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## STRENGTH ANALYSIS OF LAMELLAR GRAPHITE CAST IRON IN THE «CARBON (C) – CARBON EQUIVALENT ( $C_{EQ}$ )» FACTOR SPACE IN THE RANGE OF $C = (3,425–3,563) \%$ AND $C_{EQ} = (4,214–4,372) \%$

Для області факторного простору «вуглецевий еквівалент (Секв) – вміст вуглецю (С)» побудовано працездатний аналітичний опис впливу обраних входних змінних на межу міцності чавуну і досліджена отримана поверхня відгуку. Відзначено принципову можливість отримання більш точного опису поверхні відгуку. Отриманий результат може сприяти зниженню матеріаломісткості виливків і зниженню витрат на виплавку чавуну.

**Ключові слова:** чавун з пластинчастим графітом, індукційна тигельна піч, рівняння регресії, канонічне перетворення поверхні відгуку.

### 1. Introduction

Structural cast irons are unique ferrous alloys. Range of their application covers most sectors of the economy thanks to a good combination of performance and processing properties. A particular and prevailing area of structural cast iron use is engineering, the vast majority of case castings for which are made of this alloy. Moreover, cast iron processability allows to control the properties through the effective regulation of the processes of structure formation in the smelting step and the subsequent temperature-time processing. This provides the possibility of increasing the mechanical properties or imparting special properties for specific operating conditions: wear resistance, heat resistance, corrosion resistance, specific magnetic characteristics. Economic aspect is also important in the choice of cast iron as a construction material, because its production is associated with lower costs for implementation of the process as compared with steel. Of course, it is necessary to stress that the strength properties or special properties must be sufficient for the specific operating conditions.

However, despite the above advantages, it is well known the real problems associated with control of properties. They are related to multifactor impact on the structure formation processes, as well as to the uncertainty at the stage of establishing of quality dependencies in the chain «chemical composition – structure – properties». Solution of these problems also makes it possible to reveal formation mechanisms for structure and properties, physical and chemical laws of structure formation processes. The result, of course, can't be moved without change from one industry conditions to the other, but can be used at a specific adaptation to specific industrial conditions.

All these considerations suggest the relevance of research topics related to obtaining adequate models of the «chemical composition – property». On the basis of these models performance properties of cast iron can be predicted or optimal technological modes for selected criteria of its formation can be developed.

The presence of such models in the form of analytical description must also include the possibility of justification of the result obtained on the basis of studies of cast iron microstructure and related opportunities to reveal formation mechanisms of structure and properties. At the same time the process of obtaining the results can be implemented only in specific production conditions.

It is particular noted the relevance of research of chemical composition effect on the mechanical properties of cast iron from the following considerations. In circumstances where the requirements of the operation does not provide increased load and sufficient in terms of strength characteristics is to produce standard grades of cast iron, in the first place there is the problem of minimizing its manufacturing costs. At the same time, a necessity to solve the problem of alloy composition synthesis problem, which provides acceptable strength characteristics, is defining but the priority in its manufacture is to reduce the costs of the smelting process.

### 2. The object of research and its technological audit

*The object of research* is the structural lamellar graphite cast iron, where the carbon equivalent ( $C_{eq}$ ) is in the range of (4,214–4,372) % and the carbon content (C) is in the range of (3,425–3,563) %.

The choice of these ranges is due to the fact that in terms of creating more favorable conditions for graphitization process it is desirable to provide high carbon content in cast iron. C/Si optimum ratio minimizes the probability of chill formation, stimulating the crystallization process for the stable and not metastable diagram leading to the formation of cementite  $Fe_3C$  in the structure. However, increased carbon content reduces the tensile strength of iron cast, so the potential loss of strength due to increased carbon content can be offset by an addition of the minimum amounts of alloying elements that increase the rate of cast iron quality. It should be noted that addition of

ferroalloys in this case doesn't play a role in increasing the possible production cost, because on the other hand there is a possibility of metal consumption minimizing by thinning of casting walls. The latter becomes valid for cast iron strength improvement. The microstructure formation due to crystallization processes with cooling rates inherent in given manufacturing process in given production environment is also dependent on the position of the eutectic point on the phase diagram of Fe-C, i. e. on  $C_{eq}$  value. Research of a joint effect of the two input variables is important to study the mechanisms of formation of microstructure and mechanical properties of cast iron.

Technological audit was carried out in a foundry of «Kremenchuk plant of road machines» (Kremenchuk, Ukraine). Its aim was to identify the real values of tensile strength (TS, MPa) in the specified range  $C-C_{eq}$  in the implementation of the basic technology process of induction melting. Synthetic iron CЧ20 GOST 1412-85 was analyzed.

Smelting was carried out in the acid-line induction crucible furnace ICT1/0.8-M5. The main technical characteristics of the furnace are shown in Table 1.

As the raw materials were used: the return of its own production, graphite electrodes scrap and steel 1A (Cr3) scrap. Dimensions of large pieces of steel scrap didn't more the limits: maximum dimensions – 350 mm, thickness – 3,9 mm, minimum weight – 15–18 kg. Scrap fraction of graphite electrodes is in the range of 1–10 mm.

Cast iron was alloyed by a minimum number of elements – chromium (Cr) and nickel (Ni) in a ratio of Cr:Ni=2:1 that the chromium content in the finished cast iron is in the range (0,25–0,54) %. Ferromanganese ФМН-70, ferrochrome ФХ100 and ФХ200, ferronickel ФН-70 were used as ferroalloys for alloying, in accordance with the norms of consumption of charge materials, adopted in the enterprise. Titanium-cuprous cast irons were used for necessity of copper (Cu) and titanium (Ti) alloying at the rate of copper content in the final cast iron was in the range (0,1–0,5) %, and the ratio Cu:Ti=4:1 was kept. Ferrosilicon ФС75 was used as a modifier.

Burden loading was carried out in accordance with the approved technological instruction for grey cast iron smelting in induction furnaces ICT1/0.8-M5. Correctness control of the smelting process, the state of the crucible and the insulation of the inductor was carried using the

indicators of the control board instrumentation. Uploading of preheated burden materials was occurred only after upsetting of the burden in the crucible. The real time of melting start in the bottom was 10,7 minutes from the start of smelting. Inductor voltage was reduced under intensive melt stirring followed by the release of shot metal.

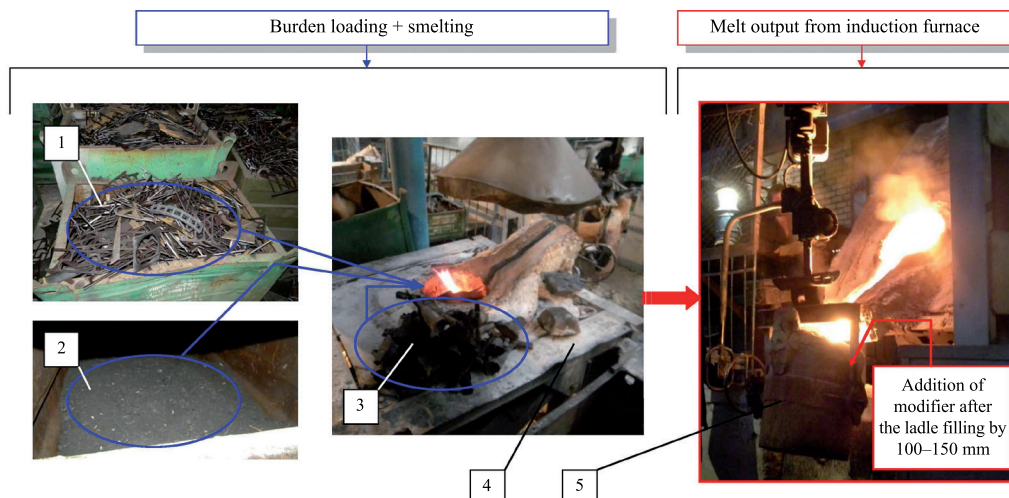
**Table 1**

The main technical characteristics of the induction furnace ICT 1/0.8-M5 for synthetic cast iron smelting

№	Technical characteristics	Norms
1	Frequency converter power, kW	800
2	Rated power, kW	823 (+20)
3	Medium frequency power, kW	785
4	Rated voltage, V	6000
5	Rated power frequency, Hz Mains Contour chain	50 1000
6	Number of phases Mains Contour chain	3 1
7	Rated metal overheat temperature, °C	1600
8	Capacity of electric furnace, t	1
9	Smelting and overheating rate, t/h	1,3
10	Specific energy consumption for smelting and overheating, kW*h/t	633
11	Cooling water consumption, m <sup>3</sup> /h	14,4

Dry sand was added for slag induction to obtain free-running slag – lime or 30 mm limestone. Melt reduction to a predetermined temperature and the chemical composition was performed after the complete smelting by heating for 5 minutes, and shut down of the furnace with delay to better redox processes in the melt. Cast iron temperature before modification was in the range 1400–1450 °C. Melt processing by modifier of 1–10 mm fraction in an amount of 0,3 % by weight of the liquid metal (3 kg per 1 ton) was performed in the ladle after the ladle filling by 100–150 mm.

Schematic diagram of the basic process of structural cast iron production in the induction furnace ICT1/0.8-M5 is shown in Fig. 1.



**Fig. 1.** Schematic diagram of structural cast iron production process in induction furnace ICT1/0.8-M5: 1 – scrap steel, 2 – graphite electrode scrap, 3 – return of its own production (sprues), 4 – induction furnace ICT1/0.8-M5, 5 – ladle

In accordance with the technological regulations of the enterprise, samples were selected for chemical analysis, and samples were filled to determine the tensile strength of cast iron in accordance with GOST 27208-87.

### 3. The aim and objectives of research

The aim of research is to describe the distribution of tensile strength values of cast iron series in the factor space  $C-C_{eq}$  in the range of  $C = (3,425-3,563) \%$  and  $C_{eq} = (4,214-4,372) \%$ . The values of Cr-Ni-Cu-Ti content in alloying complex are in narrow ranges. In accordance with this aim of research, the possibilities are opened for predicting the limit on the tensile strength of cast iron for the described area  $C-C_{eq}$ , the calculation of the optimal burden composition and to identify the mechanisms of formation of properties for the described chemical composition range.

To achieve this aim, there are the next objectives.

1. Build a workable analytical description of the impact of the selected input variables on the tensile strength of cast iron.

2. Study the response surface and identify the most informative point of the factor space for further detailed investigation of the microstructure in these points.

### 4. Research of existing solutions of the problem

Research of cast iron properties as a structural material is traditionally held in different directions, among which are:

- The study of cast iron microstructure and its impact on certain performance characteristics [1-4].
- The study of the modifier influence on the formation of cast iron microstructure [5, 6].
- Study of the influence of the chemical composition and solidification process on the microstructure and properties of cast iron [7-9].

In [1-3] the effect of the size and geometry of the graphite on the formation of iron tightness is studied. It must have an increased tightness in addition to the strength characteristics in accordance with operating requirements. Given that the key to the formation of the properties of cast iron is, according to the authors of these works, the distance between the graphite plates, as well as their size, we can talk about a certain correlation, tightness and strength. However, such correlations are not given, and quantitative descriptions relate to the impact of considered input variables relevant only to tightness performances. Nevertheless, the obtained results are quite valuable, that allow to develop this research to expand the output variables – cast iron properties.

In [4] the effect of microstructural irregularities on the fatigue strength of cast iron is studied: graphite, casting defects, metal matrix structure. The focus is made on the identification and the ability to predict the spread of the lower spread limit of fatigue strength, which has particular importance from a practical point of view. This method allows, according to the authors of [4], to predict the estimate of the output characteristics on the basis of information about the microstructural irregularities and loading conditions.

The studies, which are described in [5], have established qualitative impact of new SiC-containing modifier

in conjunction with the ferrosilicon FeSi75 on graphite morphology, matrix structure and mechanical properties of cast iron. There is the fact of improving the properties of molten iron from the original non-inoculated state. The authors of this study highlight the possibility of formation of a large number of microzones with high carbon content and silicon concentration that promotes favorable course of graphitization process. However, such assessment is made only on a qualitative level. Investigation of the effect of modifier composition in the mold on the microstructure and fatigue strength of castings made of cast iron EN GJS 700-2 is described in [6], in which the authors conclude about the effectiveness of the modification in accordance with effect on alloy microstructure. The problem is addressed mainly in the plane of the materials science approach.

Ultimate Tensile Strength (UTS), depending on the carbon content, chemical composition and solidification rate is studied in [7]. It is particular noted the author's systematic approach, considering the problems from different points of view – for reasons of analytical description of the impact of these technological parameters on UTS and materials science approach. In the first case the priority has research based on the use of regression analysis methods or modified Griffiths and Hall-Petch equations. In the second case it is the transformation of austenite during cooling and formation of the metal matrix. The authors noted, in particular, the dominant parameter that can be used to determine the tensile strength is a characteristic distance between the pearlite grains. These results can be extended to the whole range of carbon content from eutectic to hypoeutectic composition, solidifying at different cooling rates typical for thin-walled and thick-walled complex shaped castings.

Effect of alloying parameters, together with the regulation of C/Si ratio in the cast iron, on the microstructure and mechanical properties of cast iron is described in [9]. It describes a significant impact of modifier in combination with antimony (Sb) on output cast iron characteristics. It is stressed that Sb plays an important role in controlling the morphology of graphite. However, the results of this article are related to ductile iron. As for the possibility of extending the conclusions for lamellar graphite cast iron, additional research is needed.

Attempts to obtain the analytical descriptions of the effect of chemical composition on the properties of the alloy are described in [10-13]. In [10-11] the results are described for building of regression equations of impact of carbon, carbon equivalent, alloying systems Cr-Ni and Cr-Ni-Cu on the tensile strength (TS) and hardness (HB). In [12, 13] an application of methods for building of «structure – property» models under uncertainty is studied.

Described works suggest a fragmentation of research, which is manifested in attempts by various researchers consider the problem only on one side, attracting «narrow» research methods. As for the impact of cast iron chemical composition in a particular range of variation of the input variables  $C-C_{eq}$  on the tensile strength, corresponding works hasn't been found. Therefore, to solve practical problems of rational choice of the chemical composition of cast iron as a structural material, which allows to further minimize the cost of its production, it is necessary to carry out special industrial research.

## 5. Methods of research

According to the results of industrial tests, the table of input and output variables is formed. Its fragment is shown in Table 2. The initial sample consisted of 200 series of smelting.

**Table 2**

The fragment of input data table for modeling the influence of the chemical composition of the structural lamellar graphite cast iron on tensile strength

№	Input variables				Output variable
	C, %	C <sub>eq</sub> , %	Cr, %	Cu, %	TS, MPa
	x <sub>1</sub>	x <sub>2</sub>	x <sub>3</sub>	x <sub>4</sub>	γ <sub>1</sub>
1	3,63	4,387	0,32	0,19	190
2	3,57	4,282	0,3	0,15	190
3	3,58	4,323	0,42	0,21	190
4	3,57	4,284	0,32	0,12	180
5	3,7	4,531	0,26	0,19	190
6	3,68	4,468	0,4	0,19	185
7	3,64	4,395	0,38	0,1	155
8	3,63	4,394	0,4	0,1	170
9	3,59	4,333	0,26	0,1	195
10	3,59	4,364	0,27	0,17	185
11	3,45	4,243	0,31	0,11	200
12	3,62	4,381	0,32	0,09	200
13	3,55	4,29	0,3	0,14	200
.	.	.	.	.	.
64	3,56	4,224	0,29	0,18	200
65	3,68	4,447	0,37	0,18	200
66	3,48	4,183	0,42	0,11	210
67	3,53	4,231	0,31	0,11	210
68	3,6	4,408	0,33	0,11	220
69	3,54	4,292	0,36	0,12	220
70	3,39	4,089	0,54	0,09	220
71	3,43	4,265	0,42	0,27	220
72	3,41	4,241	0,45	0,2	220
73	3,41	4,132	0,48	0,25	220
.	.	.	.	.	.
200	3,15	3,896	0,33	0,3	300

On the basis of generated input data table (Table 2) into the sample to build a mathematical model:  $TS = f(C, C_{eq})$  are selected only rows with  $C = (3,425-3,563) \%$  and  $C_{eq} = (4,214-4,372) \%$ . Normalization of values of input variables is carried out according to the formula:

$$x_{inorm} = \frac{x_i^* - \bar{x}_i}{I_i}, \quad (1)$$

where  $x_{inorm}$  – normalized value of the input variables, index  $i$  takes the values  $i = 1$  for C, and  $i = 2$  for  $C_{eq}$ ,  $x_i^*$  – natural values of the input variables,  $\bar{x}_i$  – average values of the input variables,  $I_i$  – variability intervals of the input variables,  $I_i = x_i^{\max} - \bar{x}_i = \bar{x}_i - x_i^{\min}$ .

Distribution of the content of alloying elements in cast iron are detected by histograms and the establishment of distribution law, which allows to identify the limits of applicability of the resulting mathematical model

$TS = f(C, C_{eq})$  and the numerical modeling results. In other words, the task is to define those content ranges of elements of Cr–Ni–Cu–Ti of alloying complex, for which the resulting model  $TS = f(C, C_{eq})$  can be considered as workable in terms of predicting the tensile strength or selection of rational technological smelting regimes on its basis. The latter also implies the ability to optimize the burden composition according to the criterion of cost minimization:

$$J = \sum_{i=1}^n C_i U_i \rightarrow \min_{\{U_i\}}, \quad i = \overline{1, n}, \quad (2)$$

where  $C_i$  – value of  $i$ -th burden material,  $U_i$  – number of  $i$ -th burden material. At the same time, with each burden material, finished cast iron obtain  $f_{ij} U_i$  number of  $j$ -th element of the chemical composition. The total number of  $j$ -th element in the chemical composition of cast iron is:

$$\sum_{i=1}^n f_{ij} U_i,$$

where  $f_j$  – the total number of  $j$ -th element of the chemical composition in the finished cast iron.

Minimization of the functional (2) is carried out under the constraints of the form [14]:

$$\sum_{i=1}^n U_i = U^*, \quad (3)$$

$$\sum_{i=1}^n f_{ij} U_i = f_j \quad (4)$$

by the methods of linear programming [15].

Possibility of building a workable mathematical model in the form of two alternatives, in terms of the description accuracy, variants of regression equation is studied:

$$y_i = a_0 + a_1 x_1 + a_2 x_2 + a_3 x_1 x_2, \quad (5)$$

$$y_i = a_0 + a_1 x_1 + a_2 x_2 + a_3 (x_1)^2 + a_4 (x_2)^2 + a_5 x_1 x_2, \quad (6)$$

where  $a_i$  – estimated coefficients.

To estimate the vectors:

$$A = \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \end{pmatrix} \quad \text{and} \quad A = \begin{pmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \end{pmatrix},$$

ordinary least squares method (OLS) are used:

$$A = (F'F)^{-1} F'Y = CF'Y. \quad (7)$$

Vector A minimizes the functional of least squares of the type:

$$J = (FA - Y)'(FA - Y), \quad (8)$$

where  $F$  – matrix having a form to obtain the equations (5) and (6), respectively:

$$F = \begin{pmatrix} 1 & x_{11} & x_{12} & x_{11}x_{12} \\ 1 & x_{21} & x_{22} & x_{21}x_{22} \\ \dots & \dots & \dots & \dots \\ 1 & x_{n1} & x_{n2} & x_{n1}x_{n2} \end{pmatrix} \text{ and}$$

$$F = \begin{pmatrix} 1 & x_{11} & x_{12} & x_{11}x_{12} & x_{11}^2 & x_{12}^2 \\ 1 & x_{21} & x_{22} & x_{21}x_{22} & x_{21}^2 & x_{22}^2 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & x_{n1} & x_{n2} & x_{n1}x_{n2} & x_{n1}^2 & x_{n2}^2 \end{pmatrix},$$

matrix of the experimental values of tensile strength,  $TS$ , MPa:

$$Y = \begin{pmatrix} y_1 \\ y_2 \\ \dots \\ y_n \end{pmatrix}.$$

In the first step of building models it is necessary to conduct filtering of the experimental data having the aim to make possible of  $F$  rows inclusion in the matrix. They are differed in  $Cu$  and  $Cr$  values. Such filtering is mandatory because it is well known that these chemical elements substantially affect the tensile strength. Exclusion of the relevant rows provides the possibility of building a workable regression equations of the form (5) or (6) in the factor space  $C-C_{eq}$  at a fixed level of  $Cr$  and  $Cu$  content in cast iron. The latter, more strictly, is a description of the form:

$$Cr \pm \frac{\varepsilon S}{\sqrt{n}} \text{ and } Cu \pm \frac{\varepsilon S}{\sqrt{n}},$$

where  $S$  – the standard deviation of  $Cr$  and  $Cu$  content, respectively, obtained by statistical processing of the experimental data. So, confidence intervals are the «fixed» values.

### 6. Research results

Fig. 2 and 3 show histograms and curves of the density distribution of chromium and copper content in cast iron, derived from statistical processing of  $n = 50$  samples obtained after the selection of rows from the original data table (Table 2), which satisfy the conditions:

$$x_1 \in [3,425; 3,563] \text{ and } x_2 \in [4,214; 4,372].$$

Some rows, containing the values of  $x_3$  and  $x_4$ , are removed from the sample, On the basis of expert assessments, because they presumably influencing the formation of the deviation from the normal distribution law. As a result, histograms and curves of the density distribution are obtained and shown in Fig. 4, 5.

Based on these results, as «fixed» values of input variables of  $Cr$  and  $Cu$  are taken  $Cr \pm 0,032 \%$ ,  $Cu \pm 0,026 \%$ . Using OLS, by the formula (7), estimation values of regression equation coefficients of the form (5) and (6) are calculated. They are, respectively:

$$A = \begin{pmatrix} 209,3051 \\ -6,14398 \\ 1,069837 \\ -15,5749 \end{pmatrix} \text{ and } A = \begin{pmatrix} 206,7276 \\ -3,87669 \\ 6,37602 \\ -16,3141 \\ 20,44989 \\ -19,2311 \end{pmatrix}.$$

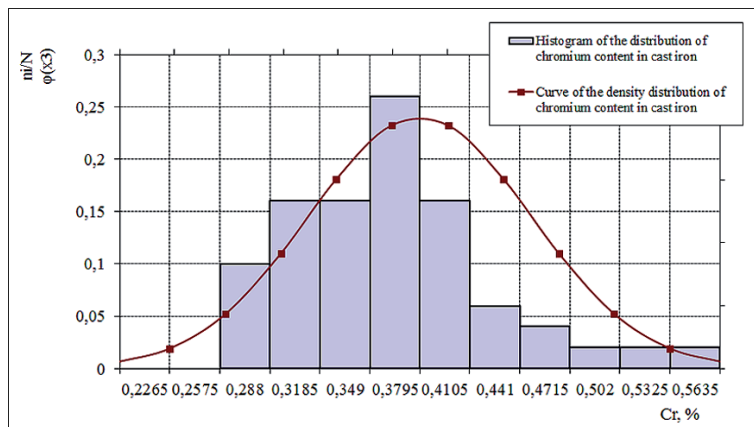


Fig. 2. Histogram and curves of the density distribution of chromium content in cast iron ( $x_3$ )

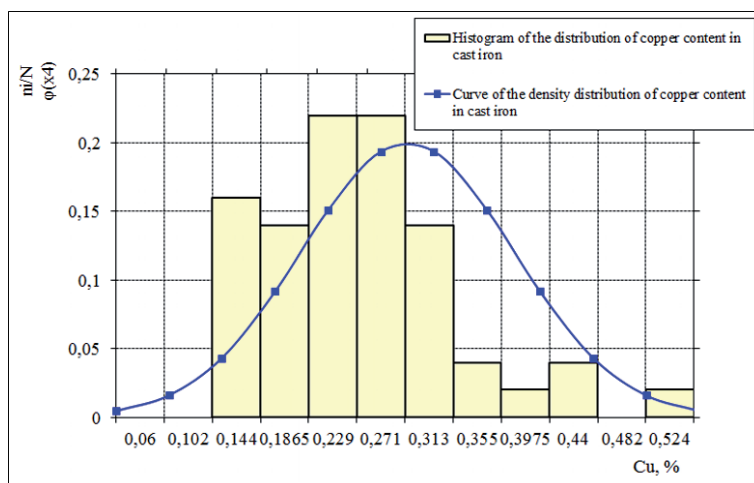


Fig. 3. Histogram and curves of the density distribution of copper content in cast iron ( $x_4$ )

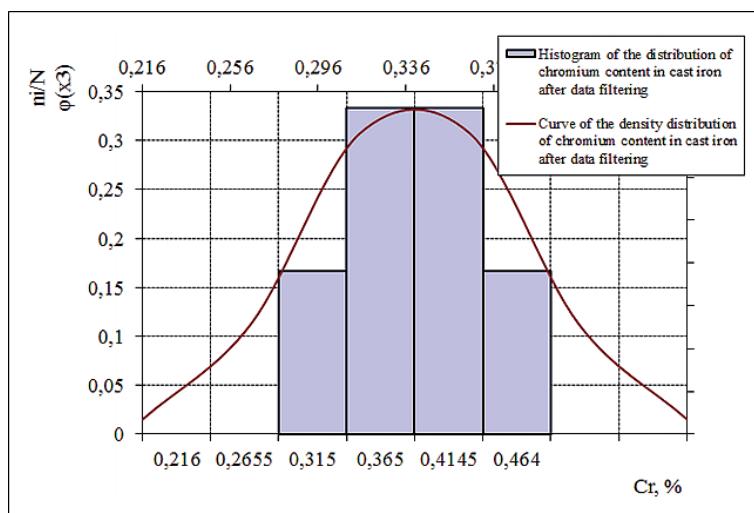


Fig. 4. Histogram and curves of the density distribution of chromium content in cast iron ( $x_3$ ) after data filtering

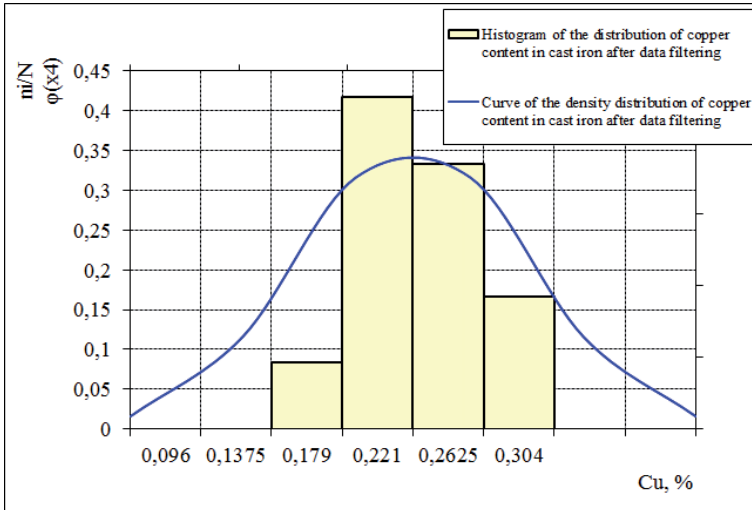


Fig. 5. Histogram and curves of the density distribution of copper content in cast iron ( $x_4$ ) after data filtering

Given the fact that the coefficients of the regression equations are estimated on the basis of passive experiment, not giving the possibility of parallel measurements of the output variable at each point of the plan, there isn't ability to check homogeneity of the design of experiment. Checking the adequacy of the model, based on the use of Fisher's  $F$ -test, or checking the hypothesis of equality of variances of the experimental error and inadequacy of the model because of this is not possible. Therefore, the potential performance of the model is evaluated on the basis of checking the number of data points belonging to a given confidence interval (Fig. 6, 7).

Fig. 6 and 7 show that the best performance provides the equation of the form (6) – if using a linear equation taking into account pair influence of the factors, confidence interval consists of 9 test points (75 %), then using a polynomial regression equation, confidence interval consists of 11 test points (92 %). Therefore, regression equation of the form (6) is taken as a workable analytical description.

Since the greatest interest is the identification of stationary points and description of kind of response surface in their neighborhood, canonical transformation of the response surface are carried out using well-known procedure [16]:

1. Determination of coordinates for fixed point  $x^*$  by solving a system of linear equations:

$$\begin{pmatrix} a_{11} & \dots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \dots & a_{mn} \end{pmatrix} \begin{pmatrix} x_1^* \\ \vdots \\ x_n^* \end{pmatrix} = - \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix}$$

2. Calculation of the target value at a fixed point:

$$y^* = a_0 + 2a'x^* + x^{*'}Ax^*$$

3. Determination of  $n$  eigenvalues  $\lambda_1, \lambda_2, \dots, \lambda_n$ :

$$\begin{vmatrix} a_{11} - \lambda & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} - \lambda & \dots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nn} - \lambda \end{vmatrix} = 0$$

$$= (-\lambda)^n + P_1\lambda^{n-1} + \dots + P_n = 0.$$

4. Writing the response surface equation in canonical form:

$$y - y^* = \lambda_1 \xi_1^2 + \lambda_2 \xi_2^2 + \dots + \lambda_n \xi_n^2.$$

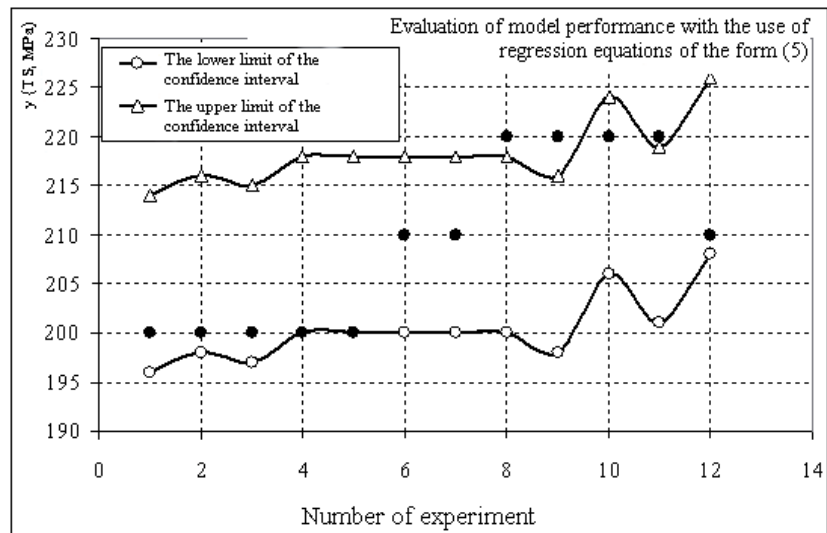


Fig. 6. The results of checking the performance of the model on the basis of linear regression equation taking into account the pair influence of the factors

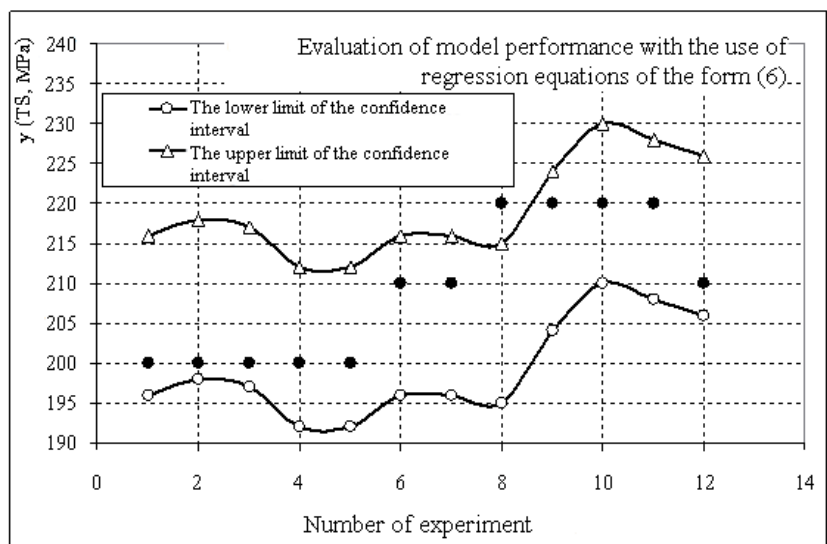


Fig. 7. The results of checking the performance of the model on the basis of polynomial regression equation

Thus, as a result of translation and rotation of axes and transition from the coordinate system  $(x_1; x_2)$  in the coordinate system  $(\xi_1; \xi_2)$ , transformation of the original response surface equation is provided:

$$y(x) = a_0 + 2a'x + x'Ax,$$

to canonical form  $y - y^* = \lambda_1 \xi_1^2 + \lambda_2 \xi_2^2 + \dots + \lambda_n \xi_n^2$ :

$$x = x^* + B\xi, \quad (9)$$

where  $B$  – the rotation matrix,  $B'B = I$  and the difference between the values of the output variable in random and fixed points is described by the equation:

$$y(\xi) - y^* = \xi' L \xi. \quad (10)$$

The following results are obtained using procedure 1–4:

$$\lambda_1 = -16,3141, \lambda_2 = 20,45, x^* = \begin{pmatrix} -0,021 \\ -0,166 \end{pmatrix},$$

$$y^* = 203,5 \text{ MPa}.$$

These normalized values of the input variables at a stationary point correspond to the actual:  $C = 3,492 \%$ ,  $C_{eq} = 4,28 \%$ .

Therefore, the equation describing the response surface in the canonical form is:

$$y - y^* = -16,3141 \xi_1^2 + 20,45 \xi_2^2. \quad (11)$$

Since the ratio of the eigenvalues in magnitude and sign determine the type of response surface and  $|\lambda_1| \neq |\lambda_2|$ ,  $\lambda_1 < 0$ ,  $\lambda_2 > 0$ , response surface is a hyperbolic paraboloid. It can be seen from the visualization results obtained by numerical calculation directly based on the equation (6) by substituting the estimates of the coefficients and the values of the input variables in the analyzed area of the factor space:  $C = (3,425-3,563) \%$  and  $C_{eq} = (4,214-4,372) \%$  (Fig. 8). The results in Fig. 8 are given for normalized values of the input variables obtained based on the formula (1).

The presence of a saddle point is an informative indicator, which suggests that the appropriate values of the input variables  $C = 3,492 \%$ ,  $C_{eq} = 4,28 \%$  at the content of alloying elements  $Cr \pm 0,032 \%$ ,  $Cu \pm 0,026 \%$  form a microstructure that ensures value of cast iron limit strength  $TS = 203 \text{ MPa}$ . In view of the resulting confidence interval, this value with a probability of 95 % is in the range of  $TS = (193-213) \text{ MPa}$ . It is obvious that obtaining such microstructure may not be desirable in the case that the technical requirements for cast iron provides production of brand CЧ20 GOST 1412-85, for which the value of the tensile strength should be in the range  $TS = (200-250) \text{ MPa}$ . Metallographic microstructure description in the saddle point is important and can be obtained by the development of modeling results.

Fig. 9 is a top view of the response surface with a stationary area and the coordinates of the point at which the metallographic analysis of shape, size, distribution and amount of graphite is carried out.

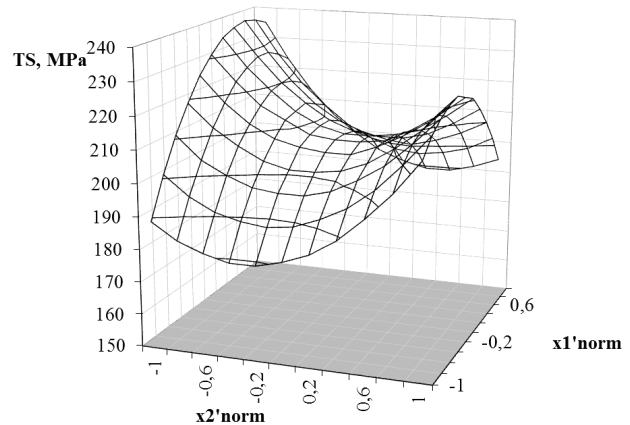


Fig. 8. Response surface describing the distribution of tensile strength values of cast iron in the factor space  $C = (3,425-3,563) \%$  and  $C_{eq} = (4,214-4,372) \%$  (input variables are defined in the normalized form)

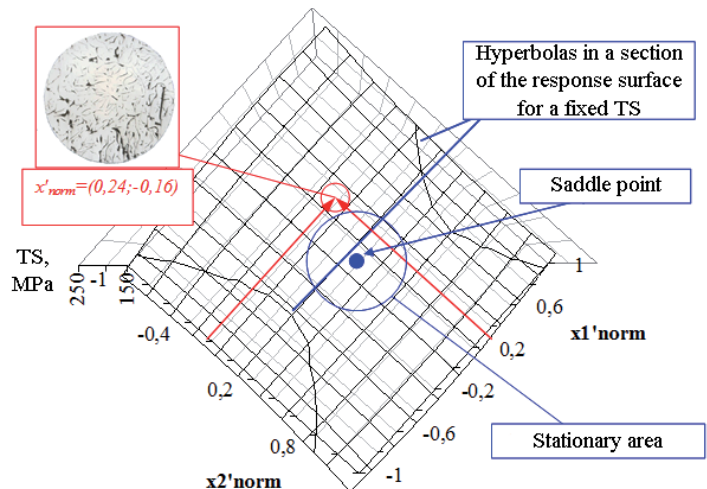


Fig. 9. Response surface  $TS = f(C, C_{eq})$  (top view) with description of the stationary area and the results of determination of the microstructure at the point of factor space with coordinates  $x'_{norm} = (0,24; -0,16)$

As follows from the obtained metallographic analysis, shape of graphite inclusions in the analyzed point of the factor space corresponds to ПГФ1, ПГФ2, graphite distribution – ПГр1, ПГр9, size – ПГд45, ПГд90, graphite amount – ПГ6. Fig. 10 shows the corresponding microstructure that is described based on the comparison with standard (GOST 3443-87), and relevant evaluation of the shape, size, distribution and amount of graphite.

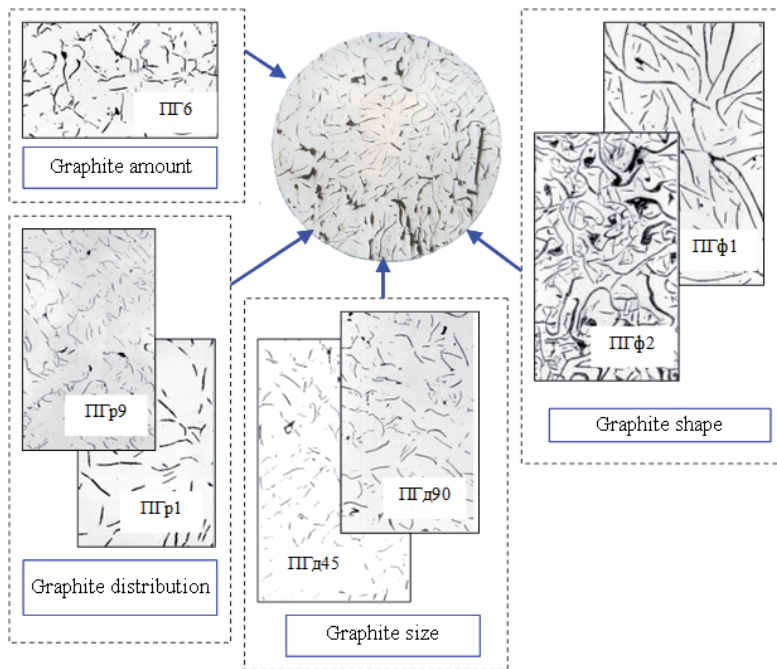
The development of obtained result can be considered a description of microstructure near the fixed point, in particular at the saddle point. Availability of data samples with such description is required when the task of further research is to establish a qualitative and quantitative effect of the microstructure parameters (shape, size, distribution and amount of graphite) on special or mechanical properties.

It should be noted that due to the evaluation specifics of the microstructure parameters based on a comparison with the sample and recommended standard, values of the input variables – graphite parameters – can be considered as fuzzy numbers. In such analysis, they have

described by the respective membership functions, such as ( $L - R$ ) type [17–19]:

$$\mu(F_{jp}) = \begin{cases} L\left(\frac{\bar{F}_{jp} - F_{jp}}{\alpha_{jp}}\right), & F_{jp} \leq \bar{F}_{jp}, \\ R\left(\frac{F_{jp} - \bar{F}_{jp}}{\beta_{jp}}\right), & F_{jp} > \bar{F}_{jp}, \end{cases} \quad (11)$$

where  $\bar{F}_{jp}$  – the value of  $p$ -th microstructure parameter (according to GOST 3443-87) in  $j$ -th metallographic analysis, which is a modal for the fuzzy number  $F_{jp}$ ,  $j = 1, 2, \dots, n$ ,  $p = 1, 2, \dots, m$ ,  $\alpha_{jp}$ ,  $\beta_{jp}$  – left and right fuzziness coefficients in the description (11).



**Fig. 10.** Description of cast iron microstructure in terms of shape, size, distribution and amount of graphite

## 7. SWOT analysis of research results

**Strengths.** The strengths of this research include the possibility to use the resulting mathematical model to address two key challenges: predicting the tensile strength for the actual chemical composition that is obtained during the melting, and the choice of the composition to provide the desired level of tensile strength. In the first case it is possible to reduce the number of laboratory tests of strength by reducing the related costs. In the second case, it is the prospect of minimizing costs for burden – reduce the cost of 1 ton of suitable casting.

**Weaknesses.** Weaknesses of this research are due to the fact that the mathematical model is based on random area of design of experiment. This means that obtained estimations of coefficients are far from optimal and there is a fundamental opportunity to improve accuracy – due to numerical building of D-optimal plan, which minimizes the volume of the dispersion ellipsoid of coefficient estimates. An alternative would be artificial orthogonalization

of full factorial experiment, inside the considered in this paper. It is possible to obtain the corresponding values of tensile strength at the points of generated orthogonal central composite design and more precise description of the response surface. However, additional smelting may be associated with significant additional costs for research. And although in this case is possible to implement an orthogonal central composite design within the existing description in accordance with given model, additional experiments can be useful to overcome a very important issue – inability to test the homogeneity of the experiment.

**Opportunities.** Additional features for use of these results in industrial applications are related to the apparent reduction of metal in the castings. This is achieved by providing a guaranteed predetermined strength in the high carbon content areas in cast iron. Due to this, on the one hand, an acceptable value of strength is ensured, and on the other – the qualitative graphitization process and reducing to a minimum the probability of cementite formation.

**Threats.** Obvious risks in use of these results relate to the following circumstance. Casting consumers prefer to buy high-quality castings of vermicular graphite gray cast iron or nodular graphite ductile cast iron. Such desire is justified, because mechanical or special properties of such cast iron are much higher. From the point of view of cast iron manufacturer if operating conditions of cast iron items are non-rigid and typical, than no need to spend extra money to improve the mechanical properties. If the manufacturer's costs are one of minimization criteria for it, the consumer is not interested in them.

Thus, SWOT analysis of research results suggests that the modeling results provide a potential opportunity to solve two problems: metal reduction in cast items and reduction of the cost of 1 ton of suitable casting. However, there are provisions to improve the quality of the results. They are associated with the ability to more

accurately determine the parameters of the mathematical model and give a more accurate description of the distribution of the tensile strength value in a given area of the factor space. It should be noted that in the preparation of this description it becomes possible to use the determination result of tensile strength at the point of the factor space, which is one of the vertices of hyperspace to build an artificial orthogonal plan in a wider region of the factor space.

## 8. Conclusions

1. It is shown that for a workable analytical description of the impact of carbon (C) and the carbon equivalent ( $C_{eq}$ ) on the value of the tensile strength of cast iron the best performance provides the polynomial regression equation. This equation structure and obtained by OLS the relevant estimation of the coefficients provide forecast accuracy, exceeding the accuracy using a linear regression equation in 1,23 times.



An existence of a saddle point is revealed on the basis of the canonical transformation of response surface. It is an informative indicator, which suggests that the respective values of the input variables  $C=3,492\%$ ,  $C_{eq}=4,28\%$  when the content of alloying elements  $Cr\pm 0,032\%$ ,  $Cu\pm 0,026\%$  form a microstructure that guarantees the value of cast iron tensile strength  $TS=203$  MPa. In view of the resulting confidence interval, this value with a probability of 95% is in the range  $TS=(193-213)$  MPa. Metallographic microstructure description at the saddle point is important and can be obtained by the development of the simulation results.

2. It is noted that obtained estimations of coefficients are far from optimal and there is a fundamental opportunity to improve accuracy – due to numerical building of D-optimal plan, which minimizes the volume of the dispersion ellipsoid of coefficient estimations. An alternative would be artificial orthogonalization of full factorial experiment, inside the considered in this work, receiving the corresponding values of tensile strength at the points of obtained orthogonal central composite design and more precise description of the response surface.

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## ИССЛЕДОВАНИЕ ПРОЧНОСТИ ЧУГУНА С ПЛАСТИНЧАТЫМ ГРАФИТОМ В ФАКТОРНОМ ПРОСТРАНСТВЕ «УГЛЕРОД (C) – УГЛЕРОДНЫЙ ЭКВИВАЛЕНТ (C<sub>ЭКВ</sub>)» В ДИАПАЗОНАХ C=(3,425–3,563) % И C<sub>ЭКВ</sub>=(4,214–4,372) %

Для области факторного пространства «углеродный эквивалент (C<sub>ЭКВ</sub>) – содержание углерода (C)» построено работоспособное аналитическое описание влияния выбранных входных переменных на предел прочности чугуна и исследована полученная поверхность отклика. Отмечена принципиальная возможность получения более точного описания поверхности отклика. Полученный результат может способствовать снижению материалоемкости отливок и снижению затрат на выплавку чугуна.

**Ключевые слова:** чугун с пластинчатым графитом, индукционная тигельная печь, уравнение регрессии, каноническое преобразование поверхности отклика.

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