Securing Visible Light Communications with Spatial Jamming

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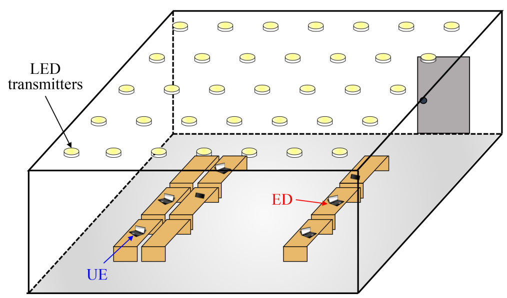


Fig. 1. An indoor VLC network consisting of multiple LED transmitters, one UE and one ED.

***Index Terms*—Physical layer security, visible light communication, jamming, stochastic geometry, secrecy rate.**

I. Introduction

Fig. 1 shows an example of an office installed with a VLC system, where multiple mobile VLC devices are present on fixed desks. Considering the typical behaviors of the office workers in which they mostly work sitting at the desks, it can be assumed that the LED transmitters know the probable and approximate locations of the mobile devices, i.e., near the desks, neither the aisle nor the rest area[1]. In other words, the LEDs transmitters are able to know the exact location and/or the CSI of the UE by utilizing a channel estimation method [1], and may have knowledge of the approximate locations of the ED. Besides, as in [2], it also can be assumed that the ED might be a registered user in the network, but, in a certain transmission session, the confidential message needs to be securely delivered from the LEDs to only the intended user; in this case, the LEDs can be assumed to know the approximate locations of the ED. Based on this assumption, in this paper, by utilizing the available information on the locations of the UE and the ED, we propose a spatial jamming strategy in which the LED jammers being located near to the ED emit random jamming signals to hinder the ED's reception of the information signal. Unlike the previously proposed jamming schemes as in [2]–[4], where all of the LEDs in the room need to participate in producing a beam-steering, in the spatial jamming strategy, only the adjacent LEDs needs to alternatively transmit a data or jamming signal, which would improve the spatial efficiency of VLC systems and permit a simpler implementation.

[1]Even in other VLC system environments, such as libraries, conference rooms, etc., the possible locations of VLC users also can be anticipated by analyzing the user behavior characteristics and the layout of the room.

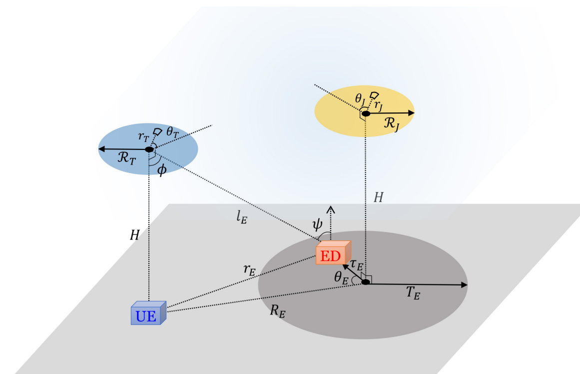


Fig. 2. Geometry of a continuous LED model with the spatial jamming scheme.

II. System Model

A. Room Configuration with the Continuous LEDs

Rather than considering the discrete LED transmitters as described in Fig. 1, in this paper, we consider a continuous LED model as described in Fig. 2, where the infinite number of LED transmitters are attached to the ceiling of the infinitely large room, and the distances among the LEDs are infinitesimal. As shown in [5], this continuum model significantly simplifies the calculation of the received optical power, while effectively approximating the real LED transmitters, i.e., the discrete model, since LED transmitters in practice are uniformly distributed to illuminate an entire room satisfying the brightness standards of the room from 400 to 1000 lux [6].

In this paper, we propose a spatial jamming strategy that utilizes the knowledge of the locations of the UE and ED. Since the channel gain in VLC systems largely depends on the distance between an LED and a PD of a receiver [7], we expect that the LEDs being located near to the UE act as information transmitters, while the LEDs being located near to the ED act as jammers. Therefore, in Fig. 2, assuming that one UE and one ED are present in the work plane (the gray plane), the LEDs transmitting data are denoted by the blue circular plane with radius $\mathcal{R}_T$, where the UE is located right below the circle. On the other hand, the LEDs emitting jamming signals are denoted by the yellow circular plane with radius $\mathcal{R}_J$, where the ED is located τ_E away from the point right below the yellow circular plane. This assumption can be justified since the information LED transmitters can be accurately selected according to the location of the UE, which is assumed to be known to the LEDs. In contrary, it is assumed that the exact location of the ED is not given to the LEDs, while only its approximate location is available; thus, selecting the jammers cannot be accurate. In addition, in this work, we assume that all of the LED transmitters can share the data and jamming signals to be transmitted by the wire cable and are capable of selectively transmitting either a data or jamming signal.

B. Received Optical Power Density Analysis

According to [7], the optical channel gain between an LED transmitter and a receiver in VLC systems can be described as

$$G = \begin{cases} \frac{(m+1)}{2\pi l^2} A_{\mathrm{RX}} \cos^m(\phi)\cos(\psi) & \text{for} \quad |\psi| \leq \Psi, \\ 0 & \text{for} \quad |\psi| > \Psi \end{cases} \tag{1}$$

where $m = -\ln(2)/\ln(\cos(\phi_{1/2}))$ is the order of *Lambertian* emission with half illuminance at $\phi_{1/2}$. l is the distance between the LED and the receiver. ϕ and ψ denote the angle of irradiance and the incidence between the transmitter and the receiver, respectively. Also, the receiver collection area is given by $A_{\mathrm{RX}} = \kappa^2 A_{\mathrm{PD}}/\sin^2(\Psi)$, where κ is the refractive index of the optical concentrator, A_{PD} is the physical area of the photodiode (PD), and Ψ is the received field of view of the PD. Moreover, as in [8], [9], if we assume that a receiver's PD faces up normal to the work plane, we can rewrite (1) as

$$G = \frac{(m+1)}{2\pi l^2} A_{\mathrm{RX}} \left(\frac{H}{l}\right)^m \left(\frac{H}{l}\right) = \varrho l^{-(m+3)} \tag{2}$$

where $\varrho = (m+1)A_{\mathrm{RX}}H^{(m+1)}/2\pi$. Note that (2) is valid only when $|\psi| \leq \Psi$ is satisfied.

In the continuum model, to deal with the infinite number of LEDs, we characterize the emitted optical power of LEDs by using the optical power density per unit of LED area P_T [W/m^2]. We assume that the information transmitter and the jammer emit signals with the same optical power density P_T. Utilizing the channel gain model provided above, firstly, the received optical power density of the data signal emitted by the information transmitters (i.e., the blue circular plane) P_D [W/m^2] at the point r_E away from the UE in the work plane can be described by

$$\begin{aligned} P_D(r_E) &= \int_0^{\mathcal{R}_T}\int_0^{2\pi} P_T \frac{(m+1)}{2\pi l^2}\left(\frac{H}{l}\right)^{(m+1)} r_T \,\mathrm{d}\theta_T \,\mathrm{d}r_T \\ &\overset{(a)}{=} \frac{P_T}{2}\left(1 + \frac{\mathcal{R}_T^2 - H^2 - r_E^2}{\sqrt{(H^2+r_E^2)^2 + 2\mathcal{R}_T^2(H^2 - r_E^2) + \mathcal{R}_T^4}}\right) \end{aligned} \tag{3}$$

where $l = \sqrt{r_T^2 + r_E^2 - 2r_T r_E \cos\theta_T + H^2}$ denotes the distance between the differential information transmitter and the ED. For (a) and the following analysis, we assume all of the LED transmitters have the *Lambertian* emission pattern with $\phi_{1/2} = 60^\circ$ $(m = 1)$. Also, note that $P_D(0)$ denotes the received optical power density of the data signal at the R_U site. Also, note r_E can be described as a function of τ_E and θ_E as $r_E = \sqrt{\tau_E^2 + R_E^2 - 2\tau_E R_E\cos\theta_E}$.

Secondly, the received optical power density of the jamming signal emitted by the jammers (i.e., the yellow circle) P_J [W/m^2] at the R_E site, which is τ_E away from the right below point of the yellow circular plane in the work plane, can be described by

$$\begin{aligned} P_J(\tau_E) &= \int_0^{\mathcal{R}_J}\int_0^{2\pi} P_T \frac{(m+1)}{2\pi l^2}\left(\frac{H}{l}\right)^{(m+1)} r_J \,\mathrm{d}\theta_J \,\mathrm{d}r_J \\ &= \frac{P_T}{2}\left(1 + \frac{\mathcal{R}_J^2 - H^2 - \tau_E^2}{\sqrt{(H^2+\tau_E^2)^2 + 2\mathcal{R}_J^2(H^2 - \tau_E^2) + \mathcal{R}_J^4}}\right) \end{aligned} \tag{4}$$

where $l = \sqrt{r_J^2 + \tau_E^2 - 2r_J\tau_E\cos\theta_J + H^2}$ denotes the distance between the differential jammer and R_E. Note that $P_J(R_E)$ denotes the received optical power density of the jamming signal at the R_U site.

C. Data and Jamming Transmission

The data signal $x(t) \in [-1,1]$ and the jamming signal $j(t)$ $[-1,1]$ in time slot t are generated from a certain real constellation, e.g., a DC-biased pulse amplitude modulation (PAM) scheme, and multiplied by a modulation index $\alpha \in [0,1]$ and a fixed bias current $I_{\mathrm{DC}} \in \mathbb{R}^+$, where $\mathbb{R}^+$ denotes the set of non-negative real-valued numbers. Note that $j(t)$ must be a random value to prevent R_E from cancelling the jamming component from the received signal. Thus, the modulated signal $s(t)$ can be described as $s(t) = \alpha I_{\mathrm{DC}}x(t)$ or $s(t) = \alpha I_{\mathrm{DC}}j(t)$. To maintain linear current-to-light conversion and avoid clipping distortion, the LED transmitter has an amplitude constraint on its input power, i.e., $s(t)$ is subject to the amplitude constraint $|s(t)| \leq \alpha I_{\mathrm{DC}}$. Therefore, the emitted optical power of each LED can be $P_{\mathrm{TX}}(t) = \eta(I_{\mathrm{DC}} + s(t))$, where η (W/V) is the current-to-light conversion efficiency[2]. Also, $\mathbb{E}[s(t)] = 0$ is assumed, the modulated signal does not affect illumination.

Utilzing the optical power densities expressions (3) and (4), the received signal voltage $y_k(t)$, where $k \in \{U, E\}$ denotes the index number of the UE and ED, respectively, can be described as

$$y_U(t) = \zeta_U P_D(0)x(t) + \zeta_U P_J(R_E)j(t) + n_U(t) \tag{5a}$$

$$y_E(t) = \zeta_E P_D(r_E)x(t) + \zeta_E P_J(\tau_E)j(t) + n_E(t) \tag{5b}$$

respectively, where $\zeta_k = \alpha A_{\mathrm{PD},k}\kappa_k^2 R_{\mathrm{rsp},k}T_k/\sin^2(\Psi_k)$. $R_{\mathrm{rsp},k}$ is the photodetector's responsivity and T_k is the transimpedance amplifier gain. Also, $n_k(t)$ signifies zero-mean additive white Gaussian noise (AWGN) with variance σ^2.

[2]The optical power density P_T for the continuum model can be linked to the emitted optical power density of each LED $P_{\mathrm{TX}}(t)$ with $P_T = \lambda_T\mathbb{E}[P_{\mathrm{TX}}(t)]$, where λ_T is the density of LED transmitters.

D. Performance Measures

For Gaussian VLC channels with amplitude constraints, we define the peak SINR, rather than the average, by assuming $x = 1$ and $j = 1$ since the channel capacity bounds of VLC systems are described as a function of the peak SINR [8], [10]. Therefore the peak SINRs at the UE and the ED can be written as

$$\gamma_U = \frac{\zeta_U^2 P_D^2(0)}{\zeta_U^2 P_J^2(R_E) + \sigma^2} \quad (6a)$$

$$\gamma_E = \frac{\zeta_E^2 P_D^2(r_E)}{\zeta_E^2 P_J^2(\tau_E) + \sigma^2} \quad (6b)$$

respectively. We use SINR to denote the peak SINR for the remainder of the paper.

The secrecy rate of the VLC channel is given by [11]

$$C_s = \max_{p_X} (\mathbb{I}(X;Y_U) - \mathbb{I}(X;Y_E)), \quad (7a)$$

$$\text{s.t.} \quad |x| \le 1 \quad (7b)$$

where p_X is the input distribution and $\mathbb{I}(\cdot;\cdot)$ denotes the mutual information. In VLC systems, since it is infeasible to derive a closed-form expression for the secrecy capacity due to the amplitude constraint [12], we provide an achievable secrecy rate expression for the proposed spatial jamming scheme. To simplify deriving a closed-form achievable secrecy rate expression, we assume that both the data signal x and the jamming signal j follow the truncated Gaussian distribution $\mathcal{N}_T(0, \sigma_T^2)$ defined over $[-1, 1]$, where $\sigma_T \in \mathbb{R}_+$, as in [4]. Its probability density function (PDF) is given by

$$f(x) = \frac{\phi\left(\frac{x}{\sigma_T}\right)}{\Phi\left(\frac{1}{\sigma_T}\right) - \Phi\left(\frac{-1}{\sigma_T}\right)} \quad (8)$$

where $\phi(\upsilon) = e^{-\upsilon^2/2}/\sqrt{2\pi}$ and $\Phi(\omega) = (1+\text{erf}(\omega/\sqrt{2}))/2$. The error function $\text{erf}(\cdot)$ is defined as $\text{erf}(\omega) = 1/\sqrt{\pi}\int_{-\omega}^{\omega} e^{-t^2}\,\text{d}t$. Note that the optimal input distribution under the amplitude constraint in VLC systems is not readily available, and the truncated Gaussian distribution was shown to outperform the uniform distribution in the terms of secrecy rates in VLC systems [4]. With this assumption, we present the following lemma, which provides an analytic achievable secrecy rate expression for the system in question.

Lemma 1. *An achievable secrecy rate for the Gaussian wiretap channel in (6) with the spatial jamming scheme can be obtained by lower-bounding the secrecy capacity in (7) to give*

$$R_s = \max\left\{\frac{1}{2}\log\left(\frac{e^{2\eta}\left(P_D^2(0) + P_J^2(r_E)\right) + C}{\varphi P_J^2(r_E) + C}\right) - \frac{1}{2}\log\left(\frac{\varphi\left(P_D^2(r_E) + P_J^2(\tau_E)\right)}{e^{2\eta} P_J^2(\tau_E)}\right), 0\right\} \quad (9)$$

where

$$\eta = \log(Z) + \frac{-\frac{1}{\sigma_T}\phi\left(\frac{-1}{\sigma_T}\right) - \frac{1}{\sigma_T}\phi\left(\frac{1}{\sigma_T}\right)}{2Z},$$

$$\varphi = 1 + \frac{\frac{-1}{\sigma_T}\phi\left(\frac{-1}{\sigma_T}\right) - \frac{1}{\sigma_T}\phi\left(\frac{1}{\sigma_T}\right)}{Z} - \left(\frac{\phi\left(\frac{-1}{\sigma_T}\right) - \phi\left(\frac{1}{\sigma_T}\right)}{Z}\right)^2,$$

$$Z = \Phi\left(\frac{1}{\sigma_T}\right) - \Phi\left(\frac{-1}{\sigma_T}\right),$$

$$C = \frac{\sigma^2}{\zeta_U^2 \sigma_T^2}.$$

Proof. See Appendix A. □

III. Spatial Jamming

In this section, we investigate the optimization problems with the spatial jamming strategy based on the SINR and the secrecy rate. In the first subsection, we analyze the optimization problem with assuming the LEDs know the exact locations of both the UE and the ED[3]. On the other hand, in the second subsection, we consider a scenario that the only the approximate or probable location of the ED, which is obtained from investigating the layout of the room and the typical behavior of the workers, is known to the LEDs.

A. One legitimate user and one known eavesdropper

In this subsection, we assume that the LEDs know the exact locations of the UE and the ED. In other words, it is assumed that τ_E and θ_E are given to the LEDs.

1) Optimization based on SINR: A natural objective of the optimization problem based on the SINR can be to maximize the SINR of the UE (6a) subject to a constraint on the SINR of the ED (6b). Given the exact locations of the UE and the ED, the optimization problem of $\mathcal{R}_T$ and $\mathcal{R}_J$ for the spatial jamming can be formulated as

$$\gamma_U^\star = \max_{\mathcal{R}_T, \mathcal{R}_J} \frac{\zeta_U^2 P_D^2(0)}{\zeta_U^2 P_J^2(R_E) + \sigma^2} \quad (10)$$
$$\text{s.t.} \begin{cases} \dfrac{\zeta_E^2 P_D^2(r_E)}{\zeta_E^2 P_J^2(\tau_E) + \sigma^2} < \rho_E \\ \mathcal{R}_T + \mathcal{R}_J \le R_E \end{cases}$$

where ρ_E is the target constraint on γ_E.

This optimization problem is a non-convex problem. However, the fact that the problem consists of two optimization variables $\mathcal{R}_T$ and $\mathcal{R}_J$ alleviates the difficulty in finding the optimal solutions. In practice, finding the optimal solutions via brute-force search of the three parameters can be executed in a second on a standard PC using MATLAB.

2) Optimization based on Secrecy Rate: From (9), we formulate the optimization problem of $\mathcal{R}_T$ and $\mathcal{R}_J$ maximizing the secrecy rate under the spatial jamming strategy as

$$R_s^\star = \max_{\mathcal{R}_T, \mathcal{R}_J} R_s \quad (11)$$
$$\text{s.t.} \ \mathcal{R}_T + \mathcal{R}_J \le R_E.$$

This optimization problem is also non-convex, however, finding the solution of the optimization problem with only two optimization variables is straightforward similarly to (10).

[3]This assumption may not be limited in practice, but it is still worthwhile to be investigated to see how the distance between the center of the jamming circle and the ED, i.e., τ_E would affect the performance of the spatial jamming.

B. One legitimate user and one random eavesdropper

In this subsection, we assume that the LEDs know the exact location of the UE and the approximate or probable location of the ED, that is, the joint probability density function (PDF) of (τ_E, θ_E) is given to the LEDs.

1) Optimization based on SINR: Without knowledge of the exact location of the ED, a natural objective is to maximize the SINR of the UE, subject to a constraint on the *average* SINR of the ED. Assuming that an ED is randomly located in a circle with radius T_E, i.e., the dark gray circle in Fig. 2, whose center point is identical to that of the yellow circle, the average SINR of an ED can be described as

$$\begin{aligned}\overline{\gamma}_E &= \mathbb{E}_{\tau_E,\theta_E}\left[\gamma_E\right] \\ &= \int_0^{T_E}\int_0^{2\pi} f_{\tau_E,\theta_E}(\tau,\theta)\frac{\zeta_E^2 P_D^2(r_E)}{\zeta_E^2 P_J^2(\tau_E)+\sigma^2}\tau\,\mathrm{d}\theta\,\mathrm{d}\tau \end{aligned} \tag{12}$$

where $f_{\tau_E,\theta_E}(\tau,\theta)$ is the joint PDF of (τ_E,θ_E) and $r_E = \sqrt{\tau^2 + R_E^2 - 2\tau R_E\cos\theta}$. For example, $f_{\tau_E,\theta_E}(\tau,\theta)$ for the uniform distribution and the bivariate normal distribution of the ED location can be provided as

$$f^{\text{unif.}}_{\tau_E,\theta_E}(\tau,\theta) = \frac{1}{\pi T_E^2} \quad \text{for} \left\{\begin{array}{l} 0 \le \tau < T_E \\ 0 \le \theta < 2\pi \end{array}\right., \tag{13a}$$

$$f^{\text{norm.}}_{\tau_E,\theta_E}(\tau,\theta) = \frac{1}{2\pi\sigma_E^2}\exp\left(\frac{-\tau^2}{2\sigma_E^2}\right) \quad \text{for} \left\{\begin{array}{l} 0 \le \tau < \infty \\ 0 \le \theta < 2\pi \end{array}\right. \tag{13b}$$

respectively, where σ_E^2 in (13b) is the variance of the ED location in a Cartesian coordinate system, i.e., $x = \tau\cos\theta$ and $y = \tau\sin\theta$ with $X \sim \mathcal{N}(0,\sigma_E)$, $Y \sim \mathcal{N}(0,\sigma_E)$, and their correlation is assumed to be as $\mathrm{cor}(X,Y) = 0$, respectively.

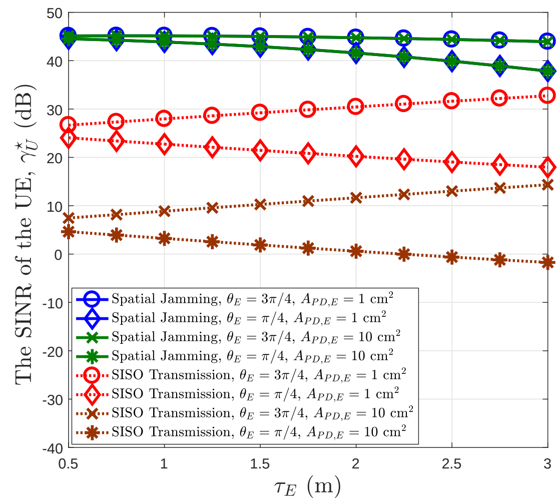
Utilizing this average SINR of the ED, we formulate the optimization problem as

$$\begin{aligned}\gamma_U^* &= \max_{\mathcal{R}_T,\mathcal{R}_J} \frac{\zeta_U^2 P_D^2(0)}{\zeta_U^2 P_J^2(R_E)+\sigma^2} \\ &\text{s.t.} \left\{\begin{array}{l}\overline{\gamma}_E < \overline{\rho}_E \\ \mathcal{R}_T + \mathcal{R}_J \le R_E\end{array}\right.\end{aligned} \tag{14}$$

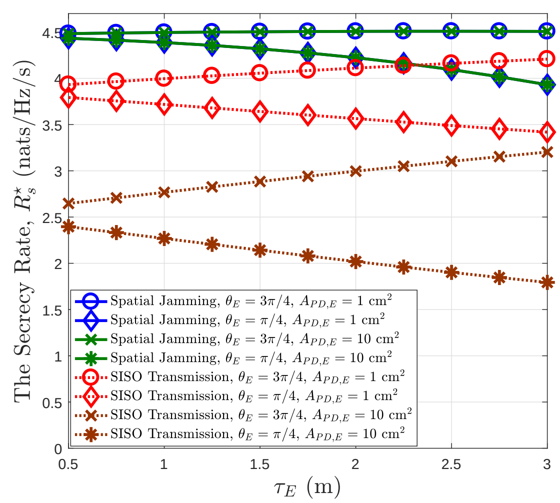
where $\overline{\rho}_E$ is the target constraint on $\overline{\gamma}_E$. Note that the optimization problem (14) is also a non-convex problem, and $\overline{\gamma}_E$ in the first constraint includes the integration to be solved numerically. However, the facts that the number of the optimization variables is only two and $\overline{\gamma}_E$ includes only two-dimensional integral enables the problem to obtain its optimal solution numerically. In practice, finding the optimal solutions via brute-force search of the three parameters can be executed in a few seconds on a standard PC using MATLAB.

2) Optimization based on Secrecy Rate: Similarly, the *average* secrecy rate can be calculated by numerically evaluating

$$\begin{aligned}\overline{R}_s &= \mathbb{E}_{\tau_E,\theta_E}\left[R_s\right] = \int_0^{T_E}\int_0^{2\pi} f_{\tau_E,\theta_E}(\tau,\theta) R_s(\tau,\theta)\,\mathrm{d}\theta\,\mathrm{d}r \\ &= \int_0^{T_E}\int_0^{2\pi} f_{\tau_E,\theta_E}(\tau,\theta)\max\left\{\frac{1}{2}\log\left(\frac{e^{2\eta}\left(P_D^2(0)+P_J^2(r_E)\right)+C}{\varphi P_J^2(r_E)+C}\right)\right. \\ &\quad \left. -\frac{1}{2}\log\left(\frac{\varphi\left(P_D^2(r_E)+P_J^2(\tau_E)\right)}{e^{2\eta}P_J^2(\tau_E)}\right), 0\right\}\mathrm{d}\theta\,\mathrm{d}r.\end{aligned} \tag{15}$$



(a) The SINR of the UE $\gamma_U^\star$ (10).



(b) The secrecy rate $R_s^\star$ (11).

Fig. 3. The optimized SINR of the UE and the secrecy rate for different locations of the ED, i.e., the knowledge of (τ_E, θ_E) is given to the LED transmitters. The result for the SISO transmission is given as a benchmark. $A_{PD,U} = 1\text{cm}^2$, $R_E = 8\text{m}^2$ and $\sigma_T = 0.62$ are used.

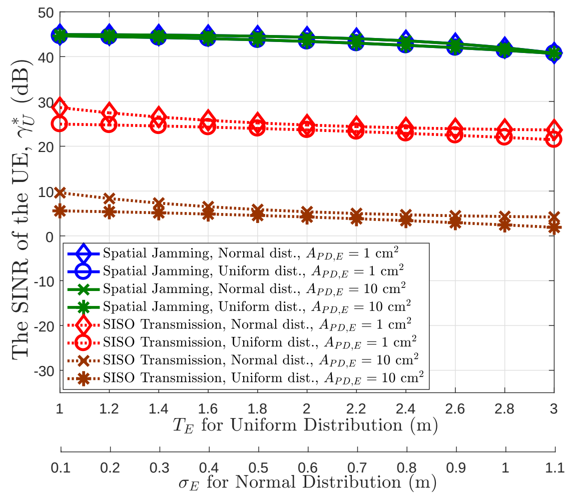
Utilizing (15), the optimization problem maximizing the *average* secrecy rate can be formulated as

$$\begin{aligned}\overline{R}_s^* &= \max_{\mathcal{R}_T,\mathcal{R}_J} \overline{R}_s \\ &\text{s.t.}\ \mathcal{R}_T + \mathcal{R}_J \le R_E.\end{aligned} \tag{16}$$

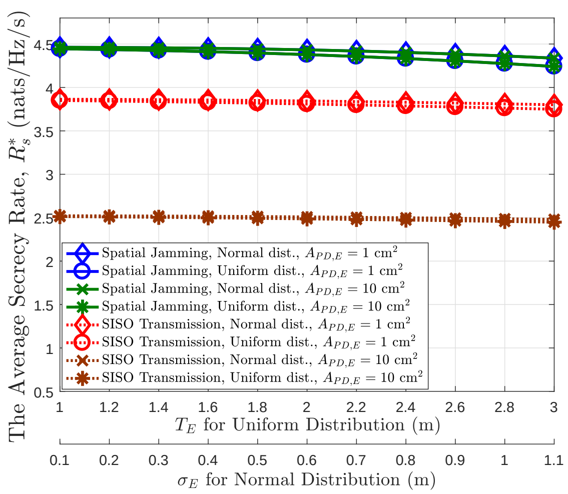
This optimization problem is also non-convex, however, due to the similar reasons related (14), finding the optimal solution is straightforward such that it can be executed in a few seconds.

IV. Numerical Result

In this section, numerical results are given to validate the performance of the proposed spatial jamming scheme.



(a) The SINR of the UE γ_U^* (14).



(b) The average secrecy rate $\overline{R}_s^*$ (16).

Fig. 4. The optimized SINR of the UE and the secrecy rate for different distribution of the ED. The uniform distribution in (13a) and the bivariate normal distribution in (13b) are used. The result for the SISO transmission is given as a benchmark. $A_{PD,U} = 1\text{cm}^2$, $R_E = 8\text{m}^2$ and $\sigma_T = 0.62$ are used.

Figs. 3(a) and (b) show the optimized SINR of the UE and the optimized secrecy rate obtained from solving (10) and (11), respectively, when the LEDs retain the knowledge of the exact locations of both the UE and the ED. The result for the SISO transmission (without jamming) [8] is given as a benchmark. When the ED moves away from the UE as well as the center of the jamming circle, i.e., when τ_E with $\theta_E = 3\pi/4$ increases, the SINR of the UE and the secrecy rate remain almost unchanged. This result comes from the fact that although the jamming signal at the ED site decreases as τ_E increases, the amount of the received data signal of the ED also decreases as r_E increases. In contrary, when the ED moves closer to the UE while moving away from the center of the jamming circle, i.e., when τ_E with $\theta_E = \pi/4$ increases, it is noted that both the SINR of the UE and the secrecy rate slightly decrease. Also, compared to the benchmark scheme, it is shown that the spatial jamming scheme outperforms the SISO transmission over the whole region of τ_E with different θ_E. Moreover, the gap between these two schemes become more significant when the physical area of the PD for the ED ($A_{\text{PD},E} = 10\text{cm}^2$) is set much larger than that of the UE ($A_{\text{PD},E} = 1\text{cm}^2$). This is because the ED can receive more data signal with using a larger PD under the SISO transmission, however, under the spatial jamming scheme, the ED receives more of the jamming signals as well as the information signals through the larger PD. As far as the jamming signal is random, the ED cannot extract only the information component from the received signal.

Figs. 4(a) and (b) show the optimized SINR of the UE and the optimized average secrecy rate obtained from solving (14) and (16), respectively, when the exact UE location and the statistical information about ED location are available. The two joint PDFs of (τ_E, θ_E) for the uniform and bivariate distributions in (13) are used. Similarly, the SISO transmission is given as a benchmark. In Figs. 4(a) and (b), it is shown that the SINR of the UE and the average secrecy rate with the spatial jamming scheme very slightly decreases as T_E and σ_E increase (i.e., the ED is more likely to be located far away from the center of the jamming circle). This result validates that even when the LEDs do not know the exact location of the ED, the proposed spatial jamming scheme effectively suppress the information reception of the ED, which improves the SINR of the UE and the average secrecy rate.

Appendix A
Derivation of the Secrecy Rate with the Spatial Jamming

A lower bound on the secrecy rate of (7) can be obtained as follows

$$\begin{aligned} C_s &= \max_{p_X, p_J} \left(\mathbb{I}(X;Y_U) - \mathbb{I}(X;Y_E)\right) \\ &\overset{(a)}{\geq} \mathbb{I}(X;Y_U) - \mathbb{I}(X;Y_E) \\ &\overset{(b)}{\geq} \mathbb{I}(X;Y_U) - \mathbb{I}(X;V_E) \\ &= \mathbb{h}(Y_U) - \mathbb{h}(Y_U|X) - \mathbb{h}(V_E) + \mathbb{h}(V_E|X) \end{aligned} \tag{17}$$

where $\mathbb{h}(\cdot)$ denotes differential entropy and $V_E = \zeta_E P_D(r_E)X + \zeta_E P_J(\tau_E)J$. (a) follows from dropping the maximization by choosing a truncated Gaussian distribution on p_X and p_J, and (b) follows from the data-processing inequality, i.e., $Y_E = g(V_E) = V_E + N_E$. Firstly, we lower-bound $\mathbb{h}(Y_U)$ by using the entropy-power inequality as

$$\begin{aligned} \mathbb{h}(Y_U) &\geq \frac{1}{2}\log\left(e^{2\mathbb{h}(\zeta_U P_D(0)X)} + e^{2\mathbb{h}(\zeta_U P_J(R_E)J)} + e^{2\mathbb{h}(N_U)}\right) \\ &= \frac{1}{2}\log\left(2\pi e\left(\sigma_T^2 e^{2\eta}\zeta_U^2\left(P_D^2(0) + P_J^2(R_E)\right) + \sigma^2\right)\right) \end{aligned} \tag{18}$$

where (18) follows from the facts that

$$\mathbb{h}\left(\zeta_U P_D(0)X\right) = \log\left(\zeta_U P_D(0)\right) + \frac{1}{2}\log\left(2\pi e\sigma_T^2\right) + \eta,$$

$$\mathbb{h}\left(\zeta_U P_J(R_E) J\right) = \log\left(\zeta_U P_J(R_E)\right) + \frac{1}{2}\log\left(2\pi e \sigma_T^2\right) + \eta,$$

$$\mathbb{h}\left(N_U\right) = \frac{1}{2}\log 2\pi e \sigma^2.$$

Then, we upper-bound $\mathbb{h}(Y_U|X)$ and $\mathbb{h}(V_E)$ as

$$\begin{aligned}\mathbb{h}\left(Y_U|X\right) &= \mathbb{h}\left(Y_U - \zeta_U P_D(0) X|X\right) = \mathbb{h}\left(\zeta_U P_J(R_E) J + N_U\right) \\ &\leq \frac{1}{2}\log 2\pi e\left(\sigma_T^2 \zeta_U^2 \varphi P_J^2(R_E) + \sigma^2\right) \end{aligned} \tag{20}$$

$$\begin{aligned}\mathbb{h}\left(V_E\right) &= \mathbb{h}\left(\zeta_E P_D(r_E) X + \zeta_E P_J(\tau_E) J\right) \\ &\leq \frac{1}{2}\log 2\pi e\left(\sigma_T^2 \zeta_E^2 \varphi\left(P_D^2(r_E) + P_J^2(\tau_E)\right)\right) \end{aligned} \tag{21}$$

by using the differential entropy of Gaussian random variables with variances $\mathbb{var}\{\zeta_U P_J(R_E) J + N_U\}$ and $\mathbb{var}\{\zeta_E P_D(r_E) X + \zeta_E P_J(\tau_E) J\}$, respectively. Lastly, we have

$$\begin{aligned}\mathbb{h}\left(V_E|X\right) &= \mathbb{h}\left(V_E - \zeta_E P_D(r_E) X|X\right) = \mathbb{h}\left(\zeta_E P_J(\tau_E) J\right) \\ &= \frac{1}{2}\log\left(2\pi e \sigma_T^2 \zeta_E^2 P_J^2(\tau_E)\right) + \eta. \end{aligned} \tag{22}$$

Plugging (18), (20), (21) and (22) into (17) yields the secrecy rate for the spatial jamming technique in (10).

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