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Neutrosophic Goal Programming Approach to A Green Supplier Selection Model with Quantity Discount

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Abstract: In this study, we have proposed a supplier selection problem with the goals of minimizing the net cost, minimizing the net rejections, minimizing the net late deliveries, and minimizing the net green house gas emission subject to realistic constraints like suppliers' capacity, buyer's demand etc. Due to uncertainty, the buyer's demand is fuzzy in nature and can be represented as a triangular neutrosophic number. We have also considered that quantity discounts are provided by the suppliers. The weights for different criteria are calculated using neutrosophic analytical hierarchy process. The neutrosophic goal programming approach has been applied in this article for solving the proposed supplier selection problem. An illustration has been given with comparison between fuzzy goal programming approach to demonstrate the effectiveness of the proposed model.

Keywords: Supplier selection; Quantity discounts; Green house gas; Neutrosophic goal programming; Triangular neutrosophic number; Neutrosophic analytical hierarchy process

1. Introduction

The supplier selection problem (SSP) is the problem of determining the right suppliers and their quota allocations. In designing a supply chain, a decision maker needs to consider decisions regarding the selection of the right suppliers and their quota allocation (Kumar, Vrat, & Shankar, 2004). Dickson(Dickson, 1966) was the first to identify 23 different criteria for various supplier selection problems. According to him quality was the most important criterion while delivery, price, geographical location and capacity were also very important factors in the supplier selection process. Weber and Current(Weber & Current, 1993) took a multi-objective approach to solve a supplier selection problem where net price, net late deliveries, net rejected unit delivered were minimized subject to a constant demand and capacity constraint. Kumar et al.(Kumar et al., 2004) applied fuzzy goal programming to solve a similar problem as Weber and Current(Weber & Current, 1993) with some additional constrains such as budget restriction for each retailer, supplier's quota flexibility etc. Wang
and Yang(Wang & Yang, 2009) considered quantity discount in supplier selection problem and applied fuzzy goal programming to find out a compromise solution. They also used analytical hierarchy process (AHP) to find out weights of different goals. Shaw et al.(Shaw, Shankar, Yadav, & Thakur, 2012) developed a supplier selection model with the amount of carbon emission by the suppliers as an objective function. They used fuzzy AHP to figure out weights for different objective functions. They also considered the aggregate demand as a fuzzy triangular number. To solve the problem, they also used fuzzy goal programming approach. Abdel-Basset et al.(Abdel-Basset,

Manogaran, Gamal, & Smarandache, 2018) used neutrosophic set for decision making and evaluation method to analyze and determine the factors influencing the selection of supply chain management suppliers. Gamal et al.(Gamal, Ismail, & Smarandache, 2018) used Multi-Objective Optimization on the basis of Ratio Analysis with the help of neutrosophic trapezoidal number to a supplier selection problem.

Zadeh(Zadeh, 1965) was the first to introduce the concept of fuzzy set. Bellman and Zadeh(Bellman & Zadeh, 1970) demonstrated decision making in fuzzy systems. Zimmermann(Zimmermann, 1978) applied the fuzzy set theory concept with some suitable membership functions to solve linear programming problem with several objective functions. Atanassov(Atanassov, 1986) developed the idea of intuitionistic fuzzy set, which is characterized by the membership degree as well as non-membership degree such that the sum of these two values is less than equal to one. Angelov(Angelov, 1997) gave the idea of optimization in intuitionistic fuzzy environment. In this article, he maximized the degree of acceptance of intuitionistic fuzzy objective(s) and minimized the degree of rejection of intuitionistic fuzzy objectives subject to the constraints of the problem.

Intuitionistic fuzzy sets cannot handle when indeterminate information is present in the concerned problem. In decision making theory, sometimes decision makers find it hard to decide due to presence of indeterminate information in the problem. So generalization of the concept of intuitionistic fuzzy sets was needed. So, Smarandache(Smarandache, 1999) incorporated the concept of indeterminacy by adding another independent membership function called as indeterminacy membership along with truth and falsity membership functions. Hezam et al.(Hezam, Abdel-Baset, & Smarandache, 2015) used neutrosophic theory in multi-objective linear programming problem. M. Hezam et al.(M. Hezam, Smarandache, & Abdel-Baset, 2016) introduced goal programming to neutrosophic fuzzy environment. In that paper, they established two models to solve an optimization problem. Here, they maximized truth and indeterminacy membership function and minimized the falsity membership function. Pramanik(Pramanik, 2016) also presented a neutrosophic linear goal programming problem. But instead of maximizing the indeterminacy membership function, he minimized it along with maximizing truth membership function and minimizing the falsity membership function. He also pointed out that minimizing the indeterminacy membership function is decision maker's best option. Islam and Kundu(Islam & Kundu, 2018) developed the geometric goal programming in neutrosophic environment and applied it to a Bridge Network Reliability Model. Islam and Ray(Islam & Ray, 2018) applied neutrosophic goal programming in multi-objective portfolio selection model. Rizk-Allah et al.(Rizk-Allah, Hassanien, & Elhoseny, 2018) used neutrosophic goal programming in a multi-objective transportation problem. (Abdel-Basset, Saleh, Gamal, & Smarandache, 2019) used type 2 neutrosophic number in supplier selection model. Plithogenic decision-making approach has been applied in selecting supply chain sustainability metrics in (Abdel-Basset, Mohamed, Zaied, & Smarandache, 2019).

Neutrosophic theory has been applied to internet of things (IoT) in (Abdel-Basset, Nabeeh, El-Ghareeb, & Aboelfetouh, 2019; Nabeeh, Abdel-Basset, El-Ghareeb, & Aboelfetouh, 2019). In (Abdel-Basset, El-hoseny, Gamal, & Smarandache, 2019; Abdel-Basset, Manogaran, Gamal, & Chang, 2019) neutrosophic theory has been applied in medical sciences.

As much as we know, neutrosophic goal programming has never been used before in a supplier selection problem. Also, there have not been many studies, in which quantity discounts offered by the suppliers. Our objective in this study is to give a computational algorithm for solving multi-objective supplier selection problem with quantity discount with the help of neutrosophic goal programming and neutrosophic analytical hierarchy process. The rest of the article is organized as follows: Section 2 presents some assumptions, notations and model description. Section 3 discusses some preliminaries and the neutrosophic analytical hierarchy process. Section 4 presents the fuzzified version of our model. Section 5 presents the computational algorithm. Section 6 provides a numerical example with comparison between neutrosophic goal programming approach and fuzzy goal programming approach. Finally, Section 7 gives some conclusions regarding the effectiveness of our proposed model.

2. Supplier Selection Model

A Supplier Selection Problem (SSP) is a very important problem for most of the manufacturing firms. The main goal of an SSP is to identify the supplier who has the most potential to meet the firm's demands with minimizing different costs for the firm in the process. An SSP is typically a multi-objective problem. Also, mostly it has conflicting goals. The assumptions and notations for our model are as follow:

2.1. Assumptions

- Single type of item is considered.
- Quantity discounts are offered by the suppliers.
- No shortage of the item is permitted for any supplier.

2.2. Notations

2.2.1. Index

- i: index for suppliers, $\forall$ i = 1,2,…, n
- m(i): number of quantity ranges in supplier-i's price level
- j: index for price level for the suppliers, $\forall$ 1,2,...,m(i)
- k: index for objective functions,

2.2.2. Decision Variables

- x_{ij}:ordered quantity for the supplier-i at the price level j
- $y_{ij}: \begin{cases} 1 & \{\text{if supplier} - \text{i is selected at price level j}\} \\ 0 & \text{otherwise} \end{cases}$

2.2.3. Parameters

D: aggregate demand of the item over a fixed planning period

a_{ij}: j^{th} price level for supplier-i

p_{ij}: the unit price of the supplier-i at price level j

η_i: percentage of units delivered late by the supplier-i

ϑ_i : percentage of rejected units delivered by supplier-i

g_i: green house gas emission (GHGE) for product supplied by supplier i.

n: number of suppliers

C_i: maximum capacity of supplier-i

B_i: budget allocated to supplier-i

2.3. Model Description and Formulation:

In this article, we study the case in which a single firm buys raw materials or semi-products from n-suppliers. Suppliers sell the products at different prices and emit different amount of greenhouse gases. The suppliers may deliver some rejected items and also they may fail to deliver in time as agreed before by the both parties. The firm requires to minimize the above mentioned costs and shortcomings. Hence a multi-objective linear programming problem has been formed to find out the optimal purchasing quantity from each supplier for the firm.

A multi-objective linear programming problem(MOLP) is of the form,

Maximize $Z_k(x_i) = [Z_1(x_i), Z_2(x_i), \ldots\ldots, Z_K(x_i)]$, k=1,2,3,...,K

Minimize $Y_l(x_i) = [Y_1(x_i), Y_2(x_i), \ldots\ldots, Y_L(x_i)]$, l=1,2,...,L

subject to,

$f_m(x_i) \le a_m$, m=1,2,...,M

$g_t(x_i) = b_t$, t=1,2,...,T

$h_o(x_i) \ge c_o$, o=1,2,...,O

$x_i \in X$, X is the solution space. Now, the multi-objective linear programming problem for this supplier selection problem (MOLP-SSP) is,

Minimize $Z_1(x_{ij}) =$

$\Sigma_{i=1}^{n}\Sigma_{j=1}^{m(i)} p_{ij}.x_{ij}$ mizeZ_1(x_ij)=?Σ_i=1^n?Σ_j=1^m(i)p_ij.x_ij (2.1)

Minimize $Z_2(x_{ij}) =$

$\Sigma_{i=1}^{n}\eta_i.\Sigma_{j=1}^{m(i)} x_{ij}$ mizeZ_2(x_ij)=?Σ_i=1^nη_i.?Σ_j=1^m(i)x_ij (2.2)

Minimize $Z_3(x_{ij}) =$

$\Sigma_{i=1}^{n}\vartheta_i.\Sigma_{j=1}^{m(i)} x_{ij}$ mizeZ_3(x_ij)=?Σ_i=1^nϑ_i.?Σ_j=1^m(i)x_ij (2.3)

Minimize $Z_4(x_{ij}) =$

$\Sigma_{i=1}^{n} g_i.\Sigma_{j=1}^{m(i)} x_{ij}$ mizeZ_4(x_ij)=?Σ_i=1^ng_i.?Σ_j=1^m(i)x_ij (2.4)

$$\Sigma_{i=1}^{n}\Sigma_{j=1}^{m(i)} x_{ij} = D, \quad (2.5)$$

$$\Sigma_{j=1}^{m(i)} x_{ij} \le C_i, \quad \text{for } i = 1,2,\ldots,n, \quad (2.6)$$

$$y_{ij} = \begin{pmatrix} 1 & if & x_{ij} > 0 \\ 0 & if & x_{ij} = 0 \end{pmatrix}, \quad \text{for } i = 1,2,\ldots,n \text{ and } j = 1,2,\ldots,m(i), \quad (2.7)$$

$$a_{ij-1}y_{ij-1} \le x_{ij} < a_{ij}y_{ij}, \quad \text{for } i = 1,2,\ldots,n \text{ and } j = 1,2,\ldots,m(i), \quad (2.8)$$

$$\Sigma_{j=1}^{m(i)} y_{ij} \le 1, \quad \text{fori} = 1,2,\ldots,n, \quad (2.9)$$

$$\Sigma_{j=1}^{m(i)} p_{ij}.x_{ij} \le B_i, \quad \text{fori} = 1,2,\ldots,n, \quad (2.10)$$

$$x_{ij} \ge 0, \quad i = 1,2,\ldots,n \text{ and } j = 1,2,\ldots,m(i). \quad (2.11)$$

• Objective function (2.1) minimizes the total cost for the purchased items.

• Objective function (2.2) minimizes the net number of late delivered items from the suppliers.

• Objective function (2.3) minimizes the total number of rejected items from the suppliers.

• Objective function (2.4) minimizes the total amount of green house gas emission by the suppliers.

• The constraint (2.5) ensures that the overall demand is met for the firm.

• The constraint (2.6) puts restrictions on the capacities of the suppliers.

• The constraint (2.7) ensures the binary nature of the supplier selection decision.

• The constraint (2.8) is a quantity range constraint to meet the number of quantity ranges in a supplier's price level.

• The constraint (2.9) guarantees that at most one price level per supplier can be chosen.

• The constraint (2.10) prevents negative orders.

• The constraint (2.11) puts restrictions on the budget amount allocated to the suppliers.

In a real life problem of supplier selection, there are many elements, which can not be known properly and they create vagueness in the decision environment. This vagueness cannot be translated perfectly by a deterministic model. Therefore, the deterministic models are not suited for real life problems ((Kumar et al., 2004; Shaw et al., 2012)). For example, the predicted aggregate demand may not be accurate. So, the aggregate demand can be taken as a triangular neutrosophic number. Also, the objective functions for the firm are conflicting in nature because e.g. one supplier may charge less for the items but it may also deliver a lot of rejected/unusable items. So, the firm will want to find a compromise solution. Hence neutrosophic goal programming has been used in this study to find out the optimal trade-off for the firm.

3. Preliminaries

3.1. *Some Definitions*

Definition 3.1.1 (Fuzzy sets): *As in* (Zadeh, 1965) *, a fuzzy set* $\tilde{A}$ *in a universe of discourse X is defined as the ordered pairs* $\tilde{A} = \{(x, M_{\tilde{A}}(x)) : x \in X\}$ *where* $M_{\tilde{A}}: X \rightarrow [0,1]$ *is a function known as the membership function of the set* $\tilde{A}$. $M_{\tilde{A}}(x)$ *is the degree of membership of* $x \in X$ *in the fuzzy set* $\tilde{A}$*. Higher value of* $M_{\tilde{A}}(x)$ *indicates a higher degree of membership in* $\tilde{A}$.

Definition 3.1.2. (Neutrosophic sets): *As in* (Smarandache, 1999)*, let X be a universe of discourse and let* $x \in X$*. A neutrosophic set A in X is characterized by a truth-membership function* $T_A(x)$*, an indeterminacy-membership function* $I_A(x)$ *, and a falsity- membership function* $F_A(x)$ *, where* $T_A(x), I_A(x), F_A(x) \in (0,1), \forall x \in X$ *and* $0^+ \leq supT_A(x) + supI_A(x) + supF_A(x) \leq 3^-$.

Definition 3.1.3. (Single valued neutrosophic sets): *According to* (Haibin, Smarandache, Zhang, & Sunderraman, 2010)*, if X is a universe of discourse and if* $x \in X$*, a single valued neutrosophic set A is characterized by a truth-membership function* $T_A(x)$*, an indeterminacy-membership function* $I_A(x)$*, and a falsity- membership function* $F_A(x)$ *, where* $T_A(x), I_A(x), F_A(x) \in [0,1], \forall x \in X$ *and* $0 \leq supT_A(x) + supI_A(x) + supF_A(x) \leq 3$pT_A(x)+supI_A(x)+supF_A(x)≤3.

Definition 3.1.4. (Intersection of two Single valued neutrosophic number): *As in* (Salama & Alblowi, 2012) *, the intersection of two single valued neutrosophic sets A and B is a single valued neutrosophic set C, written as* $C = A \cap B$B *its truth, indeterminacy and falsity membership functions are given by,*

$$T_C(x) = min(T_A(x), T_B(x)), \tag{3.1}$$

$$I_C(x) = max(I_A(x), I_B(x)), \tag{3.2}$$

$$F_C(x) = max(F_A(x), F_B(x)) \tag{3.3}$$

for all x in X.

Definition 3.1.5. (Triangular neutrosophic numbers) *As in* (Abdel-Basset, Mohamed, Zhou, & M. Hezam, 2017)*, a triangular neutrosophic number is a special kind of neutrosophic set on the real number set* $\mathbb{R}$ *denoted as* $\tilde{a} =< (a_1, b_1, c_1); \widetilde{\delta_a}, \widetilde{\theta_a}, \widetilde{\lambda_a} >$*,where* $\widetilde{\delta_a}, \widetilde{\theta_a}, \widetilde{\lambda_a} \in [0,1]$*. The truth-membership, indeterminacy-membership and falsity-membership functions are defined as follows:*

$$T_{\tilde{a}}(x) = \begin{cases} \frac{(x-a_1)\widetilde{\delta_a}}{b_1-a_1}, & if \ a_1 \leq x \leq b_1 \\ \widetilde{\delta_a}, & if \ x = b_1 \\ \frac{(c_1-x)\widetilde{\delta_a}}{(c_1-b_1)}, & if \ b_1 < x \leq c_1 \\ 0 \ , otherwise & \end{cases} \tag{3.4}$$

$$I_{\tilde{a}}(x) = \begin{cases} \frac{b_1 - x + \widetilde{\theta_a}(x - a_1)}{b_1 - a_1} , & if\ \ a_1 \le x \le b_1 \\ \widetilde{\theta_a} , & if\ \ x = b_1 \\ \frac{x - b_1 + \tilde{\theta}_a(c_1 - x)}{c_1 - b_1} , & if\ \ b_1 < x \le c_1 \\ 1\ , otherwise & \end{cases} \tag{3.5}$$

$$F_{\tilde{a}}(x) = \begin{cases} \frac{b_1 - x + \widetilde{\lambda_a}(x - a_1)}{b_1 - a_1} , & if\ \ a_1 \le x \le b_1 \\ \widetilde{\lambda_a} , & if\ \ x = b_1 \\ \frac{x - b_1 + \tilde{\lambda}_a(c_1 - x)}{c_1 - b_1} , & if\ \ b_1 < x \le c_1 \\ 1\ , otherwise & \end{cases} \tag{3.6}$$

where $\delta_a, \theta_a, \lambda_a$ are the maximum truth-membership degree, minimum indeterminacy-membership degree and minimum falsity-membership degree respectively.

3.2. *Neutrosophic Goal Programming Technique*

A minimizing type multi-objective linear programming is of the form,

$$\begin{aligned} &min\ [Z_1(x), Z_2(x), \dots, Z_K(x)] \\ &g_t(x) \le b_t, \mathrm{t} = 1,2, \dots, \mathrm{T} \end{aligned} \tag{3.7}$$

Let, the fuzzy goal for each objective function be denoted as G_k for all k=1,2,...,K and the fuzzy constraints be denoted as C_t for all t=1,2,...,T. Then, the neutrosophic decision set D^N, which is a conjunction of neutrosophic objectives and constraints, is defined by,

$$D^N = (\cap_1^K G_K)(\cap_1^T C_T) = (x, T_{D^n}, I_{D^n}, F_{D^n}) \tag{3.8}$$

$$T_{D^n} = min(T_{G_1}(x), T_{G_2}(x), \dots, T_{C_k}(x); T_{C_1}(x), T_{C_2}(x), \dots, T_{C_k}(x)), \forall x \in X \tag{3.9}$$

$$I_{D^n} = max(I_{G_1}(x), I_{G_2}(x), \dots, I_{C_k}(x); I_{C_1}(x), I_{C_2}(x), \dots, I_{C_k}(x)), \forall x \in X \tag{3.10}$$

$$F_{D^n} = max(F_{G_1}(x), F_{G_2}(x), \dots, F_{C_k}(x); F_{C_1}(x), F_{C_2}(x), \dots, F_{C_k}(x)), \forall x \in X \tag{3.11}$$

, where $T_{D^n}, I_{D^n}, F_{D^n}$ are truth, indeterminacy and falsity membership function of the neutrosophic decision set D^N respectively. Now the transformed linear programming problem of the problem in eq. (3.7) can be written as the following crisp programming problem,

$$\begin{aligned} &\mathrm{min}\ \ (1-\alpha) + && \gamma + \beta \\ &\text{subject to,} && \\ &T_{D^n}(X) && \ge \alpha \\ &I_{D^n}(x) && \le \gamma \\ &F_{D^n}(X) && \le \beta \\ &0 \le \alpha + && \beta + \gamma \le 3 \\ &\alpha && \ge \beta \\ &\alpha && \ge \gamma \\ &\alpha, \beta, \gamma && \in [0,1] \end{aligned} \tag{3.12}$$

3.3. *Neutrosophic Analytical Hierarchy Process*

The analytical hierarchy process was first introduced by Saaty(Saaty, 1980). The process has been applied to a wide variety of decision making problems. It also gives a structured method for determining the weights of criteria. The Neutrosophic Analytical Hierarchy Process(NAHP) was introduced by Abdel-Basset et al.(Abdel-Basset et al., 2017) The process of calculating weight criteria by means of NAHP is described below briefly:

- A pairwise comparison matrix based on relative importance of each criterion is formed. If A=($\widetilde{a_{ij}}$) represents the matrix then, $\tilde{a}ij$ is a neutrosophic triangular number.

• We take $\widetilde{a_{ij}} = \tilde{1}$ if i and j are equally important, $\widetilde{a_{ij}} = \tilde{3}$ if i is moderately important than j, $\widetilde{a_{ij}} = \tilde{5}$ if i is strongly important than j, $\widetilde{a_{ij}} = \tilde{7}$ if i is very strongly important than j, $\widetilde{a_{ij}} = \tilde{9}$ if i is extremely important than j. We may also take $\tilde{a} = \tilde{2}, \tilde{4}, \tilde{6}$ *or* $\tilde{8}$ for different importance.

• Next, the neutrosophic pair-wise comparison matrix is transformed into a deterministic pair-wise comparison matrix, using the following equations: if $\tilde{a} =< (a_1, b_1, c_1); \widetilde{\delta_a}, \widetilde{\theta_a}, \widetilde{\lambda_a} >$ be a single valued triangular neutrosophic number then

$$\begin{aligned} s_{ij} &= \frac{(a_1+b_1+c_1)(2+\widetilde{\delta_a}-\widetilde{\theta_a}-\widetilde{\lambda_a})}{16} \\ \widetilde{a_{ij}} &= s_{ij} \\ \widetilde{a_{ji}} &= \frac{1}{s_{ij}} \end{aligned} \tag{3.13}$$

•After forming the deterministic matrix, each column entries are normalized by dividing each entry by column sum.

• Then, we average each row to get the required weights(w_l).

• Finally, we check the consistency of the comparison matrix with the help of consistency index (CI) and consistency ratio (CR) ((Abdel-Basset et al., 2017; Saaty, 1980)):

$$\begin{aligned} CI &= \frac{\lambda_{max}-n}{n-1} \\ CR &= \frac{CI}{RI} \end{aligned} \tag{3.14}$$

where n is the number of items being compared, and RI is the consistency index of a randomly generated pair-wise comparison matrix of similar size *(Saaty, 1980)*. If CR<0.1, the comparison matrix is consistent.

4. Fuzzy Supplier Selection Model

In this model, the decision maker/ firm tries to achieve a certain goal for each objective function. The goals are a fuzzy in nature. As well as, we assumed in this study demand cannot be known precisely. So, the aggregate demand is also fuzzy in nature. After fuzzification, the eqs. (2.1) to (2.11) can be represented as follows:

Find x_{ij} to satisfy,

$$\begin{aligned} Z_k(x_{ij}) &\cong \widetilde{Z_k} && \text{for } k = 1,2,3,4 \\ \Sigma_{i=1}^{n}\Sigma_{j=1}^{m(i)} x_{ij} &\cong \widetilde{D}, \\ \Sigma_{j=1}^{m(i)} x_{ij} &\leq C_i, && \text{for } i = 1,2,\ldots,n, \\ y_{ij} &= \begin{pmatrix} 1 & if & x_{ij} > 0 \\ 0 & if & x_{ij} = 0 \end{pmatrix}, && \text{for } i = 1,2,\ldots,n \text{ and } j = 1,2,\ldots,m(i), \\ a_{ij-1}y_{ij-1} &\leq x_{ij} < a_{ij}y_{ij}, && \text{for } i = 1,2,\ldots,n \text{ and } j = 1,2,\ldots,m(i), \\ \Sigma_{j=1}^{m(i)} y_{ij} &\leq 1, && \text{fori} = 1,2,\ldots,n, \\ \Sigma_{j=1}^{m(i)} p_{ij}.x_{ij} &\leq B_i. \\ x_{ij} &\geq 0, && i = 1,2,\ldots,n \text{ and } j = 1,2,\ldots,m(i). \end{aligned} \tag{4.1}$$

where $\widetilde{Z_k}$ is the aspiration level for each objective and $\widetilde{D}$ is the fuzzified demand. Hence, the aggregate demand can be taken as fuzzy triangular number or triangular neutrosophic number.

5. Computational Algorithm

In this study, NAHP and neutrosophic goal programming approach has been used to solve the problem. The solution steps to solve this model are as follows:

Step 1: Firstly, identification of supplier selection criteria with multi-supplier quantity discounts is done.

Step 2: A panel of experts in the fields of supply chain and operations is formed. To get the weights(w_l) for different criteria they are asked to fill a nine-point-scale questionnaire to form the pairwise comparison matrix using eq. (3.13). Then, consistency property of each expert's comparison results must be checked using eq. (3.14). If it is not consistent they are ask to fill the questionnaire again. They are also asked to approximate the market demand and how much it may fluctuate.

Step 3: Objective functions for the Supplier selection model are formed. These objective functions are purchasing cost, total amount of rejected items, total amount of late deliveries and the total amount of green- house gas emitted by the suppliers.

Step 4: Each objective is solved dismissing the other objective functions subject to the constrains and using the approximate demand as predicted by the experts in step 2. Using the values of all objective function at each ideal solution, pay-off matrix can be formulated as follows:

$$\begin{pmatrix} Z_1(x_{ij}^1) & Z_2(x_{ij}^1) & Z_3(x_{ij}^1) & Z_4(x_{ij}^1) \\ Z_1(x_{ij}^2) & Z_2(x_{ij}^2) & Z_3(x_{ij}^2) & Z_4(x_{ij}^2) \\ Z_1(x_{ij}^3) & Z_2(x_{ij}^3) & Z_3(x_{ij}^3) & Z_4(x_{ij}^3) \\ Z_1(x_{ij}^4) & Z_2(x_{ij}^4) & Z_3(x_{ij}^4) & Z_4(x_{ij}^4) \end{pmatrix}, \text{where } x_{ij}^k \text{ for k} = 1,2,3,4 \text{ is the ideal solution for } Z_k$$

Step 5: For each objective function Z_k the lower bound L_k, which is the aspiration level ($\widetilde{Z_k}$) and the upper bound U_k are formed as: $L_k = \widetilde{Z_k} = min_k(Z_k(x_{ij}^k))$ and $U_k = max_k(Z_k(x_{ij}^k))$ for k=1,2,3,4.

Step 6: The bounds for the neutrosophic environment can be calculated as follows:

$U_k^T = U_k, L_k^T = L_k$, for truth membership function (5.1)

$U_k^I = U_k, L_k^I = L_k + s_k(U_k - L_k)$, for indeterminacy membership function (5.2)

$U_k^F = U_k, L_k^F = L_k + t_k(U_k - L_k)$, for falsity membership function (5.3)

, where $s_k, t_k \in (0,1)$.

Step 7: For the objective functions the truth, indeterminacy and falsity membership functions are formed as follow:

$$T_k(Z_k(x_{ij})) = \begin{cases} 1 & , \text{if } Z_k(x_{ij}) \le L_k^T \\ \frac{U_k^T - Z_k(x_{ij})}{U_k^T - L_k^T} & , \text{if } L_k^T \le Z_k(x_{ij}) \le U_k^T \\ 0 & , \text{if } Z_k(x_{ij}) \ge U_k^T \end{cases} \quad (5.4)$$

$$I_k(Z_k(x_{ij})) = \begin{cases} 0 & , \text{if } Z_k(x_{ij}) \le L_k^I \\ \frac{Z_k(x_{ij}) - L_k^I}{U_k^I - L_k^I} & , \text{if } L_k^I \le Z_k(x_{ij}) \le U_k^I \\ 1 & , \text{if } Z_k(x_{ij}) \ge U_k^I \end{cases} \quad (5.5)$$

$$F_k(Z_k(x_{ij})) = \begin{cases} 0 & , \text{if } Z_k(x_{ij}) \le L_k^F \\ \frac{Z_k(x_{ij}) - L_k^F}{U_k^F - L_k^F} & , \text{if } L_k^F \le Z_k(x_{ij}) \le U_k^F \\ 1 & , \text{if } Z_k(x_{ij}) \ge U_k^F \end{cases} \quad (5.6)$$

Step 8: Using the information in Step 2, a neutrosophic triangular number is formed for the aggregate demand as: $\widetilde{D} =< (D_1, D_2, D_3); \widetilde{\delta_a}, \widetilde{\theta_a}, \widetilde{\lambda_a} >, where\ \widetilde{\delta_a}, \widetilde{\theta_a}, \widetilde{\lambda_a} \in [0,1]$ and the values of D_1, D_2, D_3 are given by the experts. The truth, indeterminacy and falsity membership functions are denoted by $T_{\widetilde{D}}(D), I_{\widetilde{D}}(D)$ and $F_{\widetilde{D}}(D)$ respectively and can be calculated using equations (3.4)-(3.6).

Step 9: Now modifying the neutrosophic goal programming technique which was described in section 3.2, the problem in eq. (4.1) can be written as the following crisp programming problem,

$$min\ \Sigma_{l=1}^{5} w_l((1-\alpha_l) + (\gamma_l) + \beta_l)$$?Σ_l=1^5w_l((1-α_l)+(γ_l)+β_l)

subject to,

$$\begin{array}{llll} T_k(Z_k(x_{ij})) & \geq \alpha_k, & \Sigma_{j=1}^{m(i)} x_{ij} & \leq C_i, \\ I_k(Z_k(x_{ij})) & \leq \gamma_k, & y_{ij} & = \begin{cases} 1 & if \;\; x_{ij} > 0 \\ 0 & if \;\; x_{ij} = 0 \end{cases}, \\ F_k(Z_k(x_{ij})) & \leq \beta_k, & a_{ij-1} y_{ij-1} & \leq x_{ij} < a_{ij} y_{ij}, \\ T_{\tilde{D}}(D) & \geq \alpha_5, & \Sigma_{j=1}^{m(i)} y_{ij} & \leq 1, \\ I_{\tilde{D}}(D) & \leq \gamma_5, & x_{ij} & \geq 0, \\ F_{\tilde{D}}(D) & \leq \beta_5, & \Sigma_{j=1}^{m(i)} p_{ij}.x_{ij} & \leq B_i, \\ 0 \leq \alpha_l + & \beta_l + \gamma_l \leq 3, & \alpha_l & \geq \gamma_l, \\ \alpha_l & \geq \beta_l, & \alpha_l, \beta_l, \gamma_l & \in [0,1] \end{array} \quad (5.7)$$

,for all i=1,2,...,n, j=1,2,...,m(i), k=1,2,3,4,l=1,2,3,4,5.

Step 10: Finally, use LINGO software to get the results.

6. Numerical Example

The following example shows the usefulness of the proposed model. Here, considering the same weights for the objectives, we have done a comparative study between Fuzzy Goal Programming(FGP) approach and Neutrosophic Goal Programming (NGP) approach for our model. The weights have been calculated by using NAHP. Here Six suppliers have been considered in the evaluation process. Most of the data used in this example have been derived from the articles (Wang & Yang, 2009; Weber & Desai, 1996). A panel of experts (as in Step 2 of section5) will predict the aggregate demand and how much it will fluctuate as oppose to in those above studies where the aggregate demand has been taken as a fixed number. The data which is given by those experts will be used to calculate the triangular neutrosophic number and fuzzy triangular number for the aggregate demand. Moreover, there is no consideration of greenhouse gas emission for the suppliers in those studies. We assumed the amount of greenhouse gas emission for the suppliers for the example.

Table 1: supplier quantity discounts.

Supplier-i	a_{i0}	p_{i1}	a_{i1}(K)	p_{i2}	a_{i2}(K)	p_{i3}	a_{i3}(M)	p_{i4}
1	0	0.2020	50	0.1990	100	0.1980	1	0.1958
2	0	0.1900	10	0.1890	200	0.1881	-	-
3	0	0.2350	10	0.2300	100	0.2250	1	0.2204
4	0	0.2200	20	0.2150	500	0.2100	2	0.2081
5	0	0.2250	50	0.2200	500	0.2150	1	0.2118
6	0	0.2200	10	0.2170	500	0.2140	1	0.2096

Table 2: supplier source data.

	suppliers					
	1	2	3	4	5	6
Rejection rate(%)	1.2	0.8	0.0	2.1	2.3	1.2
Late delivery rate(%)	5.0	7.0	0.0	0.0	3.0	4.0
GHGE(kg)	0.1	0.2	0.25	0.15	0.3	0.1
Capacity(C_i)	2.4 M	360 K	2.783 M	3.0 M	2.966 M	2.5 M
Budget constraint(B_i)($)	600000	100000	650000	500000	500000	300000

Table 3: Comparison matrix

	Cost	Lead time	Quality	GHGE	Demand
Cost	$\tilde{1}$	$\tilde{2}$	$\tilde{3}^{-1}$	$\tilde{6}^{-1}$	$\tilde{5}^{-1}$
Lead time	$\tilde{2}^{-1}$	$\tilde{1}$	$\tilde{5}^{-1}$	$\tilde{8}^{-1}$	$\tilde{1}$
Quality	$\tilde{3}$	$\tilde{5}$	$\tilde{1}$	$\tilde{3}^{-1}$	$\tilde{2}^{-1}$
GHGE	$\tilde{6}$	$\tilde{8}$	$\tilde{3}$	$\tilde{1}$	$\tilde{3}^{-1}$
Demand	$\tilde{5}$	$\tilde{1}$	$\tilde{2}$	$\tilde{3}$	$\tilde{1}$

The suppliers provide quantity discounts with the anticipation that the firm will increase order quantity in each order, thereby reducing the supplier's order processing cost. The data for quantity discounts are given in table 1. The data for other parameters are given in table 2. The comparison matrix for the criteria given in table 3.

The objective functions are,

$$\begin{aligned}
Z_1 = {} & 0.202x_{11} + 0.199x_{12} + 0.198x_{13} + 0.1958x_{14} + 0.19x_{21} + 0.189x_{22} + 0.1881x_{23} + 0.235x_{31} + \\
& 0.23x_{32} + 0.225x_{33} + 0.2204x_{34} + 0.22x_{41} + 0.215x_{42} + 0.21x_{43} + 0.2081x_{44} + 0.225x_{51} + \\
& 0.22x_{52} + 0.215x_{53} + 0.2118x_{54} + 0.22x_{61} + 0.217x_{62} + 0.214x_{63} + 0.2096x_{64} \\
Z_2 = {} & 0.05(x_{11} + x_{12} + x_{13} + x_{14}) + 0.07(x_{21} + x_{22} + x_{23}) + \\
& 0.03(x_{51} + x_{52} + x_{53} + x_{54}) + 0.04(x_{61} + x_{62} + x_{63} + x_{64}) \\
Z_3 = {} & 0.012(x_{11} + x_{12} + x_{13} + x_{14}) + 0.008(x_{21} + x_{22} + x_{23}) + 0.021(x_{41} + x_{42} + x_{43} + x_{44}) + \\
& 0.023(x_{51} + x_{52} + x_{53} + x_{54}) + 0.012(x_{61} + x_{62} + x_{63} + x_{64}) \\
Z_4 = {} & 0.1(x_{11} + x_{12} + x_{13} + x_{14}) + 0.2(x_{21} + x_{22} + x_{23}) + 0.25(x_{31} + x_{32} + x_{33} + x_{34}) + \\
& 0.15(x_{41} + x_{42} + x_{43} + x_{44}) + 0.3(x_{51} + x_{52} + x_{53} + x_{54}) + 0.1(x_{61} + x_{62} + x_{63} + x_{64})
\end{aligned} \tag{6.1}$$

Subject to the constraints,

$$\begin{aligned}
& x11 + x12 + x13 + x14 \le 2400K, \quad x21 + x22 + x23 \le 360K \quad x31 + x32 + x33 + x34 \le 2783K \\
& x41 + x42 + x43 + x44 \le 3000K, \quad x51 + x52 + x53 + x54 \le 2966K, \quad x61 + x62 + x63 + x64 \le 2500K
\end{aligned} \tag{6.2}$$

$$
\begin{array}{llllll}
y_{ij} = \begin{cases} 1 & if \;\; x_{ij} > 0 \\ 0 & if \;\; x_{ij} = 0 \end{cases}, & & \Sigma_{j=1}^{m(i)} y_{ij} & \leq 1, & 0 \leq x_{11} < & 50000y_{11}, \\
50000y_{11} \leq x_{12} & < 100000y_{12} & 100000y_{12} \leq x_{13} & < 1000000y_{13}, & x_{14} \geq & 1000000y_{14}, \\
0 \leq x_{21} & < 10000y_{21}, & 10000y_{21} \leq x_{22} & < 200000y_{22}, & x_{23} \geq & 200000y_{23}, \\
0 \leq x_{31} & < 10000y_{31}, & 10000y_{31} \leq x_{32} & < 100000y_{32}, & 100000y_{32} \leq x_{33} < & 1000000y_{33}, \\
x_{34} & \geq 1000000y_{34}, & 0 \leq x_{41} & < 20000y_{41}, & 20000y_{41} \leq x_{42} < & 500000y_{42}, \\
500000y_{42} \leq x_{43} & < 2000000y_{43}, & x_{44} & \geq 2000000y_{44}, & 0 \leq x_{51} < & 50000y_{51}, \\
50000y_{51} \leq x_{52} & < 500000y_{52}, & 500000y_{52} \leq x_{53} & < 1000000y_{53}, & x_{54} \geq & 1000000y_{54}, \\
0 \leq x_{61} & < 10000y_{61}, & 10000y_{61} \leq x_{62} & < 500000y_{62}, & 500000y_{62} \leq x_{63} < & 1000000y_{63}, \\
x_{64} & \geq 1000000y_{64}, & x_{ij} \geq & 0. & &
\end{array}
\qquad (6.3)
$$

$$
\begin{array}{rl}
0.202x_{11} + 0.199x_{12} + 0.198x_{13} + 0.1958x_{14} \leq & 600000 \\
0.19x_{21} + 0.189x_{22} + 0.1881x_{23} \leq & 100000 \\
0.235x_{31} + 0.23x_{32} + 0.225x_{33} + 0.2204x_{34} \leq & 650000 \\
0.22x_{41} + 0.215x_{42} + 0.21x_{43} + 0.2081x_{44} \leq & 500000 \\
0.225x_{51} + 0.22x_{52} + 0.215x_{53} + 0.2118x_{54} \leq & 500000 \\
0.22x_{61} + 0.217x_{62} + 0.214x_{63} + 0.2096x_{64} \leq & 300000
\end{array}
\qquad (6.4)
$$

$$
\begin{array}{rl}
D = & x_{11} + x_{12} + x_{13} + x_{14} + x_{21} + x_{22} + x_{23} + x_{31} + x_{32} + x_{33} + x_{34} + \\
& x_{41} + x_{42} + x_{43} + x_{44} + x_{51} + x_{52} + x_{53} + x_{54} + x_{61} + x_{62} + x_{63} + x_{64}.
\end{array}
\qquad (6.5)
$$

To find the weights for different objective functions we have taken $\tilde{1}$=<(0.6,1,5);(0.9,0.2,0.3)>, $\tilde{2}$=<(1,2,6);(0.8,0.4,0.2)>, $\tilde{3}$=<(0,3,9)(0.6,0.3,0.2)>, $\tilde{5}$=<(2,5,10);(0.6,0.3,0.2)>,$\tilde{6}$=<(2,6,9);(0.7,0.5,0.1)>, $\tilde{8}$=<(3,8,11);(0.7,0.5,0.1)>. From the discussions in section 3.3, we have the following weights: $w_1 = 0.126469, w_2 = 0.131538, w_3 = 0.207651, w_4 = 0.272911, w_5 = 0.26143$. For these set of weights we get CI=0.0540024. RI equal to 1.12 for five criteria, which is derived from (Saaty, Vargas, & others, 2006). So, we have CR=.0482164<0.1 and hence the consistency property holds. We calculate the aspiration levels for each objective function, dismissing other objective functions. From eqs. (5.1) to (5.3) for $s_k = .3, t_k = .2, \forall k = 1,2,3,4,$ we can calculate the bounds for truth, indeterminacy and falsity membership functions. The results are given in table 4. Here, the aggregate demand is taken as fuzzy triangular number for the FGP approach and triangular neutrosophic number for the NGP approach. We are Using LINGO to get the results which are given in table 5 and table 6.

Table 4: Bounds of each objective function, dismissing other objectives.

	Z_1	Z_2	Z_3	Z_4
$L_k = L_k^T$	2221790	170620	119367	1644500
$U_k = U_k^T$	2293665.6	321100	182870	2239650
L_k^I	2243352.68	215764	138417.9	1823045
U_k^I	2293665.6	321100	182870	2239650
L_k^F	2236165.12	200716	132067.6	1763530
U_k^F	2293665.6	321100	182870	2239650

For the FGP approach the demand is predicted to be 10900000 and assumed to vary between 10500000 and 12000000. The FGP approach can be written as (Similarly as (Shaw et al., 2012; Wang & Yang, 2009)),

$$
\begin{aligned}
&\max\ \Sigma_{l=1}^{5} w_l \lambda_l \\
&\text{subject to,} \\
&\frac{2293665.6 - Z_1}{2293665.6 - 2221790} \geq \lambda_1, \\
&\frac{321100 - Z_2}{321100 - 170620} \geq \lambda_2, \\
&\frac{182870 - Z_3}{182870 - 119367} \geq \lambda_3, \\
&\frac{2239650 - Z_4}{2239650 - 1644500} \geq \lambda_4, \\
&\frac{12000000 - D}{1100000} \geq \lambda_5, \\
&\frac{D - 10500000}{400000} \geq \lambda_5,
\end{aligned}
\tag{6.6}
$$

where Z_1, Z_2, Z_3, Z_4, D are given in eqs. (6.1) and (6.5), along with the constraints in eqs. (6.2) to (6.4).

For the NGP approach, we take $D_1 = 10500000, D_2 = 10900000, D_3 = 12000000, \delta_D = .99, \theta_D = .3, \lambda_D = .01$. One can calculate easily the truth, indeterminacy, falsity membership functions for $\tilde{D}$ and the objective functions using eqs. (3.4), (3.5), (3.6) and (5.1), (5.2), (5.3) and table 4 respectively. The NGP approach is given as follow (5.7):

$$
\begin{aligned}
&\min\ \Sigma_{l=1}^{5} w_l((1-\alpha_l) + (\gamma_l) + \beta_l) \\
&\text{subject to the constrains,}
\end{aligned}
$$

$$
\begin{array}{llllllll}
\frac{2293665.6 - Z_1}{71875.6} & \geq \alpha_1 & \frac{Z_1 - 2243352.68}{50312.9} & \leq \gamma_1 & \frac{Z_1 - 2236165.12}{57500.5} & \leq \beta_1 \\
\frac{321100 - Z_2}{150480} & \geq \alpha_2 & \frac{Z_2 - 215764}{105336.} & \leq \gamma_2 & \frac{Z_2 - 200716}{120384} & \leq \beta_2 \\
\frac{182870 - Z_3}{63503} & \geq \alpha_3 & \frac{Z_3 - 138417.9}{44452.1} & \leq \gamma_3 & \frac{Z_3 - 132067.6}{50802.4} & \leq \beta_3 \\
\frac{2239650 - Z4}{595150} & \geq \alpha_4 & \frac{Z_4 - 1823045}{416605} & \leq \gamma_4 & \frac{Z_4 - 1763530}{476120} & \leq \beta_4 \\
\frac{(D - 10500000).99}{400000} & \geq \alpha_5 & \frac{(12000000 - D).99}{1100000} & \geq \alpha_5 & \frac{7750000 - 0.7D}{400000} & \leq \gamma_5 \\
\frac{0.7D - 7300000}{1100000} & \leq \gamma_5 & \frac{9850000 - 0.9D}{400000} & \leq \beta_5 & \frac{0.9D - 9700000}{1100000} & \leq \beta_5
\end{array}
\tag{6.7}
$$

where Z_1, Z_2, Z_3, Z_4, D are given in eqs. (6.1) and (6.5), along with the constraints in eqs. (6.2) to (6.4).

Table 5:

	Z_1	Z_2	Z_3	Z_4
FGP approach (6.6)	2273582.988	248142.2467	134341.3432	1968186.806
NGP approach(with weights(6.7))	2243352.680	243860.3333	131058.5429	1925367.672
NGP approach(without weights (3.12)	2258260.159	245971.8743	132677.3910	1946483.082

Table 6:

	x_1	x_2	x_3	x_4	x_5	x_6
FGP approach (6.6)	2400000	360000	2783000	2402691	**1523011**	1431297
NGP approach(with weights(6.7))	2400000	360000	2783000	2402691	**1380280**	1431297
NGP approach(without weights (3.12)	2400000	360000	2783000	2402691	**1450665**	1431297

Table 7:

Weights	Z_1	Z_2	Z_3	Z_4
$w_1 = 0.1, w_2 = 0.3, w_3 = 0.2, w_4 = 0.2, w_5 = 0.2$	2236165.120	227233.7668	134751.5086	1939102.007
$w_1 = 0.15, w_2 = 0.25, w_3 = 0.1, w_4 = 0.2, w_5 = 0.3$	2243352.680	243860.3333	131058.5429	1925367.672
$w_1 = 0.1, w_2 = 0.1, w_3 = 0.1, w_4 = 0.3, w_5 = 0.4$	2273582.988	248142.2467	134341.3432	1968186.806

As it can be seen in table 5, the NGP approach (with weights) yields the best result among other methods for each objective function for the chosen weights. Finally, we provide the results of the proposed NGP approach for different weights. The results are given in table 7.

7. Conclusion

On its own, a supplier selection problem in a quantity discount environment is a very complicated task. Also, there may exist vagueness and imprecision in the goals of the decision maker and market demand. To approximate the imprecise aggregate demand, we have used the triangular neutrosophic numbers and to deal with the vagueness we have used neutrosophic goal programming. The proposed generalized models can deal with imprecise market demand as well as the vagueness present in the goals of the decision maker. As oppose to the studies that already exist, our study also includes the case where the decision maker cannot decide about the goals with certainty, by including indeterminacy membership function. As shown in the numerical example, neutrosophic goal programming method yield better value for the objective functions than the fuzzy goal programming method for the given weights.

This study has been done assuming that no shortages are allowed. We also assumed that a single type of item is being supplied.

The proposed model can be expanded if we assume shortages are allowed as well as multi-item are consided . The proposed model can be solved using particle swarm optimization.

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