INFLUENCE OF THE SINUSOIDAL AND GAUSSIAN NOISES IN THE ESTIMATION OF THE EEG FRACTAL DIMENSION

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ABSTRACT

EEG signals corresponding to different psychophysiological conditions can be characterized by their fractal dimension (D). The noises present on the recording can affect the estimation of such a dimension. In this work we analyse the behaviour of two D estimators in case of different kinds (gaussian and sinusoidal) and amplitudes of noise. The EEG fractal dimension seems to be strongly compromised by gaussian noise greater than about 3-4%, of the EEG rms value, while a 50Hz noise of about 10% of the EEG rms signal is necessary to produce estimation errors greater than 10%. A dependence on the sampling frequency of the D estimation is also pointed out.

1 INTRODUCTION

Recent studies [1-3] have shown that the analysis of the fractal dimension, D, of a biological signal (like EEG, ECG or Speech) in the time domain can be successfully applied to identify different situations (i.e. voiced vs. unvoiced speech or epileptic seizures vs. rest condition in EEG or different arrhythmia types in ECG recordings) in a more immediate way with respect to other traditional methods (as for example those based on the power spectrum or the coherence function).

The advantages of this methodology are due both to the nonlinear origin of the analysed signals and to the fact that the traditional spectral analyses frequently do not consider the phase behaviour. Hence, they lose some information contained in the signal that the fractal analysis on the contrary utilizes [2]. Moreover, to obtain a reliable estimation of the power spectrum, the algorithms need a large number of points and averaged values on some intervals [4]. Furthermore, because of the utilized sampling frequencies, the examined intervals result too large both to identify transient events and to satisfy the stationarity hypothesis of the signal.

On the other hand, among the several methods [5-8] used to estimate the fractal dimension, there are a few that are able to obtain reliable values with a limited number of points [7,8] thus allowing to identify events of brief duration also. The fractal dimension of a signal is a measure of fragmentation, and its estimation is altered by the presence of noise. Since biological signals are frequently affected by noise introduced both by the measuring instrumentation and by undesired biological events, the D estimation can result incorrect.

In this work the influence of gaussian white noise as well as of sinusoidal noise (line noise) on the fractal dimension estimation has been examined so as to evaluate the robustness and the reliability of the different methods. To this aim first we added the noises to synthetic fractal signals of fixed dimension and successively to EEG recordings.

2 MATERIALS AND METHODS

To evaluate the fractal dimension we used the morphological approach described by Maragos [7] and the graph dimension estimation suggested by Higuchi [8] by using a maximum interval time (kmax) equal to 5. The synthetic signals were obtained from a Fractional Brownian motion (FBm) generated by means of the random midpoint displacement technique and from the Weierstrass function [9]; D ranged between 1.1 and 1.9.

The algorithms were applied both to the synthetic and to the EEG signals, the latter recorded from distinct sites of the scalp in some normal and epileptic subjects during different states (open and closed eyes, epileptic seizure, etc.). We added to each signal either a pure sinusoidal noise (at 50Hz) or a gaussian white noise. The noise rms amplitude was varied from 0 to 20% of the signal rms amplitude to take into account the highest possible noises present in the EEG recordings.

The estimated D was the mean value of the fractal dimension obtained on 10 successive signal segments of 512 points each (with a 12% overlapping), equivalent to 1 sec. The EEG segments (sampled at 512Hz) were selected in such a way as to have very low noise and to maintain the stationarity for the whole duration of the analyzed tract.

3 RESULTS AND DISCUSSION

The fractal dimension evaluated on the synthetic signals, with the highest values of 50Hz noise added, asymptotically tended to a value of about 1.24, when Fs was 512Hz, whatever the selected D was (fig.2b). Since a pure sinusoidal continuous signal has an integer dimension equal to 1 and the process to estimate D could be affected by the number of samples present in each cycle, first we examined the dependence on the sampling frequency of the D evaluation with the two methods. In both cases (fig.1) as the ratio between the sinusoidal frequency, F, and the sampling frequency, Fs, increases, the D value increases too with an estimation error lower than 10% for F/Fs lower than 0.06. Then, to correctly estimate the fractal dimension of a sinusoidal wave, a sampling frequency much larger than the Nyquist one is necessary (or a number of points per cycle greater than about 16).

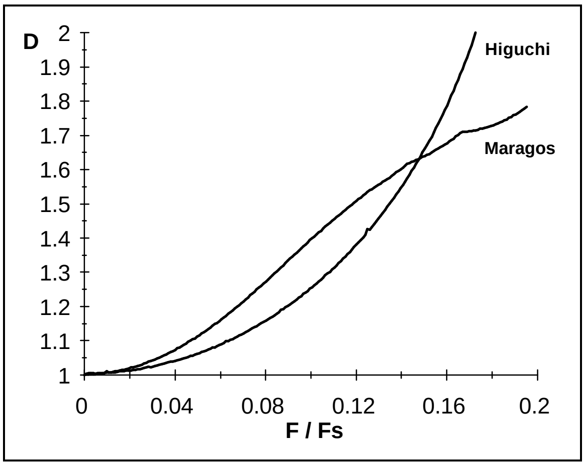
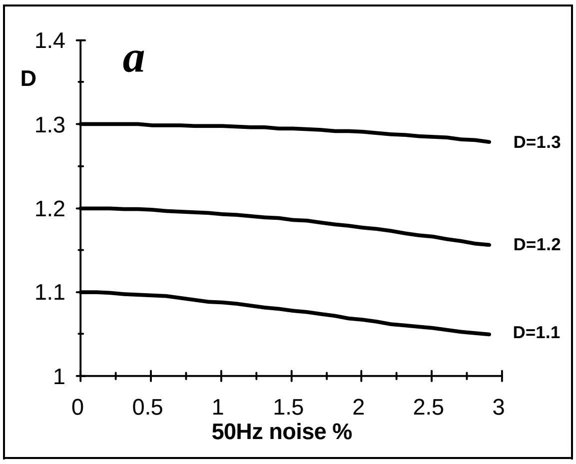


Fig.1 Fractal dimension (D) of a sinusoidal signal vs. the ratio between the sinusoidal frequency (F) and the sampling frequency (Fs), evaluated with the two algorithms. The true value of D is 1.

Fig.2a-b shows the influence on D of the 50Hz sinusoidal noise added to the Weierstrass synthetic function when the sampling frequency changes from 2560Hz to 512Hz. The results are similar to those



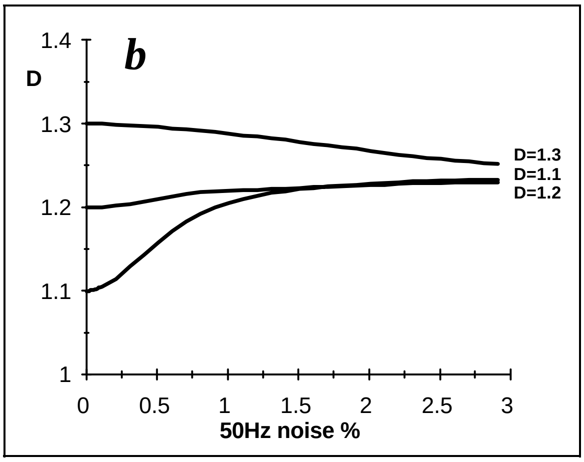


Fig.2a-b D values estimated from the three Weierstrass synthetic signals (with D set to 1.1, 1.2 and 1.3) vs. % of the 50Hz sinusoidal noise added. The sampling frequency was fixed to 2560Hz (a) and to 512Hz (b) corresponding to about 51 and 10 points per cycle respectively.

obtained by using the FBm signal and indicate that the more the noise increases and the less the Fs is, the more the D estimation is affected by the noise.

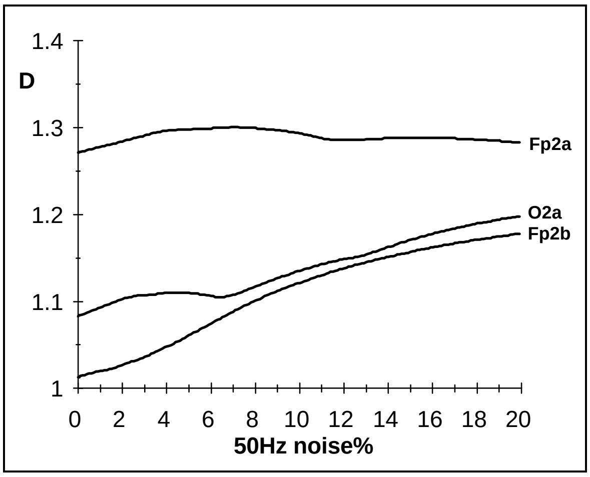


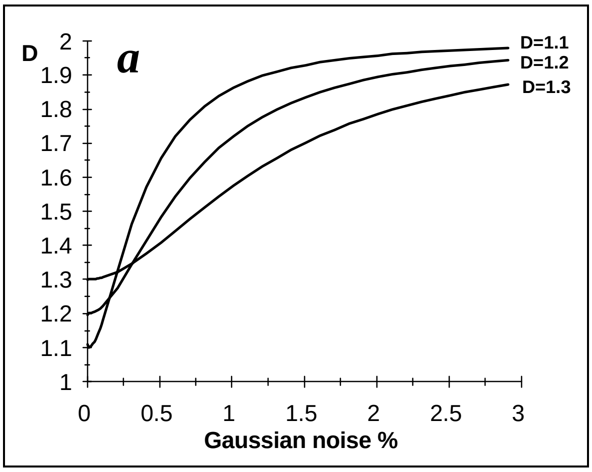
Fig.3 D values of the EEG signal recorded from FP2 and O2 derivations vs. % of the 50Hz sinusoidal noise added. The sampling frequency was fixed to 512Hz. Fp2a and O2a are derived from a normal subject with open eyes; Fp2b from an epileptic subject during a spike and wave epileptic seizure.

In any case, the D value tends to the one that the sinusoid alone should have at the corresponding sampling frequency (i.e. D=1.02 at Fs=2560Hz; D=1.24 at Fs=512Hz, see fig.1). So, a noise of 1% produces at Fs=512Hz an estimation error greater than 10% with respect to a true D value of 1.1 (fig.2b); this fact is still more relevant because the error is not equal for all the possible D values, thus producing a reduction of the possible differences present in the signal.

In particular, this fact is important in the EEG signal analysis where just the differences among various psychophysiological states have to be recognized. For example in fig.3 the Fp2a signal, recorded in a normal subject, must be separated from the Fp2b one, recorded in an epileptic subject. The difference and the consequent ability to distinguish between the two conditions decrease as the noise increases; however the EEG seems to be less sensitive (about 20 times) to the 50Hz (line) noise than the synthetic signals.

It can be concluded that to obtain reliable estimates of D, able to maintain a sufficient dissimilarity among different situations, a high signal to noise ratio as well as a high Fs (about 8 times the Nyquist frequency) are requested.

The influence of the gaussian white noise on the D estimation is similar to that of the sinusoidal noise (fig4a-b). As the noise increases, the D value tends to 2, which is the fractal dimension of white noise. Yet,



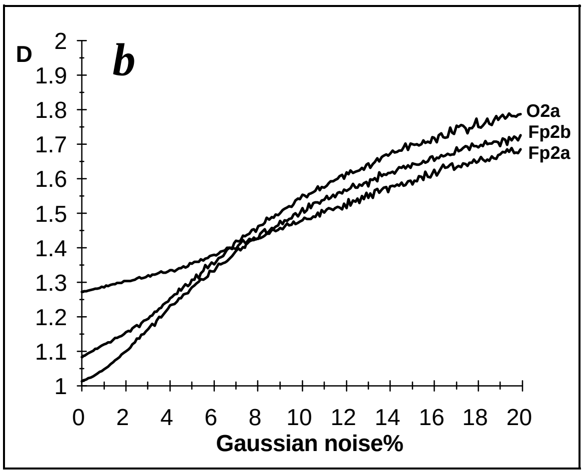


Fig.4a-b D values of the Weierstrass synthetic signal (a) and of the EEG (b) vs. % of the gaussian white noise added. The sampling frequency was fixed to 512Hz. The EEG derivations are the same as in fig.3.

this tendency is more sudden than in the 50Hz noise case, especially for the lowest true D values. This error is larger for small true D values and it makes the fractal dimension to be unsuitable for the identification of different conditions when the rms of the white noise is greater than 0.3% of the corresponding signal.

This fact is evident also in the EEG case, even if it happens at a higher noise amplitude (3-4%).

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