Multivariate Global Sensitivity Analysis of Rocking Responses of Shallow Foundations under Controlled Rocking

Aria Fathi, Ph.D. Research Associate, S.M.ASCE[1], Mehran Mazari, Ph.D., A.M.ASCE[2], Mahdi Saghafi, Ph.D. Research Associate, S.M.ASCE[3]

[1]Center for Transportation Infrastructure Systems (CTIS), The University of Texas at El Paso, 500 W. University Ave., El Paso, TX 79968; e-mail: afathi@miners.utep.edu
[2]Department of Civil Engineering, California State University Los Angeles, 5151 State University Drive, Los Angeles, CA 90032; e-mail: mmazari2@calstatela.edu
[3]Center for Transportation Infrastructure Systems (CTIS), The University of Texas at El Paso, 500 W. University Ave., El Paso, TX 79968; e-mail: msaghafi@miners.utep.edu

ABSTRACT

A strong input energy, e.g., earthquake, may cause a partial separation of foundation from the underneath geomaterials. The concept of rocking behavior of shallow foundations has become popular in performance-based design (PBD) earthquake geotechnical specifications as an energy-dissipation mechanism. The two main performance indicators of the soil-structure (SS) systems during rocking are: (1) the moment capacity of the assembled SS system that gradually mobilized under cyclic loading, and, (2) the dissipation of energy governed by the rotational moment hysteresis loops. The aim of this study is to evaluate the rocking responses of slender high-rise structures using global sensitivity analysis (GSA) methods. For this purpose, a Finite Element (FE) analysis database was generated using a wide range of geomaterials with different stiffness properties. The two rocking responses, mobilized moment and dissipated energy obtained from the FE model, can be attributed to variations of the input parameters such as structure dimensions, geomaterials properties, and rotation of building. To avoid the implementation of time-consuming FE analyses, Random Forest (RF) metamodels were developed using the synthetic database for the rocking responses. This paper discusses two different GSA methods including Elementary Effects (EE) and Sobol's method to assess the impact of input parameters on the models. The results show that both methods are efficient in evaluating the impact of input parameters on the responses. EE requires a smaller number of generated samples and less computational effort. However, Sobol's method is more efficient in measuring the joint effects despite the higher computational cost.

Keywords: Shallow Foundations, Rocking Responses, Random Forest (RF), Machine Learning (ML), Global Sensitivity Analysis (GSA), Elementary Effects (EEs), Sobol's method, Morris's method.

INTRODUCTION

The partial separation, or uplift, of one side of a foundation due to large inertial and eccentric forces induced by strong vibrations, is normally accompanied by a significant change of shear stress in the opposite side of the foundation during rocking. Accumulation of permanent displacement appears because of an increase in compound shear and normal stresses at both sides of the foundation. The rocking behavior of shallow foundations is one of the major concepts that has drawn an increasing attention within the past decades as a performance enhancement tool in earthquake geotechnical design specifications. A number of these studies that discuss the foundation failure for the rocking structures can be found in Housner (1963), Haeri and Fathi (2015), and Gajan et al. (2005).

Several studies have focused on the nonlinear load-displacement behavior of shallow foundations under both static and dynamic loading conditions (Anastasopoulos, 2011; Jafarzadeh et al. 2012; Ibsen et al. 2015; Abadi et al. 2015; Rashidi and Haeri, 2017; Khosravi et al. 2017; Larsen et al. 2017; and Mousavi and Ghayoomi 2018). For instance, Ayoubi and Pak (2017) used this framework to study the settlement of shallow foundations on a liquefiable soil and derived a practical equation to estimate settlement of a building during an earthquake. Moreover, Gajan and Kutter (2008) conducted several centrifuge tests for different types of soils (including clay and sand) to assess the rocking behavior of shallow foundations subjected to low frequency cyclic movement. They indicated that the contact area of foundations can be correlated with moment capacity, energy dissipation, and permanent settlement measured during rocking. In a recent study, Fathi et al. (2018) developed a database of finite element (FE) analyses including a wide range of stiffness values and structure dimensions to evaluate the soil-structure systems subjected to slow cyclic loading. They showed that mobilized moment is closely related to the magnitude of uplift so that the mobilized moment can increase with an increase in foundation uplift at a certain rotation amplitude.

To understand the rocking behavior of shallow foundations, the relationships between input and output variable(s) are required. Sensitivity analysis (SA) is a commonly used technique to identify the most contributing model parameters. Generally, SA includes local and global approaches. Local SA (LSA) evaluates the variation of model response by changing one parameter while other parameters are fixed at a certain value. Global SA (GSA) investigates the changes of model response by varying all parameters at the same time. However, GSA methods are computationally cost-intensive compared to the LSA techniques (Saltelli et al. 1999).

The application of GSA in identifying the principle input variables, that control the responses of complex nonlinear relationships, have been extensively studied in the literature (Saltelli et al. 1999 and 2008; Zamanian 2016). Only a few studies have addressed the application of GSA in the SSI context. Zoutat et al. (2018) employed the Sobol's variance-based GSA (Sobol 1993) to estimate the contribution of the input variables to the lateral displacement of buildings considering SSI and damping as the controlling factors.

The main goal of this study is focused on a global evaluation/sensitivity analysis of rocking responses of shallow foundations (i.e., mobilized moment and damping ratio). Two GSA methods, Sobol and Elementary Effects, were employed to investigate the impact of input parameters on the rocking responses of shallow foundations. A finite element model (Fathi et al. 2018) was developed for estimation of rocking behavior of shallow foundations. The synthetic database included a wide range of geomaterial properties and different building dimensions. A metamodel, using the synthetic database, was constructed to perform the GSA. The following sections include the details of the FE model followed by the GSA methods used in this study. The construction of metamodels is followed by inspecting the impact of input parameters on the rocking responses.

FINITE ELEMENT MODELING OF SHALLOW FOUNDATIONS ROCKING

A wide range of sandy materials with different stiffness values (i.e., very loose to very dense) was employed to simulate the soil system combined with a range of structural dimensions, using a dynamic FE model in ABAQUS®. A nonlinear elastic-perfect plastic behavior was selected to simulate the geomaterial behavior under static and cyclic loading. An iterative process was employed to adjust the geomaterial stiffness parameters, using different stress and strain levels, during the FE analysis so that the geomaterials can behave nonlinearly prior to the Mohr-Coulomb failure criteria. The assembled foundation-superstructure system was subjected to a slow lateral cyclic loading (displacement-controlled) at its center of gravity (Figure 1a). The applied cyclic loading provided the shallow foundation with three ranges of rotation through the time history of loading (Figure 1b). The simulated soil-structure system was developed using plain strain elements. To avoid the reflection of loading waves, nonreflective boundaries were selected. The applied rotations are 0.0015, 0.005, and 0.015 radians. The rocking responses (mobilized moment, energy dissipation, and permanent displacement) were recorded at certain rotations. A metamodel was then constructed from the results of the FE model.

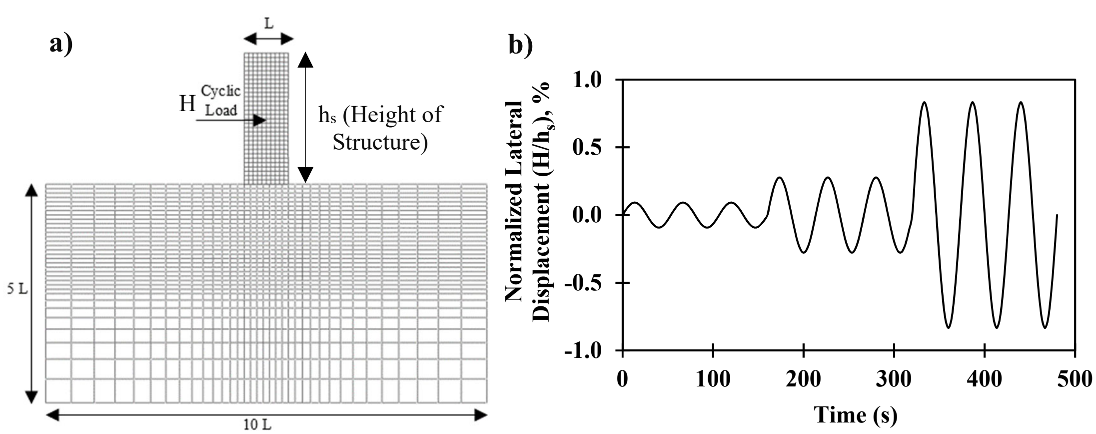

Figure 1. (a) FE model of the soil-structure system; and (b) the applied lateral slow cyclic loading (Fathi et al. 2018).

A nonlinear model proposed by Seed et al. (1986) was employed to simulate the dynamic shear moduli of granular soils as follows:

$$G_{max} = 218.82\ K_{2(\max)}(\sigma')^{0.5} \quad (1)$$

where G_{max} is maximum shear modulus, $K_{2(max)}$ is a laboratory shear modulus coefficient measured at low strain level which varies between 30 and 75 for sandy material, and σ' is mean effective principal stress which is defined as:

$$\sigma' = \frac{\sigma_1' + \sigma_2' + \sigma_3'}{3} \quad (2)$$

In this study, 400 FE scenarios were simulated. Input parameters were randomly generated for each soil property within the defined ranges for the soil and structure systems as listed in Table 1. These values were then randomly chosen for the assembly of the soil-structure systems.

Table 1. Range of soil-structure system properties.

Structure Features	Symbol	Range of Values
Weight	W	19.6 – 58.8 MN
Height of structure	h_s	30 – 60 m
Length	L	20 m
Soil Properties		
shear modulus coefficient	$K_{2(max)}$	30 – 75
Cohesion	C	15 kPa
Angle of internal friction	φ	38
Dilatation angle	ψ	3
Poisson's ratio	ν	0.3

GLOBAL SENSITIVITY ANALYSIS

Most GSA techniques are based on Monte-Carlo (MC) simulation. MC is a mathematical approach which is employed to generate random variables for risk and uncertainty assessment of a specific system (Saltelli et al. 2008). Two GSA methods are more common: (1) variance-based techniques such as the Sobol' method (Sobol 1993) and Fourier Amplitude Sensitivity Test (FAST) (Cukier et al. 1973); (2) global screening methods such as Elementary Effects, also known as Morris method, and Latin Hypercube One-factor-At-a-Time (LH-OAT) (Yang. 2011). In this study, both Sobol and Morris methods are adopted.

Sobol's Method. The Sobol's method is a variance-decomposition GSA that can be performed for monotonic and nonmonotonic models (Sobol 1993). Sobol's method can be utilized to compute main, S_i, and total sensitivity, ST_i, indices by evaluating a multidimensional integral via the Monte Carlo method (Saltelli et al. 2008). Total sensitivity index includes the main effect of input parameter, X_i, and its interaction with other variable(s). Assuming $Y = f(X_1, X_2, \ldots, X_N)$ as a deterministic model, where Y is a scalar output and X_i variables are N statistically independent input variables, the variance of Y can be decomposed into the following form:

$$V(Y) = \sum_i V_i + \sum_i \sum_{j>i} V_{ij} + \sum_i \sum_{j>i} \sum_{k>j} V_{ijk} + \cdots + V_{123\ldots N} \quad (3)$$

where $V_i = V(E(Y|X_j))$ is the first order partial variance that represents the direct influence of Xi on the output; and V_{ij} is the second order partial variance. The V_i parameter accounts for the average reduction of output variance resulting from keeping X_i as a constant value within its defined range. In other words, the individual contribution of each Xi to the total variance $V(Y)$ is estimated by Vi. The second order partial variance V_{ij} represents the second order interaction between X_i and X_j on $V(Y)$. The higher order partial variances for quantifying higher order interactions can be mentioned likewise. Sobol's sensitivity indices (i.e., first, second, third and higher orders) can be determined by normalizing the partial variances V_i, V_{ij}, V_{ijk} with respect to the total variance $V(Y)$ as follows (Saltelli et al. 2008):

$$\mathrm{S_i} = \frac{\mathrm{V_i}}{\mathrm{V(Y)}},\ \mathrm{S_{ij}} = \frac{\mathrm{V_{ij}}}{\mathrm{V(Y)}}, \mathrm{S_{ijk}} = \frac{\mathrm{V_{ijk}}}{\mathrm{V(Y)}} \tag{4}$$

If there is a considerable difference between the first-order and total indices, the interaction between the parameters is considered significant.

Elementary Effects (EE) Morris Method. The elementary effects (EE) is an easy-to-implement and robust method for screening the effects of input parameters on the model response(s) (Morris, 1991). The nonlinear relationships between the input and output variable(s) can be measured using EE. In this method, the input parameter that generates a large variation in the model response(s) is the one that affects the output the most. The Morris' method can be defined as follows:

$$EE_i = \frac{Y(X_1,\ldots,X_{i-1},X_i+\Delta,\ldots,X_N) - Y(X_1,X_2,\ldots,X_N)}{\Delta} \tag{5}$$

where Δ is a number in the set $\{1/(\rho - 1), \ldots , 1\text{-}1/(\rho - 1)\}$; $X_1, X_2, \ldots, X_N$ are the input parameters vary in an N-dimensional space, and ρ is the number of levels that input space is discretized by which. Saltelli et al. (2008) suggest an even number for ρ to avoid an unbiased probability for each value. For this study, $\rho = 20$ was defined as the number of levels for each sensitivity test, doing so, Δ can take any discretized value from the set $\{0.0526, 0.1053, \ldots, 0.95\}$ in the interval [0; 1]. The impact of variation in an input parameter can be assessed by EE using the value of Δ. Morris' sensitivity indices are comprised of mean, μ_i, and standard deviation, σ_i, of elementary effects (EE_i). μ_i accounts for the main impact of input parameter on the response(s). σ_i shows the interaction effect of different parameters. In other words, the higher the aforementioned indices, the greater are the impact of the parameter on the model response (Saltelli et al. 2008).

CONSTRUCTION OF METAMODELS

Metamodels are extensively employed for the parametrization between the input and output variables. These models are used to assemble a sufficient number of surrogate models to avoid performing the computationally cost-intensive experimental or numerical simulations. In other words, the metamodels are developed to forecast the responses using a new generation of input variables upon establishing relationships between the inputs and the corresponding output(s). Several studies have attempted to address function fitting and model prediction using machine learning (ML) techniques such as artificial neural network (ANN), support vector machine (SVM), and random forest (e.g., Mazari and Rodriguez 2016; and Ashtiani et al. 2018).

To develop the surrogate models of rocking moment capacity and damping ratio, random forest (RF) was employed as the proposed ML technique in this study. The RF technique was first introduced by Breiman et al. (2001). RFs are often used to solve complex nonlinear problems. This method has the capability for estimating nonlinear interaction between the input and output variables. For the purpose of model prediction, RF is constructed using a bagging process which mainly contains bootstrapped of several regression trees (Breiman et al. 2001).

The mobilized moment ($M_{mobilized}$, MN.m) and damping ratio (ξ, %) were defined as a function of rotation (θ, radian), stiffness parameter ($k_{2(max)}$), height of structure (h, m), weight of structure (W, MN), and ratio of contact area (η), i.e., the contact area of the foundation during rocking to the actual area of the foundation; the general forms of the predictive functions are as follows:

$$M_{mobilized} = f\left(\theta, k_{2(max)}, h, W, \eta\right) \tag{6}$$

$$\xi = f\left(\theta, k_{2(max)}, h, W, \eta\right) \tag{7}$$

Root Mean Squared Error ($RMSE$), was used as the fitness function for the development of metamodels. $RMSE$ is calculated as follows:

$$RMSE = \sqrt{\sum_{i=1}^{n} \frac{(x_i - y_i)^2}{n}} \tag{8}$$

where x_i is the measured value, y_i is the predicted value, and n is the number of samples. To predict the rocking responses, the synthetic database, including 400 cases at different rotations (0.0015, 0.005, and 0.015 rad.)—1200 samples in total—was utilized. The database was randomly divided into the training and testing subsets. For this purpose, 80% of the database (960 cases) was assigned to the training set and the rest (240 FE scenarios) were assigned to the testing set. Figures 2 and 3 demonstrate the comparison of random forest-predicted models with the simulated FE models for mobilized moment and damping ratio, respectively. An accurate estimate of the rocking responses can be generated by RF models as judged by high coefficient of determination, R^2, and low values of ($RMSE$) for both training and testing datasets (Figures 2 and 3).

RESULTS AND DISCUSSIONS

The results of elementary effects (EEs) for rocking behavior of shallow foundations are presented using convergence plots and scatter graphs on the (μ_i, σ_i) plane as introduced by Morris (1991). In this study, 3000 samples were randomly selected, and thereafter, rocking responses were predicted using the constructed metamodels. Figure 4 shows the convergence plots of average EEs, μ_i, obtained from 3000 evaluations. μ_i, represents the main impact of each parameter on the output values. The input variables θ, rotation, and $k_{2(max)}$, stiffness parameter, affect the rocking responses, i.e., mobilized moment and damping ratio, the most as shown in Figure 4. Figure 5 illustrates the main and total effects of each parameter on the above-mentioned responses.

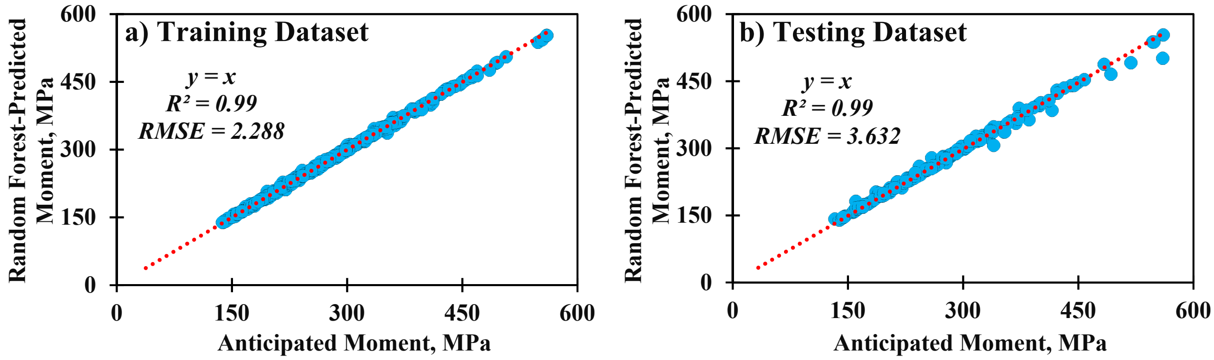

Figure 2. Comparison of random forest-predicted model and anticipated FE model for the mobilized moment: (a) training dataset and (b) testing data.

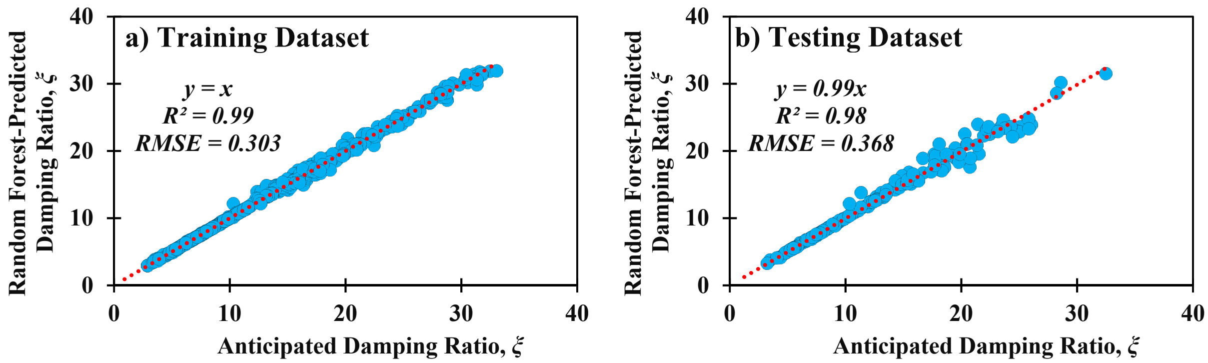

Figure 3. Comparison of random forest-predicted model and anticipated FE model for the damping ratio: (a) training dataset and (b) testing data.

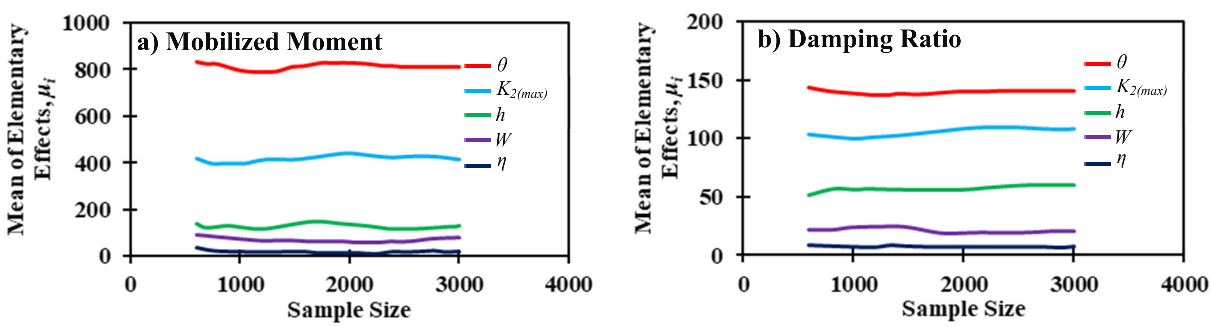

Figure 4. Mean, μ_i, of elementary effects evaluated by the number of evaluation for (a) mobilized moment; and (b) damping ratio.

Generally, the graph (μ_i, σ_i) can be interpreted as one of the following: (1) The input parameter has minor impact on the output if both μ_i and σ_i are small values; (2) there is a linear relationship between the input parameter and output if μ_i is a large number and σ_i is a small one; (3) there is a nonlinear relationship between the input and output parameters in conjunction with a strong interaction between the input parameters with a small number of μ_i and a large number of σ_i; and (4) and finally, the main and interaction effects are significantly high if both μ_i and σ_i are large (Saltelli et al. 2008).

There is a general consensus that an adequate number of samples can help to implement reliable and unbiased Sobol sensitivity analysis. To compute the first order (main effect) and total sensitivity indices, 10,000 model evaluations were performed using the constructed metamodels. The sensitivity indices using the Sobol's method are shown in Figure 6. Similar to elementary effects, θ and $k_{2(max)}$ are the factors affect the mobilized moment the most, respectively (Figure 6a). While the stiffness parameter has the most impact on the damping ratio, and the second most influential factor is parameter rotation as shown in Figure 6b. The contact area ratio, η, has no significant impact on the responses as found in all the analyses.

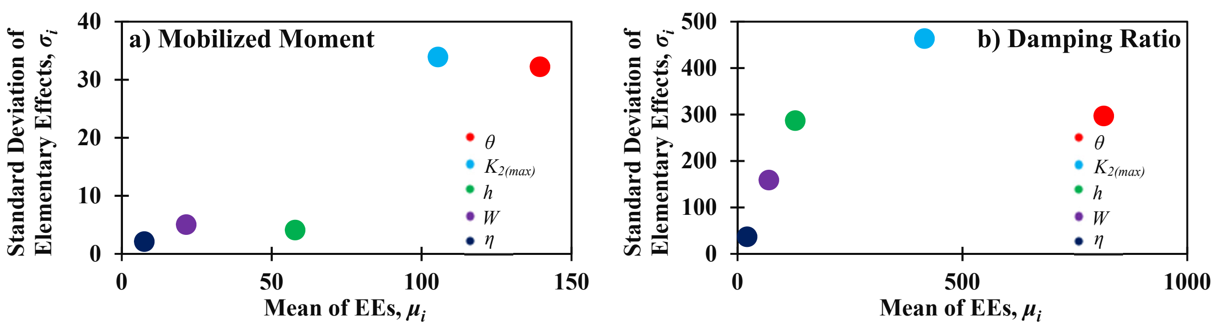

Figure 5. Mean (μ_i) of elementary effects vs. standard deviation (σ_i) of elementary effects for (a) mobilized moment; and (b) damping ratio.

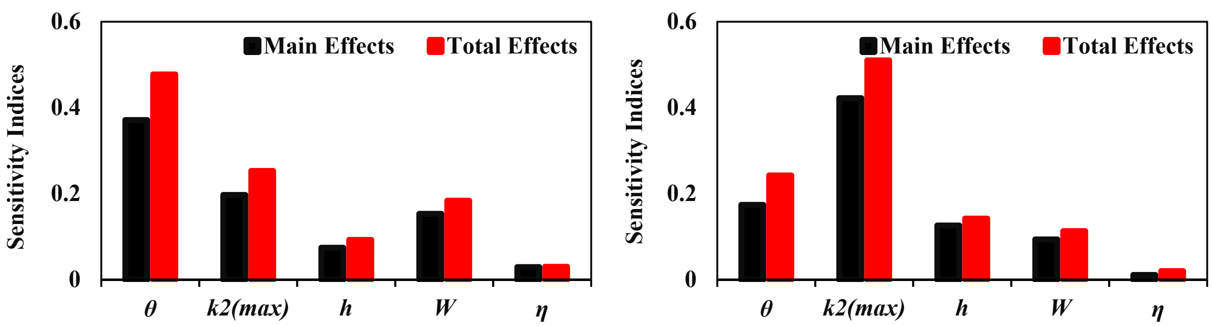

Figure 6. Sobol's sensitivity indices for (a) mobilized moment; and (b) damping ratio.

SUMMARY AND CONCLUSIONS

The analysis of rocking behavior of shallow foundations has demonstrated that the rocking behavior can be taken into account as an energy dissipation mechanism for the buildings. Moreover, the two responses, i.e., mobilized moment and damping ratio, can be considered as performance indicators of soil-structure systems during rocking. Since the rocking of shallow foundations is a complex problem, implementation of global sensitivity analysis (GSA) might be required to get a deep insight into such responses.

To generate a database including rocking responses of shallow foundations, a dynamic finite element (FE) model was deployed. The database was developed based on a wide range of geomaterials with different stiffness values associated with high-rise structures with different dimensions. Since GSA evaluation requires a huge number of evaluations (samples), random forest (RF) metamodels were developed using the generated database to avoid implementation of time-consuming FE analysis. The prediction power of the metamodels for rocking responses was reasonably well as assessed by the low *RMSE* and high coefficient of determination, R^2. For this study, two different GSA methods including Sobol and elementary effects (method of Morris) were deployed. The GSA methods show that the rotation and geomaterials stiffness affect the mobilized moment and damping ratio the most. Sobol's method was found to be more efficient to calculate the joint effects; however, the EE requires less computational effort.

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