an effective length $\ell_e(\varphi)$ in PG divided by the real length in Eq. 12 of the cord traversing the sphere at angle $\varphi$ from the origin O in Fig. 2:

$$\ell_e(\varphi) = \frac{1}{k}\left[1 - \exp\left(-2kr\sqrt{a^2 - \sin^2\varphi}\right)\right] \tag{47}$$

This is a contracted or compressed length, with which we may construct a virtual volume (body) by replacing the points defined by Eq. 17 with new ones defined by

$$u_{e2} = u_1 + \ell_e \tag{48}$$

We may refer to these shapes as *gravitoids*, which are helpful for our theoretical understanding of the underlying workings of PG. Further details and analysis is provided in Appendix A.

We may refer to the above parameter $q$ as *contraction factor*, because it is equal to the ratio of an effective contracted length over the real length.

Note 1: The current practice to find the mass of a planet is to place an artificial satellite around it and measure the period and radius of orbit. However, we now find that the actual mass still remains unknown by such measurements. This is not a trivial finding.

Note 2: For small values of $k$ or $R$, we revert to Newtonian mechanics, which can also be seen by expanding the exponential to a Taylor series $e^x = 1 + x + \dfrac{x^2}{2} + \dfrac{x^3}{6}$.

$$A_{R_{kR \to 0}} = \frac{4}{3} kR(1 - kR)_{kR \to 0} \approx \frac{4}{3} kR \tag{49}$$

$$\text{i.e. for very small } kR: \ \pi A_R = \frac{4}{3} \pi kR = f_{gNewton} \tag{50}$$

the latter reproducing Eq. 25 for $f_g$ in Newton derivation at $r = R$.

## 5.2 Universal gravitational "constant" in any absorption regime vs. a new cosmic constant

We note that in Eq. 25 the multiplier preceding the factor $1/r^2$ divided by $k$ provides the volume $V$ of the gravitating sphere. Likewise, in Eq. 38, the multiplier preceding $1/r^2$ divided by $k$ also provides an *effective* sphere volume $V_e$ with the same center:

$$V_e \equiv \frac{\pi A}{k} \tag{51}$$

The real volume, real density and real mass are designated by $V$, $\rho$, and $M$. The measured (effective, measured, or apparent) density $\rho_e$ is the effective mass $M_e$ divided by the real volume

$$\rho_e = \frac{M_e}{V} = \frac{\rho V_e}{V} \tag{52}$$

and

$$\rho_e V = \rho V_e = M_e \tag{53}$$

also

$$\frac{\rho}{\rho_e} = \frac{V}{V_e} = \frac{M}{M_e} \tag{54}$$

We can now continue from Eq. 38 by multiplying with $V \rho_e$ both numerator and denominator as follows:

$$f_g = \frac{\pi A}{V \rho_e} \frac{V \rho_e}{r^2} = \frac{k V_e}{V \rho_e} \frac{M_e}{r^2} = \frac{k V_e}{V_e \rho} \frac{M_e}{r^2} = \frac{k}{\rho} \frac{M_e}{r^2} \tag{55}$$

which is identical to Eq. 26, except that we use the real density and not the effective (fictitious) one used (or implied) in Newton's equation. Based on this, we can repeat the same steps to establish the force on a testing mass and derive an identical form of equation as in 33

$$g = \frac{\mathrm{d}F}{\mathrm{d}m} = \frac{\dfrac{J_g}{c} \mathrm{d}S \cdot k'x}{\rho' \mathrm{d}S \cdot x} = \frac{J_0}{c} \frac{k}{\rho} \frac{M_e}{r^2} \frac{k'}{\rho'} = \frac{J_0}{c} \Lambda^2 \frac{M_e}{r^2} \tag{56}$$

but where again we use the real density of the gravitating body in $\Lambda$. We repeat the same equations in order to stress that they are different in the meaning of $\rho$ and $\Lambda$, whereby we derive the same expression for the universal gravitational constant:

$$G = \frac{J_0}{c} \Lambda^2 \tag{57}$$

This is the same equation arrived at for weak gravion absorption, so that $\Lambda$ is the new universal constant for the cosmos. From this and the known density of a given mass, we derive the absorption coefficient $k$. The universal constant $G$ is proportional to the ratio of $k/\rho$ squared, where $\rho$ is the real density. For very low values of $k$ the real density becomes very close to or is indistinguishable from the measured (effective) density. From this, we learn that $G$ is constant only to the extent that $J_0/c$ is constant in the neighboring universe. As pointed out earlier, $\Lambda$ expresses the number of scattering events per unit length per unit mass
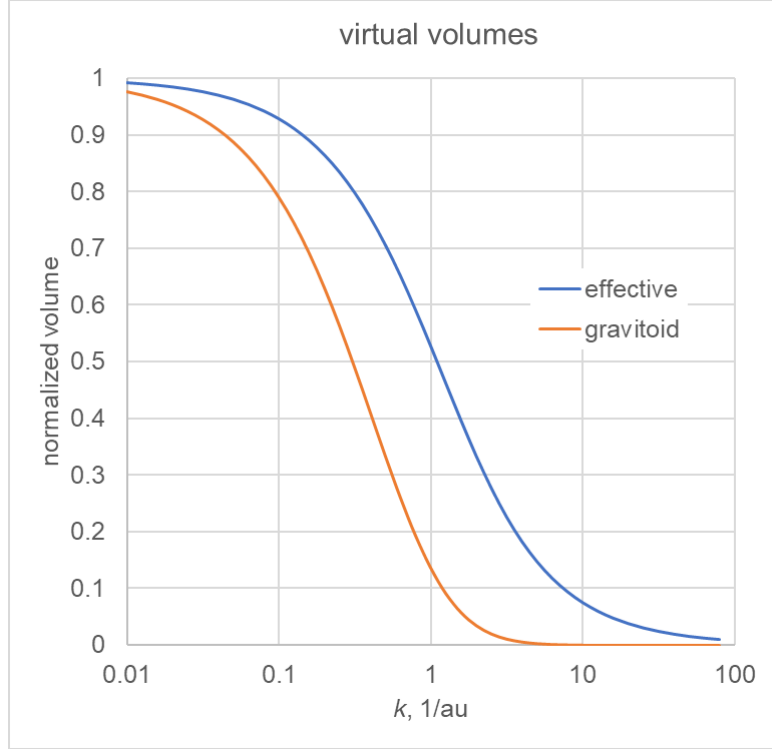
Figure 6: *Virtual volumes, gravitoid and effective, normalized over the real spherical volume.*

density anywhere and provides a more tangible constant parameter to have. Thus, $G$ may be found to be relatively more variable than previously suspected, according to further investigations by PG.

The effective volume introduced above is plotted in Fig. 6 against $k$ after it is normalized over the real volume. As expected, it coincides with the real volume (at very low $k$), but then monotonically decreases to a vanishing value at very large $k$.

## 5.3 Maximum universal acceleration

We can try to use known values of planet parameters to derive the $\Lambda$, $k$, $\rho$ and $J_0$. Basically, we need to know the flux intensity $J_0$, or the absorption coefficient $k$, on which all other parameters depend. Conversely, from the known physical parameters of a planet, we may assume values for $J_0$ in any given range and derive the other new parameters of $\Lambda$, $k$ and $\rho$ as a function of $J_0$. In practice, we may proceed as follows:

The acceleration of gravity $g_R$ at the surface of a sphere, i.e. at $r = R$, is given by Eq. 41 as:

$$g_R = \frac{J_0}{c}\Lambda\frac{\pi A}{R^2} = \frac{\pi J_0}{c}\Lambda A_R = \frac{\pi G}{\Lambda}A_R \tag{58}$$

From Eqs. 41, 44 and Fig. 3, there is a maximum possible acceleration $g_{maximum} \equiv g_0$ in the surrounding universe to be manifested on the surface of a star (sphere) with sufficiently large product $kR$, i.e. with $A_R = 1$, given by any of the following equations:

$$g_0 = \pi\frac{J_0}{c}\Lambda = \frac{\pi G}{\Lambda} = \frac{\pi\rho G}{k} \tag{59}$$

In subsequent work, we will be using values of $g_0$ in a tentative range to obtain an idea of the expected magnitude of various parameters and anticipated measurements. That is, until we establish the actual value of $g_0$, we may obtain the new constant $\Lambda$ and hence $k$ from the known density of a mass, for any given value $g_0$.

It is useful to write Eqs. 56 and 58 correspondingly as:

$$g = G\frac{M_e}{r^2} = \frac{G}{\Lambda}\frac{\pi A}{r^2} = g_0\frac{A}{r^2} \tag{60}$$

$$g_R = g_0 A_R \tag{61}$$

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Now, given the measured acceleration $g_R$ on the surface of a spherical body, we can find $k$ by solving the equation below:

$$g_0 A_R - g_R = g_0 \left[ 1 - \frac{1}{2k^2 R^2} + \frac{\exp(-2kR)(2kR+1)}{2k^2 R^2} \right] - g_R = 0 \tag{62}$$

as a function of $g_0$. Then, for any given $g_0$, we can find in turn $\rho$, $\Lambda$ and $J_0$ from Eq. 59. We will return to the question of $g_0$ in a subsequent section

## 5.4 Commonality and departure between Newton and PG

It is not fortuitous that both Newtonian and PG mechanics share a common limiting case but depart thereafter. Let us start from the derivation of the volume of the sphere as seen from point O in Fig. 2. For the elementary volume we have

$$dV = d\Omega u^2 du \tag{63}$$

which multiplied by the density $\rho$ gives the elementary mass and, divided by the inverse square distance, yields the Newtonian acceleration:

$$dg_{Newton} = \frac{d\Omega u^2 du}{u^2} \rho \tag{64}$$

In PG, we use the factor $f_g$ from which we obtain the same elementary acceleration by :

$$dg_{PG} = \frac{g_0}{\pi} d\Omega k du \tag{65}$$

which is identical to Newton above except for the proportionality constants. This initial similarity is not trivial, because it explains the fundamental difference at the root of the two theories (approaches), as we increase the absorption coefficient: In Newton, it is given that the acceleration is inversely proportional to the square of distance, whilst in PG this is a consequence of the solid angle (geometry) incorporating the inverse square distance relationship. In Newton, this is the result of an assumed radiance (of gravity) emanating from the elementary mass, whilst in PG the same field emanates from the radiance of the surrounding universe. Whilst the analogy might seem trivial simply shifting the problem of origin of the elusive gravity from the inside to the outside of a given mass, the consequences diverge from the two approaches as we increase the absorption coefficient to any level. That is, when considering very large masses or densities, Newton and PG provide very different solutions and outcomes: Newton provides a linear cumulative radiance of gravity by simple summation of all the constituent masses/volumes, whilst PG allows for shadowing (shielding) of the universal radiance traversing the mass, which, in turn, results in an asymptotic limit to the total shielding and hence to the total acceleration or force. We may think of this limit as effectively integrating the Newtonian law linearly but within contracting upper limits of a volume per Eq. 65, which defines the said gravitoid. This shape would produce the same Newtonian force with a mass having the actual (real) density. The above integration has been performed numerically and potted against $k$ in Fig. 6 after it is normalized by dividing by the sphere volume, as was done for the effective spherical volume defined by Eq. 51. For comparison, this is also plotted against $k$ in Fig. 6. We note that it is generally lying above the gravitoid, as it should, because it is further away from the gravitoid. They both have the same real density and both yield the correct value of acceleration for the real gravitating sphere.

Hence, these are important findings for cosmological considerations in relation to what happens as we keep adding mass (accretion) to a star (dwarfs, black holes, etc.). We will discuss this again later.

An important conclusion here is that there is more mass in the universe than Newton's Law measures. This is a form of dark matter but not exactly in the sense considered by existing theories to date, in accounting for the observed celestial motions. We now find weaker forces, not greater. However, the greater forces, if needed, may be accounted for by forces originating from the outside now predicted from PG theory, not from the inside anticipated by Newton. At very large distances, forces are exerted by the gravions in the universe, so there is no need to attribute them to an attraction by dark matter. However, dark matter should assume a different meaning now by the shadowing effect (gravitational shielding) in PG. Thus, breaking up a planet to dust would appear to create new matter (out of shadow - see redistribution of density in later Section 9), which gives a kind of credence to the creationist theory of matter, except that no new matter actually is created other than new matter coming out of the shadows (literally). All this and more creates new understanding and new physics that will become clearer as we develop and prove the novel PG presented in this work. As we investigate next, the bigG is a function of the gravion density in the universe, which

should vary between regions inside a galaxy and in intergalactic regions. So, if we need extra forces, these may arise from the variation of bigG alone. The sum total of the effects caused by hidden masses and the variation of bigG might explain or replace the hypothesized dark matter and dark energy of current theories. PG may offer the new physics needed.

## 5.5   Summary of new parameters and relationships

We have already expressed various relationships in alternative forms, which we may further re-arrange for easy reference in later derivations here or elsewhere as follows: By combining Eqs. 41 and 56 we derive:

$$\pi A = \Lambda M_e \tag{66}$$

and

$$\Lambda = \frac{\pi A}{M_e} = \frac{\pi}{M_e} \left[ R^2 - \frac{1}{2k^2} + \frac{\exp(-2kR)(2kR+1)}{2k^2} \right] = \frac{\pi R^2 A_R}{M_e} \tag{67}$$

We obtain a further insight of the above parameters by re-writing the above as

$$A_R = \Lambda \frac{M_e}{\pi R^2} = \Lambda \lambda_e \tag{68}$$

by defining an effective mass-thickness $\lambda_e$ (or area-density) with

$$\lambda_e = \frac{M_e}{\pi R^2} \tag{69}$$

The neighborhood prevailing gravion pressure $p_g$ should be handy to have (per Eq. 57) as

$$p_g = \frac{J_0}{c} = \frac{G}{\Lambda^2} \tag{70}$$

If two spheres (planets) 1 and 2 have equal surface acceleration $g_R$, it follows from Eq. 58 that the product $kR$ for both spheres is the same

$$k_1 R_1 = k_2 R_2 \tag{71}$$

Also we have the universal (cosmic) constancy for $\Lambda$ giving:

$$\Lambda = \frac{k_1}{\rho_1} = \frac{k_2}{\rho_2} = \frac{\pi G}{g_0} = \text{constant} \tag{72}$$

so that we obtain

$$\rho_1 R_1 = \rho_2 R_2 \tag{73}$$

The above equations apply in PG theory with real densities $\rho_1$ and $\rho_2$. In Newtonian mechanics, we similarly obtain for the effective (apparent) densities $\rho_{e1}$ and $\rho_{e2}$, i.e. if the $g_R$ is equal for both spheres (at their surface):

$$g_R = \frac{4}{3} G R_1 \rho_{e1} = \frac{4}{3} G R_2 \rho_{e2} \tag{74}$$

$$\rho_{e1} R_1 = \rho_{e2} R_2 \tag{75}$$

We obtain the ratios of real to effective densities as

$$\frac{\rho_1}{\rho_{e1}} = \frac{\rho_2}{\rho_{e2}} \tag{76}$$

From a given value for $g_0$, we find the corresponding $k$ from Eq. 62 and then $\rho$ from Eqs. 59:

$$\rho = g_0 \frac{k}{\pi G} \tag{77}$$

and then the ratio $\rho/\rho_e$ from the known effective (measured) density. This ratio is also provided directly from:

$$\frac{\rho}{\rho_e} = \frac{4}{3} k R \frac{g_0}{g_R} \tag{78}$$

We also derive relationships including the frequently encountered factor $A_R$:

$$M_e = \frac{\pi \rho R^2}{k} A_R \tag{79}$$

$$V_e = \frac{\pi R^2}{k} A_R \tag{80}$$

Finally, it is important to note that the parameter $g_0$ or the factor $f_g$ yield the acceleration $g$, via Eqs. 43, 59 and 60 in a simple form by a summary of equations:

$$g = f_g \frac{g_0}{\pi} = f_g \frac{J_0}{c} \Lambda = f_g \frac{G}{\Lambda} = g_0 \frac{A}{r^2} = g_0 \frac{A_R}{n_R^2} \tag{81}$$

The maximum (or limiting) universal constant $g_0$ now takes on a tangible significance in establishing the quantitative relationships in PG, and it may substitute the constant $G$ accordingly. We should stress that new universal constant of the cosmos $\Lambda$ is given by Eq. 67, which in words states that the hitherto universal constant $G$ is proportional to the new universal constant (maximum acceleration) $g_0$ both with reference to a region of the universe, so that together they yield the cosmic (overall universal) constant $\Lambda$. It should be noted that we attempt to distinguish the term "universal" from the term "cosmic" with reference to the neighboring universe or to the "entire" universe (=cosmos).

With the new parameters now introduced, it is useful to re-write Eq. 45 of the ratios of accelerations at the surface of a sphere, or the contraction factor $q$, as a function of tentative values for $g_0$:

$$q = \frac{g_{PG}}{g_{Newton}} = \frac{3A_R}{4kR} = \frac{3A_R}{4\pi G \rho R} g_0 \tag{82}$$

This factor can be retrospectively inserted in other relations involving the ratio of parameters corresponding to Newton and PG, as will also be used in later derivations.

# 6  Force between two spherical masses

For the force between two spherical masses, we can formulate the problem entirely from gravion absorption considerations, carry out four integrations and produce the force law, as we did for the acceleration at a point for a single sphere. This would be an independent way, from first principles, to derive the required relationship. However, we can still arrive at the same desired result in a much simpler as follows:

Since we already have established the relationships between all the parameters needed for the PG force equations, we can apply a "reverse engineering" approach. Now, in the knowledge that Newton is correct except for the masses used, we can start with the Newtonian law of force by using the effective masses provided by PG theory together with preceding equations between various parameters:

$$F = G \frac{M_{e1} M_{e2}}{r^2} = G \frac{\pi A_1}{\Lambda} \frac{\pi A_2}{\Lambda} \frac{1}{r^2} = \frac{J_0}{c} \frac{\pi A_1 \pi A_2}{r^2} = \frac{g_0}{\Lambda} \frac{\pi A_1 A_2}{r^2} \tag{83}$$

This is consistent with our hitherto understanding of the meaning of the parameters involved. The importance is that Newton's law now involves the effective masses $M_{e1}$ and $M_{e2}$, not the real masses assumed, but not used, in prior mechanics. The above equations is a far reaching conclusion. Now we can write, or start with the PG force law as

$$F = \pi \frac{g_0}{\Lambda} \frac{A_1 A_2}{r^2} \tag{84}$$

where we do not need the masses, but equivalently we need the more intrinsic parameter of absorption coefficients (relating to mass), the radii (geometry), the new cosmic constant $\Lambda$ and the prevailing maximum acceleration $g_0$ in the neighboring universe, or equivalently the pressure $J_0/c$ exerted by the radiant energy in our neighborhood. We may further rearrange the above to provide a more tangible idea of how the force is derived by

$$F = p_g A_{R_1} A_{R_2} \frac{\pi R_1^2 \pi R_2^2}{r^2} \tag{85}$$

which states that the force is proportional to the pressure exerted by the gravions times the absorptivities times the cross-sections of the spheres while still being inversely proportional to the square of the distance. It seems like we can separate one group of factors pertaining to geometry alone and another group of factors
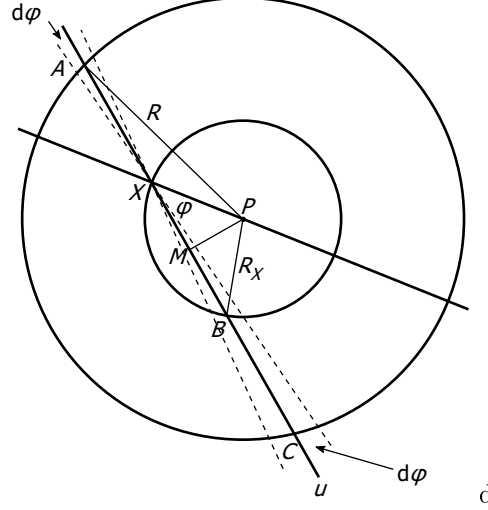
Figure 7: *Derivation of the internal field of a uniform density sphere.*

pertaining to matter (energy) and its interactions involving the local system of two masses interacting with the universal pressure of gravions. The two masses do nothing by themselves except for the mediating flow of gravions.

Therefore, the above equations provide variant expressions of the law of gravitational force in the new physics of PG. They are particularly appealing by their consistency and symmetry of parameters beyond Newtonian physics.

The mathematical derivation from first principles of the radiant intensity absorption involving multiple integrations involves a number of simplifications and cancellation of terms by appropriate choice of a reference system of co-ordinates. The multiple integrations may also be done by numerical means via relatively simple Python codes run in parallel to shorten the computation times. Integrals involving absorption along lines crossing a single sphere cancel out leaving only integrals crossing both spheres simultaneously. This work has been omitted from the present report to avoid needless congestion that could potentially distract from the important finding above and beyond. This section may be expanded to form a self contained chapter with sufficient detail to qualify for publication, which, however, can await at least for an initial response by the established scientific community on the current report as is.

IMPORTANT: The new gravitational law in PG expressed by Eq. 85 has far more repercussions than being a simple substitution of old parameters with new. It states that the force between two masses is not simply proportional to the effective masses but proportional to the associated absoptivities of the masses. This means that non-spherical masses exert a different force for different relative orientation of the same masses (bodies). If, as they move towards or away from each other they change orientation, they also change gravitational absorptivity and hence the trajectories would not be as expected from Newtonian mechanics. The dependence of acceleration on the density distribution will become more clear in Section 9. Further analysis is presented under the Equivalence Principle in Section 12.1.

# 7   Internal spherical field

So far, we have examined the field generated externally to a spherical body, but we now proceed to find the field also inside the sphere. With reference to Fig. 7, the acceleration at any point X inside a sphere with radius $AP = R$ is provided by integrating the absorption along the lengths of mass inside the differential solid angles indicated on either side of the point X along any direction of the line $u$. We note that the absorption length $XA = BC$ leaving only the length $XB$ to yield a net absorption, which is the same as that of the sphere with radius $R_X$ crossing the point X. Therefore, we have the same situation as that experienced by Newtonian mechanics, in that a hollow sphere would exert zero force inside its cavity. Now, the acceleration at this internal point is given by (see Eq. 81)

$$g_X = g_{0X}\left(1 - \frac{1}{2k^2 R_X} + \frac{\exp(-2kR_X) \cdot (2kR_X + 1)}{2k^2 R_X}\right) \equiv g_{0X} A_{R_X} \tag{86}$$

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where $A_{R_X}$ is the familiar $A_R$ factor at the surface of the internal sphere with radius $PB = R_X$ and $g_{0X} < g_0$ due to the shielding of the outer layer from X.

We can find $g_{0X}$ by resorting to the usual absorption factor $f_{gX}$ at point X by the following steps:

The exponential absorption factor in the direction XBC is

$$1 - \exp\left(-k \cdot XB(\varphi) + k \cdot BC(\varphi)\right)$$

and in the direction XA is

$$1 - \exp\left(-k \cdot XA(\varphi)\right)$$

so that we take their difference in the integral:

$$f_{gX} = \int\limits_0^{\pi/2} 2\pi \sin\varphi \cos\varphi \mathrm{d}\varphi \cdot \left[\exp\left(-k \cdot XA(\varphi)\right) - \exp\left(-k \cdot XB(\varphi) + k \cdot BC(\varphi)\right)\right] \tag{87}$$

and integrate with respect to angle $\varphi$ from 0 to $\pi/2$ as can be seen in the referenced diagram.

From the geometry shown and M being the mid-point of the cord AC, we find and replace the lengths accordingly with:

$$XB(\varphi) = 2R_X \cos\varphi$$

$$XA(\varphi) = BC(\varphi) = \sqrt{R^2 - (R_X \sin\varphi)^2} - R_X \cos\varphi$$

to obtain the integral formula:

$$f_{gX} = \int\limits_0^{\pi/2} 2\pi \sin\varphi \cos\varphi \mathrm{d}\varphi \cdot$$

$$\left[\exp\left(-k\sqrt{R^2 - (R_X \sin\varphi)^2} + kR_X \cos\varphi\right) - \exp\left(-k\sqrt{R^2 - (R_X \sin\varphi)^2} - kR_X \cos\varphi\right)\right] \tag{88}$$

From this found, we can derive the acceleration at X by the factor $g_0/\pi$ and equate it to its value given above by Eq. 86:

$$g_X = \frac{g_0}{\pi} f_{gX} = g_{0X} A_{R_X} \tag{89}$$

from which we can find the relationship between the internal $g_{0X}$ and external $g_0$ .

$$g_{0X} = \frac{g_0 f_{gX}}{\pi A_{R_X}} \tag{90}$$

The expected Newtonian acceleration at X is given by:

$$g_{XN} = \frac{4}{3}\pi G \rho_e R_X \tag{91}$$

No analytical relationship was found for $f_{gX}$, so that we may resort to numerical means for this parameter. For practical application, we also need to see the difference of PG from Newton acceleration against various depths from the surface of the Earth by replacing the internal radius as a function of depth.

$$R_X = R - depth \tag{92}$$

We present some values as in Table 1 for Earth by using average values for density and absorption coefficient taken from the Table 3 as used for various planets in the following section. Tentatively, we initially use the value $g_0 = 1000$ m/s$^2$. The results provide the expected deviation of measurements from Newtonian physics at various depths, if the Earth's crust had uniform density and a spherical shape. We can do measurements in a very deep mine or in a deep ocean, however, we would need to re-calculate the local acceleration in both Newton and PG cases. In practice, measurements of this kind would be complicated by influences of the local variations of density and time dependent fluctuations of the local acceleration, but the given table provides a first idea of the order of magnitude of expected deviation from Newton for a prospective careful experiment. It seems that these deviations should be measurable by a sensitive gravimeter

| $depth=$ | 2500 | 5000 | 7500 | 10000 | 12500 |
|---|---|---|---|---|---|
| $g_N=$ | 9.816404 | 9.812551 | 9.808697 | 9.804844 | 9.800990 |
| $g_0$ | $\Delta g$ | $\Delta g$ | $\Delta g$ | $\Delta g$ | $\Delta g$ |
| 300 | 7.29E-05 | 1.45E-04 | 2.18E-04 | 2.90E-04 | 3.61E-04 |
| 500 | 4.32E-05 | 8.62E-05 | 1.29E-04 | 1.72E-04 | 2.14E-04 |
| 1000 | 2.14E-05 | 4.27E-05 | 6.39E-05 | 8.50E-05 | 1.06E-04 |
| 2000 | 1.07E-05 | 2.13E-05 | 3.18E-05 | 4.23E-05 | 5.28E-05 |
| 5000 | 4.25E-06 | 8.48E-06 | 1.27E-05 | 1.69E-05 | 2.11E-05 |
| 10000 | 2.12E-06 | 4.24E-06 | 6.34E-06 | 8.43E-06 | 1.05E-05 |
| 20000 | 1.06E-06 | 2.12E-06 | 3.17E-06 | 4.21E-06 | 5.26E-06 |
| 30000 | 7.08E-07 | 1.41E-06 | 2.11E-06 | 2.81E-06 | 3.50E-06 |
| 50000 | 4.24E-07 | 8.47E-07 | 1.27E-06 | 1.69E-06 | 2.10E-06 |

Table 1: *Difference of acceleration $\Delta g$ between Newton and PG at various depths in Earth.*
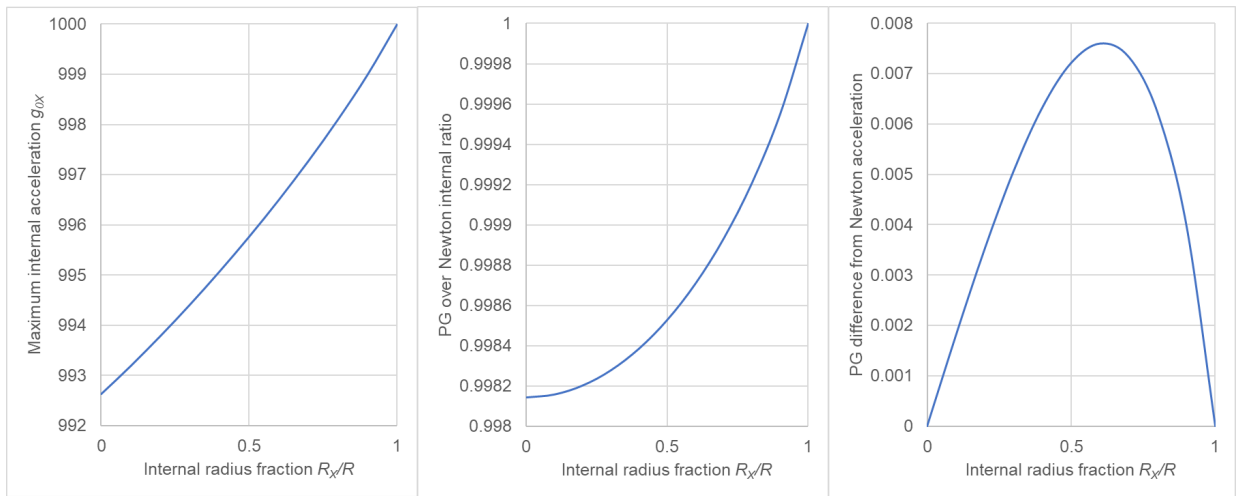


Figure 8: *Internal maximum acceleration $g_{0X}$, internal ratio of PG over Newton accelerations and internal difference of PG from Newton acceleration in Earth.*

with sufficient confidence if $g_0$ has a sufficiently low value. In turn, by establishing true measurements of the acceleration at various depths, we can deduce the unknown parameter $g_0$ .

For theoretical considerations, we can also see the variation of maximum internal acceleration $g_{0X}$, the ratio of $g_{0X}/g_{NX}$ (PG/Newton) and the difference of PG from Newton acceleration in Fig. 8 at any depth (fractional radius) again for a tentative external $g_0$ =1000 m/s$^2$ in the case of planet Earth using the same parameters.

Establishing the variation of the maximum acceleration factor $g_0$ inside a planet, it also suggests that this parameter may not be so constant even in our relatively "small" area of the universe even inside the heliosphere, since there is a significant mass within the heliosphere itself, whilst our planets are just internal points within this sphere. This might explain the Pioneer anomaly for the deviation of gravity measurements from expected values from Newtonian mechanics. This then points also to the alternative possibility of purposefully sending a spacecraft to more accurately measure the same effect while eliminating (preventing) other already proposed explanations.

## 7.1 The Greenland experiment anomaly

Shortly before publication of the manuscript of this work, it has been accidentally found a report on "*The Greenland Gravitational Constant Experiment*" Zumberge *et al.* (1990) dealing exactly with the measurement of gravity in a bore hole in the ice-sheet. A deviation (shortfall) from Newton has been found in the range of between 1-4 mGal. This report appears particularly comprehensive in dealing with all possible sources of error and still found to establish a gravitational anomaly that cannot be explained by known theory stating in the abstract that: "*An anomalous variation in gravity totaling 3.87 mGal (3.87 x 10$^{-5}$ m/s$^2$) in a depth interval of 1460 m was observed. This may be attributed either to a breakdown of Newtonian gravity or to unexpected density variations in the rock below the ice*" . Although these measurements cannot be used "as is" to do any quantitative connection to the PG predictions in this Section, we do note that the order of magnitudes match well with those of Table 8. This is particularly encouraging to organize a similar experiment, perhaps, best suited in an ocean, where the local variations of gravity may be less or more easily predictable and the depth measurements about one order of magnitude greater.

However, variant reports by Zumberge and coworkers have failed to reproduce this anomaly at various oceanic depths, which might be attributed to either (a) the experimental error involved overwhelms the anticipated effect, which is of the order of magnitude tentatively deduced from the Allais effect per following Section 11.4, or (b) the mathematical treatment used for the Newtonian derivation may need reworking, else the direct method used in PG computations needs to be applied for the specific mass distribution at the location tested and integrated with the whole planet.

# 8 Application to the solar system

We can tentatively apply the equations of PG so far, like in Sections 5.3 and 5.5, to the solar system by assuming values of the maximum prevailing acceleration $g_0$ in our area of the universe.

We first plot the density ratios for many bodies of the solar system in Fig. 9 in the given hypothetical range of values for $g_0$ between 300-50000 m/s$^2$. These graphs show that we get practically identical curves for Mercury and Mars having close to same surface accelerations, whilst all else are proportionally separated in accordance to their surface gravity. The graphs indicate the degree of departure from real densities depending on the chosen value of $g_0$

For better appreciation of the magnitudes involved, numerical results are also presented, as example, for three bodies (Sun, Earth and Moon) in Table 2 in the same range of $g_0$. Some typical values of $\Lambda$ for the same range of $g_0$ are also given being universal for all bodies per Eq. 72.

For any given set of mass and radius of each planet, we have derived the corresponding surface acceleration and effective density rather than using random (published) values from different sources. This is necessitated by the need to be consistent and accurate in these calculations and avoid discrepancies. These parameters are sensitive to very small changes of the input data. Small bodies are even more sensitive and round-off errors in the calculations are significant. Excel sheets were used for formatting the plotted figures, which initially necessitated the use of an "Add-in" (xlPrecision) to increase accuracy beyond 15 decimal places. Likewise, in a later use of Python code to reproduce the same output, we had to use increased computational precision to avoid serious round-off errors. In Table 3, we provide the initial data used for various bodies here and in all calculations elsewhere in this report. We also quote in parenthesis some variant values of the surface acceleration that were found from different sources for comparison, but not applied.

In all above, the derived and used parameters are based on the average density of the chosen bodies, which strictly speaking cannot produce the correct (actual) PG result, unless we knew in advance the radial
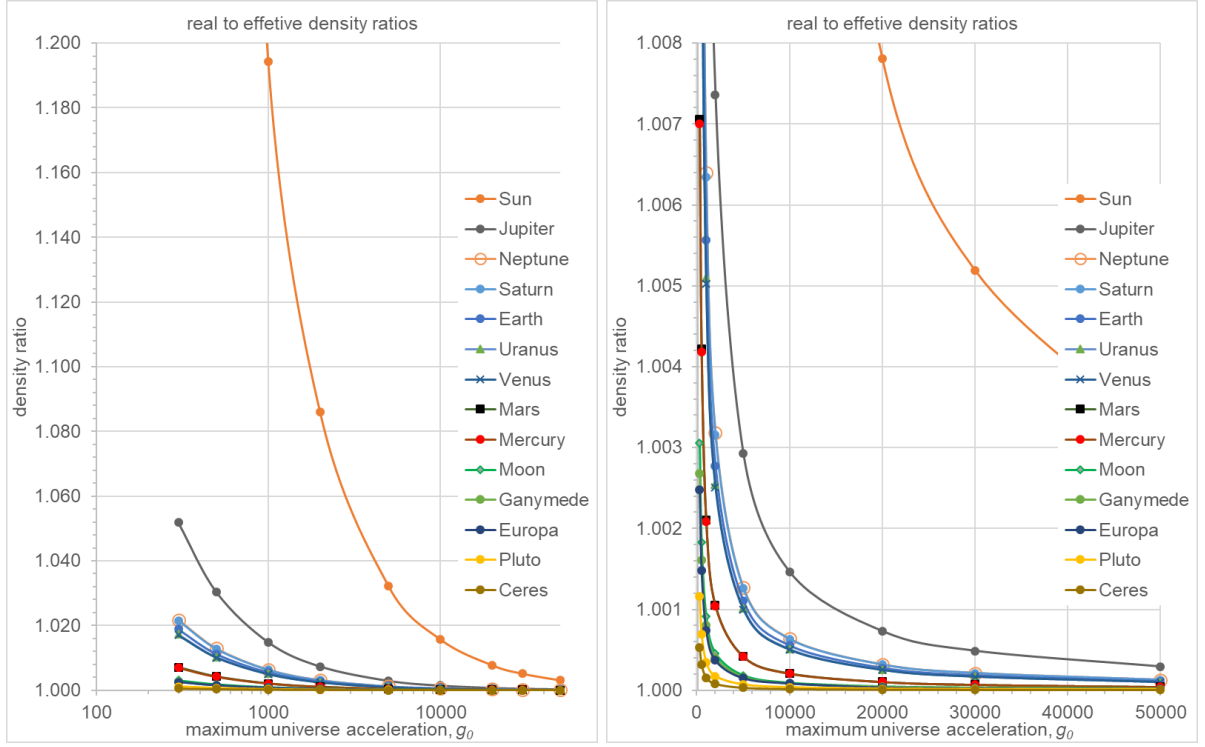
Figure 9: *Ratio of real to effective (measured) densities for planets, moons and the Sun.*

| | Sun | | Earth | | Moon | | |
|---|---|---|---|---|---|---|---|
| $g_0$, m/s$^2$ | k, 1/m, | $\rho/\rho_e$ | k, 1/m | $\rho/\rho_e$ | k, 1/m | $\rho/\rho_e$ | $\Lambda$, m$^2$/kg |
| 300 | 3.42E-09 | 3.4640 | 3.93E-09 | 1.01886 | 2.35E-09 | 1.003060 | 6.99E-13 |
| 500 | 9.11E-10 | 1.5366 | 2.34E-09 | 1.01121 | 1.41E-09 | 1.001833 | 4.19E-13 |
| 1000 | 3.54E-10 | 1.1943 | 1.16E-09 | 1.00556 | 7.02E-10 | 1.000915 | 2.10E-13 |
| 2000 | 1.61E-10 | 1.0860 | 5.80E-10 | 1.00277 | 3.51E-10 | 1.000457 | 1.05E-13 |
| 5000 | 6.12E-11 | 1.0322 | 2.31E-10 | 1.00111 | 1.40E-10 | 1.000183 | 4.19E-14 |
| 10000 | 3.01E-11 | 1.0158 | 1.16E-10 | 1.00055 | 7.02E-11 | 1.000091 | 2.10E-14 |
| 20000 | 1.49E-11 | 1.0078 | 5.78E-11 | 1.00028 | 3.51E-11 | 1.000046 | 1.05E-14 |
| 30000 | 9.94E-12 | 1.0052 | 3.85E-11 | 1.00018 | 2.34E-11 | 1.000030 | 6.99E-15 |
| 50000 | 5.95E-12 | 1.0031 | 2.31E-11 | 1.00011 | 1.40E-11 | 1.000018 | 4.19E-15 |

Table 2: *Calculated absorption coefficient k and ratio of real $\rho$ over effective $\rho_e$ density for the Sun, Earth and Moon in an assumed range of $g_0$ values.*

| planet | radius $R$ | mass $M_e$ | density $\rho_e$ | $g_R$ (other) |
|---|---|---|---|---|
| Sun | 6.95E+08 | 1.989E+30 | 1.41446E03 | 274.825 (273.7) |
| Jupiter | 6.9911E7 | 1.8982E27 | 1.326E3 | 25.9204 (24.79) |
| Neptune | 2.4622E7 | 1.02413E26 | 1.6379344E3 | 11.27456624 (11.15) |
| Saturn | 5.8232E7 | 5.6834E26 | 6.87123E2 | 11.1860(10.44) |
| Earth | 6.371E6 | 5.97237E24 | 5.5136E03 | 9.82026 (9.807) |
| Uranus | 2.5362E7 | 8.6810E25 | 1.27037E3 | 9.00729 (8.69) |
| Venus | 6.0518E6 | 4.8675E24 | 5.243E3 | 8.87009 (8.87) |
| Mars | 3.3895E6 | 6.4171E23 | 3.93408E03 | 3.727854(3.720) |
| Mercury | 2.4397E6 | 3.3011E23 | 5.42701E3 | 3.70150 (3.7) |
| Moon | 1.73700E+06 | 7.34767E+22 | 3.34705E03 | 1.62533 (1.625) |
| Ganymede | 2.634E6 | 1.4819E23 | 1.93590E3 | 1.42554 (1.428) |
| Europa | 1.560E6 | 4.799844E22 | 3.01832E03 | 1.316343805 (1.315) |
| Pluto | 1.1883E6 | 1.303E22 | 1.85386E03 | 0.615862 (0.62) |
| Ceres | 4.730E05 | 9.393E20 | 2.161E3 | 0.280203 (0.28) |
| Callisto | 2.4103E6 | 1.075938E23 | 1.8344 | 1.235 |

Table 3: *Numerical constants of planets, moons, and the Sun used in calculations of preceding tables and graphs.*

density distribution for any given body. However, we obtain some first order of magnitude idea of the new important parameters introduced in this work. It should be noted that the density ratios approach unity as we increase $g_0$.

## 8.1 Further analysis

To better understand the meaning of the real density expected for a planet, we can plot what the acceleration on the surface would be if the measured (effective) density were used as the real density. Let's use the data for the Sun given by Table 3 and plot $g_R$ against $g_0$ in Fig. 10 using Eqs. 58 and 59. We note that the Sun's real acceleration is approached asymptotically at very high values of $g_0$. The latter is as expected, because increasing $g_0$ decreases $k$, which makes the PG value to become Newtonian, i.e. to reduce to $g_R =$274.825 m/s$^2$ as given in Table 3. The same can be deduced by taking the limit of Eq. 58 as $k \to 0$.

We have already found that by increasing the radius of a planet by adding mass at constant density, the surface acceleration reaches a saturation limiting value, namely, $g_0$, i.e. when $A_R$ becomes unity. This is at variance with Newtonian prediction of infinity by Eq. 74.

Likewise, with increasing the density by keeping the radius constant, the Newtonian prediction is infinity. However, in PG the factor $A_R$ being a function of the product $kR$ becomes a product also of $\Lambda \rho R$ meaning that $A_R \to 1$ by increasing $\rho$ with constant $R$ and $\Lambda$. Similarly, by shrinking a star (sphere) with constant mass, we obtain unity for $A_R$ as the density becomes fast very large (the density being inversely proportional to the third power of radius). In other words, the eventual surface acceleration reaches the saturation value of $g_0$ in clear distinction from Newtonian mechanics.

Last in this connection, we should also consider what happens at a fixed point in space away from a sphere (star), when the sphere shrinks with constant mass. By Newton, the acceleration remains constant at that point, but by PG this is not the case: The acceleration monotonically becomes smaller, due to self shadowing ($k$ increases much faster than the radius) by

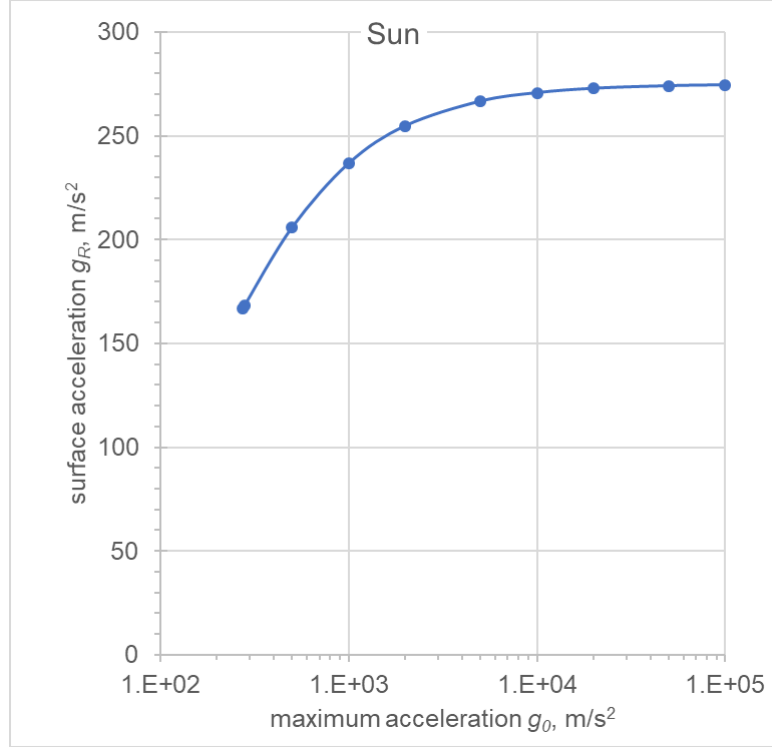$$g_{fixed\_r} = g_0 \frac{\pi A}{r^2} \tag{93}$$

noting that $A$ varies as:

Figure 10: *Expected surface acceleration on the surface of the Sun against maximum $g_0$ using the measured density as real density.*

$$A = R^2 \left[ 1 - \frac{R^4}{2C^2} + \frac{R^4}{2C^2} \exp(-2C/R^2) \cdot (2C/R^2 + 1) \right] \tag{94}$$

where the constant $C$ is defined during the $k$ substitution below:

$$k = \frac{3GM}{g_0 R^3} \equiv \frac{C}{R^3} \tag{95}$$

Noted also that the PG equation of acceleration reduces to Newton's equation, as expected, for very small values of $k$:

$$g_A = g_0 A_{A_{k \to 0}} = g_0 \frac{4}{3} kR = \frac{4}{3}\pi G \rho R = G \frac{M}{R^2} \tag{96}$$

Furthermore, we can substitute $k$ accordingly and find $g_A$ for a white dwarf and a neutron star. The extreme accelerations reported for these bodies pose for now a serious question on whether PG could ever be directly measurable or detectable if $g_0$ needs to be too high. This would constitute a new serious challenge for PG by not being able to detect it experimentally, unless those extremely high values of acceleration are generated by yet another type of push particle. We will discuss this issue again in Part 2 of this report.

# 9   Concentric spheres with different densities

We now consider the case of two concentric spheres of different density as depicted in Fig. 11. The inner sphere has a radius $R_1$ with density $\rho_1$, mass $M_1$ and absorption coefficient $k_1$, and the outer sphere has a radius $R_2$ with density $\rho_2$, mass $M_2$, and absorption coefficient $k_2$. There are two cases of PG absorption, namely, one along a typical cord AB traversing only the outer sphere, and another traversing segment CD of the outer sphere then a cord DE of the inner sphere and then segment EF of the outer sphere again.

To find the acceleration at point P being at a distance $r = PQ$, we follow the integration steps as in the first place for PG ( Eq. 37), but for the two parts described above:

Part one involves integration in the angle between $\varphi_1$ and $\varphi_2$ for the outer spherical layer.
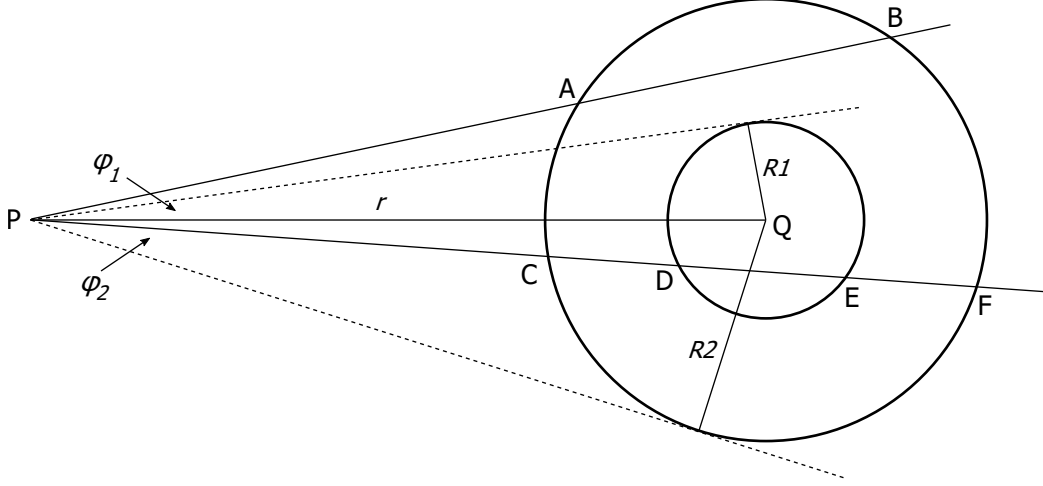
Figure 11: *Derivation of acceleration from concentric spheres with different densities.*

$$f_{g2} = 2\pi \left[ \frac{\sin^2 \varphi}{2} - \frac{\exp\left(-2kr\sqrt{a^2 - \sin^2 \varphi}\right)\left(2kr\sqrt{a^2 - \sin^2 \varphi} + 1\right)}{4k_2^2 r^2} \right]_{\varphi_1}^{\varphi_2} \tag{97}$$

where

$$a = \frac{R_2}{r} = \sin \varphi_2 \tag{98}$$

After substituting the integration limits, we get a familiar relationship as follows:

$$f_{g2} = \pi \left[ R_2^2 - R_1^2 - \frac{1}{2k_2^2} + \frac{\exp\left(-2k_2\sqrt{R_2^2 - R_1^2}\right)\left(2k_2\sqrt{R_2^2 - R_1^2} + 1\right)}{2k_2^2} \right] \frac{1}{r^2} \tag{99}$$

Part two then involves the following steps starting with the general PG Eq. 37, where we have for the inner sphere

$$a = \frac{R_1}{r} = \sin \varphi_1 \tag{100}$$

and need to replace the exponential having length $\ell$ in the exponent with three exponential factors corresponding to the three consecutive absorption layers (lengths) in EF, DE and CD:

$$f_{g1} = \int_0^{\varphi_0} 2\pi \sin \varphi \cos \varphi \mathrm{d}\varphi \cdot \left[1 - \exp(-k_2 \cdot EF(\varphi)) \cdot \exp(-k_1 \cdot DE(\varphi)) \cdot \exp(-k_2 \cdot CD(\varphi))\right] \tag{101}$$

That is

$$f_{g1} = \int_0^{\varphi_0} 2\pi \sin \varphi \cos \varphi \mathrm{d}\varphi \cdot \left[1 - \exp\left(-2k_2 \cdot EF(\varphi) - k_1 \cdot DE(\varphi)\right)\right] \tag{102}$$

Because

$$2EF = CF - DE \tag{103}$$

and using Eq. 12 for each of the spheres, we can easily replace with:

$$f_{g1} = \int_0^{\varphi_0} 2\pi \sin \varphi \cos \varphi \mathrm{d}\varphi \cdot \left[1 - \exp\left(-2k_2 r \sqrt{a_2^2 - \sin^2 \varphi} - 2\left(k_1 - k_2\right) r \sqrt{a_1^2 - \sin^2 \varphi}\right)\right] \tag{104}$$

| Jupiter | radius $R$ | mass $M_e$ | density $\rho_e$ |
|---|---|---|---|
| whole planet | 6.9911E7 | 1.8982E27 | 1326 |
| core | 1.6E7 | 4.289E26 | 25000 |
| outer | 6.9911E7 | 1.4693E27 | 1039 |

| $g_0$ | $g_{RPG}$ | $g_R - g_{RPG}$ | $\rho/\rho_e$ |
|---|---|---|---|
| 300 | 18.97555828 | 6.944841717 | 1.051873063 |
| 500 | 18.87716456 | 7.043235439 | 1.030304147 |
| 1000 | 18.80444874 | 7.115951257 | 1.014860051 |
| 2000 | 18.76839103 | 7.152008972 | 1.007359332 |
| 5000 | 18.74684295 | 7.17355705 | 1.002927051 |
| 10000 | 18.73967389 | 7.180726112 | 1.001460768 |
| 20000 | 18.73609184 | 7.184308161 | 1.000729697 |
| 30000 | 18.73489819 | 7.185501814 | 1.000486312 |
| 50000 | 18.73394339 | 7.186456606 | 1.000291714 |

Table 4: *A two-layered sphere model of Jupiter with same real mass redistributed to the corresponding radii provided; Surface accelerations with PG and difference from Newton in a range of $g_0$ values.*

for which unfortunately the anti-derivative could not be found analytically. The total acceleration is given by the usual factor as:

$$g = g_0(f_{g1} + f_{g2})/\pi \qquad (105)$$

As usual, we equate $r = R_2$, when we need to find the acceleration $g_R$ at the surface of a sphere.

We may appreciate the relative magnitudes involved, if we were to take, for example, Jupiter as consisting of two concentric spheres with the tentative (arbitrary) parameters provided in Table 4. Jupiter's core constitution is uncertain, so that the values are only indicative for the present purposes and chosen among various values in the literature (https://sciencing.com/jupiters-core-vs-earths-core-21848.html). The Jupiter mass is actually layered with variable densities, but the best we can demonstrate at this stage is to start with a uniform mass equal to the total one actually measured ($M_e$) from its corresponding acceleration $g_R = 25.92$ m/s$^2$ (already used here). We then find the real mass $M$, as we did for various planets before, by first solving the equation of the parameter $A_R$ for $k$ with any given $g_0$, from which we establish the real density $\rho$ and density ratio $\rho/\rho_e$. Next, we redistribute this mass in the two spheres in the same proportion as initially provided in this table, namely 0.2259599008534 fraction of the total is compressed inside the inner sphere (core) and the remainder fraction is contained by the outer spherical layer. The real densities $\rho_1$ and $\rho_2$ in the two layers are readily found, from which the corresponding parameters $k_1$ and $k_2$ are calculated and used in Eq. 11. The results for the acceleration on the surface of the planet are given in numerical form in Table 4 again as a function in the typical range of $g_0$.

In Newtonian mechanics, the redistribution would have no effect on the surface acceleration $g_R$, but in PG the surface acceleration $g_{RPG}$ is very different, as we can see it is significant. The lower values obtained from PG indicate that the final actual densities should be increased in order to yield the real measured surface acceleration. In other words, there is a significant amount of hidden mass by the mere fact of having a dense core over and above (in addition to) the hidden mass also present in a uniform distribution. This is important, which means that any attempt to redistribute the mass of Jupiter along the radius should take into account the new physics revealed by PG. This also means that all previous calculations assuming an average constant density for the planets produces only approximate results. The difference becomes more important with the increase of the planet or star size. Noted also that the main (dominant) component of the PG acceleration comes from the diluted outer layer mass for the chosen mass redistribution - if the two components are considered separately. Hence in general, all prior attempts dealing with assumed mass and mass distributions should be re-appraised accordingly. In fact, artificial satellites orbiting Jupiter have reported anomalous orbits with a noticeable wobble, which may be attributed to Moons of Jupiter being
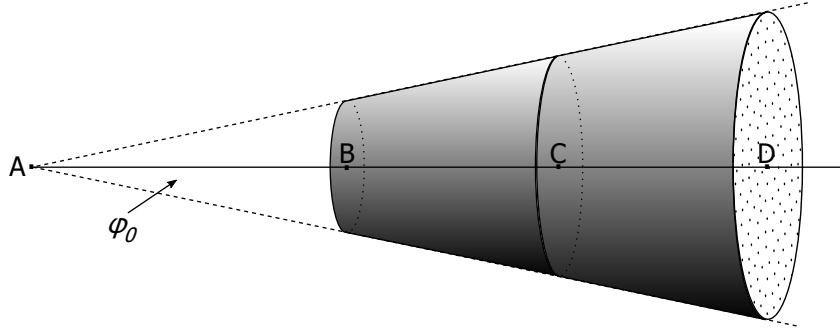
Figure 12: *Coaxial truncated spherical cones (sections) with fixed and equal height.*
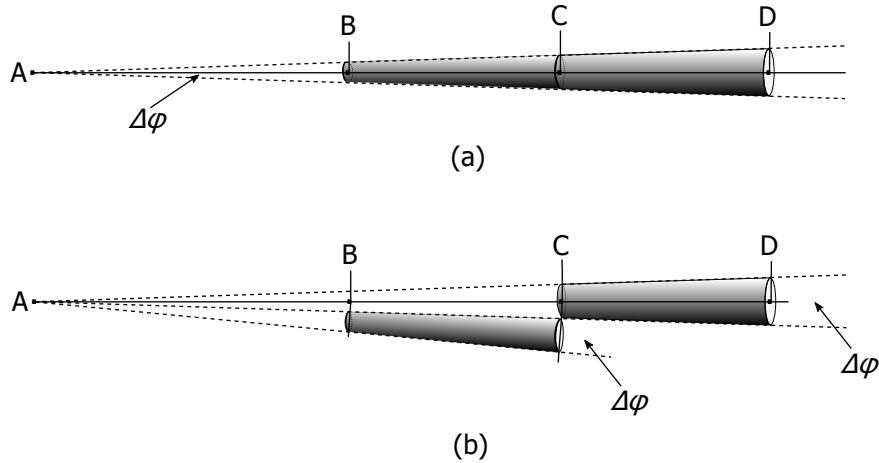


Figure 13: *Elementary truncated cones of equal height in series (a) and in parallel (b).*

shadowed by a different core density from the outer planet. We now have a new basis to re-evaluate and explain many phenomena already on record.

We may generalize and conclude that the radial distribution of density in a spherical body is critical in the generation of acceleration at the surface of the sphere and beyond according to PG, whereas this distribution makes no difference in the Newtonian acceleration lumping the mass at the center of gravity (i.e. center of the sphere). In an arbitrary shape with an arbitrary density distribution then, the only correct way is to derive the acceleration and force by integration of the gravion absorption around three coordinate axes yielding the three components of the vector of acceleration.

# 10    The superposition principle revisited and revised

The superposition principle, also known as superposition property, states that, for all linear systems, the net response caused by two or more stimuli is the sum of the responses that would have been caused by each stimulus individually. This applies to Newtonian gravity. However, this is not valid in general PG, unless the absorption coefficient $k$ is relatively small.

Let us now consider Fig. 12, where we draw two co-axial truncated cones subtending the same solid angle at point A with semi-angle $\varphi_0$ and with equal height, namely, $BC = CD = \ell$. Each material cone creates the same amount of gravion shadowing, if considered separately, i.e. without the presence of the other. In other words, PG provides an insight first with an immediate result that all truncated cones of constant angle and equal height will produce the same acceleration of gravity regardless of their distance from a common convergence point; this result can be derived at without any computation or integration of the elementary masses constituting these shapes.

However, when they act in series as depicted, the inner (nearest to A) cone is shadowed by the outer one and absorbs a lesser amount from the decreased output of gravion intensity by the outer cone. In the special case where the absorption is linear, which is the case when $k$ is sufficiently small, then we can superpose their separate absorption like in Newtonian superposition of gravity.

Now, we consider the general case of PG again in Fig. 13(a), which is essentially the same as the previous figure but the truncated cones subtend a very small angle $\Delta\varphi$, which allows the shifting of the inner cone as in 13(b) by the same small angle without practically changing the direction of the vector of shadowing (acceleration), i.e. both are considered to retain the same direction at point A. By this, we get a simplified derivation in the case of exponential absorption of gravions (i.e. general PG) below.

Each truncated cone constitutes a material layer with thickness $\ell$ and absorption coefficient $k$, so that the transmitted intensity is given by Eq. 6. When the layers are in series as shown in (a), the total absorption through the double thickness is

$$\Delta J_{series} = \Delta J_0 \left(1 - \exp(-k2\ell)\right) \tag{106}$$

However, if the inner cone is shifted as in (b) with the vectors of acceleration practically lined up, we can add them numerically for the total absorption according to Eq. 6 as

$$\Delta J_{parallel} = \Delta J_0 2 \left(1 - \exp(-k\ell)\right) \tag{107}$$

The difference between these cases then becomes

$$\Delta J_{parallel} - \Delta J_{series} = \Delta J_0 \left(1 + \exp(-k2\ell) - 2\exp(-k\ell)\right) \tag{108}$$

which is a positive number and indicates that the total shadowing (acceleration) by the two layers is stronger when they are in parallel than when in series by one shielding the other.

The same can be expressed also in terms of absorption fractions:

$$f_{parallel} - f_{series} = 1 + \exp(-k2\ell) - 2\exp(-k\ell) \tag{109}$$

and in terms of acceleration:

$$\Delta g = g_{parallel} - g_{series} = g_0 \left(1 + \exp(-k2\ell) - 2\exp(-k\ell)\right) \tag{110}$$

# 11 On direct measurement of PG

## 11.1 Sphere

We can inquire about the difference of acceleration derived by Newton and PG on the surface of a sphere of known density to determine if it is practically possible to detect and measure the new PG parameters directly. If the Newton acceleration on the surface is $g_{RN}$ and the PG acceleration $g_R$, their difference is given by

$$\Delta g = g_{RN} - g_R = \frac{4}{3}\pi G\rho R - g_0 A_R \tag{111}$$

for which we need $k$ in $A_R$ given from Eq. 59 as

$$k = \frac{\pi G\rho}{g_0} \tag{112}$$

from assumed values of $g_0$ and the real density of the sphere. We can plot the difference like we plotted the ratio of accelerations in Fig. 5, but we prefer to see directly some numerical outputs in Table 5 by choosing, say steel with $\rho = 7500$ kg/m$^3$.

We may also further work on the equation above to produce:

$$\Delta g = g_0 \left(\frac{4}{3}kR - A_R\right) \tag{113}$$

which is a function of the product $kR$.

For very high $kR$, the difference is very high, but for very small $kR$ the difference is very small but finite. By expanding the exponential to a Taylor series $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$ and taking the limit for small $kR$, we obtain for the difference of accelerations

$$A_{R_{kR\to 0}} = \frac{1}{6}kR \left(8 - 6kR + 4k^2R^2\right) \tag{114}$$

| $R=$ | 10 | 100 | 1000 | 10000 | 100000 |
|------|-----|------|------|-------|--------|
| g$_N=$ | 2.097E-05 | 2.097E-04 | 2.097E-03 | 2.097E-02 | 2.097E-01 |
| g$_0$ | $\Delta g$ | $\Delta g$ | $\Delta g$ | $\Delta g$ | $\Delta g$ |
| 300 | 8.243E-13 | 8.243E-11 | 8.243E-09 | 8.243E-07 | 8.243E-05 |
| 500 | 4.946E-13 | 4.946E-11 | 4.946E-09 | 4.946E-07 | 4.946E-05 |
| 1000 | 2.473E-13 | 2.473E-11 | 2.473E-09 | 2.473E-07 | 2.473E-05 |
| 2000 | 1.236E-13 | 1.236E-11 | 1.236E-09 | 1.236E-07 | 1.236E-05 |
| 5000 | 4.946E-14 | 4.946E-12 | 4.946E-10 | 4.946E-08 | 4.946E-06 |
| 10000 | 2.473E-14 | 2.473E-12 | 2.473E-10 | 2.473E-08 | 2.473E-06 |
| 20000 | 1.236E-14 | 1.236E-12 | 1.236E-10 | 1.236E-08 | 1.236E-06 |
| 30000 | 8.243E-15 | 8.243E-13 | 8.243E-11 | 8.243E-09 | 8.243E-07 |
| 50000 | 4.946E-15 | 4.946E-13 | 4.946E-11 | 4.946E-09 | 4.946E-07 |

Table 5: *Difference of acceleration between Newton and PG on the surface of an iron sphere with density 7500 kg/m$^3$.*

$$\Delta g_{kR \to 0} = \frac{1}{3} g_0 k^2 R^2 (3 - 2kR)_{kR \to 0} = g_0 k^2 R^2 \tag{115}$$

For the numerical example of the table, we see that we could have used Eq. 115 for small $kR$, which provides that the difference is proportional to $g_0$ and to the square of the radius of the sphere. The practical outcome is that, for the smallest sphere, we would need an extremely sensitive gravimeter with an accuracy up to 10 orders of magnitude smaller than the expected Newtonian value. The situation improves fast as we increase the sphere diameter, except that such spheres are out of any practical use. The situation improves with decrease of $g_0$.

## 11.2 Cone

We have further investigated whether the same reference spherical masses used above, if reshaped properly, they could yield any improved (i.e. greater) difference between Newton and PG for a possible measurement from a known density mass. This has been investigated for truncated and spherical cones with negative results (i.e. no improvement). However, interestingly enough it was found that there is an optimum cone angle yielding maximum acceleration difference at their apex, but still very close to, (but less than) the spherical shape. There is no need to present these results at present in order to give priority to more mundane issues below.

## 11.3 Cube

Perhaps, a large steel (or other heavy material) cube shape might be more feasible to construct by bricks, which would reduce cost by later disassembling and re-use of the steel material. The Newtonian gravitational fields has already been provided analytically by Chappel *et al.* (2012). Measurements of some gravity contour (or point) around the cube may be done with the most sensitive gravimeter to investigate possible "anomaly". With a positive outcome, we can then calculate the corresponding PG gravity contour (or point) by integrating the shading of gravions per established theory. From the known density, we will then be able to directly derive all other PG parameters.

## 11.4 The Allais effect

The previous finding on gravity superposition in PG can be used for explaining the known "Allais effect" recorded during total eclipses of the Sun. According to this, the gravity on Earth is increased during the eclipse, namely, the Moon+Sun have less attraction on Earth during an eclipse than just before or after the eclipse.

Qualitatively, we can say that when the Moon stands between the Sun and the Earth, it shields the shadowing of the Sun reducing the sum total of the shadows of the Moon and Sun separately prior to them being aligned. The principle of this effect, now for spherical bodies, is derived quantitatively below by PG theory.

The derivation is facilitated, since by coincidence the Sun and the Moon subtend practically about the same average solid angle of 0.53 and 0.52 degrees, respectively. Based on Fig. 14, we use $R_s$ for the Sun radius and $R_m$ for the Moon radius, located at distances $PQ = r_s$ and $PQ' = r_m$ from point Q.

When the two spheres are lined up, then the integral of their PG gravity acceleration is given by

$$f_g = \int_0^{\varphi_0} 2\pi \sin\varphi \cos\varphi \mathrm{d}\varphi \cdot \left[1 - \exp\left(-k_s \ell_s(\varphi) - k_m \ell_m(\varphi)\right)\right] \tag{116}$$

where we have simply added the lined-up cord lengths $AB = \ell_s$ and $A'B' = \ell_m$ of the Sun and the Moon. Using the established lengths for these cords, we substitute as follows:

$$f_g = \int_0^{\varphi_0} 2\pi \sin\varphi \cos\varphi \mathrm{d}\varphi \cdot \left[1 - \exp\left(-2k_s r_s \sqrt{a_s^2 - \sin^2\varphi} - 2k_m r_m \sqrt{a_m^2 - \sin^2\varphi}\right)\right] \tag{117}$$

However, because both spheres are taken to subtend equal angles, we have

$$a_1 = \frac{R_m}{r_m} = \frac{R_s}{r_s} = a_2 = \sin\varphi_0 = a \tag{118}$$

and the integrand is simplified below:

$$f_g = 2\pi \int_0^{\varphi_0} \sin\varphi \cos\varphi \mathrm{d}\varphi \cdot \left[1 - \exp\left(-2\left(k_s r_s + k_m r_m\right)\sqrt{a^2 - \sin^2\varphi}\right)\right] \tag{119}$$

which is of the same form as Eq. 36 by setting $kr = k_s r_s + k_m r_m$. Thus, we obtain from

$$f_g = 2\pi \left[\frac{\sin^2\varphi}{2} - \frac{\exp\left(-2kr\sqrt{a^2 - \sin^2\varphi}\right)\left(2kr\sqrt{a^2 - \sin^2\varphi} + 1\right)}{4k^2 r^2}\right]_0^{\varphi_0} \tag{120}$$

the accelerating absorption fraction

$$f_g = 2\pi \left[\frac{\sin^2\varphi}{2} - \frac{\exp\left(-2\left(k_s r_s + k_m r_m\right)\sqrt{a^2 - \sin^2\varphi}\right)\left(2\left(k_s r_s + k_m r_m\right)\sqrt{a^2 - \sin^2\varphi} + 1\right)}{4\left(k_s r_s + k_m r_m\right)^2}\right]_0^{\varphi_0} \tag{121}$$

By substituting the limits of integration and in view of Eq. 118, we get

$$f_g = \pi \left[\frac{R_s^2}{r_s^2} - \frac{1}{2\left(k_s r_s + k_m r_m\right)^2} + \frac{\exp\left(-2\left(k_s R_s + k_m R_m\right)\right) \cdot \left(2\left(k_s R_s + k_m R_m\right) + 1\right)}{2\left(k_s r_s + k_m r_m\right)^2}\right] \tag{122}$$

The above result applies during the eclipse, i.e. when the spheres are in "series".

When the spheres are in "parallel" to each other, i.e. just before or just after the eclipse, we use the form of Eq. 38 to sum the separate contributions of each as

$$f_g = \pi \left[R_s^2 - \frac{1}{2k_s^2} + \frac{\exp(-2k_s R_s) \cdot (2k_s R_s + 1)}{2k_s^2}\right]\frac{1}{r_s^2} + \pi \left[R_m^2 - \frac{1}{2k_m^2} + \frac{\exp(-2k_m R_m) \cdot (2k_m R_m + 1)}{2k_m^2}\right]\frac{1}{r_m^2} \tag{123}$$

The Allais effect then should appear as the difference of acceleration in the above positions (equations)

$$g_{Allais} = g_{parallel} - g_{series} \tag{124}$$

where we have multiplied Eqs. 122 and 123 by $G/\Lambda$ to obtain the factor $g_0$ and hence the corresponding accelerations.

We can plot calculated values of $g_{Allais}$ against $g_0$ to establish for which values of $g_0$ we find the measured $g_{Allais}$. However, because we are dealing with very small numbers in these calculations, it is necessary not
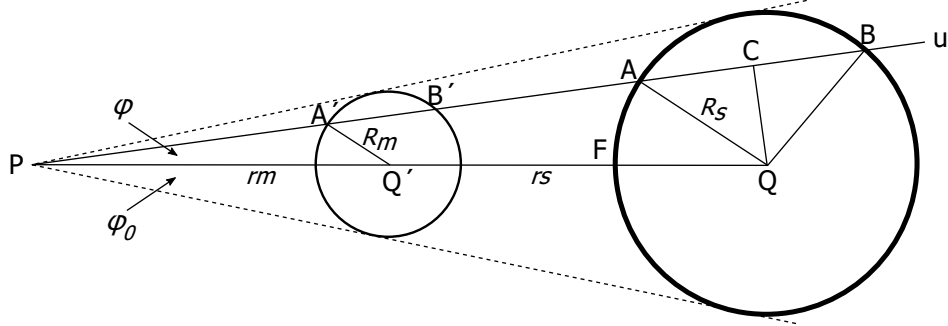
29

Figure 14: *PG diagram during Sun-Moon eclipse.*

| $g_0$, m/s$^2$ | k, 1/m, Sun | ρ , kg/m$^3$ | $\rho/\rho_e$ |
|---|---|---|---|
| 300 | 5.865327998802280E-09 | 8392.131445 | 5.459718 |
| 500 | 1.027835151863200E-09 | 2451.050109 | 1.594594 |
| 1000 | 3.894839466528660E-10 | 1857.583228 | 1.208499 |
| 2000 | 1.758800091947190E-10 | 1677.664808 | 1.091448 |
| 5000 | 6.665703761053060E-11 | 1589.551972 | 1.034124 |
| 10000 | 3.276671652759630E-11 | 1562.757685 | 1.016692 |
| 20000 | 1.624742838322570E-11 | 1549.791755 | 1.008257 |
| 30000 | 1.080184118890250E-11 | 1545.531147 | 1.005485 |
| 50000 | 6.466902304965150E-12 | 1542.144338 | 1.003282 |

Table 6: *Variation of Sun parameters against increasing values of $g_0$ with adjusted radius $R = 6.76002E08$ m corresponding to effective density $\rho_e = 1537.0998312$ kg/m$^3$ and $g_R = 290.489290112956$, but the same mass $M_e = 1.989E30$*

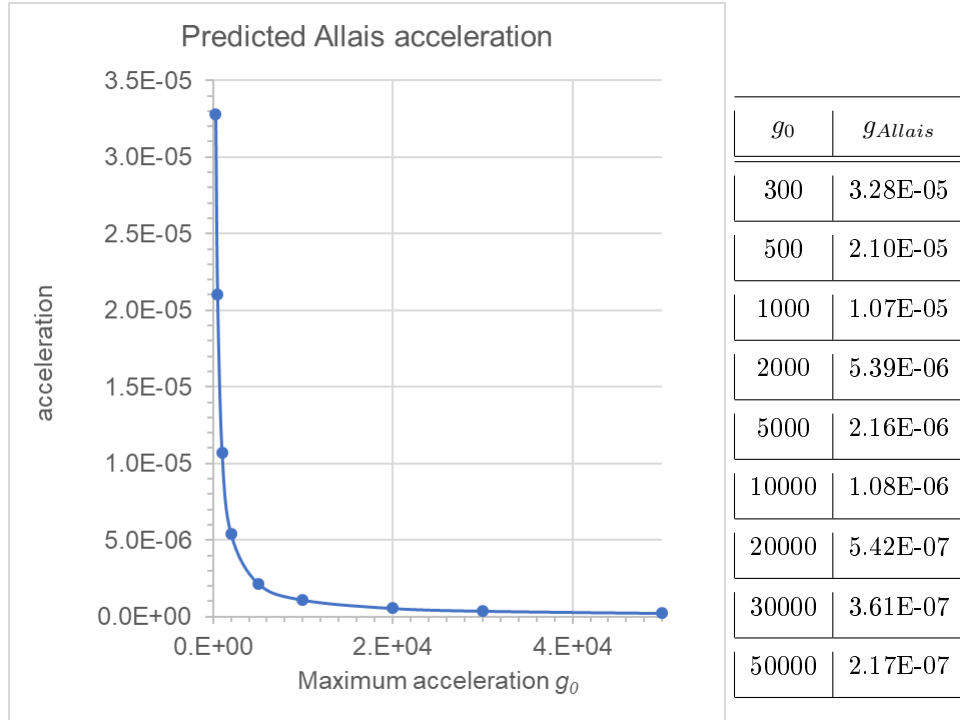| $g_0$ | $g_{Allais}$ |
|---|---|
| 300 | 3.28E-05 |
| 500 | 2.10E-05 |
| 1000 | 1.07E-05 |
| 2000 | 5.39E-06 |
| 5000 | 2.16E-06 |
| 10000 | 1.08E-06 |
| 20000 | 5.42E-07 |
| 30000 | 3.61E-07 |
| 50000 | 2.17E-07 |

Figure 15: *Predicted variation of the Allais effect acceleration vs. assumed values of $g_0$.*

only to use increased accuracy in the mathematical tools employed and also to eliminate the small but significant difference arising from the fact that the Moon and Sun do not subtend exactly equal angles. In other words, it was found that the small difference of the actual subtended angles overwhelms the Allais effect altogether by resulting in significant inconsistencies in the calculations. In order to test the validity of the principle, at least, we need to take the real values for one sphere and project (adjust) slightly the other so as to satisfy the condition of Eqs. 118. This is done only to be able to use the derived Allais equations above, in order to demonstrate the effect; otherwise we would be faced with considerable complexity to use PG with the actual angles, which is not needed for the present purposes. A high accuracy theoretical derivation would also require the availability of experimental measurement of the Allais effect with commensurate accuracy. Unfortunately, such measurements do not exist, because the Allais effect has been reported with significant inconsistencies. We believe that these inconsistencies are caused by the interference of various other effects on Earth, such as tidal effects and others (may be atmospheric, etc.) as well as variability of the time and position during this event. In view of these practical difficulties, it would be in vain to apply derived formulas for a direct practical outcome, except that we can use them to establish the principle and an order of magnitude. Refinement of the theory and practice of the Allais phenomenon is left for future work. Therefore, we opted to adjust the radius of the Sun to match the subtended angle of the Moon, while keeping its mass constant. We have done this and present the numerical values in Table 6, as opposed to the values presented in Table 2. We can then use these values of $k$ for the Sun and the corresponding values from Table 3 for the Moon in Eq. 124, which we plot in Fig. 15 with tabulated numerical values of the same graph in the inserted table.

From a paper by Lorenzen (2017), we obtain an average value $g_{Allais} = 3.5E-7 \text{ m/s}^2$, which corresponds to $g_0 = 30800$ m/s. If that were to be correct, then we would have derived the fundamental constant $g_0$ for our neighborhood universe. However, this is only a tentative value, most likely to be revised later. It may be that similar measurements taken from the Moon during a solar eclipse by the Earth could provide more reliable values; clearly, in that case, we should derive another equation taking into account the actual solid angles subtended by Earth and the Sun at the Moon.

With the proposed tests, PG could be verified but not disproved in the event of a very high level of $g_0$ reducing the effect beyond the measuring ability of our instruments at present. It is hoped that "anomalies" of Newtonian mechanics will be firmly measured and established to provide an affirmation of PG once and for all in the near future.

## 11.5  Other verification means for PG

We have already proposed measurements of the internal field of Earth's crust, variation of gravity in the heliosphere and other methods above. Another test may be by using very sensitive gravimeters to measure the variation of gravity on Earth during a 24 hour rotation preferably during a new Moon (or better before, during and after a solar (better total) eclipse), whereby the Earth is shadowing the Sun+Moon system overnight. The deviation from predicted values using Newtonian mechanics should provide an indication and perhaps an evaluation of the PG parameters.

Alternatively, a similar to the previous observation could be made by the variation of the orbit of an Earth satellite during a new Moon (or better before, during and after a solar (better total) eclipse, whereby the Earth is shadowing the Sun and Moon systems during the night passage of the satellite, presuming that the effect on the orbit could be measurable. Could then such a variation explain the variations (wobble) observed by Juno during orbiting Jupiter?

# 12  The Equivalence Principle

We can easily reproduce Newton's attraction force by push gravity and hopefully all other observed relationships (a task by later work). Furthermore, under the understanding of PG, we can now say that the well known equivalence principle (EP) is not violated. In fact, it is better explained as an identical process in the two systems being referred to, namely, one in a gravitational field and another accelerated by an equal force in space outside the gravitational field. That is, whether a mass is pushed "by hand" (or pulled via a rope in an elevator in free space), or the same mass is pushed by gravions by an equal force, then the outcome should be the same, namely, the mass will travel distances proportional to the square of time $(t^2)$. Push gravity creates a force by streaming gravions through the entire mass dragging every mass element concurrently, the sum total being a force no different from a push (pull) force acting by a spring with measurable deformation on a solid mass (or an imaginary accelerating force experienced inside an elevator in space). The gravitational push force is distributed throughout the mass, whilst the spring force acts on the external surface of a **rigid** (for argument's sake) mass and indirectly transmitted and distributed to all body elements producing an identical outcome. Then the same mass being acted by an equal force would accelerate by the same amount, i.e. we would measure distances proportional to the square of time, from the moment the mass is set free to travel (in free space or in the neighborhood of the gravitating body). If the mass is held stationary by some stationary "wall", then the mass experiences the force (by the gravions or the spring) without moving (like pushing on or pushed by a stationary wall). The gravion force appears as a mysterious attraction force by Newton, which necessitated the adoption an "equivalence principle" to explain the observable equal outcomes by the same mass acted upon by the Earth's gravity, or by " the rope on an enclosed elevator encompassing the same total mass". With the insight readily provided by PG (streaming gravions), the "equivalence principle" need not be a "principle" at all any more; it is just an identity as seen by PG, **it is** the same thing.

The Equivalence Principle (EP) is a mere and easily understood consequence of the hidden reality of the PG gravity principle, hence there is no need to postulate the EP any longer. The self-shadowing (shielding) causing an underestimation of the real mass does not refute the above understanding: To the extent that part of the mass is shielded from the action of gravions, if we push it "by hand" by the same force, as Newton would have us to use, then we would correspondingly measure the same distances. The actual mass (bigger than the apparent one) would be acted upon with an equal force, in both cases, of a falling body due to gravity or moving in space outside gravity. In both systems (cases) the same force acts on the same mass being real or effective, producing the same outcome.

Summary: PG does not require an equivalence principle, since everything exists in a real "elevator" being pushed by streaming gravions, not requiring a fictitious (gedanken) second elevator as theorized to date.

## 12.1  Falling bodies and Flyby anomaly

From the above description and understanding of the EP (per author) and if there is no distinction between effective and real mass (as per PG), then it follows that the gravitational and inertial mass are equivalent. The latter then forms an alternative form of the Principle. However such an equivalence of masses is clearly at variance with PG: As understood and described above, the effective mass corresponds to the gravitational mass, the force from which is transmitted to the real (entire) mass of the body, i.e. to the inertial mass of the body.

| $g_0$ | PG over Newton velocity ratios |
|-------|--------------------------------|
| 300   | 0.999999980343 |
| 500   | 0.999999988206 |
| 1000  | 0.999999994103 |
| 2000  | 0.999999997051 |
| 5000  | 0.999999998821 |
| 10000 | 0.999999999410 |
| 20000 | 0.999999999705 |
| 30000 | 0.999999999803 |
| 50000 | 0.99999999882 |

Table 7: *Ratio of falling velocities by PG over Newton.*

Thus, applying the PG parameters as developed so far, let us designate by $M_e$ the mass of the gravitating very large body (sphere), so that it is considered stationary, when other very much smaller bodies with effective mass $m_e$ fall to it. We consider only the case, where any final velocities achieved remain far from being relativistic, so that the assumed steady state of PG is retained. We can use the effective masses as in Newtonian mechanics for the potential energy $GM_e m_e/r$ around the gravitating mass. Likewise, we integrate for the the work done by the resulting gravitational force on the total (real/inertial) mass $m$ to obtain the additional kinetic energy as the body falls from point (radius) $r_1$ to $r_2$ and apply the conservation of energy equation:

$$\frac{1}{2}mu_{PG}^2 = \frac{GM_e}{r_1}m_e - \frac{GM_e}{r_2}m_e \tag{125}$$

from which the final velocity $u_{PG}$ is given by

$$u_{PG} = \sqrt{2GM_e\left(\frac{1}{r_1} - \frac{1}{r_2}\right)\frac{m_e}{m}} \tag{126}$$

It there is no distinction between the two masses above, then by Newtonian mechanics, the corresponding final velocity $u_N$ would be

$$u_N = \sqrt{2GM_e\left(\frac{1}{r_1} - \frac{1}{r_2}\right)} \tag{127}$$

The ratio between the above two velocities is then immediately obtained as

$$\frac{u_{PG}}{u_N} = \sqrt{\frac{m_e}{m}} \tag{128}$$

and from Section 5.5, we substitute the ratio of masses to obtain:

$$\frac{u_{PG}}{u_N} = \sqrt{\frac{3A_R g_0}{4\pi G\rho R}} = \sqrt{q} \tag{129}$$

as a function of the unknown parameter $g_0$. The ratio of masses in Eq. 128 is independent of the gravitating center (body) and it is equal to the accelerations ratio given by 82 on the moving (gravitated) sphere, or to the contraction factor $q$. We already listed the difference of the two velocities in 5 for steel spheres. We repeat the same but for the above ratios of a steel sphere with radius $R = 10$ m and density $\rho = 7500$ kg/m$^3$ in the typical range of $g_0$ in Table 7.

We can apply the above for typical flyby velocities and find that the difference between the above velocities is of the order of mm/s. This is consistent with observed flyby anomalies and it might help further explain them, i.e. in addition or in lieu of various proposed explanations. The Oumuamua anomaly (Bialy & Loeb, 2018) might be another candidate to re-examine as a flyby effect.

Theoretically, a spacecraft on an elliptical orbit could experience a greater force on its inbound direction than on its outbound one by changing direction of its disk-like (for example) shape thus exhibiting a greater effective mass in one part of the orbit than in the other. This would result in incremental accretion of energy

until it can reach escape velocity and then repeat the same process around a bigger planet (e.g. Jupiter) or the Sun. Similarly, mass distribution in a fan-like configuration might optimize the flyby effect by opening and closing the fan accordingly. In Section 11.2, we report that there is an optimum angle for spherical or truncated cone shape, whilst other shapes may be further investigated later. This might have little practical application, but it remains to be seen, if there is some benefit in furthering such an investigation.

In general, this effect says that a steel ball and a feather do not fall at the same speed inside a vacuum chamber: Let us consider a flat feather falling with its plane parallel or vertical to the direction of the gravitational field. In both cases it has the same real mass but different effective mass. When it falls with its plane vector parallel to the field, the effective mass is greater than when it falls with its plane vector normal to the field. In both cases we have the same object (mass) and the same inertial mass. Therefore, it will fall faster in the first case than in the second. The maximum speed (and acceleration) will be when the effective mass is practically equal to the real mass, i.e. when the feather can be spread out as much as possible (e.g. by thinning it down). Let us then consider a steel sphere and a very thin steel disk of the same mass; we can achieve this by first using the sphere and then the same object is flattened out to a very thin disk. Like with the feather, the steel sphere will achieve a slower final velocity than the same mass in the shape of disk. Now, the fine steel disk and the fine feather will fall at the same speed if they are both thin enough and in accordance with Newtonian mechanics, because they both use the real mass. However, the steel sphere will be slower than the feather, because the sphere displays an effective mass further away from its real mass than the feather does. The effect of orientation of a falling body is thus a new finding by PG, an extremely small effect to measure in the laboratory, but it may become cumulative and observable during a fall towards a planet or star from a significant distance.

The flyby difference (referred to as an anomaly to date) might be used purposefully for the measurement of the unknown value $g_0$ in our solar system. Furthermore, the presented perceptions on EP itself from the perspective of PG theory might help us better understand the Principle and its implications in past and future physics.

# Part Two (2)

The following presentation is an integral part of the whole report, but it is separated out because it contains a significant amount of speculative theory, which may have to be revised or rejected without affecting the preceding Part One (1). Part 1 should remain valid at least as a mathematical development of PG based on a set of postulates, barring inadvertent but rectifiable errors. In Part Two (2) specifically, it is proposed that general push gravity can be extended by borrowing the mathematical tools of general relativity, or that the two theories may complement each other. However, no concrete step has been undertaken yet, whilst it is hoped that this would be achieved better by experts in GR. It is further attempted to apply PG in astrophysics in the case of white dwarfs, neutron stars and black holes, but again it is hoped that this task would be best undertaken by others. The application of PG in particle physics and cosmology is barely but humbly mentioned in the hope that it might spark another discussion and research for a unified field theory and a theory of everything: It makes sense to assume that all fields are created by particles, now by push particles, i.e. all with a common denominator as the only way to achieve unity.

## 13    Response to criticisms

As mentioned at the outset of this report, there are numerous objections to the idea of push gravity since the original proposal by Fatio. This has applied to all hitherto variants of PG, but it is hoped that all these objections may be overcome in part by the preceding findings and in part by some new arguments and models presented in this Part 2. Most of the objections may be overcome without further ado, but the main problem of energy absorption and mass accretion can only be tackled speculatively at this point, if we have to face the dilemma of abandoning the preceding findings, or advancing forward on those findings. The best known objections are discussed next.

### 13.1    Weak absorption, range and gravitational shielding

Whilst early conceptions of push gravity maintained that it was mandatory to assume very minimal absorption of gravions in order to avoid the objectionable gravitational shielding, it is exactly the opposite consideration that frees push gravity and explains some of its intrinsic workings. Gravitational shielding or self-shadowing by mass is now at the core of the workings and understanding of PG. This is not something to object to, because via and by its presence we can actually derive the gravitational law, in fact, in a new form that can account for a lot of missing information in Newtonian mechanics including singularities.

### 13.2    Equivalence Principle

The allegation that PG would violate the Equivalence Principle must have been a misconception in view of our previous explanation. PG actually frees us from having to resort to the Equivalence Principle, which arose out of the need to understand the nature of force initially perceived as arising either from a gravitational field or from a moving mass under acceleration by an applied external force. PG finds no distinction between these two kinds of force, as the flow of gravions produces the same force in both situations (systems). For the first time, we have a tangible explanation of the **phenomenon** of equivalence of force. The gravitational force experienced by a body as attraction (pull) is actually a push force, namely, the sum total of all elementary push forces distributed in the bulk of the mass and arising from absorbed gravions. The latter force is of the same nature as the push force applied to a hypothetical elevator in free space, inside which we would experience an equivalent force. This equivalent force need not be such (equivalent) axiomatically, because it is "prima-facie" push in nature. Hence the Equivalence Principle per se vanishes without ever being in conflict with PG.

    Nevertheless, if the Principle emanates (or is based upon) the equivalence of the gravitating mass to the inertial mass, then PG is clearly at variance. However, PG quantifies the variance as being extremely small to easily detect in the laboratory, but, hopefully, big enough to measure in flyby experiments. If the EP is described as equivalence of masses (gravitational and inertial) then PG clearly violates it. Conversely, if PG provides the true relationship between the two masses, i.e. effective and real masses of the same body, then EP violates nature.

    At any rate, EP is arbitrary in the first place and redundant under PG.

## 13.3   Theory of relativity

It has been argued that PG is incompatible with the established theories of relativity. It is often argued that since the general theory of relativity (GR) is continually verified by contemporary measurements with great accuracy, PG not emanating from within GR must be wrong. However, the counter-argument may be that PG is a re-appraisal of classical Newtonian mechanics, upon which to build and extend the current relational developments of relativity. PG explains the generation of a gravitational field around a mass that presumably can be observed and measured identically with existing data. We would suggest that it is prejudicial to think that PG has to arise out of (or fit in) GR, whilst the opposite might be true. Therefore, the two theories may not have to be in conflict upon closer examination.

If gravions travel with the speed of light, then in the steady state, they establish a pushing field that fictitiously appears as an attractive field around the shadowing (gravitating) mass. This field is being established at the speed of light without emanating from the mass, but rather emanating from the surrounding universe. If the mass starts moving at speeds comparable to the speed of gravions (and light), then there will be a disturbance of the surrounding shadowing or warped space (field) due to a time lag that propagates at the gravion speed. This disturbance would be consistent with the gravitational waves scientists are trying to detect.

An analogy may be found in solid state physics near a PN-junction, where "holes" are formed from the absence of electrons on one side of the junction with an equal amount of excess electrons on the other side. These holes are treated, or behave, like a positive current of charge moving in the opposite direction to electrons with negative current. GR then is like it is treating gravity as gravion-holes apparently emanating from (or associated with) the mass, whilst in reality it is the real gravions (particles) moving in the opposite direction towards the gravitating mass that should be considered, or equivalently considered. The end result (force and acceleration) appears to be the same by both approaches. Both ways of creating a field around a mass presumably create identical apparent outcomes.

We propose then that the gravitational field described by PG and the field described by GR are quantitatively identical at every point around a stationary material body. The difference is that PG tells us how/why this is formed (i.e. its origins), whilst GR remains mute about the origin of the same field, but it yields verifiable measurements, anyway. The latter is sometimes described like "GR generates correct results for the wrong reasons". However, knowing the origins of gravity is a fundamental difference between GR and PG that could get us over the existing barriers in physics.

When we start applying PG to moving bodies with significant speed relative to the speed of gravions, then we may be able to borrow the mathematical tools already developed for relativity, special and general, to describe the same resulting effects and measurements. There is probably no restriction to the importation of Special Relativity as is. The mathematical derivations and achievements of GR may also be transferred and used in PG, in particular as they relate space and time. This transfer might be particularly useful where GR actually succeeds and discarded where GR fails (e.g. at very long distances, etc). The present work has only dealt with PG in the steady state without ever involving time effects yet. Therefore, it might be premature to argue that the two theories are in conflict.

Arguments of the type, for example, that because the Mercurial precession can be explained by GR is proof and manifestation of the success of GR should by no means be used to oppose PG. The same fields being established also by PG should arrive at the same outcomes. In fact, PG provides a new framework to re-appraise the contributions of other planets on Mercury's precession by expanding classical mechanics with PG, which may produce a further refinement of the same calculations taking into account the real density and mass distributions of all the planets contributing to this precession.

If at first sight the above assertions might seem simplistic, it is because there is a large volume of phenomena to be understood under PG, before we make further assertions. For example, could the temporal part of the metric in relativity, which determines the rate at which clocks tick and is responsible for Newtonian gravity, relate to the rate of gravion flux intensity? Could the increase of mass of a moving body as it approaches the speed of light be tied and explained in the new terms of real and effective mass? Should we, perhaps, re-appraise the meaning of inertial mass in conjunction with the meaning of matter and "stuff"? In general, could the theoretical concepts of relativity achieve an embodiment in PG theory? These important questions may now be better understood hoping to further an inquiry into the new PG theory.

## 13.4   Drag

It has been argued that push particles (original ultramundane corpuscles) would introduce a drag force on the orbiting Earth, eventually slowing down the planet to ever closer orbits around the Sun. This would indeed be a consequence, if the particles were acting like classical mechanical balls. However, the gravions are relativistic with no difference in speed relative to the planet motion. Gravions are not expected to make

a difference over any effects already experienced with photons over the broadest spectrum of wavelengths originating from outer space.

## 13.5   Superluminal speed

During the early stages of push gravity theories, the hypothetical corpuscles were required to have some superluminal speed to reduce the expected drag to a practically ineffective minimum. However, this is not required after the advent of relativity and in the light of the present arguments.

## 13.6   Orbital aberration

It has been further argued that PG would introduce orbital aberration due to the finite speed of gravity created by gravions. This aberration would tend to accelerate an orbiting body away from the other, unless gravity propagates much faster than the speed of light, or must not be a purely central force. It has been further argued that the sane finite speed of gravity problem is almost exactly canceled by the mathematics in GR. Now, it is not clear why PG cannot overcome this problem in the same way, if GR can. It is proposed that we may continue to use and adapt aspects and derivations of GR, or postulate an equivalence between GR and PG (at least in part), until it can be finally clarified if this is at all appropriate, or under what conditions.

Nevertheless, recent measurements report that planetary orbits are widening faster than if this were solely through the Sun losing mass by radiating energy. This results in an anomalous increase of the astronomical unit, which might then be explained by the above PG criticism pending further analysis of the situation.

## 13.7   Energy and mass considerations

Basically, the most serious criticism arises from the need that the gravions must be absorbed in order to produce a force, but the amount of energy absorption would then be so high as to be unsustainable by the gravitating body. This is the main reason, for which notables like Kelvin, Maxwell and Poincaré (Wikipedia contributors, 2018a; Poincare, 1908), after initial consideration, moved away from PG. There is no obvious or immediate solution to this major problem haunting any PG theory. For this reason, we based the entire development of PG on the assumption that the absorbed energy is somehow re-emitted. However, until some experiments provide encouragement at least, we are entitled to speculate with some improved models in continuation to previous attempts to overcome this hurdle. Let's first formulate the energy absorption problem based on derivations in Part 1.

We find the total energy passing per unit surface area of a sphere and absorbed by the bulk in the sphere, which is given by the previously defined $J_a$ (not $J_g$)

$$J_a = J_0 f_a = J_0 \int_0^{\varphi_0} 2\pi \sin\varphi \mathrm{d}\varphi \cdot [1 - \exp(-k\ell(\varphi))] \tag{130}$$

$$J_a = 2\pi J_0 \int_0^{\varphi_0} \sin\varphi \left[1 - \exp\left(-2kr\sqrt{a^2 - \sin^2\varphi}\right)\right] \mathrm{d}\varphi \tag{131}$$

At the surface of the sphere ( $r = R$) for the absorbed flux density per unit area $J_{aR}$ we have

$$J_{aR} = 2\pi J_0 \int_0^{\varphi_0} \sin\varphi \left[1 - \exp\left(-2kR\sqrt{1 - \sin^2\varphi}\right)\right] \mathrm{d}\varphi \tag{132}$$

which can be integrated analytically as

$$J_{aR} = 2\pi J_0 \left[-\cos\varphi - \frac{\exp(-2kR\cos\varphi)}{2kR}\right]_0^{\varphi_0} \tag{133}$$

and finally obtain

$$J_{aR} = 2\pi J_0 \left[1 - \frac{1}{2kR} + \frac{\exp(-2kR)}{2kR}\right] \tag{134}$$

The above provides the absorbed density flux per unit area from all directions inside a hemispherical solid angle. Thus, we multiply by the surface area of the sphere to obtain the total absorbed density flux, i.e. the total energy per unit time, or power $W$ as:

$$W = 8\pi^2 J_0 \left[ 1 - \frac{1}{2kR} + \frac{\exp(-2kR)}{2kR} \right] R^2 \equiv 8\pi^2 J_0 A_{aR} R^2 \tag{135}$$

defining a new absorption factor for PG by

$$A_{aR} \equiv 1 - \frac{1}{2kR} + \frac{\exp(-2kR)}{2kR} \tag{136}$$

Since we also have $J_0$ by Eqs. 59 and 70, finally we obtain :

$$W = 8\pi^2 \frac{cG}{\Lambda^2} A_a R^2 = 8\pi \frac{cg_0}{\Lambda} A_{aR} R^2 \tag{137}$$

from which we can readily calculate the energy absorption rate if we know $k$, or $\Lambda$, or $g_0$. We can initially use some values for $g_0$, or equivalently for the other parameters, and find tentative orders of magnitude for the total absorbed energy, or the total energy per unit mass (kg). We may initially use a simplified form at very small values of $kR$ as in Newtonian situation. Expanding the exponential term to a Taylor series using the first four terms, we obtain for the absorption factor $A_{aR}$:

$$A_{aR} = kR \left( 1 - \frac{3}{4} kR \right) \approx kR \tag{138}$$

and

$$W = 8\pi \frac{cg_0}{\Lambda} kR^3 = 8\pi cg_0 \rho R^3 \tag{139}$$

The rate of absorption per unit mass $W_M$ of the sphere is then provided by the simple expression:

$$W_M = \frac{2}{3} cg_0 \tag{140}$$

Numerically, if $c = 3 \times 10^8$ m/s, we have

$$W_M = 2 \times 10^8 g_0 \tag{141}$$

from which, depending on the prevailing $g_0$, we find the absorbed energy. With a moderate level of $g_0 = 10^4$ m/s$^2$, we would get $2 \times 10^{12}$ J/s/kg. From $E = mc^2$, that energy dissipation, if not re-emitted and if not converted to heat, it would accrete a mass of about $2 \times 10^{-5}$ kg for every kg of the sphere (Earth) every second. This is clearly an enormous amount of energy (mass) that cannot be accounted for by our experience on the planet. An early criticism leveled against PG claimed that the absorbing mass would be doubling every second, if to avoid heat dissipation. This criticism is generally valid even with our much lower accretion rate found above.

The above derivations are the most telling reason for the rejection of PG, which has been the case to date. Therefore, this constitutes a critical point whether to continue with this theory or come to an end of this investigation once more. The present author is of the opinion to persist in finding some way(s) to push through this barrier, literally. That is because the preceding findings have produced a system of consistent outcomes with Newtonian mechanics as the limiting case, and because it promises to resolve many other cosmological problems on a new basis. We may recall an analog situation early in the $20^{th}$ century, when the orbiting electrons should be emitting electromagnetic radiation, the lack of which did not deter the then visionary scientists to introduce and accept the orbital model of the atom. Thus, instead of rejecting the PG theory, we may have to accept that the dissipated energy by gravions manages somehow to escape out of the absorbing mass in a different form of radiating particles. A new motto could then be "**what goes in must come out**", but catchphrases don't make science on their own, unless they are confirmed without leading to another impasse: The above demand allegedly leads to another violation, namely, of the second law of thermodynamics, an objection discussed separately next.

## 13.8   Second law of thermodynamics

It has been argued that the gravions, if re-emitted as different particles to carry away the dissipated energy, would violate the second law of thermodynamics, which was the reason for rejecting the re-emission of particles/energy as initially (tentatively) proposed but abandoned by notables such as Kelvin, Poincaré, Lorentz and Thomson (Wikipedia contributors, 2018a). However, if we look closer at the intrinsic meaning of this law, it may not necessarily be violated overall. This arises from the fact that the law relates to the

most probable state of a closed system having the maximum entropy. The entropy $S$ relates with the number of accessible states $\Omega$ via

$$\Omega = \exp(S/k) \tag{142}$$

($k$ here is the Boltzmann constant) and the probability $P$ of finding the system in that state is

$$P \sim \Omega = \exp(S/k) \tag{143}$$

Now, when the system has a relatively small number of accessible states, the fluctuations can be very frequent, wide and repeatable, i.e. recurrence of unstable states may be quite feasible within the time scale of gravion frequency absorption. The consequence of this is that the system can often be found "momentarily" in a state of decreased entropy favoring the emission of some augmented (with accreted mass) particle out of the system. This happens when by random redistribution of mass and energy within the subsystem generates a sub-particle capable of overcoming the constraints that keep the subsystem together. When enough quantitative material and energy accumulation has occurred (accreted), the subsystem bounces emitting a new particle, all of this on an extremely short time-scale (appearing to us). The particles of the subsystem co-operate to get rid of and push out one of their own members every-now-and-then, often or not in the time scale of the subsystem. In other words, the second law of thermodynamics does not prevent us from accepting that matter/energy can be re-emitted after a number of trial fluctuations following a certain number of gravion absorption inside a proton, electron, neutron or any other nuclear, sub-nuclear, or elementary particle (subsystem). Thus, what was initially conceived by the critics as thermal dissipation inside matter in general, it will not appear as known chemical (molecular) heat that would melt and evaporate the planet. It would only appear as internal energy of a particle that is not thermally coupled with an atom or molecule via some sort of recoil action during the said re-emission. The re-coil produced by the proposed re-emission is taken up and averaged out by the subsystem behaving under the established quantum mechanical laws. In fact, it might be that the underlying mechanism of quantum mechanical randomness may be caused exactly by such re-coil of the subsystems of particles. Electrons and nuclear particles move about randomly per quantum mechanics. This model further assumes that the re-emitted particles are also penetrating the surrounding matter out of the planet with a long enough mean free path as not to heat the planet catastrophically but not long enough as to act like gravions in generating gravity (i.e. canceling out gravity). It is only the very long mean free paths of gravions that generate gravity among planets and stars, while the second generation emitted particles, as proposed here, behave like a diffusing gas out of the planet, perhaps, with some but not catastrophic heat dissipation. It may be that part of, if not all, the heat in the core of planets is generated by this mechanism in an analogous way, in addition to, or in lieu of, the heat being produced by radioactivity per prevailing theories.

We need not at this point specify the exact nature of the particles being re-emitted, other than for them to be able to carry away the absorbed gravion energy, or a critical part thereof. It is left for further investigation by particle and nuclear physics to establish if any of the known particles qualifies to play this role, as for example, neutrinos might (or might not) serve this purpose. Alternatively, we may build on a new model to describe the properties and consequences of this second generation of particles emanating from the primary gravion flow.

In support of the above general proposal, we may site a similar situation that explains radioactivity. Particles can rearrange in the nucleus, or change from one type to another statistically over time. Random quantum fluctuations can promote relaxation to a lower energy state and decay via quantum tunneling. Radioactive decay half-life varies over many orders of magnitude on a timescale down to $10^{-23}$ seconds (Wikipedia contributors, 2019d). In our proposed analog, we may envisage all sub-nuclear particles including protons, electrons, positrons, etc. to undergo such statistical fluctuations inside themselves at even extremely smaller time scales beyond the range of our measuring instruments, in effect, appearing like providing a continuous absorption of gravions and re-emission of secondary type-II particles diffusing in the surrounding material space without causing further gravitation or catastrophic heat. This continuous absorption then is tantamount to a continuous push without the feared catastrophic melting down.

The above proposed model should not be less plausible than the latest quantum fluctuation theories (Wikipedia contributors, 2019c). It is in accord with the fluctuation theorem and the ongoing discussion, research and experiments relating to Maxwell's demon.

Thus, the present framework in understanding gravity should not be inconsistent with modern theories. Quantum field theory is about very small stuff, small particles (the standard model). Gluons bind quarks together. Quantum gravity considers loops of gravitational force, then we get knots, loop quantum gravity and time disappears (problem of frozen time). These quantum states of space fluctuate, fluctuations in the quantum states of space create the appearance of time. These loops exist on the scale of Plank length. A proton contains $10^{65}$ quantum volumes, whilst gravitons is said to carry the force of gravity by exchanging

them, (the photon caries the electromagnetic force, so the graviton carries the gravitational force), but gravitons are thought to be pseudo-force particles according to loop theory. The quantum nature of space does not allow singularities, whilst the universe did not come about with a bang but with a big bounce [Jim Baggott: https://www.youtube.com/watch?v=dW7J49UTns8]. All these latest conceptions might be further adjusted and advanced by the new understanding of PG, so that our approach should not be less plausible than all these other modern models and proposals. In fact, PG seems to be consistent with the above theories so that PG may act as a resolution by binding together of the best of elements in those theories.

# 14　Momentum or push gravity as the universal and unifying cause of all types of acceleration (force)

If gravity is finally proven to be caused by gravions under the working of PG, then it could be a logical conclusion that all forces may be attributable to a similar cause, albeit by different kind of push particles. What would then be needed is that each kind of particles have a mean free path much longer than the dimensions of the masses (particles) acted upon. This requirement is already fulfilled for planets and stars by gravions, to which we may also refer as the first type-I push particles. The force is then generated by the law of conservation of momentum and energy. This momentum force is well established in physics as it is also a tangible phenomenon, i.e. understood by common experience. We may then extend the proposal to apply to all kinds of force fields regardless of the size of the field generating body. This may be a sensible proposition, because size should not be an obstacle at least for all experimentally known particles. Given that the size of an atom is of the order of $\approx 10^{-9}$ m, we still have 16 orders of magnitude to reach the Planck particle, the length $\ell_P$ of which is defined as:

$$\ell_P = \sqrt{\frac{\hbar G}{c^3}} \tag{144}$$

where $\hbar$ is the reduced Planck constant. The Planck length is about $10^{-20}$ times the diameter of a proton.

Thus, nucleons may be maintained by their own surrounding push (momentum) particles, the nucleus may be maintained by yet another kind of push particles, and so on for atoms, each group maintained by their own associated type of momentum particles. The universal situation may be that the space is filled by particles with a wide range of sizes (energies/wavelengths) corresponding to an equally wide range of mean free paths, acting as push particles to their matching (corresponding) relatively much bigger particles (bodies). The entire universe may then be thought as an agglomeration of varying concentrations of matter automatically sorting out themselves by the surrounding push particles. This proposition may then constitute a likely basis for a unification of all force fields in the cosmos from the smallest to the largest phenomena. Le Sage made a similar attempt to account for forces of different chemical strengths, by the existence of different species of "ultramundane" corpuscles of different sizes, whilst all this should be reconsidered and re-appraised in the light of modern particle physics, quantum mechanics, relativity and astrophysics.

Ultimately and inexorably, however, the above model only shifts the problem to what keeps the "ultimate" mysterious particle as a unity (re gravion), if not for an attractive force, according to Kant's philosophical reasoning. However, the lack of understanding of the nature of an ultimate particle is not yet reason good enough to reject a possible unifying model that allows us to concentrate our attention more to a smaller "area" of the cosmos that underlies as a common denominator to all other processes.

From the above broad model, we may narrow down the cosmological questions to assuming the existence of types of particles corresponding at least to the known force fields. Thus, gravions are type-I push particles that mediate the gravitational force, type-II push particles are those mediating the electrical field forces, type-III those mediating the nuclear forces, etc. Already, in quantum theory the electric field is thought to be due to a continuous stream of exchange of photons (say, here type-II particles). Dibrov (2011) believes that the core of electrons and positrons remains stable by pressure of the bombardment of "fations". The electron, in his proposed model, "*as against the static Abraham−Lorentz electron, is the dynamic object transforming the gravitational field energy into the energy of the electric field, and periodically exploding up.*" However, he probably means something very different to our proposed model in this report, because he talks about charge already being present in the electron, and he only tries to justify the re-emission of the "fation" energy in the form of electric field sub-particles. Considering various parameters quantitatively and his main conclusions, it is clear that his theory is not consistent with our findings. For example, "the active mass is not equal to the passive mass", he discovers a "violation of equivalence principle for the electron" and that the "gravitation constant G is not equal to the actual one", to mention a few aspects of his push gravity theory that are in
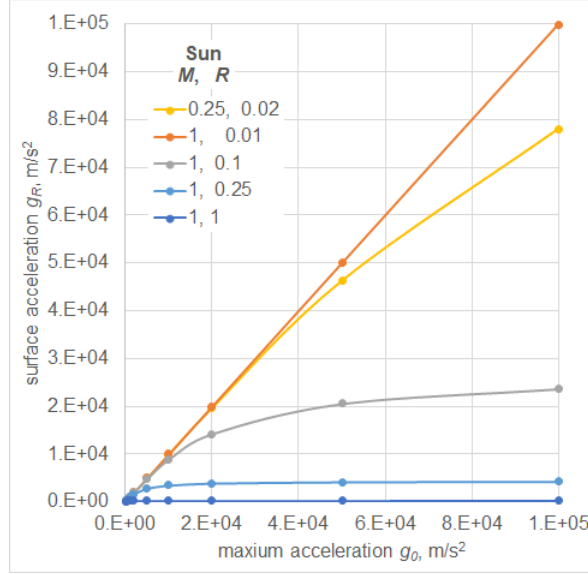
Figure 16: *Surface $g_R$ against maximum $g_0$ for stars with mass and radius (M, R) in units of Sun mass and Sun radius.*

clear variance to the ideas proposed herewith. Nevertheless, an "exploding electron" seems consistent with re-emission of absorbed gravion energy.

We may go on to elaborate on our general proposal (model). For example, the type-II particles, in particular, may be subdivided either in two sub-classes responsible for the positive and negative electrical force (such as opposite spin or an as yet unknown attribute), or may emanate from two complementary types of matter organization at the electron and positron level. The emanated energy (type-II) carriers exit as a result of the absorbed type-I push particles by protons and electrons, and so on and so forth.

In summary, for a field unification theory, it is logical and consistent to envisage and assume that all force fields are created by particles including gravity. This general idea of the underlying particles for all fields is then greatly facilitated by a push/momentum mechanism in a PG framework advanced in this report.

# 15    White dwarfs, neutron stars and black holes

It is reported that the gravitational field on a white dwarf is of the order of $10^6$ m/s$^2$, whilst that on a neutron star is of the order of $10^{12}$ m/s$^2$, and much greater on black holes. If these extreme accelerations are caused by gravions (in that case being the universal cause of all gravitational fields), then it might be unlikely that we can practically detect them directly by the methods proposed here, because of the need for extremely sensitive gravimeters. However, if it were found that the maximum $g_0$ is, say, around 30000 m/s$^2$ by some careful measurement, then we would be faced to explain the super high values of acceleration on white dwarfs, neutron stars and black holes. Then, one possible explanation would be to assume that those extreme accelerations may be caused by different, more energetic types of push particles. Before we further speculate on these other hypothetical types of push particle, let us apply a little further the already found PG relationships below.

We continue our investigation from where we left off in Section 8.1. By increasing the maximum prevailing acceleration $g_0$ in the neighboring universe, we inversely decrease the corresponding $k$ (see Eq. 59) by

$$k = \frac{\pi G \rho}{g_0} = \frac{3GM}{4g_0 R^3} \tag{145}$$

so that by keeping the mass and radius of a star constant, the PG equation is reduced to the value provided by Newton, namely, to $\frac{4}{3}\pi G\rho R = GM/R^2$, which is the saturation (asymptotic) value(s) observed in Fig. 10, when $k$ becomes sufficiently small.

Let us now see the values of surface acceleration $g_R$ against the prevailing maximum acceleration $g_0$ possible in a particular space of the universe for stars having various combinations of masses $M$ with radii $R$. This is shown in Fig. 16, where both mass and radius are expressed in units of Sun ($\odot$) mass and radius. The masses used are those of the apparent Sun mass but taken to be the real mass of a hypothetical star as a first approximation to get a feel of the situation. Then as expected, the pair $(M, R)=(1, 1)$ reproduces
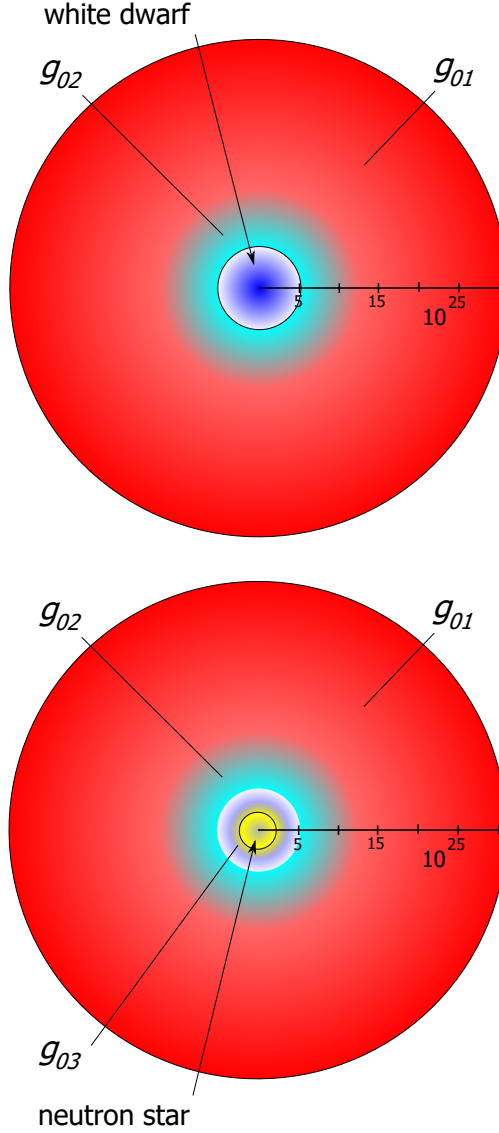
41

Figure 17: *Diagrammatic perception of a white dwarf (above) with its surrounding PG type-II ($g_{02}$) field inside the universal type-I ($g_{01}$) field, and a neutron star (below) with its surrounding PG type-III field ($g_{03}$) inside a dwarf type-II ($g_{02}$) field inside the universal type-I ($g_{01}$) field; the scales in m are logarithmic and approximate.*

the curve in Fig. 10 very close to the abscissa (visibly touching it) with an asymptotic value approaching the Newtonian value of surface acceleration of our Sun. The additional curves now show the outcomes of different values of the pair $(M, R)$, which can be understood by the above Eq. 145: For any fixed, $g_0$ and $M$, the value of $k$ increases very fast with a decrease of radius, which forces the surface acceleration to be well below the saturation values is reached, as noted by the curves on the figure. When $R$ reduces below a sufficiently low value, $k$ becomes so large that the factor $A_R$ in PG becomes unity and $g = g_0$, which is the straight line at unity slope in the figure.

In the event that we can safely measure $g_0$ and find that this is not as high as on the surface of a dwarf, however, it may be sufficient to trigger gravitational collapse in the presence of a critical mass. After the collapse, a white dwarf is formed that may be sustained also by push particles of a different kind (type-II). Likewise, upon formation of a neutron star, the forces holding it together may further be provided by push particles of a third kind (type-III) as they evolve upon the onset of a further collapse. This proposal forms initially a qualitative model, which is depicted with some hypothetical quantitative dimensions in logarithmic scales (powers of 10) in Fig. 17. The validity of such a hypothesis should by all means be cross-examined against existing data and theories in astrophysics to be further refined or even rejected, if not appropriate.

For the general reader and to better describe the proposed model here, it is helpful to summarize the current understanding of these dense bodies by conveniently referring to a brief description provided in

relevant articles by Wikipedia contributors (2019f). The summary descriptions below are needed to precede a new idea here attempting to connect the neutron star field with the atomic nuclear field, both unified under the proposed PG field model.

A white dwarf is a very dense stellar core remnant composed mainly of electron-degenerate matter. It has a mass like the Sun with a volume like the Earth. Because it no longer undergoes fusion reactions, it has no source of energy, so that it cannot support itself by fusion heat against gravitational collapse. It is supported by electron degeneracy pressure and is extremely dense. Accretion takes place by accumulating particles into a massive object, typically gaseous matter. Galaxies, stars, and planets, are formed by accretion processes. Neutrinos are radiated by white dwarfs through the Urca process (Wikipedia contributors, 2019e), which is a neutrino-emitting process playing a central role in the cooling of neutron stars.... White dwarfs have masses from about 0.07 to 10 M⊙.

An astronomical body can collapse by its own gravity drawing matter inward toward its center. Gravitational collapse is a fundamental mechanism for structure formation in the universe. It can all start from relatively smooth distribution of matter gradually collapsing to form pockets of higher density, stars and planets, stellar groups and clusters of galaxies.

A giant star with a total of between 10 and 29 solar masses collapses to a neutron star (Wikipedia contributors, 2019b). Other than black holes, neutron stars are the smallest and densest stars with a radius on the order of 10 km and a mass less than 2.16 solar masses. They are produced from the supernova explosion of a massive star, and together with gravitational collapse achieve the density of atomic nuclei.

Binary systems of neutron stars can undergo accretion making the system bright in X-rays and a source of short-duration gamma-ray bursts, as well as produce gravitational disturbance. At soaring temperatures, electrons and protons combine to form neutrons via electron capture, releasing a flood of neutrinos. It is important for our model proposed here to quote verbatim from Wikipedia the following: *"When densities reach nuclear density of $4 \times 10^{17}$ kg/m$^3$, neutron degeneracy pressure halts the contraction. The in-falling outer envelope of the star is halted and flung outwards by a flux of neutrinos produced in the creation of the neutrons, becoming a supernova. The remnant left is a neutron star. If the remnant has a mass greater than about 3 M⊙, it collapses further to become a black hole".*

*The temperature inside a newly formed neutron star is from around $10^{11}$ to $10^{12}$ Kelvin. However, the huge number of neutrinos it emits carry away so much energy that the temperature of an isolated neutron star falls within a few years to around $10^6$ kelvin. At this lower temperature, most of the light generated by a neutron star is in X-rays.*

*A neutron star has some of the properties of an atomic nucleus, including density (within an order of magnitude) and being composed of nucleons. In popular scientific writing, neutron stars are therefore sometimes described as 'giant nuclei'. However, in other respects, neutron stars and atomic nuclei are quite different. A nucleus is held together by the strong interaction, whereas a neutron star is held together by gravity. The density of a nucleus is uniform, while neutron stars are predicted to consist of multiple layers with varying compositions and densities."*

It is the above last statement that we can seize upon to support the PG model here, namely, we say here **that a nucleus and a neutron star are both held by <u>the same force</u>: That force is the pressure exerted by the presumed type-III push particles**. We propose that the strong nuclear interaction is no different from the neutron star gravity, namely, both being created by push particles of the same type. In consequence of this model, the space around any atomic nucleus inside the electron orbitals is occupied by push particles holding the nucleus together. The current understanding is that this space seems relatively more "empty" than the interplanetary space, so there is nothing weird about our hypothesis that push particles small enough occupy this space fulfilling the requirements of PG with regard to mean free path and absorption coefficient of the nucleus. There is plenty of "room" for such superfine particles on the scale all way down to the Plank length. It may turn out that these push particles are x-rays and gamma-rays of sufficiently short wavelength, which would be consistent with the strong x-ray emission by neutron stars. That may also provide the existence/mechanism of x-ray emission by the orbital electrons in atoms adjusting to different energy levels, as well as somehow explain the original mystery of stable electron orbitals of accelerating orbital charges. Thus, the atom is a micro-neutron star created from (after) breaking down a neutron star. We might want to call the corresponding type-III push particles *neutrions* (neutron + ιόν) in analogy to gravions. However, in proposing this model, it may not be clear how to differentiate between a nucleus and a neutron particle, so that we may have to refine the various distinctions of push particles mediating strong and weak interactions and all other sub-nuclear forces. The proposed model is only a general approach towards a unification of fields, which requires the cooperation of particle physics and astrophysics.

The above proposed scheme for neutron stars and atoms may not be acting alone, as it requires the simultaneous cooperation of a type-II push particles holding electrons and nuclei together in the atom. In a

similar fashion, white dwarfs are the plasma state by free electrons and nuclei having released their mediating binding (type-II) particles around the white dwarf. These mediating particles responsible for the appearance of electric field might be called *electrions* (from electricity + ιόν). "Neutrions" and "electrions" are finally redistributed after explosion to form atoms.

At any rate, the above general model could be described in more specific terms of particle physics such as: Gluons participate in the strong interaction in addition to mediating it. This is unlike the photon, which mediate the electromagnetic interaction but lacks an electric charge. Gluons also share this property of being confined within hadrons. One consequence is that gluons are not directly involved in the nuclear forces between hadrons. The force mediators for these are other hadrons called mesons. Although in the normal phase of QCD single gluons may not travel freely, it is predicted that there exist hadrons that are formed entirely of gluons — called glueballs. There are also conjectures about other exotic hadrons in which real gluons (as opposed to virtual ones found in ordinary hadrons) would be primary constituents.

The above intermittent extracts from established theories and observations from astrophysics and particle physics serve only to stimulate further discussion, one way or another, that could involve the push theory principle consistent with the findings of this report.

# 16   Discussion

An extended discussion of push gravity has been further prepared in relation to important topics such as the expanding universe, the gravitational law, perpetual motion universe, galaxies and the inertia-mass-matter relationship. However, they are omitted from the present version of the report: These major topics are already fraught with controversies and they are better left to be dealt with by the experts or supplemented by a later version of this work.

An interesting exposition of push gravity is presented by Thomas (2014). This provides a good philosophical basis of the concept of push/shadow gravity and a motive for further investigation. However, the positive aspects of PG should not be diminished by possible failure of certain interpretations of important issues. For example, the referenced gravitons are thought (proposed therein) to be strings (as in string theory), which may or may not be proven correct, so that PG should not be bound by such specific technical claims. The Allais effect is attributed to some sort of lensing mechanism of the gravitons around the Moon, but we have showed that the effect can be readily interpreted and even measured by the PG derivations in Part 1. These and other speculative technical interpretations, if found incorrect, should by no means reflect on PG in general. We have now tried to create an alternative paradigm of PG by building the mathematics on a set of postulates in order to arrive at the established laws of physics and beyond.

The present author's main expertise lies outside the fields directly pertaining to this report. As a result, Part 2, in particular, may not be as authoritative as it should be, whilst Part 1 could be seen as an attempt to produce and report new data and evidence in support of a long standing hypothesis to explain gravity. It is hoped that others may use and apply the latter findings in a better way, or as they see fit. In this context, the primary aim would have been achieved, namely, to place PG within the mainstream of physics. For the latter, it would be an even greater achievement, if work is undertaken to test the veracity of the present findings within the programs of various institutions and organizations. Should an affirmative finding be achieved, then PG could immediately find its rightful place in science. At any rate, the present author should be excused for possible "collateral" errors, whilst attention to the novel disclosures may not diminish.

If there is a "theory of everything", then gravions could provide an underlying mechanism not only in gravity but also in quantum mechanics. It may be that we can make one step closer to a better philosophical understanding of the cosmos, if we can grasp the nature of the gravion, perhaps, as being the embodiment of the coexistence of opposites in a perpetual flow of the universe expounded by Heraclitus (Wikipedia contributors, 2019a).

# 17   Conclusion

An attempt has been made to modify and advance the old principle of push gravity theory to a stage where gravity may be seen from a totally novel perspective. It constitutes a daring step, because it challenges and potentially provokes a re-consideration of long standing ideas and principles. This has already required a daunting determination especially as it comes from a non-expert in the field of gravity.

The basic new element is the use of a gravity particle absorption coefficient that is not limited only to very low values as in prior PG theory. The consequences of that can be dramatic.

The theory of PG has now been brought to a stage ready for verification with several proposed tests and methods. Should these tests yield a positive outcome, they could provide explanation to many outstanding

issues in science. Otherwise, the test may prove insufficient pending further instrument refinements. Alternatively, if one produces sufficient evidence to reject PG once and for all, that would compel science to concentrate on other pathways as it does even more. At any rate, it should be appreciated that the proposed tests are inexpensive at least in relative terms for many organization to engage.

In summary, new work provides sufficient evidence for a genuine re-appraisal of push gravity. A novel quantitative theory has been advanced on the basis of a set of primary principles (postulates), from which the derivation of classical acceleration and force by stationary massive bodies in the steady state is possible. In contrast to prior conceptions, it is shown that the absorption of gravity particles by matter need not be extremely weak and linear, in order to derive and explain the observed classical laws of gravity. Any value of the absorption coefficient by a uniform spherical mass produces a gravitational field obeying the inverse square of distance law. The gravitational constant (big G), is itself a function of the ratio of the absorption coefficient over the density of matter. The latter ratio now becomes the new universal constant of the cosmos, whilst G can vary in different locations of the universe. The measured mass of planets and stars is only an effective or apparent mass actually smaller than the real mass due to a self-shadowing or shielding effect of the absorption of gravitational particles. Any given mass appears quantitatively different depending on its spatial distribution. We now find that Newton's gravitational law uses only the apparent (or effective) masses with a potentially variable G, but the inverse square distance relationship is preserved in the cosmos. The radiant flux of energetic particles being uniform over a region of space creates a maximum acceleration of gravity for all material bodies in that region, so that any further mass accretion over a certain upper limit does not create additional acceleration; this limit is reached when practically all gravitational particles are absorbed (saturation state) by the massive body above a saturation mass. The latter limit should be measurable, for which some tentative situations and experiments are proposed for prospective experiments and tests. The internal field of a spherical mass and the external field of a two layered sphere have been derived. The superposition principle of gravity fields has been reformulated and the Allais effect explained. The equivalence principle can now be properly understood and explained in a way that the principle per se becomes redundant. For moving bodies, the established relationships from special and general relativity may continue to operate within the gravitational fields created by push particles, but may need to be adapted and re-aligned within the greater framework of push gravity principles operating at any distance.

An attempt is made to overcome the main remaining objection of presumed catastrophic thermal accretion of absorbed particles. A further attempt is made also for the push-gravity principles to explain the vastly higher intensity gravitational fields of white dwarfs, neutron stars and black holes. It is proposed that the field of white dwarf stars is created also by push particles but of a different kind, namely, by those responsible for mediating the electric field. In the same way, the field of neutron stars is created by yet a third kind of push particles, namely, those responsible for mediating the nuclear field. In general, push particles may exist with different energy (or mass) having different mean free paths as they traverse different concentrations of masses (black holes, neutron stars, dwarfs, stars, planets, ordinary masses, atoms, nuclei, protons and all the known or unknown sub-nuclear particles). The invariable principle of momentum transfer (push) by particles directly relating to their absorption rate by the various concentrations (density) of masses could be the basis and the starting principle for a prospective unification theory of everything. The first part of this report, if verified, should create the basis for new physics across many fields of physical science.

If there is a "theory of everything", then gravions could provide an underlying mechanism not only in gravity but also in quantum mechanics. It may be that we can make one step closer to a better philosophical understanding of the cosmos, if we can grasp the nature of the gravion, perhaps, as being the embodiment of the coexistence of opposites in a perpetual flow of the universe.

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# APPENDIX

## A   Gravitoids

Let us be reminded that a prolate spheroid is a surface of revolution obtained by rotating an ellipse about its major axis, whereas an oblate spheroid is obtained by rotating about its minor axis. It is well known that spheroids acting as gravitational bodies would produce precession or regression of the elliptical orbit of a planet around it. This arises by the gravitational force being slightly greater or lesser than the inverse of the square of the distance. In other words, it is the distance (not the mass) responsible for these phenomena (here, we are not referring to the relativistic cause of the extra Mercurial precession). It is interesting to examine and clarify what happens with PG theory in this connection via the following observations.

Now, by virtue of Eq. 38, the self-shadowing effect produces a gravitational force (acceleration) less than the value expected from simple Newtonian attraction by a sphere. By increasing $k$ with all else constant,
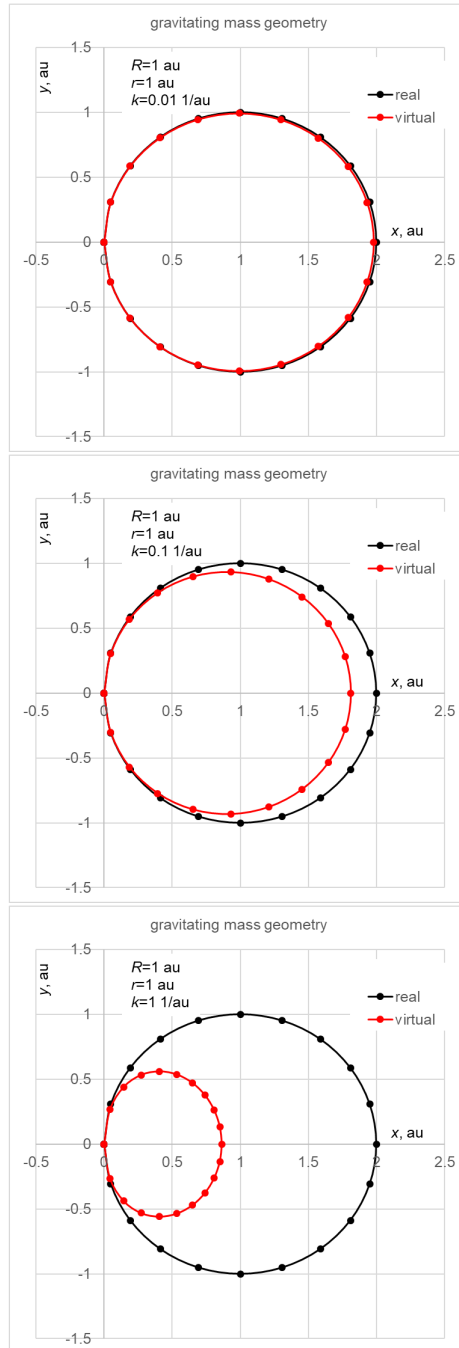
Figure 18: Real sphere in black, gravitoid (virtual) shape in red for three values of $k$ and $r = R = 1$.
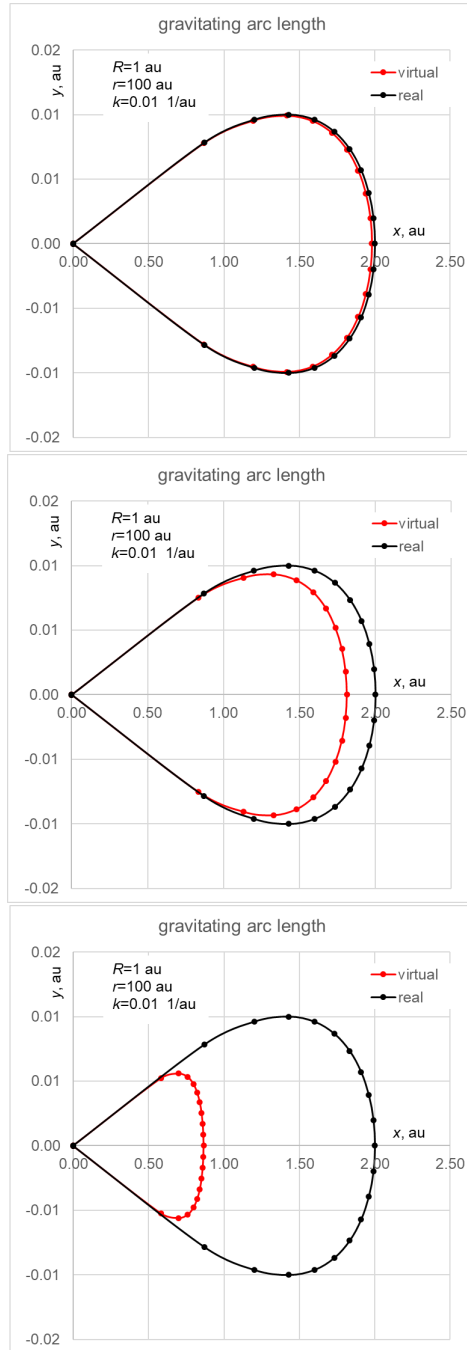
Figure 19: Arc length for real sphere in black and gravitoid (virtual) in red for three values of $k$ at $n_R = 100$.

the force increases in proportion to it (or the density) by Newtonian mechanics, but to a lesser degree by PG theory, on account of the exponential decrease along a cord (straight line) of the sphere in Fig. 2. That is, the Newtonian length that would produce an attractive force effectively contracts (shrinks) to produce the correct force. Each elemental component is then equivalent to having a lesser length at a given density, whilst in reality there is additional mass for the remainder of the length of the cord. This becomes, in effect, a virtual mass distributed inside a spheroid-like shape. It may initially look like an oblate spheroid, but it has a peculiar shape dependent on the distance OP and $k$. For a sufficiently long distance OP (i.e. $r$), the lines u are nearly parallel (very small angles $\varphi$), whilst as we approach the surface at point C, the same lines radiate at large angles $\varphi$, and the shape becomes more like a compressed egg along its axis while being inflated at right angles. We can see these and other effects by plotting the corresponding lengths and shapes quantitatively for two positions of the point O, namely, at the surface of the sphere and at a relatively long distance of $n_R = 100$.

On the surface of the sphere, we show pairs of the cord lengths and body shapes between Newtonian and virtual PG cases for three different values of $k$, in Fig, 18. To clarify, because point O lies on the surface of the sphere, any distance from the fixed point O to any other point on the surface defines the cord length, via which we also plot the sphere. Thus, these graphs show simultaneously both the cords and volumes of revolution corresponding to the real sphere and virtual shape yielded by PG. As expected, for $k < 0.01$, PG shapes become gradually indistinguishable from Newton. Otherwise, the difference increases significantly.

Next, we plot the virtual cord lengths for a sphere with unity radius from a distance $r = 100$ units. Planet Mercury approximately has this distance from the Sun at its aphelion. We consider again three values for $k = 0.01$, $k = 0.1$ and $k = 1$ in Fig. 19 together with the same **real** cord lengths of the same sphere (in black). We have used the same Eqs. 12 and 47.

Finally, we can visualize the corresponding virtual shapes of the sphere (here, like the Sun) from the same distance of 100 sphere radii (as from Mercury) with the same corresponding values of $k$ in Fig. 20. This is obtained by adding the PG cord length by 47 to the corresponding $u_1$ provided by Eq. 16, i.e. we use the virtual end points of $u_{e2}$ in PG given by Eq. 48.

The above spheroid-like shapes are bounded by the red lines together with overlapping black lines on the left. We note that a shallow dimple appears on the far side, the depth of which increases as we further increase $k$, effectively producing a *dimpled spheroid-like* shape.

As previously noted, the real shapes (and sizes) of a sphere effectively act as some peculiar virtual shapes, fictitious and invisible, for which we may collectively use (coin) the new term *gravitoids*. Their mass may be used with linear absorption as in Newton's law to yield the force as predicted by PG.

Below, we also present the analytical expressions already used to plot these gravitoids in Fig. 6 and discussed in Section 5.4. We follow the steps in finding the volume of a sphere to illustrate the point of deviation (departure) between the two approaches:

$$V_{sphere} = \int_0^{\varphi_0} \int_{u_1}^{u_2} 2\pi \sin\varphi \mathrm{d}\varphi \centerdot u^2 \mathrm{d}u = \frac{2\pi}{3} \int_0^{\varphi_0} \left[\sin\varphi \mathrm{d}\varphi \centerdot u^3\right]_{u_1}^{u_2}$$

$$= \frac{2\pi r^3}{3} \int_0^{\varphi_0} \sin\varphi \mathrm{d}\varphi \left[\left(\cos\varphi + \sqrt{a^2 - \sin^2\varphi}\right)^3 - \left(\cos\varphi - \sqrt{a^2 - \sin^2\varphi}\right)^3\right] \quad (146)$$

and by using the limits in Eqs. 16 and 17, it finally yields the expected result:

$$V_{sphere} = \frac{2\pi r^3}{3} \left[-2\cos\varphi \left(\cos^2\varphi + a^2 - 1\right)^{3/2}\right]_0^{\varphi_0} = \frac{4\pi R^3}{3} \quad (147)$$

Similarly, starting from the same elementary volume equation

$$V_{gravitoid} = \int_0^{\varphi_0} \int_{u_1}^{u_2} 2\pi \sin\varphi \mathrm{d}\varphi \centerdot u^2 \mathrm{d}u = \frac{2\pi}{3} \int_0^{\varphi_0} \left[\sin\varphi \mathrm{d}\varphi \centerdot u^3\right]_{u_1}^{u_2} \quad (148)$$
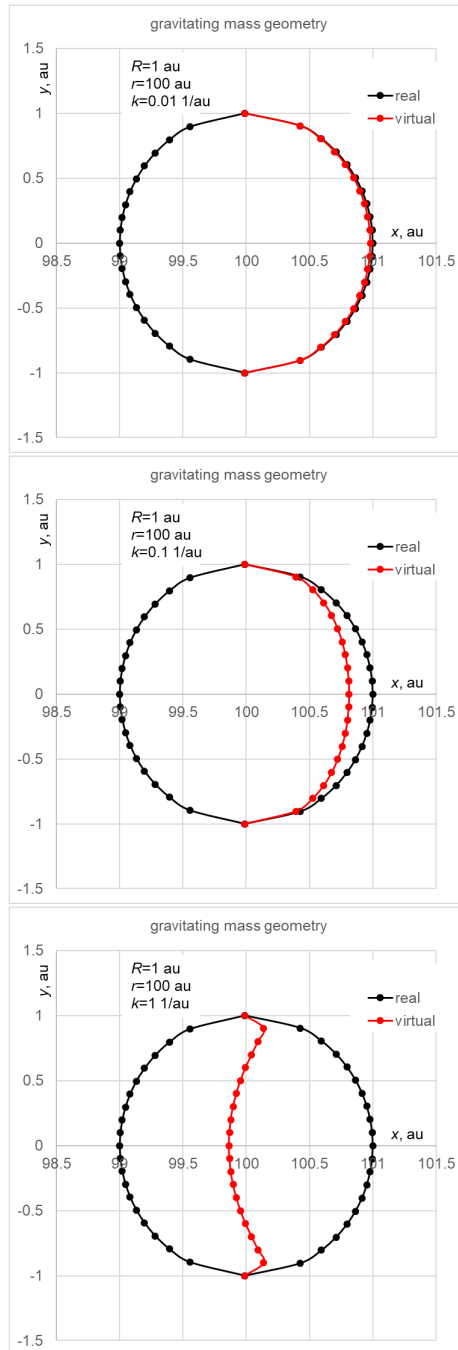
but using the limits in Eqs. 16 and 48 we obtain:

Figure 20: Real sphere in black, gravitoids (virtual) shape in red (together with black left of red line) for three values of $k$ and $n_R = 100$.

$$V_{gravitoid} = \frac{2\pi}{3} \int\limits_0^{\varphi_0} \sin\varphi \mathrm{d}\varphi.$$

$$\left[ \left( r\left(\cos\varphi - \sqrt{a^2 - \sin^2\varphi}\right) + \frac{1}{k} - \frac{1}{k}\exp\left(-2kr\sqrt{a^2 - \sin^2\varphi}\right)\right)^3 - \left(r\left(\cos\varphi - \sqrt{a^2 - \sin^2\varphi}\right)\right)^3\right] \tag{149}$$

by which we finish up with a different curve shape for the volume of the gravitoid. This shape filled with the actual (real) density may be used with Newton's law to reproduce the same force yielded by PG. The above examples simply illustrate that the initial common integration for a volume diverges on account of the different integration limits in the corresponding theories of Newton and PG. They illustrate the formal relationships between the two theories. The above integration has been performed numerically and plotted against $k$ in Fig. 6 after it is normalized by dividing by the sphere in Eq. 147, as was done for the effective spherical volume defined by Eq. 51.

We note that the effective volume generally lies above the gravitoid, as it should, because it is further away from the gravitoid relative to the reference point O. If they both contain the same real density matter, then both yield the correct value of acceleration by applying Newton's equation. We further note that the gravitoid volume (effective mass) increases, as we move away from the gravitating mass (e.g. compare the obvious corresponding sizes provided by Figs. 18 and 20). However, this does not affect the inverse of $r^2$ dependency, because this effective mass increase is compensated by the integration to a lower upper limit of angle (i.e. over a smaller angle range). For a possible precession to be generated, we need to consider the time effects also in PG as in the corresponding GR theory.

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