OPPORTUNITIES FOR YEAR-ONE CHILDREN TO ACQUIRE FOUNDATIONAL NUMBER SENSE: COMPARING ENGLISH AND SWEDISH ADAPTATIONS OF THE SAME SINGAPORE TEXTBOOK

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We compare adaptations of a Singaporean year-one mathematics textbook for use in England and Sweden respectively. The texts were analysed in two different ways against the eight dimensions of Foundational Number Sense (FoNS), a set of core competences that the literature has shown to be necessary for year-one children's later mathematical learning. The first analysis, based on frequencies, showed that neither adaptation incorporated any opportunities for children to acquire the two FoNS competence relating to estimation and number patterns respectively. They also showed that the English adaptation comprised significantly more tasks than the Swedish, particularly with respect to systematic counting, where the former comprised 26% more tasks than the latter. The second analysis, based on moving averages, showed that across five of the six FoNS categories for which there were data, the temporal location and emphases of FoNS-related learning were comparable, with, in particular, no such opportunities after the mid-point of the school year in either book. However, the English adaptation's presentation of systematic counting, occurring at various points throughout the school year, was substantially different from the Swedish adaptation, highlighting differences due, we speculate, to interpretations of local didactical traditions.

INTRODUCTION

For many teachers of mathematics, irrespective of where they work, the textbook they use is not only the major resource for lesson planning and the provision of tasks for students but also the means by which the curriculum within which they work is realised and the determinant of what students learn (Tarr, Cháves, R. Reys & B. Reys, 2006). That said, the analysis of textbooks is probabilistic in the sense that teachers make decisions as to how they use any book, leaving the analytical question "what would students learn if their mathematics classes were to cover all the textbook sections in the order given? What would students learn if they had to solve all the exercises in the textbook?" (Mesa, 2004, pp. 255–256). Moreover, in those cultures in which textbooks are unregulated, typically leading to a plethora of choice for teachers, students may receive very different opportunities to learn (Huntley & Terrell, 2014; Tarr et al., 2006). Thus, the reasons for analysing textbooks are varied and include, acknowledging the huge sums of money spent on producing and purchasing them, concerns about value for money (Harel & Wilson, 2011) and their being fit for purpose (Huntley & Terrell, 2014; Tarr et al., 2006). More recently, in part motivating this paper, research has been driven by scholars' desires to better understand the functioning of educational systems more successful than their own (Ding, 2016; Li, Chen & An, 2009; Yang, R. Reys & Wu, 2010). That said, again part motivating this study, while some "effort has been put into content analysis and exploring the ways in which textbooks are used in classrooms… very few mathematics education researchers have taken a really close look at what is in the textbooks, with the focus on how the material is presented and what kind of learning may be

implied" (Kajander & Lovric, 2009, p.174). Moreover, while textbook analysis is an increasingly popular undertaking, studies focusing on year-one children are rare. In England, these children are aged 5 and in Sweden 7.

In this paper we compare two adaptations of a popular Singaporean mathematics textbook written for year-one children. These are English adaptation, Maths – No Problem (hereafter MNP), and the Swedish adaptation, Singma. The analyses are framed theoretically by the lens of foundational number sense (FoNS), a set of eight number-related competences, based solely in the integer range 0-20), that research has shown to underpin year-one children's later mathematical learning (Andrews & Sayers, 2015). Acknowledging that all humans (and many other species) are born with number-related insights concerning quantity discrimination (Lipton & Spelke 2005) and that curricula typically expect students to develop the number sense "required by all adults regardless of their occupation" (McIntosh, B. Reys & R. Reys, 1992, p. 3), FoNS, which requires instruction, is intended to provide the foundations of the bridge between the two. The initial aim of the project team, by means of a systematic review of the literature, was to identify a set of curriculum independent competences that would be simple to operationalise in different cultural contexts. Moreover, its origins in the international literature makes the FoNS framework an appropriate tool for comparing textbooks and their presentation of key number-related competences.

Earlier FoNS-related analyses have compared the English version of the Singaporean textbook, MNP, with other texts used in England (Petersson, Sayers, Rosenqvist & Andrews, under review) and the Swedish version, Singma, with other texts used in Sweden (Sayers, Petersson, Rosenqvist & Andrews, under review). The results of these studies have highlighted the extent to which the Singapore import differs in its emphases from books authored by English and Swedish colleagues respectively. Indeed, both analyses allude to the problematic nature of textbook importation and the didactical challenges teachers must face in order to use them successfully. Moreover, since the production of textbooks is unregulated in both England and Sweden, there is no official expectation that textbooks should explicitly address the particular expectations of the two countries' curricula. Thus, assuming that an importer would wish to retain the integrity of the original work, it would seem reasonable to expect the two adaptations to match each other closely. This paper, therefore, is framed by the following question: How are FoNS-related learning opportunities manifested in the two independent translations of the same textbook? Each adaptation is subjected to two analyses, each drawing on different forms of task distribution. In so doing, we acknowledge Rezat's (2006, p. 482) position that a mathematics textbook "is historically developed, culturally formed, produced for certain ends and used with particular intentions". That is, despite English publishers' expectations that purchasers of their Singapore adaptations should attend induction courses, any textbook is clearly a product of the culture and curriculum in which it was written with no obvious guarantee that it would function adequately in another context.

METHODS

Two adaptations of the same Singaporean textbook, one from England (MNP) and one from Sweden (Singma), were identified for analytical purposes. With respect to both adaptations, all materials intended for the use of year-one children were coded, each by at least two

members of the project team, for FoNS-related learning opportunities. In this way, each task was coded as a series of 1s and 0s, according to the presence or absence of the eight FoNS categories. Throughout, the focus of the analyses was solely on tasks that expected action on the part of the student. Thus, explanatory worked examples were included but all tasks in teacher guides were excluded. Other studies have counted the number of pages devoted to the content under scrutiny, arguing that since "pages consisting of tasks for the students to solve contain many similar tasks… the result of counting the number of tasks… would probably not differ much from the result obtained by counting whole pages" (Bråting, Madej & Hemmi, 2019). Our view is that because textbooks differ greatly in the ways in which mathematics is presented, some comprising very dense pages and others not (Haggarty & Pepin, 2002), counting tasks is more likely to yield an accurate representation of the opportunities given to children, particularly when we are comparing adaptations of the same book.

In addition to simple frequency analyses, whereby each occurrence of each category was counted, a moving average was calculated for each code as it occurred in each book. This approach is typically used to analyse trends in, for example, temperature over time, while eliminating any undue influence of outliers (Fan & Yao, 2003, p. 9). In similar vein, the use of moving averages with textbooks, whereby data are successive tasks, should offer a clear indication of a textbook's sequential emphases. In this way, single data points are replaced by the arithmetical mean of a sequence of data points, drawn from before, including, and after the point in question. This process smooths out short-term fluctuations in time series so that longer-term patterns become more visible and the influence of outliers is eliminated. Mathematically, a moving average means substituting a single data point (t_k, y_k) with $(t_k, \hat{y}_k)$, where $\hat{y}_k$ is the arithmetic mean of its neighbouring data points y_j as in equation 1. Importantly, if the time period selected for the moving average is too short, then its associated graph becomes noisy and trends may be lost. Similarly, if the time period is too long then important details may be lost (Wakaura & Ogata, 2007). Thus, the choice of time interval is key to the successful use of the approach.

$$\hat{y}_k = \sum_{j=k-n}^{k+n} \frac{1}{2n+1} y_j$$

Equation 1

Of particular interest to the analyst is the size of the divisor, $2n + 1$, which represents the total number of data points included in the calculation and is dependent on the time period chosen for the calculation. That is, $2n + 1$ refers to the original point, y_k, and its $2n$ neighbouring data points, *n* before and *n* after. In the context of a mathematics textbook, the width $2n + 1$ of this window could be the number of tasks that an average student is expected to cover each day, or each week or each month and this choice depends on the research question. Thus, one moving average window could be $\frac{all\ items\ in\ a\ book\ series\ for\ one\ year}{40\ school\ weeks}$, roughly corresponding to a single week's workload across the school year. This means that wherever the moving average diagram shows 'over zero', then the pupil would have met that coded property during that week. In this paper, we have selected a window to represent the likely material a student would encounter during one week.

RESULTS

Table 1 summarises the eight FoNS categories and presents the frequencies of each category in each of the two books. Interestingly, despite the research-led identification of the FoNS codes, the figures show that neither book includes any opportunities for children to engage with estimation or number patterns. Also, were the two adaptations to be exact replicas of the original the two sets of figures would be the same. This is clearly not the case, with, in general terms, MNP comprising more than 15% more tasks overall than Singma. Indeed, across the six FoNS categories for which evidence is available, MNP has more tasks than Singma, ranging from almost 26% more tasks focused on systematic counting to just under 2% for tasks focused on simple arithmetic operations. A chi square test confirmed ($p<0.0005$) the statistical significance of the differences between the two sets of frequencies.

Table 1: Summaries of the eight FoNS categories and the frequencies for each in each book

	FoNS Characteristic	Pupils are encouraged (in the range 0-20) to	Singma	MNP	% change
1	Number recognition	Identify, name and write particular number symbols	614	685	11.6
2	Systematic counting	Count systematically, forwards and backwards, from arbitrary starting points	214	269	25.7
3	Number and quantity	Understand the one-to-one correspondence between number and quantity	335	371	10.7
4	Quantity discrimination	Compare magnitudes and deploy language like ‘bigger than’ or ‘smaller than’	110	120	9.1
5	Different representations	Recognise and make connections between different representations of number	346	370	6.9
6	Estimation	Estimate, whether it be the size of a set or an object	0	0	
7	Simple arithmetic	Perform simple addition and subtraction operations	415	423	1.9
8	Number patterns	Recognise and extend number patterns, identify a missing number	0	0	
		Total tasks per book	1694	1955	15.4

Of course, frequencies alone offer but one perspective on the content of a textbook, typically offering no indication as to the location of different forms of task in the narrative of the whole year’s study. To address this, we turn to moving averages based on a one-week time period.

The graphs shown in figure 1 are, effectively, indistinguishable. Both begin the school year with repeated emphases on tasks involving number recognition, followed by a fallow period

and a second, equally strong emphasis ending around four months into the school year. After this, neither book offers any further number recognition-related opportunities.

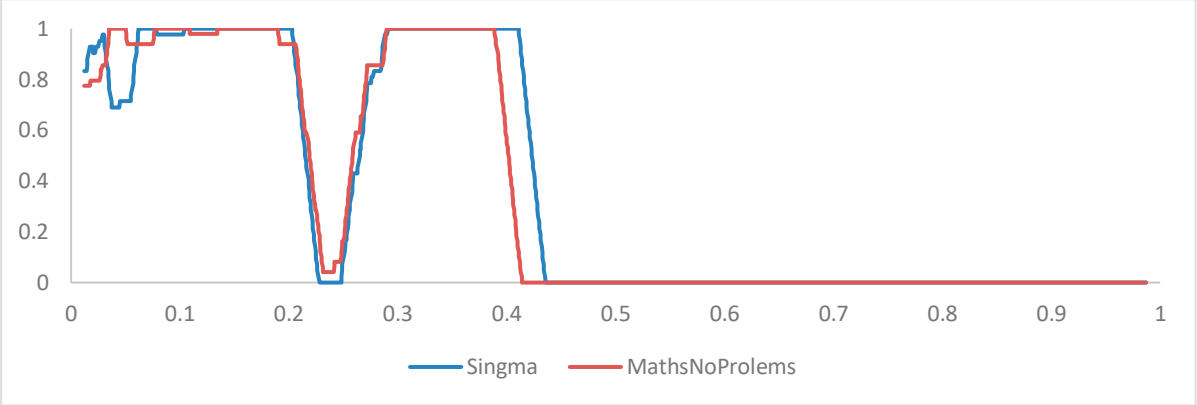

Figure 1: Graphs of the moving averages for FoNS 1 number recognition

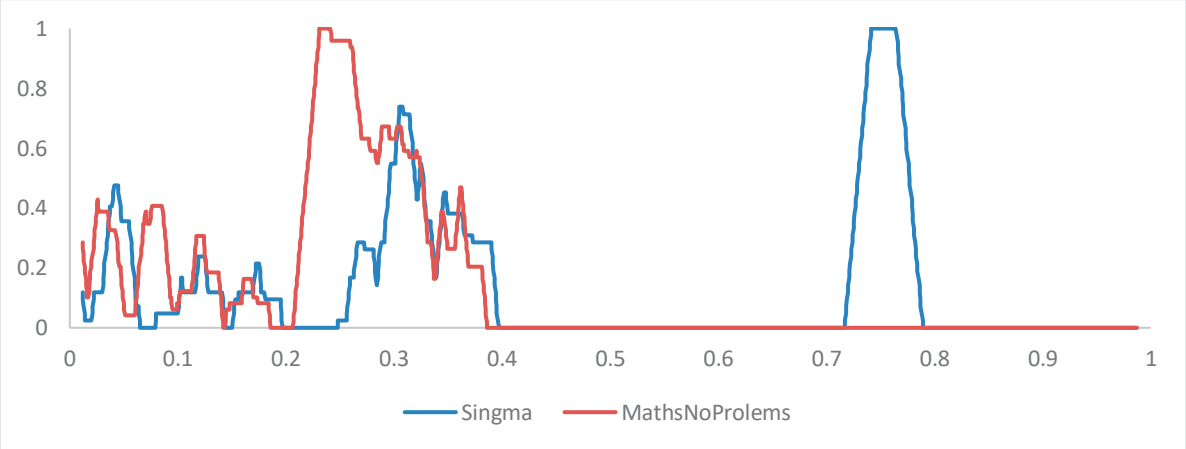

Figure 2: Graphs of the moving averages for FoNS 2 systematic counting

Unlike the close resonance of the graphs for FoNS 1, number recognition, the two books offered different emphases with respect to systematic counting. On the one hand, MNP begins with four short periods of limited emphasis before, after around two months, a final strong emphasis that gradually diminishes towards the four-month mark. On the other hand, the first four months of Singma mirror those of MNP, albeit with consistently lower emphases. The major difference is the spike during the eighth month, whereby a strong emphasis, stronger than at any other time of the year, emerges. Indeed, it is the only occasion that either of the two books offers any FoNS-related opportunities after the midpoint of the school year. What makes these differences particularly interesting is that the two strong spikes reflect when the two books introduce the vocabulary of ordinality; early in MNP and late in Singma.

The graphs of figure 3 show broadly similar trends. Both books end any opportunities for tasks related to the relationship between number and quantity around four months into the school year. That said, the broad patterns are similar, with early high levels of emphasis followed by a second period with slightly lower emphases. The remaining three figures, 4, 5 and 6, show similar trends with respect to the opportunities presented to children concerning quantity discrimination, different representations of number and simple arithmetical operations respectively.

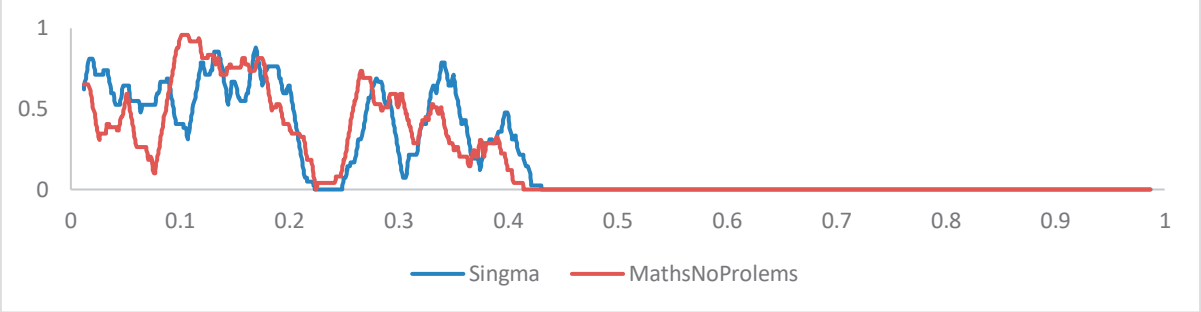

Figure 3: Graphs of the moving averages for FoNS 3 relationship between number and quantity

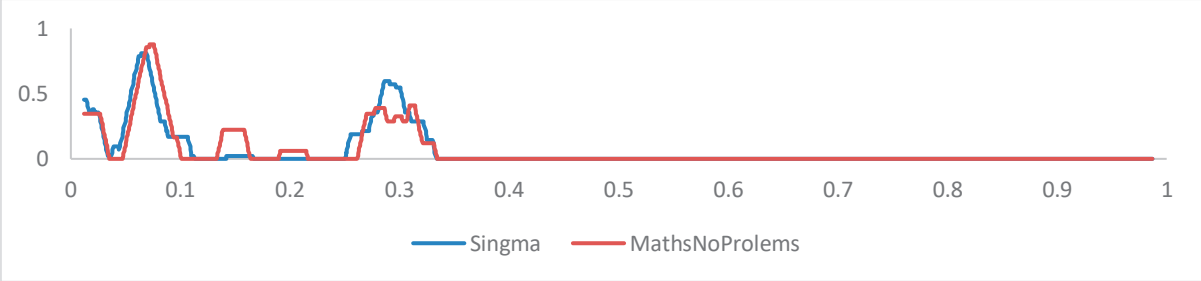

Figure 4: Graphs of the moving averages for FoNS 4 quantity discrimination

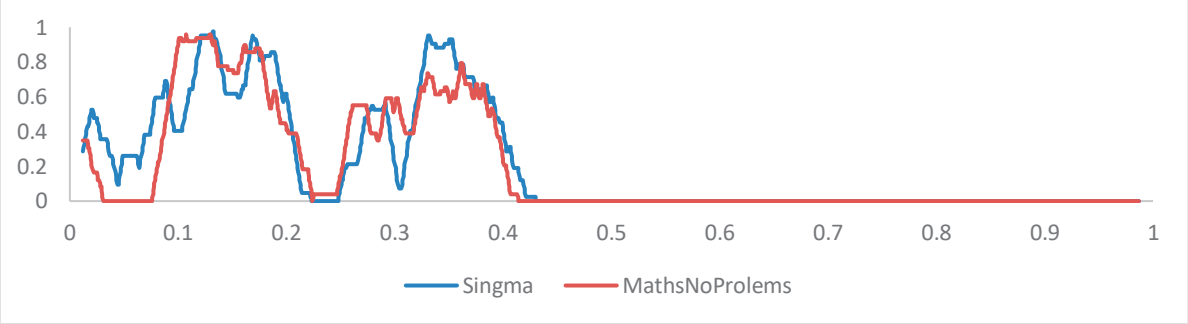

Figure 5: Graphs of the moving averages for FoNS 5 different representations of number

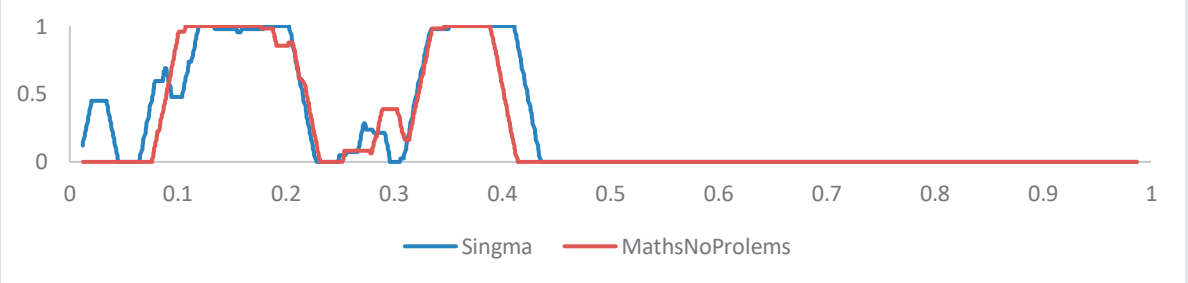

Figure 6: Graphs of the moving averages for FoNS 7 simple arithmetical operations

DISCUSSION

Our goal for this paper was to compare how two adaptations, one English and one Swedish, of the same Singaporean textbook structure year-one children's opportunities to acquire foundational number sense (FoNS). FoNS, which literature has shown to form the basis of later mathematical learning, is an eight dimensional set of competences necessary for year-one children, irrespective of their cultural or curricular traditions. If the two adaptations were merely translations, then it would be reasonable to expect the English and Swedish versions to

comprise the same tasks. The two analyses presented above offer some interesting insights into nature of these two adaptations. First, neither book acknowledges the importance of two FoNS categories, omissions that may compromise later mathematical learning. These are estimation (Libertus, Feigenson & Halberda, 2013) and number patterns (Lembke & Foegen 2009).

Table 2. Order systematic counting-related content in Singma and MNP

Singma	MNP
Number track, range [0 - 10]	Number track, range [0 - 10]
Add or subtract by counting [0 - 10]	Add or subtract by counting [0 - 10]
Number track, range [11 - 20]	Ordinal vocabulary
Add or subtract by counting [11 - 20]	Number track, range [11 - 20]
Ordinal vocabulary	Add or subtract by counting [11 - 20]

Second, with the exception of simple arithmetical operations, MNP comprises significantly more tasks across all FoNS categories than Singma, which is interestingly odd in light of our earlier analyses showing that MNP comprised 29% more tasks than the English-authored textbook with which it was compared (Petersson et al. under review) and Singma comprised 36% fewer tasks than the Swedish-authored textbook with which it was compared (Sayers et al., under review). That is, the adapters seem to have very different views, in relation to the typical textbooks of their country, with regard to the sufficiency of the tasks presented in their adaptations. Third, with a single exception, although on this occasion it was systematic counting, the moving averages showed that despite differences in frequencies, the structures of the two textbooks were remarkably similar, with almost identical emphases over the course of the school year. Fourth, with respect to systematic counting, sub-topics were ordered differently in the two books, as shown in table 2. Here the vocabulary of ordinality occurs at different times; after all counting-related material in Singma and at the midpoint, of all the counting material in MNP. Indeed, the spike shown in the second half of the Singma school year was due to the introduction of numbers greater than 10, which all occurred in the first half the year in Singma. To conclude, the two adaptations, while broadly adopting the same structure, differ in a number of respects due, we speculate, to authors' culturally situated interpretations of the number-related curriculum requirements of the two countries' curricula and expectations of learner readiness (Rezat, 2006).

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