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Ŧ - ROUGH FUZZY IDEALS OF Γ-SEMIGROUPS

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Abstract:
The aim of this paper is to introduce the notion of, Ŧ -rough fuzzy ideals, Ŧ -rough fuzzy bi -ideals in Γ-semigroups and studied some of its properties.

Index Terms: Γ-semigroups, Ŧ -rough fuzzy ideals, Ŧ -rough fuzzy bi -ideals

1. Introduction:

The fundamental concept of Γ-semigroup was introduced by Sen [12]. Many researchers have worked on Γ-semigroup and its sub structures. Many Classical notion of semigroups have be extended to Γ-semigroups by Sha [10,11]. The notion of rough set was introduced by Pawlak [8]. Rough set theory, a new mathematical approach to deal with inexact, uncertain or vague. Knowledge has recently received wide attention on the research areas in both of the real-life applications and the theory itself. It has found practical applications in many areas such as knowledge discovery, machine learning, data analysis and so on [5-7]. Rough set theory is a mathematical frame work for dealing with uncertainty and to some extend overlapping fuzzy set theory. The concept of a fuzzy set was introduced by Zadeh [15] and it is now a rigorous ara of research with manifold applications ranging from engineering and computer science to medical diagnosis and social behaviour studies. Rosenfeild[9] introduced fuzzy group. The algebraic approach of rough set was studied by some authors, for example [1,2,3,13,14]. The notion of rough fuzzy sets was introduced by Dubois and Prade [4]. The aim of this paper is to introduce the notion of, Ŧ -rough fuzzy ideals, Ŧ -rough fuzzy bi -ideals in Γ-semigroups and studied some of its properties.

2. Preliminaries Notes:

The following definitions and preliminaries are required in the squel of our work and hence presented in brief. Let θ be a congruence relation on M, that is θ is an equivalence relation on M such that $(a,b) \in \theta \Rightarrow (a\gamma x, b\gamma x) \in \theta$ and $(x\gamma a, x\gamma b) \in \theta$ for all $a, x, b \in \Gamma$. If θ is a congruence relation on M, then for every $x \in M$, $[x]_\theta$ denotes the congruence class of x with respect to the relation θ. A congruence relation θ on M is called complete if $[a]_\theta \Gamma [b]_\theta = [a\Gamma b]_\theta$ for every $a, b \in M$.

Definition 2.1 [8] A pair (U, θ) where $U \neq \emptyset$ and θ is an equivalence relation on M is called an approximation space.

Definition 2.2 [8] For an approximation space (U, θ) is a rough approximation we mean a mapping $\theta: P(U) \to P(U) \times P(U)$ defined by for every

$$A \in P(U), \theta(A) = (\underline{\theta}(A), \overline{\theta}(A)), \text{ where } \underline{\theta}(A) = \{x \in U : [x]_\theta \subseteq A\}, \ \overline{\theta}(A) = \{x \in U : [x]_\theta \cap A \neq \emptyset\}.$$

$\underline{\theta}(A)$ is called lower rough approximation of A and $\overline{\theta}(A)$ is called upper approximation of A in (U, θ). $\theta(A)$ is said to be rough set if $\underline{\theta}(A) \neq \overline{\theta}(A)$.

Let M be a nonempty subset Γ -semigroup. A mapping $\mu: M \to [0,1]$ is called a fuzzy subset of M. If μ is a fuzzy subset of M, for $t \in [0,1]$ then the set $\mu_t = \{x \in M \mid \mu(x) \geq t\}$ is called a level subset of M with respect to a fuzzy subset of μ, A fuzzy subset $\mu: M \to [0,1]$ is a non empty fuzzy subset if μ is not a constant function. For any two fuzzy subsets μ and λ of M, $\lambda \subseteq \mu$ means $\lambda(x) \leq \mu(x)$ for all $x \in M$.

Let μ and γ be two fuzzy subsets of Γ-Semigroup M and $x, y, z \in M, \alpha \in \Gamma$. We define

$$\mu * \gamma(x) = \begin{cases} sup_{x=y\alpha z}, min\{\mu(y), \gamma(z)\}: \\ 0, \quad otherwise. \end{cases}$$

$\mu \cap \gamma(x) = min\{\mu(x), \gamma(x)\}$, for all $x \in M$.

A fuzzy subset μ of Γ -semigroup M is called a

- a fuzzy Γ -subsemigroup of M if $\mu(x\alpha y) \geq \mu(x) \wedge \mu(y)$.
- a fuzzy left(right) ideal of M if $\mu(x\alpha y) \geq \mu(y)(\mu(x))$.
- a fuzzy ideal of M if $\mu(x\alpha y) \geq \mu(x) \vee \mu(y)$.

Definition 2.3 [9] A fuzzy subset μ of M is called fuzzy bi-ideal of M, if for all $x, y \in M$ and $\gamma \in \Gamma$,

- $\mu(x\gamma y) \geq \mu(x) \wedge \mu(y)$
- $\mu * M * \mu \leq \mu$.

Definition 2.3 [9] A fuzzy subset μ of M is called fuzzy bi-ideal of M, if for all $x, y \in M$ and $\gamma \in \Gamma$,

- $\mu(x\gamma y) \geq \mu(x) \wedge \mu(y)$
- $(\mu * M) \wedge (M * \mu) \leq \mu$.

Definition 2.3 [4] Let μ be a fuzzy subset of a Γ-semigroup M. Let $\overline{\theta}(\mu)$ and $\underline{\theta}(\mu)$ be the fuzzy subsets of M defined by $\overline{\theta}(\mu)(x) = \bigvee_{a\in[x]_\theta} \mu(a)$ and $\underline{\theta}(\mu) = \bigwedge_{a\in[x]_\theta} \mu(a)$ are called, respectively, the θ-*upper* and θ-*lower approximations* of the fuzzy set μ. $\theta(\mu) = (\underline{\theta}(\mu), \overline{\theta}(\mu))$ is called a *rough fuzzy set* with respect to θ if $\underline{\theta}(\mu) \neq \overline{\theta}(\mu)$.

Definition 2.4 [3] Let A and B be two non-empty subsets and $X \subseteq B$ and $Ŧ: A \to P^*(B)$ be a set valued mapping where $P^*(B)$ denotes the set of all non-empty subsets of B. The lower inverse and upper inverse of $Ŧ$ are defined by $\overline{Ŧ}(A) = \{x \in X | Ŧ(x) \cap A \neq \emptyset\}$ and $\underline{Ŧ}(A) = \{x \in X | Ŧ(x) \subseteq A\}$.

3. Ŧ-Rough Fuzzy Ideal of Γ-Semigroups:

In this section, as a generalization of rough fuzzy ideals, we introduce the notion of Ŧ-rough fuzzy ideal of Γ-semigroup and study the properties of Ŧ-rough fuzzy ideals. Throughout this paper M denotes Γ-semigroup, unless otherwise specified.

Definition 3.1 Let M and M_1 be two Γ-semigroups and $Ŧ: M \to P'(M_1)$ be a set valued mapping. Ŧ is called a set-valued homomorphism if for all $x, y \in M$, $\gamma \in \Gamma$,

- $Ŧ(x\gamma y) = Ŧ(x)\Gamma Ŧ(y)$
- $\left(Ŧ(x)\right)^{-1} = \{a^{-1}: a \in Ŧ(x)\} = Ŧ(x^{-1})$.

Let θ be a complete congruence relation on M. Define $Ŧ: M \to P'(M_1)$ by $Ŧ(x) = [x]_\theta$ for all $x \in M$.

Definition 3.2 Let M and M_1 be two Γ-semigroups and $Ŧ: M \to P'(M_1)$ be a set-valued homomorphism. Let μ be a fuzzy subset of M_1. for every $x \in M$, we define

$$\underline{Ŧ}(\mu)(x) = \bigwedge_{a\in Ŧ(x)} \mu(a) \text{ and } \overline{Ŧ}(\mu)(x) = \bigvee_{a\in Ŧ(x)} \mu(a).$$

$\underline{Ŧ}(\mu)$ and $\overline{Ŧ}(\mu)$ are called respectively the Ŧ-rough lower and the Ŧ-rough upper fuzzy subsets of M.

Theorem 3.3:

Let M and M_1 be two Γ-semigroups and $Ŧ: M \to P'(M_1)$ be a set-valued homomorphism. Let μ and λ be two fuzzy subsets of M_1, then the following hold:

- $\underline{Ŧ}(\mu) \subseteq \mu \subseteq \overline{Ŧ}(\mu)$
- $\underline{Ŧ}(\emptyset) = \emptyset = \overline{Ŧ}(\emptyset)$
- $\left(\underline{Ŧ}(M)\right) \subseteq M \subseteq \overline{Ŧ}(M)$ and $\underline{Ŧ}(M_1) \subseteq M_1 \subseteq \overline{Ŧ}(M_1)$
- $\overline{Ŧ}(\mu \cup \lambda) = \overline{Ŧ}(\mu) \cup \overline{Ŧ}(\lambda)$
- $\underline{Ŧ}(\mu \cap \lambda) = \underline{Ŧ}(\mu) \cap \underline{Ŧ}(\lambda)$
- $\mu \subseteq \lambda$ implies $\underline{Ŧ}(\mu) \subseteq \underline{Ŧ}(\lambda)$
- $\mu \subseteq \lambda$ implies $\overline{Ŧ}(\mu) \subseteq \overline{Ŧ}(\lambda)$
- $\underline{Ŧ}(\mu) \cup \underline{Ŧ}(\lambda) \subseteq \underline{Ŧ}(\mu \cup \lambda)$
- $\overline{Ŧ}(\mu \cap \lambda) \subseteq \overline{Ŧ}(\mu) \cap \overline{Ŧ}(\lambda)$.

Theorem 3.4:

Let M and M_1 be two Γ-semigroups and $Ŧ: M \to P'(M_1)$ be a set-valued homomorphism. Let μ and λ be two fuzzy subsets of M_1, then the following hold:

- $\overline{Ŧ}(\mu) * \overline{Ŧ}(\lambda) \leq \overline{Ŧ}(\mu) \cap \overline{Ŧ}(\lambda)$
- $\underline{Ŧ}(\mu) * \underline{Ŧ}(\lambda) \leq \underline{Ŧ}(\mu) \cap \underline{Ŧ}(\lambda)$

Proof:

(i) For any $a, b, c \in M, \gamma \in \Gamma$, we have

$$\begin{aligned}
\overline{Ŧ}(\mu) * \overline{Ŧ}(\lambda)(a) &= \bigvee_{a=b\gamma c}\left[\min\{\overline{Ŧ}(\mu)(b), \overline{Ŧ}(\lambda)(c)\}\right] \\
&= \bigvee_{a=b\gamma c}\left[\min\{\bigvee_{y\in Ŧ(b)} \mu(y), \bigvee_{z\in Ŧ(c)} \lambda(z)\}\right] \\
&\leq \min\{\bigvee_{a=b\gamma c}\{\bigvee_{y\in Ŧ(b)} \mu(y), \bigvee_{z\in Ŧ(c)} \lambda(z)\}\} \\
&\leq \min\{\bigvee_{y\in Ŧ(a)} \mu(y), \bigvee_{z\in Ŧ(a)} \lambda(z)\} \\
&= \min\{\overline{Ŧ}(\mu)(a), \overline{Ŧ}(\lambda)(a)\} \\
&= \overline{Ŧ}(\mu)(a) \wedge \overline{Ŧ}(\lambda)(a)
\end{aligned}$$

Hence $\overline{Ŧ}(\mu) * \overline{Ŧ}(\lambda) \leq \overline{Ŧ}(\mu) \cap \overline{Ŧ}(\lambda)$.

(ii) The proof is similar to (i).

Theorem 3.5:

Let M and M_1 be two Γ-semigroups and $Ŧ: M \to P'(M_1)$ be a set-valued homomorphism. If μ, λ, σ are fuzzy subsets of M_1, then

- $\overline{Ŧ}(\mu) * \overline{Ŧ}(\lambda) * \overline{Ŧ}(\sigma) = \overline{Ŧ}(\mu) * \left(\overline{Ŧ}(\lambda) * \overline{Ŧ}(\sigma)\right) = \left(\overline{Ŧ}(\mu) * \overline{Ŧ}(\lambda)\right) * \overline{Ŧ}(\sigma)$
- $\underline{Ŧ}(\mu) * \underline{Ŧ}(\lambda) * \underline{Ŧ}(\sigma) = \underline{Ŧ}(\mu) * \left(\underline{Ŧ}(\lambda) * \underline{Ŧ}(\sigma)\right) = \left(\underline{Ŧ}(\mu) * \underline{Ŧ}(\lambda)\right) * \underline{Ŧ}(\sigma)$

Proof:

(i) For any $x \in M, \alpha, \beta \in \Gamma$, we have

$$\left(\overline{Ŧ}(\mu) * \overline{Ŧ}(\lambda) * \overline{Ŧ}(\sigma)\right)(x) = \vee_{x=a\alpha b}\left[\min\left\{\left(\overline{Ŧ}(\mu) * \overline{Ŧ}(\lambda)\right)(a), \overline{Ŧ}(\sigma)(b)\right\}\right]$$
$$= \vee_{x=a\alpha b}\left[\min\{\vee_{a=p\alpha q} \min\{\overline{Ŧ}(\mu)(p), \overline{Ŧ}(\lambda)(q)\}, \overline{Ŧ}(\sigma)(b)\}\right]$$
$$= \vee_{x=p\alpha q\beta b}\left[\min\{\min\{\overline{Ŧ}(\mu)(p), \overline{Ŧ}(\lambda)(q)\}, \overline{Ŧ}(\sigma)(b)\}\right]$$
$$= \vee_{x=p\alpha q\beta b}\left[\min\left\{\overline{Ŧ}(\mu)(p), \min\{\overline{Ŧ}(\lambda)(q), \overline{Ŧ}(\sigma)(b)\}\right\}\right]$$
$$= \overline{Ŧ}(\mu) * \left(\overline{Ŧ}(\lambda) * \overline{Ŧ}(\sigma)\right)(x)$$

Hence $\left(\overline{Ŧ}(\mu) * \overline{Ŧ}(\lambda) * \overline{Ŧ}(\sigma)\right)(x) = \overline{Ŧ}(\mu) * \left(\overline{Ŧ}(\lambda) * \overline{Ŧ}(\sigma)\right)(x)$.

Obviously $\overline{Ŧ}(\mu) * \overline{Ŧ}(\lambda) * \overline{Ŧ}(\sigma) = \left(\overline{Ŧ}(\mu) * \overline{Ŧ}(\lambda)\right) * \overline{Ŧ}(\sigma)$.

Therefore $\overline{Ŧ}(\mu) * \overline{Ŧ}(\lambda) * \overline{Ŧ}(\sigma) = \overline{Ŧ}(\mu) * \left(\overline{Ŧ}(\lambda) * \overline{Ŧ}(\sigma)\right) = \left(\overline{Ŧ}(\mu) * \overline{Ŧ}(\lambda)\right) * \overline{Ŧ}(\sigma)$

(ii) Proof is similar to (i).

Theorem 3.6:

Let M and M_1 be two Γ-semigroups and $Ŧ: M \to P'(M_1)$ be a set-valued homomorphism. If μ be a fuzzy subset of M_1, then

- $\overline{Ŧ}(\mu) * \overline{Ŧ}(\mu) \le \overline{Ŧ}(\mu)$
- $\underline{Ŧ}(\mu) * \underline{Ŧ}(\mu) \le \underline{Ŧ}(\mu)$

Proof:

For all $x \in M, \alpha \in \Gamma$, we have

$\overline{Ŧ}(\mu) * \overline{Ŧ}(\mu) \le \overline{Ŧ}(\mu) \cap \overline{Ŧ}(\mu)$ By Theorem 3.4(i)

$\le \overline{Ŧ}(\mu)$

Hence $\overline{Ŧ}(\mu) * \overline{Ŧ}(\mu) \le \overline{Ŧ}(\mu)$

(ii) $\underline{Ŧ}(\mu) * \underline{Ŧ}(\mu) \le \underline{Ŧ}(\mu) \cap \underline{Ŧ}(\mu)$ By Theorem 3.4(ii)

$\le \underline{Ŧ}(\mu)$.

Hence $\underline{Ŧ}(\mu) * \underline{Ŧ}(\mu) \le \underline{Ŧ}(\mu)$

Theorem 3.7:

Let M and M_1 be two Γ-semigroups and $Ŧ: M \to P'(M_1)$ be a set-valued homomorphism. If μ be a fuzzy subsemigroup of M_1, then $\overline{Ŧ}(\mu)$ is a fuzzy subsemigroup of M.

Proof:

(i) Let μ be a fuzzy subsemigroup of M_1. Then $\mu(x\gamma y) \ge \mu(x) \wedge \mu(y)$. For all $x, y \in M, \gamma \in \Gamma$,

$$\overline{Ŧ}(\mu)(x\gamma y) = \vee_{a \in Ŧ(x\gamma y)} \mu(a)$$
$$\ge \vee_{p \in Ŧ(x), q \in Ŧ(y)} \mu(p\gamma q)$$
$$\ge \vee_{p \in Ŧ(x), q \in Ŧ(y)} (\mu(p) \wedge \mu(q))$$
$$= \left(\vee_{p \in Ŧ(x)} \mu(p)\right) \wedge \left(\vee_{q \in Ŧ(y)} \mu(q)\right)$$
$$= \overline{Ŧ}(\mu)(x) \wedge \overline{Ŧ}(\mu)(y)$$

Hence $\overline{Ŧ}(\mu)(x\gamma y) \ge \overline{Ŧ}(\mu)(x) \wedge \overline{Ŧ}(\mu)(y)$.

Therefore $\overline{Ŧ}(\mu)$ is a fuzzy subsemigroup of M.

Theorem 3.8:

Let M and M_1 be two Γ-semigroups and $Ŧ: M \to P'(M_1)$ be a set-valued homomorphism. If μ be a fuzzy subsemigroup of M_1, then $\underline{Ŧ}(\mu)$ is a fuzzy subsemigroup of M.

Proof:

Let μ be a fuzzy subsemigroup of M_1. Then $\mu(x\gamma y) \ge \mu(x) \wedge \mu(y)$. For all $a, b \in M, \alpha \in \Gamma$,

$\underline{Ŧ}(\mu)(a\alpha b) = \wedge_{x \in Ŧ(a\alpha b)} \mu(a\alpha b)$

$\ge \wedge_{y \in Ŧ(a), z \in Ŧ(b)} \mu(y\alpha z)$

$\ge \wedge_{y \in Ŧ(a), z \in Ŧ(b)} (\mu(y) \wedge \mu(z))$

$= \left(\wedge_{y \in Ŧ(a),} \mu(y)\right) \wedge \left(\wedge_{z \in Ŧ(b)} \mu(z)\right)$

$= \underline{Ŧ}(\mu)(a) \wedge \underline{Ŧ}(\mu)(b)$

Hence $\underline{Ŧ}(\mu)(a\alpha b) \ge \underline{Ŧ}(\mu)(a) \wedge \underline{Ŧ}(\mu)(b)$.

Therefore $\underline{Ŧ}(\mu)$ is a fuzzy subsemigroup of M.

Theorem 3.9:

Let M and M_1 be two Γ-semigroups and $Ŧ: M \to P'(M_1)$ be a set-valued homomorphism. If μ be a fuzzy ideal of M_1, then $\overline{Ŧ}(\mu)$ is a fuzzy ideal of M.

Proof:

Let μ be a fuzzy ideal of M_1. Then $(x\gamma y) \geq \mu(x) \vee \mu(y)$. For all $x, y \in M, \gamma \in \Gamma$,

$\overline{Ŧ}(\mu)(x\gamma y) = \vee_{a \in Ŧ(x\gamma y)} \mu(a)$
$\geq \vee_{p \in Ŧ(x), q \in Ŧ(y)} \mu(p\gamma q)$
$\geq \vee_{p \in Ŧ(x), q \in Ŧ(y)} (\mu(p) \wedge \mu(q))$
$= (\vee_{p \in Ŧ(x)} \mu(p)) \vee (\vee_{q \in Ŧ(y)} \mu(q))$
$= \overline{Ŧ}(\mu)(x) \vee \overline{Ŧ}(\mu)(y)$

Hence $\overline{Ŧ}(\mu)(x\gamma y) \geq \overline{Ŧ}(\mu)(x) \vee \overline{Ŧ}(\mu)(y)$.
Therefore $\overline{Ŧ}(\mu)$ is a fuzzy ideal of M.

Theorem 3.10:

Let M and M_1 be two Γ-semigroups and $Ŧ: M \to P'(M_1)$ be a set-valued homomorphism. If μ be a fuzzy ideal of M_1, then $\underline{Ŧ}(\mu)$ is a fuzzy ideal of M.

Proof:

The proof is similar to the proof of Theorem 3.6.

Theorem 3.11:

Let M and M_1 be two Γ-semigroups and $Ŧ: M \to P'(M_1)$ be a set-valued homomorphism. If μ be a fuzzy left (resp right) ideal of M_1, then

- $\overline{Ŧ}(\mu) * M \leq \overline{Ŧ}(\mu)$ and $M * \overline{Ŧ}(\mu) \leq \overline{Ŧ}(\mu)$
- $\underline{Ŧ}(\mu) * M \leq \underline{Ŧ}(\mu)$ and $M * \underline{Ŧ}(\mu) \leq \underline{Ŧ}(\mu)$

Proof:

Let μ be a fuzzy left ideal of M_1.

(i) By Theorem3.9 $\overline{Ŧ}(\mu)$ is a fuzzy left ideal of M.Then for all $x, y \in M, \gamma \in \Gamma$,

$(\overline{Ŧ}(\mu) * M)(x) = \vee_{x=a\alpha b} [\min\{\overline{Ŧ}(\mu)(a), M(b)\}]$
$= \vee_{x=a\alpha b} [\min\{\overline{Ŧ}(\mu)(a), 1\}]$
$\leq \vee_{x=a\alpha b} [\overline{Ŧ}(\mu)(a)]$
$= \vee_{x=a\alpha b} \{\vee_{a \in Ŧ(y)} \mu(y)\}$
$\leq \vee_{a \in Ŧ(x)} \mu(a)$
$= \overline{Ŧ}(\mu)(x)$

Hence $\overline{Ŧ}(\mu) * M \leq \overline{Ŧ}(\mu)$.

Consider $M * \overline{Ŧ}(\mu) = \vee_{x=a\alpha b} [\min\{M(a), \overline{Ŧ}(\mu)(b)\}]$
$\leq \vee_{x=a\alpha b} [\overline{Ŧ}(\mu)(b)]$
$\leq \vee_{b \in Ŧ(x)} \mu(b)$
$= \overline{Ŧ}(\mu)(x)$

Hence $M * \overline{Ŧ}(\mu) \leq \overline{Ŧ}(\mu)$.

(ii) The proof is similar to (i).

Lemma 3.12:

Let M and M_1 be two Γ-semigroups and $Ŧ: M \to P'(M_1)$ be a set-valued homomorphism. If μ be a fuzzy ideal of M_1, then for any $t \in [0,1]$,

- $(\underline{Ŧ}(\mu))_t = \underline{Ŧ}(\mu_t)$
- $(\overline{Ŧ}(\mu))_t = \overline{Ŧ}(\mu_t)$
- $\underline{Ŧ}(\mu)_t^M = \underline{Ŧ}(\mu_t^M)$
- $\overline{Ŧ}(\mu)_t^M = \overline{Ŧ}(\mu_t^M)$.

Proof:

(i) For all $x \in M$.

$x \in (\underline{Ŧ}(\mu))_t \Leftrightarrow \underline{Ŧ}(\mu)(x) \geq t$
$\Leftrightarrow \wedge_{a \in Ŧ(x)} \mu(a) \geq t$
$\Leftrightarrow \forall a \in Ŧ(x), \mu(a) \geq t$
$\Leftrightarrow Ŧ(x) \subseteq \mu_t$
$\Leftrightarrow x \in \underline{Ŧ}(\mu_t)$.

Therefore $(\underline{Ŧ}(\mu))_t = \underline{Ŧ}(\mu_t)$

(ii) For all $a \in M$,

$$a \in \left(\overline{Ŧ}(\mu)\right)_t \Leftrightarrow \overline{Ŧ}(\mu)(a) \geq t$$
$$\Leftrightarrow \vee_{p \in Ŧ(a)} \mu(p) \geq t$$
$$\Leftrightarrow \exists\, p \in Ŧ(a), \mu(p) \geq t$$
$$\Leftrightarrow Ŧ(a) \cap \mu_t \neq \emptyset$$
$$\Leftrightarrow a \in \overline{Ŧ}(\mu_t)$$

Therefore $\left(\overline{Ŧ}(\mu)\right)_t = \overline{Ŧ}(\mu_t)$

(iii) For all $x \in M$,

$$x \in \underline{Ŧ}(\mu)_t^M \Leftrightarrow \underline{Ŧ}(\mu)(x) > t$$
$$\Leftrightarrow \wedge_{a \in Ŧ(x)} \mu(a) > t$$
$$\Leftrightarrow \forall a \in Ŧ(x), \mu(a) > t$$
$$\Leftrightarrow Ŧ(x) \subseteq \mu_t^M$$
$$\Leftrightarrow x \in \underline{Ŧ}(\mu_t^M).$$

Therefore $\underline{Ŧ}(\mu)_t^M = \underline{Ŧ}(\mu_t^M)$

(iv) For all $a \in M$,

$$a \in \overline{Ŧ}(\mu)_t^M \Leftrightarrow \overline{Ŧ}(\mu)(a) > t$$
$$\Leftrightarrow \vee_{p \in Ŧ(a)} \mu(p) > t$$
$$\Leftrightarrow \exists\, p \in Ŧ(a), \mu(p) > t$$
$$\Leftrightarrow Ŧ(a) \cap \mu_t^M \neq \emptyset$$
$$\Leftrightarrow a \in \overline{Ŧ}(\mu_t^M).$$

Therefore $\overline{Ŧ}(\mu)_t^M = \overline{Ŧ}(\mu_t^M)$.

3. Ŧ-Rough Fuzzy Bi-ideal of Γ-Semigroups:

In this section Ŧ-rough fuzzy bi-ideals and Ŧ-rough fuzzy quasi-ideals of Γ-*semigroups* are introduced and discuss some of its properties.

Theorem 4.1:

Let M and M_1 be two Γ-semigroups and $Ŧ: M \to P'(M_1)$ be a set-valued homomorphism. If μ be a fuzzy bi-ideal of M_1, then $\overline{Ŧ}(\mu)$ is a fuzzy bi-ideal of M.

Proof:

Let μ be a fuzzy bi-ideal of M_1, then for all $x, y \in M, \gamma \in \Gamma$, we have $\mu(x\gamma y) \geq \mu(x) \wedge \mu(y)$ and $\mu * M * \mu \leq \mu$.

Obviously we have $\overline{Ŧ}(\mu)(x\gamma y) \geq \overline{Ŧ}(\mu)(x) \wedge \overline{Ŧ}(\mu)(y)$.

Consider

$$\overline{Ŧ}(\mu) * M * \overline{Ŧ}(\mu) = \vee_{x=a\alpha b} \left[\min\{(\overline{Ŧ}(\mu) * M)(a), \overline{Ŧ}(\mu)(b)\}\right]$$
$$= \vee_{x=a\alpha b} \left[\min\{\vee_{a=p\gamma q} \min\{\overline{Ŧ}(\mu)(p), M(q)\}, \overline{Ŧ}(\mu)(b)\}\right]$$
$$= \vee_{x=a\alpha b} \left[\min\{\vee_{a=p\gamma q} \left[\overline{Ŧ}(\mu)(p), \overline{Ŧ}(\mu)(b)\right]\}\right]$$
$$\leq \min\{\overline{Ŧ}(\mu)(x), \overline{Ŧ}(\mu)(x)\}$$
$$\leq \overline{Ŧ}(\mu)(x)$$

Hence $\overline{Ŧ}(\mu) * M * \overline{Ŧ}(\mu) \leq \overline{Ŧ}(\mu)$.

Theorem 4.2:

Let M and M_1 be two Γ-semigroups and $Ŧ: M \to P'(M_1)$ be a set-valued homomorphism. If μ be a fuzzy bi-ideal of M_1, then $\underline{Ŧ}(\mu)$ is a fuzzy bi-ideal of M.

Proof:

Proof is similar to the proof of Theorem 4.1.

Theorem 4.3:

Let M and M_1 be two Γ-semigroups and $Ŧ: M \to P'(M_1)$ be a set-valued homomorphism. If μ be a fuzzy quasi-ideal of M_1, then $\overline{Ŧ}(\mu)$ is a fuzzy quasi-ideal of M.

Proof:

Let μ is a fuzzy quasi-ideal of M_1, then for all $x, y \in M, \gamma \in \Gamma$, $\mu(x\gamma y) \geq \mu(x) \wedge \mu(y)$ and $(\mu * M) \wedge (M * \mu) \leq \mu$.

Obviously we have $\overline{Ŧ}(\mu)(x\gamma y) \geq \overline{Ŧ}(\mu)(x) \wedge \overline{Ŧ}(\mu)(y)$

Consider $\overline{Ŧ}(\mu) * M(x) = \vee_{x=a\alpha b} \left[\min\{\overline{Ŧ}(\mu)(a), M(b)\}\right]$

$$= \vee_{x=a\alpha b} \overline{Ŧ}(\mu)(a)$$
$$\leq \overline{Ŧ}(\mu)(x)$$

These implies that $\overline{Ŧ}(\mu) * M \leq \overline{Ŧ}(\mu)$.

Similarly $M * \overline{Ŧ}(\mu) \leq \overline{Ŧ}(\mu)$.

It follows that $(\overline{Ŧ}(\mu) * M) \wedge \left(M * \overline{Ŧ}(\mu)\right) \leq \overline{Ŧ}(\mu) \wedge \overline{Ŧ}(\mu) \leq \overline{Ŧ}(\mu)$.

Therefore $\overline{Ŧ}(\mu)$ is a fuzzy quasi-ideal of M.

Theorem 4.4:

Let M and M_1 be two Γ-semigroups and $Ŧ: M \rightarrow P'(M_1)$ be a set-valued homomorphism. If μ be a fuzzy quasi-ideal of M_1, then $\underline{Ŧ}(\mu)$ is a fuzzy quasi-ideal of M.

Proof:

Proof is Similar to the proof of Theorem 4.3.

Theorem 4.5:

Let M and M_1 be two Γ-semigroups and $Ŧ: M \rightarrow P'(M_1)$ be a set-valued homomorphism. If μ be a fuzzy quasi-ideal of M_1, then $\overline{Ŧ}(\mu)$ and $\underline{Ŧ}(\mu)$ are fuzzy bi-ideal of M.

Proof:

Let μ be a fuzzy quasi-ideal of M_1. By Theorem 4.3 and 4.4 $\overline{Ŧ}(\mu)$ and $\underline{Ŧ}(\mu)$ are fuzzy quasi-ideals of M. For all $x, y \in M, \gamma \in \Gamma$, obviously we have $\overline{Ŧ}(\mu)(x\gamma y) \geq \overline{Ŧ}(\mu)(x) \wedge \overline{Ŧ}(\mu)(y)$.

$\overline{Ŧ}(\mu) * M * \overline{Ŧ}(\mu) \leq \overline{Ŧ}(\mu) * M * \mathrm{M} \leq \overline{Ŧ}(\mu) * M$, and

$\overline{Ŧ}(\mu) * M * \overline{Ŧ}(\mu) \leq \mathrm{M} * M * \overline{Ŧ}(\mu) \leq M * \overline{Ŧ}(\mu)$

It follows that

$\overline{Ŧ}(\mu) * M * \overline{Ŧ}(\mu) \leq (\overline{Ŧ}(\mu) * M) \wedge \left(M * \overline{Ŧ}(\mu)\right) \leq \overline{Ŧ}(\mu)$, [Since $\overline{Ŧ}(\mu)$ is a fuzzy quasi-ideal]

Therefore $\overline{Ŧ}(\mu)$ is a fuzzy bi-ideal of M.

Similarly we have proved $\underline{Ŧ}(\mu)$ is a fuzzy bi-ideal of M.

5. Conclusion:

Fuzzy set theory and rough set theory take into account two different aspects of uncertainty that can be encountered in real-world problems in many fields. fuzzy sets deal with the possibilities uncertainty, connected from ambiguity of information. The connection between fuzzy sets and rough set lead to various models. This paper is intended to built up a connection between rough sets, fuzzy sets and Γ-semigroups. The notion of Ŧ-rough fuzzy subsemigroups, Ŧ-rough fuzzy ideals Ŧ -rough fuzzy bi-ideals and Ŧ -rough fuzzy quasi-ideals in a Γ-semigroup are studied. We belive, this paper offered here will turn out to be more useful in the theory and applications of rough sets and fuzzy sets.

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