# Behaviour of Spectral Active Power Terms for the Swiss 15 kV 16.7 Hz Railway System

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*Abstract*—A non-negligible amount of active power is carried also by harmonic (or, in general, spectral) terms, especially for highly distorted systems, such as electrified ac railways. The paper considers the distribution and correlation of these spectral active power terms, including train operating conditions, and gives criteria to select and group harmonic power terms. Several processing and display techniques are used to this aim: scatter histograms for joint probability, correlation coefficients, V-I maps. Results are shown and discussed for the Swiss 15 kV 16.7 Hz railway, using measured pantograph voltage and current quantities.

# Keywords—Electric transportation systems, Energy consumption, Power Quality, Power system harmonics

### I. INTRODUCTION

Reactive power flow and harmonic distortion terms are responsible for power losses in the power distribution system, for which many Power Quality (PQ) standards have been prepared, in particular for LV and MV public and industrial networks. The same kind of losses occurs also in electrified traction systems, along the traction line and at substations, and impacting as well on the feeding high-voltage network upstream. This work focuses on ac electrified railways in a single-train perspective [1][2]. The reason for narrowing the analysis to ac railways is that harmonic distortion and spectral power terms of dc electrified railway system are in general much lower, thanks to the large amount of capacitance distributed in the system and the widespread use of filters at substation and on-board [3]. Energy and power are exchanged at the line-pantograph interface and at the same interface pantograph electrical quantities are measured for both PQ [2] and billing purposes [1]. It is in general agreed that distortion components carry little active power and should be negligible compared to the active power carried by the fundamental component. However, it can be demonstrated that for ac systems harmonic power terms amount a fraction of percent [3] and need suitable characterization.

First of all, the required uncertainty of the energy measurement function implemented on-board (and including the data acquisition system and the voltage and current sensors) is in the range of a fraction of percent [1].

In addition, the economic impact of an energy saving of a magnitude commensurate to the observed variability is definitely worth a better understanding of involved approximations and variability. Based on previous measurements and PQ assessment of some European railways [3][4], the magnitude of the identified harmonic power terms is indeed relevant for the uncertainty budget.

The evaluation of the pantograph harmonic power (aka "distortion power") shall consider not only the spectra of the pantograph electrical quantities  $V_p$  (voltage) and  $I_p$  (current) alone for the calculation of the spectral power terms, but also their relative frequency of occurrence and relevance with respect to the total power flow, in a (absorbed or exchanged) energy perspective. Frequency of occurrence is assessed in relation to the rolling stock operating conditions, for which usually distinction is made among acceleration, cruising, coasting and braking; it is easy to see that auxiliaries and parts of the propulsion chain are differently involved in the four modes and, as a consequence, different harmonic signatures may be identified succeeding one to the other along the train route.

Going from a single train perspective to multiple train scenarios, involving an entire supply section (and possibly overfed sections), it is acknowledged that the amount of quantities that describe the electrical behavior of rolling stock and the network increases: the phase relationship between the distortion components of different trains changes with their relative position [5][6]; similarly, the line impedance at the pantograph is also variable with frequency and train position [7], and influences the phase angle between voltage and current components. This kind of problems was already analyzed to the aim of mutual interference and coupling to signalling circuits [5]-[7]; it is believed that for power absorption assessment these phenomena represent a secondorder refinement, that would not improve significantly the accuracy of the representation.

The following analysis is carried out in a single-train perspective and is based on pantograph voltage and current quantities, as required for power and energy consumption assessment [1][3]. The spectral active power terms and their correlation with train operating conditions are considered, to shed some light on the relevant components and operating conditions, and the typical variability. Having identified the overall frequency intervals characterized by a relevant power exchange [3], the analysis is refined by analyzing the behaviour of each spectral power term in terms of the amount of transported power and its sign and the correlation with train operating conditions.

### II. POWER RELATED QUANTITIES

The selected approach for the definition of the harmonic power terms is that of the IEEE Std. 1459 [8]: the total apparent power in nonsinusoidal conditions is subdivided into active power (defined for the fundamental and the

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harmonics) and a set of distortion power terms, analytically derived from the cross-product of harmonic terms of voltage or current [9]. To this aim voltage and current vectors are decomposed into a fundamental term and an additional term  $(V_H \text{ or } I_H)$ , including the dc component and the remaining harmonics.

$$V_{H}^{2} = V_{0}^{2} + \sum_{h \neq 1} V_{h}^{2} \qquad I_{H}^{2} = I_{0}^{2} + \sum_{h \neq 1} I_{h}^{2} \qquad (1)$$

The cross product of the so expressed voltage and current vectors determines the terms that compose the generalized apparent power: fundamental power  $S_1=V_1I_1$ , current distortion power  $D_I=V_1I_H$ , voltage distortion power  $D_V=V_HI_1$ , and harmonic apparent power  $S_H=V_HI_H$ . The last three terms may be grouped to form the non-fundamental apparent power  $S_N$ , comprising harmonic and inter-harmonic terms.

$$S^{2} = (V_{1}I_{1})^{2} + (V_{1}I_{H})^{2} + (V_{H}I_{1})^{2} + (V_{H}I_{H})^{2} = S_{1}^{2} + S_{N}^{2}$$
(2)

The assessment of the relevance of such harmonic active power terms is performed by means of a fractional index  $k_h$ , weighting the active power carried by the h-th component  $P_h(t)=\text{Re}\{S_h(t)\}$  with respect to the active power at the fundamental  $P_1(t)$ :

$$k_h = P_h / P_1 \tag{3}$$

The efficiency of a spectral term to carry active power is evaluated by means of the index

$$c_h = P_h / S_h \,, \tag{4}$$

that weights the amount of active and apparent power for each h-th component (i.e. the harmonic displacement factor).

It must be noted that the measured distribution of harmonics for a given railway supply network in reality depends also on the type of train or locomotive used for the tests; different types of rolling stock are characterized by different harmonic signatures with somewhat different distributions of components. However, the standpoint is to identify a widely applicable set of rules and define the relevance of harmonic power terms and grouping criteria, minimizing the influence of the behaviour of a specific rolling stock.

Differently from the initial grouping based simply on the adjacency on the frequency axis [3], in this work the behavior of the relevant active power components is tracked over an entire run, spanning all rolling stock operating conditions. A three-step procedure is followed:

- the entire test run is processed by means of the Short Time Fourier Transform (STFT) and active power terms are calculated for the entire spectrum and entire test run;
- the most relevant active harmonic terms are identified on absolute terms (i.e. their intensity) and on a relative basis (the index  $k_h$ ) using statistic quantities (e.g. mean, median and percentiles) and correlation;
- the selected terms are then tracked again over the same test run, evaluating their correlation to the train operating conditions.

These steps are detailed in the following subsections.

#### A. Spectrum and active power terms calculation

The STFT algorithm is run using two time window lengths,  $T_{16}$  and  $T_{50}$ . The benefit of using only one *T* value for both systems is arguable: if it ensures the same frequency resolution *df*, it would not be however commensurate to the harmonics and train dynamics, being the fundamental frequency and the harmonic patterns of the two networks different. Then two time windows are selected to ensure that even and odd harmonics are clearly detectable, but fast enough to track time-varying components and to cope with local non-stationarity:  $T_{16}$ =180 ms and  $T_{50}$ =60 ms, giving 1/3 of the fundamental frequency as frequency resolution *df*, and not so dissimilar from the 200 ms requirement of EN 61000-4-7 [10] for 50 Hz distribution systems.

Data are windowed before transformation, using a Hann window that has a satisfactorily narrow resolution (the Equivalent Noise Bandwidth is 1.5 bins of the rectangular window) and low sidelobe amplitude, reducing effectively the spectral leakage [11] (equivalent windows with respect to these two parameters are the Hamming and Gaussian).

An overlap p=50% is used for better tracking vs. time; the time granularity is finer than what required for energy measurements (normal practice is using a time resolution of 1 s although the minimum required by EN 50463 [1] for energy sampling is 5 min.) and electromechanical simulation (time step in the 0.25-1 s range) [12], giving margin for averaging and sample reduction.

Pantograph voltage and current spectra V and I are then multiplied to calculate the apparent power S, from which the fundamental and the spectral active power terms  $P_h$  are extracted.

### B. Selection of terms to track and statistics

Statistics of  $P_h(t)$  and of the fractional active power  $k_h(t)$  are then calculated to guide the selection of the most relevant components (a harmonic order *h* is used for brevity to indicate the frequency  $f = h \cdot df$ ).

Since the focus is on active power, the relevant  $c_h$  values are those close to unity; for this reason the complement of  $c_h$ ,  $(1-c_h)$ , is displayed, giving continuity to the plotted curve around unity. An alternative form is to show the reciprocal  $1/c_h$ , possibly with some compression of the scale, e.g. using a log function.

Terms for which the fundamental power is less than 0.1% of the nominal power are excluded, adopting some margin for accuracy and completeness of the estimates with respect to the EN 50463-2 requirement of >0.4%  $I_n$  (sec. 4.2.3.5). Regarding the significance of a term for the power and energy assessment, it is observed that an overall harmonic power of a fraction of % is a significant scenario and that several individual harmonic power terms might contribute to it; assuming that no more than ten harmonics may be identified as relevant (confirmed by the analysis in sec. IV), a conservative threshold of 0.01% for individual harmonic power terms is set to decide on their significance.

A representation by means of correlation and trend is useful to relate power terms to operating conditions, as well as to identify grouping criteria. Statistics represent the relevance over the set of collected values, that for the used data represent commercial service conditions, including all four operating modes with timing and dynamics in line with timetable and commercial service.

### C. Tracking of spectral active power

Tracking is used to further refine the analysis and to identify specific behavior with respect to specific operating conditions. Tracking is done with respect to time, plotting the amplitude of indexes  $k_h$  and  $c_h$  together with descriptors of rolling stock operating conditions: such descriptors are at the moment represented by the tractive effort, corresponding to the fundamental active power itself.

## III. RAILWAY SYSTEMS DESCRIPTION

The 15 kV 16.7 Hz system has a highly interconnected network with supply sections longer than those of modern 2x25 kV 50 Hz systems [3]. This increases the chance of network resonances already at low frequency, causing in some cases the amplification of voltage or current distortion at specific frequency intervals and as a consequence increasing the harmonic power terms [4].

Let's consider a simplified situation where a piece of rolling stock pulls some traction characterized by some harmonic content and a quite large equivalent harmonic impedance (supported by the control compensation and by the reactance of the on-board transformer). An increase of the line impedance seen from the pantograph due to a resonance increase proportionally the voltage distortion will components, since we have assumed that the equivalent circuit of the rolling stock approaches a pure current generator. Symmetrically it is possible to consider a preexisting line voltage distortion caused by the substation or other trains on the line (values of voltage THD shown in [4] are in the range 3-4%). An anti-resonance bringing the pantograph line impedance to particularly low values will amplify the current sourced by the rolling stock that happens to operate in a nearly short circuit condition at those specific frequencies. In both situations the product of the line voltage and rolling stock current will increase, giving rise to an increase of the harmonic power terms. At resonances and anti-resonances the resulting impedance of the traction line (as for all reactive resonant circuits) becomes almost resistive, increase significantly the amount of active power characterizing the observed harmonic power.

The variation of line impedance between resonances and anti-resonance may be about one order of magnitude or larger, depending on the amount of damping, e.g. the total load pulled by the trains in the same supply section.

#### IV. RESULTS FOR THE 15 KV 16.7 HZ SYSTEM

The run is characterized by two intense accelerations at about 100 and 300 s, several braking phases interleaved with coasting/cruising phases, and a time interval between about 800 and 900 s of stationary conditions. Please, see Fig. 1 for the active and reactive power and the rms current curves calculated at the fundamental frequency.

The spectrum of the exchanged active power  $P_h$  is characterized by a set of low frequency components in general related to the so called network distortion commented in the previous section. At high frequency, some variability is due to the network resonances/anti-resonances, amplifying some voltage or current components, respectively [4] (see Fig. 2). Low-order characteristic harmonics are visible up to about 300 Hz, followed by the patterns of the four-quadrant converters. The particularly intense third harmonic (50 Hz) is not only a network distortion harmonic (external), but it is also due to the auxiliaries and services on-board working at 50 Hz, rather than 16.7 Hz, and thus representing a source of current with phase and direction unrelated from the traction effort, and thus weakly dependent on the overall train operating conditions. This is easily demonstrated by taking the correlation coefficient between the  $k_h$  index at 50 Hz and the active power at the fundamental  $P_1$ , as shown later in Table I.



Fig. 1. (a) Fundamental active power  $P_1$  (red, left) and rms current (black, right); (b) Fundamental active  $P_1$  (red) and reactive  $Q_1$  (blue) power.



Fig. 2. Harmonic active power  $P_h$  vs. frequency and time in dB (10log<sub>10</sub>).

To analyze the correlation between harmonics (and thus the possibility of grouping), besides a straightforward comparison of plots vs. time, scatter plots and correlation coefficient are used.

The selection of the  $P_h$  terms for successive analysis is done based on the STFT in Fig. 2, identifying the hottest components, at least for a significant part of the run: the color scale was arranged to have a dark red/brown color for all points with  $P_h>6$  kW (38 dB), compatible with the threshold of 0.01% preliminarily set in the Introduction.

In the following figures (Fig. 3, 4 and 5) the  $k_h$  index is shown in dB, calculated using the  $20\log_{10}()$  expression, being  $k_h$  a ratio, so a pure number.



The maximum values of  $k_h$  for the 3rd harmonic,  $k_h(3f_1)$ , are particularly large, but occur at very low levels of active power  $P_1$ ; for exchanged levels of  $P_1$  above about 0.5 MW, the value of  $k_h(3f_1)$  is below -40 dB (1%); for all the considered values of the fundamental active power, it remains above 0.1%, so it is definitely a significant component to monitor. It may also be seen that  $k_h(5f_1)$ , although smaller, spreads to larger values of  $P_1$  and remains for more than 90% of the time above the 0.01% threshold (-80 dB). The successive terms  $k_h(7f_1)$  and  $k_h(9f_1)$  are slightly smaller with smaller spread, but in any case they are above the 0.01% threshold for about 50% of the time for  $P_1$  values up to 2 MW. These four components are at various extents all significant and must be part of the set of components monitored and tracked for power and energy absorption assessment purpose.



Fig. 3.  $k_h$  index (expressed in dB, 20log<sub>10</sub>) vs. fundamental active power: 3rd, 5th and 7th low-order harmonics.

Fig. 4.  $k_h$  index (expressed in dB, 20log<sub>10</sub>) vs. fundamental active power: 39th, 43th and 55th 4QC converter harmonics.



Fig. 5.  $k_h$  index (expressed in dB, 20log<sub>10</sub>) vs. fundamental active power: 79th (87th), 81st (83rd, 85th) and 119th (121st) 4QC converter harmonics.

Some high-order harmonic power terms are shown in Fig. 4 and 5, having selected harmonic orders that belong to patterns similar to switching emissions of the on-board fourquadrant converters (4QC), namely Pulse Width Modulation emissions with sideband lines located around the typical switching frequencies of rolling stock in service on the Swiss network. It is evident that the amount of active power is much smaller than for low-order harmonics, and that values above the 0.01% threshold occur only below 0.5 MW of exchanged active power.

Long term tracking and comparison of high-order components may be not trivial, if the frequency stability of the fundamental for 16.7 Hz networks is considered [13][14]: as per [13] the frequency bias for the 120th harmonic in the Swiss network is 5 Hz that is equal to the frequency bin used

for this analysis, i.e. one third of the fundamental; standard deviation is 2.8 Hz, so affecting the uncertainty of the estimate. This result was obtained for a long run of 400 km across Switzerland from Zurich to Brig.

To verify if different spectral components belong to the same source, and whether they are caused by the train under test or by nearby trains, correlation coefficients between pairs of  $k_h$  (Table I) may be used, as well as VI maps (Fig. 6).

TABLE I.	CORRELATION OF $K_H$ COEFFICIENTS
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Harm. order	Harm. order	Corr. coeff.
79, 87	81, 83, 85	0.30
81, 83, 85	81, 83, 85	1.0
79	87	-0.40
119	121	0.67
53	55	0.77
53	57	0.57
55	57	0.40
3	5, 7, 9, 11	-0.3 to -0.9
5	7, 9	0.60
7	9	0.91
3	81, 83, 85	-0.40
3	79, 87	-0.04
1 <sup>a</sup>	3, 5, 7, 9, 11, 13	-0.03 to -0.06

<sup>a</sup> "1" indicates the active power at the fundamental  $P_1$ , higher harmonic orders indicate the  $k_h$  coefficient.

Additional information may come from the analysis of X-Y maps, where one quantity is plotted against the other at a given frequency (see Fig. 6), resulting in a Lissajous (or Bowditch) curve. For voltage  $V_h$  vs. current  $I_h$  the plot can be interpreted as the dynamic impedance. If we consider  $Q_h$  vs.  $P_h$ , that are two integral quantities, the relevant feature is the angle (the harmonic displacement factor), shown in Fig. 7 in polar coordinates.



Fig. 6. V-I maps: voltage [Vrms] vs. current [Arms] for various components; cruising operating condition, 541 to 567 s.





By inspection of Fig. 6 almost linear plots for components between  $79f_1$  and  $87f_1$  are observed, indicating that the train is almost a constant passive load over that time interval, that allow to conclude that they are not generated by on-board systems. For the 3<sup>rd</sup> component (50 Hz), that we already recognized as an internally generated component, the plot is quite different with a more complex pattern spanning over a rectangular area. Components between 7th and 13th have only a roughly linear behavior, with a thick plot that might be caused by some variability. Components 39<sup>th</sup>, 43<sup>rd</sup>, 53<sup>rd</sup>, 55<sup>th</sup>, 57<sup>th</sup>, 119<sup>th</sup> and 121<sup>st</sup> have all shapes that suggest an on-board origin, although by coincidence a nearby train with variable operating condition might be the source reflected into our train replicating attenuated the changes of the V and I components as a passive load. Fig. 7 gives additional information: the  $11^{\text{th}}$  harmonic has a much more complex pattern, with reverse of power flow and capacitive/inductive behavior. The 119<sup>th</sup> and 121<sup>st</sup> components also have a reverse power flow.

#### V. CONCLUSIONS

The behavior of harmonic power terms has been considered for an example of a 16.7 Hz system, the Swiss railway network. Such terms could bring a significant amount of active power, comparable or larger than the uncertainty of the estimate required by CENELEC standards [1]. The assessment is complicated by various necessities: evaluating the amount of active power in absolute terms and as a fraction of the fundamental power, correlate the behavior of spectral power terms to the rolling stock operating conditions, verify common behavior as for components belonging to the same distortion pattern or emitted by the same source, in particular recognizing if they emitted by the train under test or if the latter is only a passive load for distortion caused by external sources.

Experimental data measured on a 25 minute run along one of the main Swiss lines were processed by means of Short Time Fourier Transform, calculating then the  $k_h$  index for active power terms, joint probability histograms, correlation coefficients and V-I maps, in order not only to quantify the amount of harmonic active power for the various spectral terms, but also to investigate the correlation with the train operating conditions. The difficulty is evident as soon as details of V-I or Q-P maps are inspected, with different behaviors for different time points along the same interval assigned to a supposedly unique operating condition, or for different time intervals with in principle "identical" operating conditions.

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