Neutronic Simulation of Fuel Assembly Vibrations in a Nuclear Reactor

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CORTEX
Core monitoring techniques and experimental validation and demonstration

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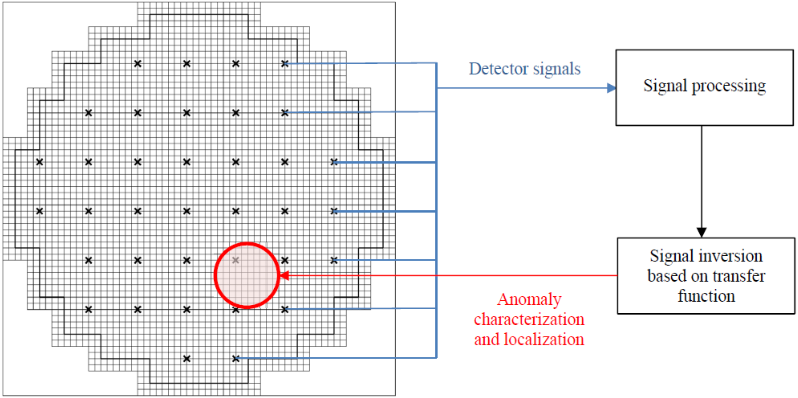
Contents

- This work is part of the European project **CORTEX**:

  Core Monitoring Techniques & Experimental Validation and Demonstration for Improved Safety.

- The strategic objectives are:
  - To develop **simulation tools** to model the stationary fluctuations effect in power reactors with a high level of fidelity.
  - Validation of the simulation tools using **reactor experiments** specifically designed for neutron noise analysis applications.
  - To develop **machine learning** methodologies for recovering the anomaly responsible for the observed fluctuations.
  - To combine the modelling capabilities and the signal analysis techniques into tools that can be directly used **for core diagnostics**.

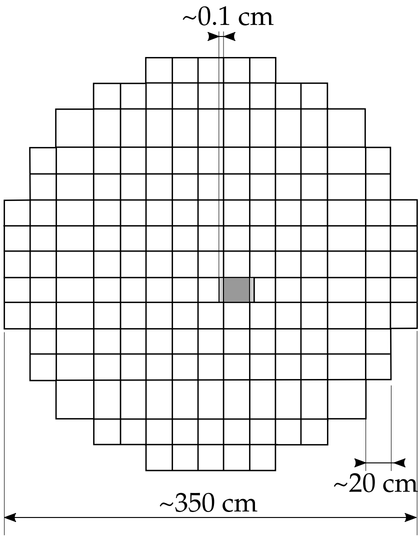
- Concept of CORTEX project.



- **Subtask 1.2.2:** Development of a new time-domain FEM kinetic code. It must solve the neutron distribution inside a nuclear reactor with **vibrating fuel assemblies**.
- The oscillation of an assembly has an amplitude between 0.3 and 5 mm and a frequency from 0.8 to 25 Hz. Best case estimates are computed for 1 mm amplitude and 1 Hz.

Spatial scales of the problem

- Different scales of the problem



- The neutron diffusion equation in the two energy groups approximation is considered

$$[v^{-1}]\frac{\partial \Phi}{\partial t} = -L\Phi + (1-\beta)M\phi + \sum_{p=1}^{N_p} \lambda_p \chi C_p$$

$$\frac{\partial C_p}{\partial t} = \beta_p \left(\nu\Sigma_{f1}, \quad \nu\Sigma_{f2}\right), \qquad p = 1, \ldots, N_p$$

where

$$L = \begin{pmatrix} -\vec{\nabla}(D_1\vec{\nabla}) + \Sigma_{a1} + \Sigma_{12} & 0 \\ -\Sigma_{12} & -\vec{\nabla}(D_2\vec{\nabla}) + \Sigma_{a2} \end{pmatrix}, \quad \Phi = \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix},$$

$$M = \begin{pmatrix} \nu\Sigma_{f1} & \nu\Sigma_{f2} \\ 0 & 0 \end{pmatrix}, \quad [v^{-1}] = \begin{pmatrix} \frac{1}{v_1} & 0 \\ 0 & \frac{1}{v_2} \end{pmatrix}, \quad \chi = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

Finite Element Method Discretization

- A high order finite element method for reactors with any type of geometry is developed.
- We obtain a discrete version of the neutron diffusion equation.

$$
\begin{aligned}
[\tilde{v}^{-1}]\frac{\partial \tilde{\Phi}}{\partial t} &= -\, L\tilde{\Phi} + (1-\beta)M\tilde{\Phi} + \sum_{p=1}^{N_p} \lambda_p X C_p, \\
P\frac{\partial C_p}{dt} &= \beta_p \left(M_{11}, \quad M_{12} \right) \tilde{\Phi}, \qquad p = 1, \ldots, N_p,
\end{aligned}
$$

Time dependent system

- The time dependent problem is reduced to solve the system

$$T^{n+1}\tilde{\Phi}^{n+1} = R^n\Phi^n + \sum_{p=1}^{N_p} \lambda_p e^{-\lambda_p \Delta t} X C_p^n,$$

where

$$T^{n+1} = \frac{1}{\Delta t}[\tilde{v}^{-1}] + L^{n+1} - \hat{a}M^{n+1},$$

$$R^n = \frac{1}{\Delta t}[\tilde{v}^{-1}],$$

$$\hat{a} = 1 - \beta + \sum_{p=1}^{N_p} \beta_p \left(1 - e^{-\lambda_p \Delta t}\right).$$

- The neutron precursors term is integrated with an **explicit scheme**:

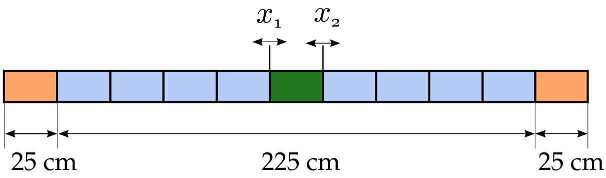
$$PC_p^{n+1} = PC_p^n e^{-\lambda_p \Delta t} + \frac{\beta_p}{\lambda_p}(1 - e^{-\lambda \Delta t}) \left(M_{11}^{(n+1)},\ \ M_{12}^{(n+1)}\right) \Phi^{n+1}$$

Time discretization

- This system of equations is large and sparse and has to be solved for each new time step.
- The BICGSTAB method with an incomplete LU preconditioner is chosen to solve this linear system.
- To improve the ILU preconditioner calculation an inverse Cuthill-Mckee reordering is used.

Numerical Results: 1D benchmark

- To test the numerical tools developed a simple one dimensional benchmark is defined.
- We consider an oscillation of 1 mm at 1 Hz.

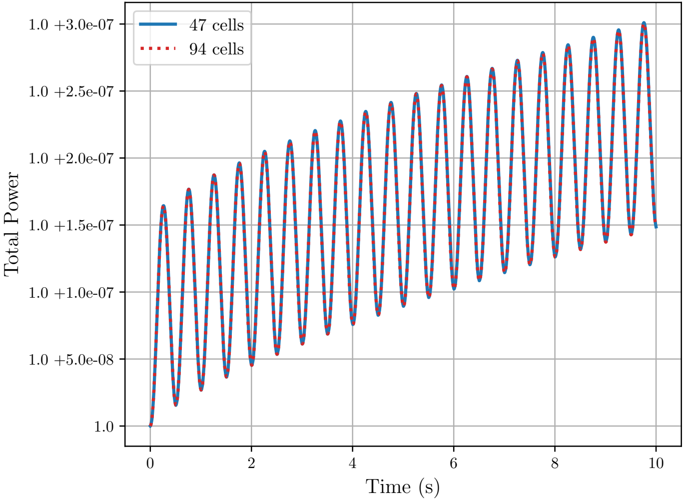


- The oscillation is treated as a movement of an homogeneous material inside the next assemblies.
- The oscillation is defined as

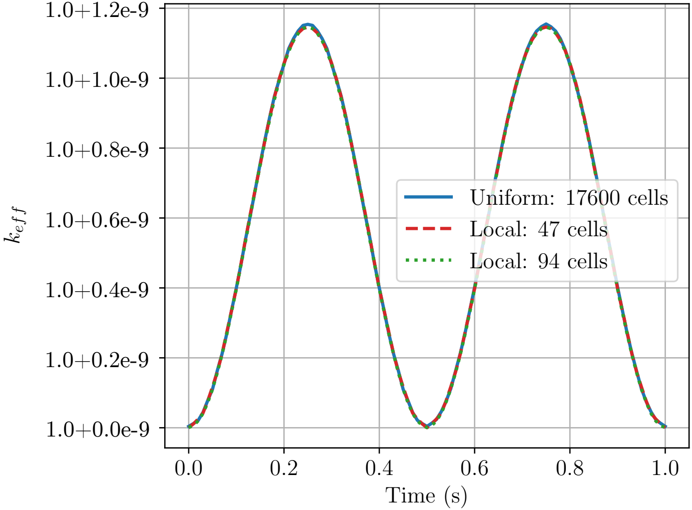
$$b_i(t) = b_{i0} + A\sin(\omega_p t)$$

Total neutron power

- Total neutron power along 10 periods.

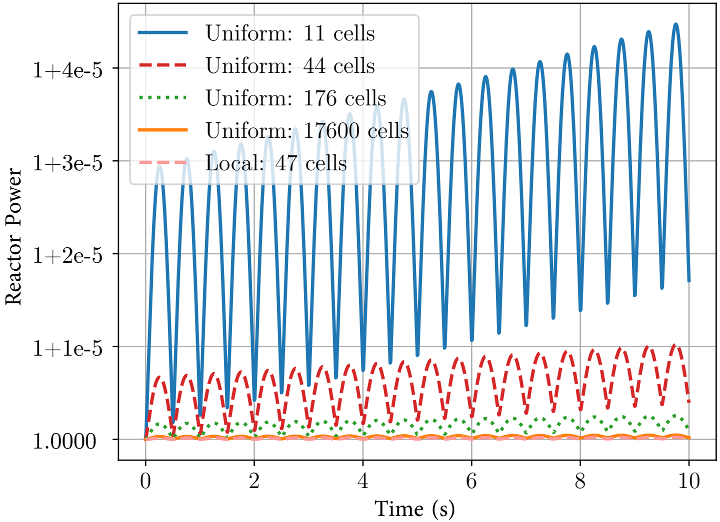


- Multiplicative factor along one period.



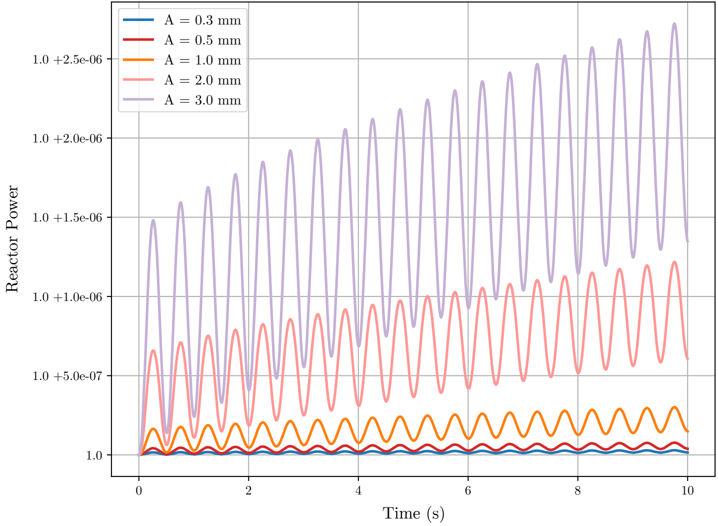
Wrong results

- A spatial discretization not fine enough leads to **wrong** results.

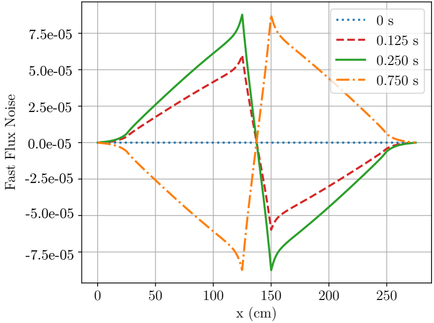


Total power evolution

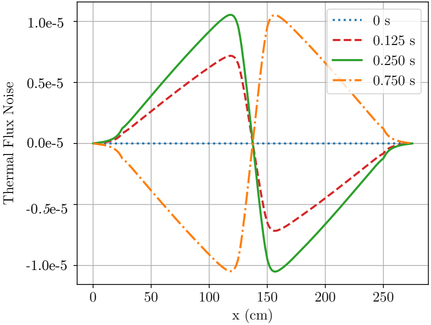
- Total neutron power evolution for different oscillation amplitudes.



- Spatial-dependence of neutron noise oscillations:



(a) Fast Flux



(b) Thermal Flux

Frequency-domain neutron noise equation

- To compare spatial results, the **noise theory** in the frequency domain is utilized. **CORE SIM code** is used in this way.

Demazière C. (2011), CORE SIM: A multi-purpose neutronic tool for research and education, *Annals of Nuclear Energy*, (38) 2698–2718.

- Time dependent parameters are split into mean values and fluctuations

$$\phi(x,\, t) = \phi_0(x) + \delta\phi(x,\, t),$$
$$\Sigma(x,\, t) = \Sigma_0(x) + \delta\Sigma(x,\, t).$$

- Fluctuations are assumed to be small compared to mean values:

$$\delta\phi(x, t) \ll \phi_0(x)$$

and stationary

$$\langle\delta\phi(x, t)\rangle = \phi_0(x)$$

- CORE SIM only allows 1 precursors group.

Frequency-domain neutron noise equation

- Performing a Fourier transform and neglecting second-order terms, the neutron noise diffusion equation reads as

$$\left(\vec{\nabla} D \vec{\nabla} + \Sigma_{\mathrm{dyn}}\right) \begin{pmatrix} \delta\hat{\phi}_1 \\ \delta\hat{\phi}_2 \end{pmatrix} = \phi_r \, \delta\Sigma_{12} + \phi_a \begin{pmatrix} \delta\Sigma_{a1} \\ \delta\Sigma_{a2} \end{pmatrix} + \phi_f \begin{pmatrix} \delta\nu\Sigma_{f1} \\ \delta\nu\Sigma_{f2} \end{pmatrix},$$

where

$$\Sigma_{\mathrm{dyn}} = \begin{pmatrix} -\Sigma_1 & \nu\Sigma_{f2}\left(1 - \frac{i\omega\beta_{\mathrm{eff}}}{i\omega+\lambda_{\mathrm{eff}}}\right) \\ \Sigma_{12} & \left(\Sigma_{a2} + \frac{i\omega}{\mathrm{v}_2}\right) \end{pmatrix}, \qquad \phi_r = \begin{pmatrix} \phi_1 \\ -\phi_1 \end{pmatrix},$$

$$\phi_a = \begin{pmatrix} \phi_1 & 0 \\ 0 & \phi_2 \end{pmatrix}, \qquad \phi_f = \begin{pmatrix} -\phi_1\left(1 - \frac{i\omega\beta_{\mathrm{eff}}}{i\omega+\lambda_{\mathrm{eff}}}\right) & -\phi_2\left(1 - \frac{i\omega\beta_{\mathrm{eff}}}{i\omega+\lambda_{\mathrm{eff}}}\right) \\ 0 & 0 \end{pmatrix}$$

- The Fourier transform of the temporal flux noise have been defined

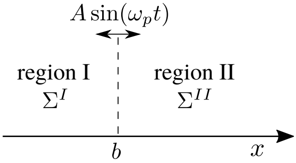
$$\delta\hat{\phi}(x,\, \omega) = \mathcal{F}[\delta\phi(x,\, t)]$$

- **First order noise theory** considers that the neutron noise is monochromatic against monochromatic cross sections perturbations.

Vibrating fuel assemblies model

- A vibrating assembly is described with two **in-phase moving interfaces**.

$$b(t) = b_0 + A\sin(\omega_p t)$$



- The cross section at the interface $x = b(t)$ are described as

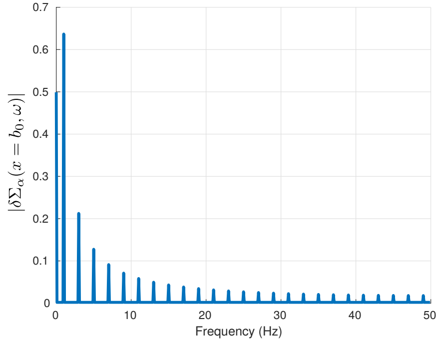
$$\Sigma_\alpha = (1 - \mathcal{H}(x - b(t)))\Sigma_\alpha^{\mathrm{I}} + \mathcal{H}(x - b(t))\Sigma_\alpha^{\mathrm{II}}$$

- Using one-term Taylor expansion:

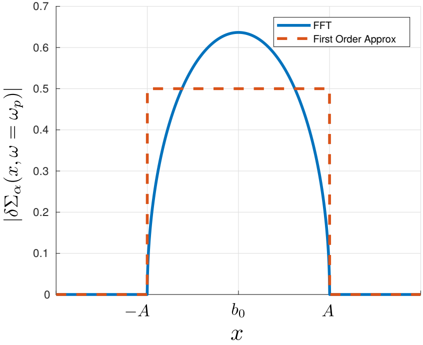
$$\delta\Sigma_\alpha(x,\omega) = -\frac{1}{2}j\left(\Sigma_\alpha^I - \Sigma_\alpha^{II}\right) \times \delta(x - b_0)\delta(\omega - \omega_p)$$

Vibrating fuel assemblies model

- Since in CORE SIM the perturbation is introduced node-wise, one could assume that perturbed region is $x \in [b_0 - DX,\ b_0 + DX]$ with a perturbation value of $\delta\Sigma_\alpha(x,\omega) = -\frac{1}{2} j \frac{A}{DX}\left(\Sigma_\alpha^{I} - \Sigma_\alpha^{II}\right)$.
- To check the validity of the first order approximation a numerical FFT of time domain cross section is performed.

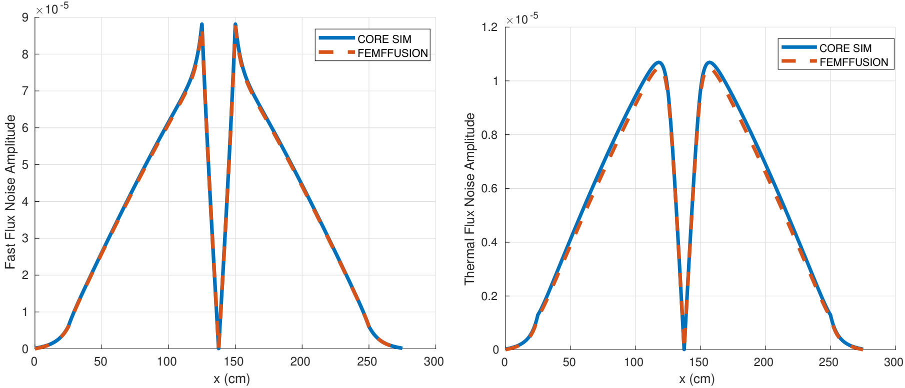


(a) Spectrum of the FFT at b_0



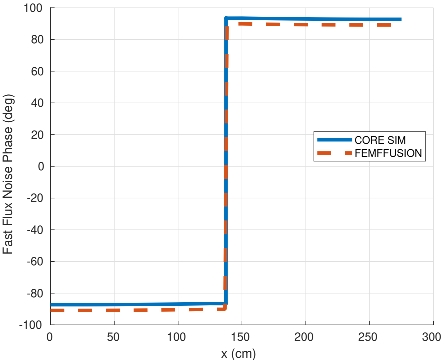
(b) Perturbation amplitude at ω_p

- Comparison of amplitudes

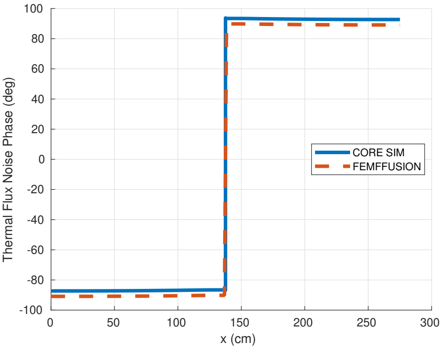


(a) Fast Flux (b) Thermal Flux

- Comparison of phase

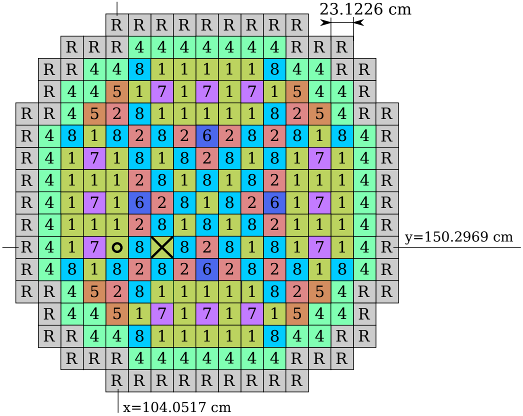


(a) Fast Flux



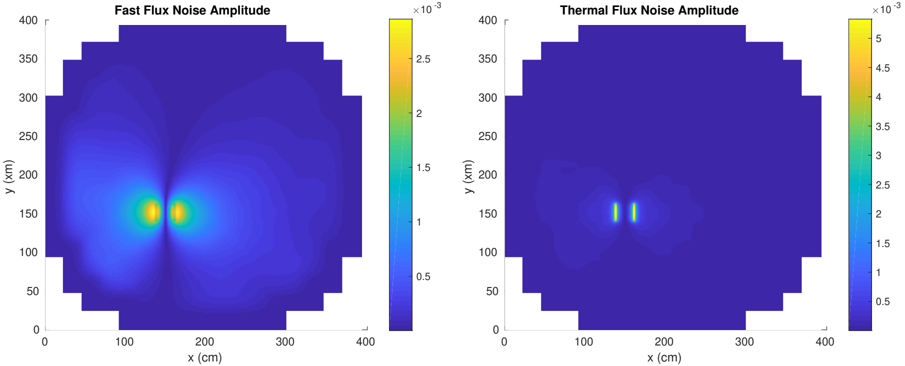
(b) Thermal Flux

- Now we discuss a 2D benchmark
- We use a classic 2D-BIBLIS benchmark, where assembly ($\times$) is selected to vibrate in x direction.



- Also, a detector ($\circ$) is assumed to be at $x = 104$ cm, $y = 150$ cm.

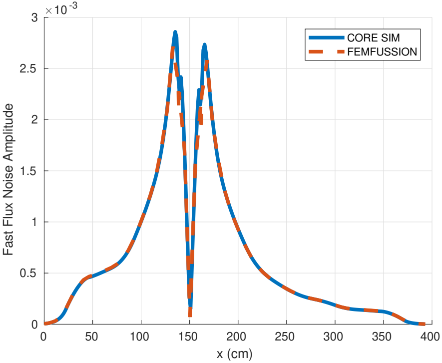
- Noise amplitude with *FEMFFUSION*



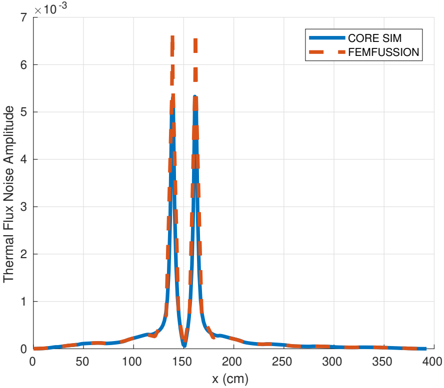
(a) Fast Flux

(b) Thermal Flux

- Comparison of amplitudes at $y = 150.2969$ cm.

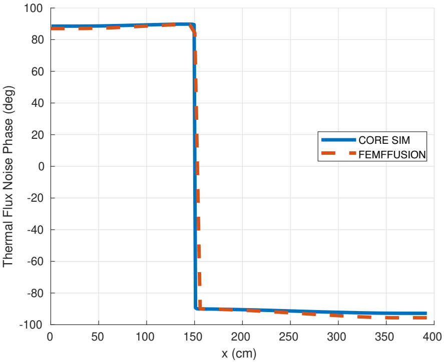


(a) Fast Flux

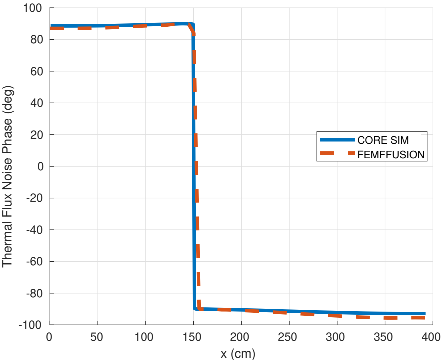


(b) Thermal Flux

- Comparison of phase at $y = 150.2969$ cm

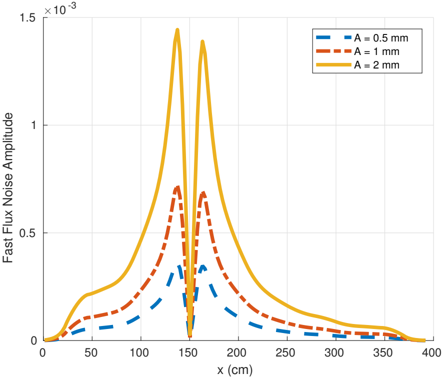


(a) Fast Flux

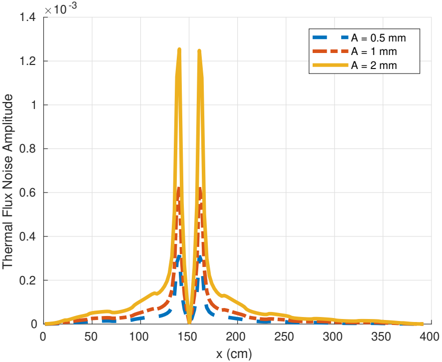


(b) Thermal Flux

- Neutron noise result for different **vibration amplitudes**



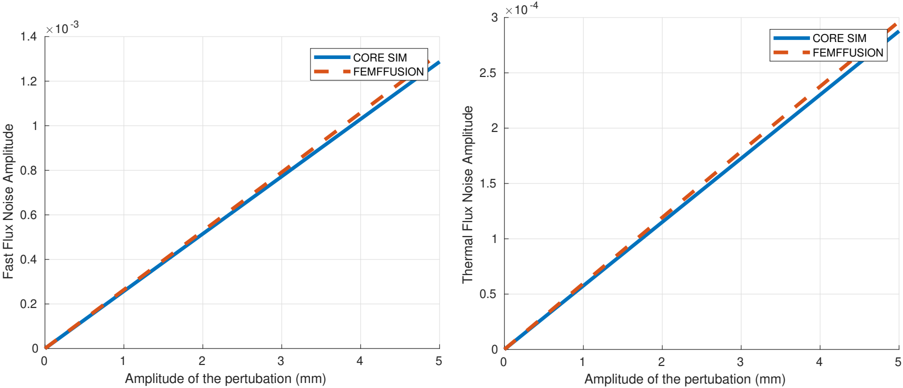
(a) Fast Flux



(b) Thermal Flux

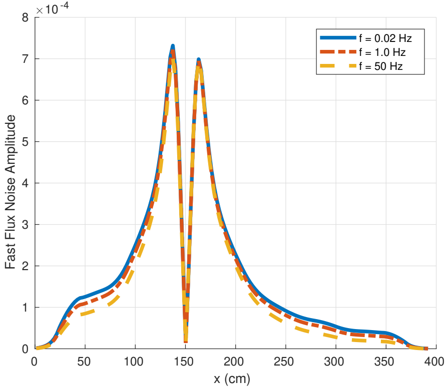
2D Vibration Amplitude Analysis

- Vibration amplitude and noise at the detector ($x = 104$, $y = 150$) cm

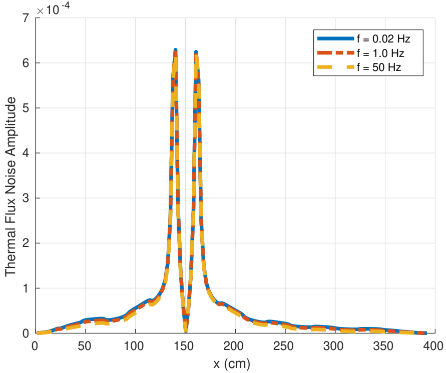


(a) Fast Flux (b) Thermal Flux

- Neutron noise result for different **vibration frequencies**

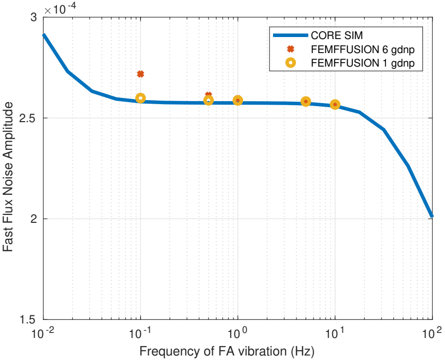


(a) Fast Flux

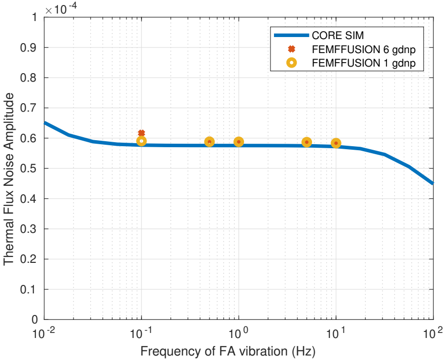


(b) Thermal Flux

- Vibration frequency and noise at the detector $(x = 104,\ y = 150)$ cm



(a) Fast Flux



(b) Thermal Flux

Conclusion

- The current research is an attempt to understand the coupling mechanism between the **mechanical vibration** of fuel assemblies and the generated **neutron noise**.
- The problem combines different spatial scales. This implies that we need to work with a very high precision.
- Numerical results show two different effects:
  - A slow variation of the power due to a change in the criticality of the system is observed. This effect is small and will be compensated by the thermal-hydraulic coupling.
  - A same frequency oscillation of the neutron flux as the FA vibration.
- The **time-domain solution** is compared with the **frequency-domain** solution obtaining a close match. The frequency domain analysis takes much less computational load than the time domain analysis.
- **Experimental results** in the framework of the CORTEX project are expected validate the simulations in on going works.

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under grant agreement No 754316. The content in this presentation reflects only the views of the authors.
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