Small scale Suspended Interferometer for Ponderomotive Squeezing (SIPS) as test bench of the EPR squeezer for Advanced Virgo

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April 17, 2020

Abstract

We are developing a small-scale interferometer with monolithic suspension of test masses (SIPS), that will be sensitive to the quantum radiation pressure noise in the audio frequency band of gravitational wave detectors. In the same time, a table-top experiment for the frequency dependent squeezing generation, through the Einstein Podolsky Rosen (EPR) principle, is under construction. SIPS interferometer, being designed to achieve the quantum radiation pressure noise limit exploiting high finesse optical cavities, turns out to be a suitable test bench for the EPR technique, before a possible integration in Virgo for broadband quantum noise reduction. In this work, we describe the concept of the SIPS experiment, the most important noises affecting this setup, and we briefly present the current status and the plan for the integration with the EPR squeezing experiment.

DOI: 10.5281/zenodo.3569196

1 Introduction

In 2015, after many years of R&D efforts done by the LIGO-Virgo collaboration for the upgrade of ground based gravitational wave detectors to the second generation, for the first time it has been possible a direct observation of a gravitational wave (GW) event [1]. In the following years, many other GW events have been detected in coincidence by Virgo and the two LIGO detectors [2]. Nevertheless, the present generation of detectors is facing the limit in sensitivity due to the quantum nature of light: the so-called Standard Quantum Limit (SQL) [3, 4]. Therefore, any upgrade aiming at increasing the sensitivity should imply broadband quantum noise reduction techniques. Virgo and LIGO already adopt frequency independent squeezers, which have successfully demonstrated to reduce the quantum noise in the high frequency range, where shot noise is dominant (above 200Hz) [5, 6]. In the very near future, GW detectors will be also limited by quantum noise in the low frequency range (below 100Hz), where quantum radiation pressure noise becomes dominant. Therefore, it is crucial to develop table-top experiments, such as SIPS, aiming at broadband quantum noise reduction in the audio band (10Hz-1kHz), exploiting the so called-frequency dependent squeezing (FDS). At present, among the GW community, the common followed path to produce frequency dependent squeezing consists in the use of a detuned filter cavity to experience the rotation of the squeezing quadrature in function of the frequency [7, 8]. Hence, in the very next future, Virgo and LIGO will be able to produce broadband squeezing thanks to the introduction of high-finesse optical filter cavities of hundreds of meters length. Nevertheless, many R&D works have been done on this field. An interesting alternative to the use of filter cavities, is based on the EPR technique [9, 10], which exploits the generation of two entangled beams where one has the same frequency of the carrier laser inside the GW detector, and the other is detuned such to experience its travel along the GW interferometer as the effect of a detuned filter cavity. The main advantage of an EPR squeezing setup consists in the compactness with respect to the need of deep infrastructure interventions for the realisation of the hundreds of meters long filters cavities near GW detectors arms [15]. The optomechanical coupling, in an optical cavity, between a suspended mirror and vacuum fields fluctuations, the so-called optical spring effect, is also one of the most interesting alternatives to produce FDS through the ponderomotive squeezing [11, 12]. This constitutes the main scientific motivation which leads us to develop the experiment presented in this paper: a small-scale Suspended Interferometer for Ponderomotive Squeezing (SIPS). SIPS is a table-top experiment designed to be sensitive to the quantum radiation pressure noise in the audio frequency band of GW detectors (10Hz-1kHz). In order to be able to produce FDS in the audio frequency band, seismic noise at low frequency must be suppressed. This can be possible by suspending the whole interferometer to a chain of mechanical filters like a superattenuator of Virgo [13, 14] . In the same time, we are developing a table-top experiment for the FDS generation through the Einstein Rosen Podolsky (EPR) principle [15]. These two experiments are growing up in parallel, and EPR technique needs to be tested in an high finesse optical cavity achieving the radiation pressure noise limit. Therefore, SIPS turns out to be a suitable test bench for the EPR technique, before a possible integration of this setup in Virgo for the broadband quantum noise reduction.

2 Suspended Interferometer for Ponderomotive Squeezing

The Standard Quantum Limit can be seen as a statement of the Heisenberg principle, where the uncertainties on the two quadratures of the quantized electromagnetic field of laser light, amplitude X_1 and phase X_2, correspond to radiation pressure noise (RPN) and shot noise (SN), respectively [16]. Quantum noise is given by the uncorrelated sum of RPN and SN, and SQL is the minimum of QN for a given value of laser power. SN and RPN together enforces SQL only if they are uncorrelated, then, in principle, SQL can be overcome by creating correlations between these two noises [3]. When the two uncertainties are equal, we are in presence of a coherent state, but squeezing one quadrature at the expenses of the other quadrature, we can reduce one of the two contributions of quantum noise.

Such kind of state is called squeezed state. If a phase-squeezed state is injected at the dark (detection) port of an interferometer for GW detection, it reduces quantum shot noise at high frequencies without requiring higher laser power [5, 6, 18]. Nowadays, squeezers based on Optical Parametric Oscillator cavities are the most efficient ones [17, 18], even if affected by photo-thermal fluctuations at low frequency. One of the most promising alternative is Ponderomotive Squeezing (PS) generated in a RPN-limited cavity with mirrors suspended in order to have the proper pendulum resonance below the GW detectors typical frequency band. The effect of radiation pressure can be exploited in optomechanical cavities to generate a quantum correlation between phase (SN) and amplitude noises (RPN), in the so-called ponderomotive technique to achieve squeezing of light [11, 12]. The feasibility of an optomechanical quantum-correlated system, such as the optical spring, has been demonstrated in the past years with small membrane resonators in the MHz region [19] and more recently (2019) with micro-resonators [20] at lower frequencies. Present technology assures the possibility to efficiently filter out seismic noise [14, 13]. Then, it has been possible to design a macroscopic optomechanical cavity, with suspended mirror, dominated by the quantum radiation pressure noise. This is feasible with a mirror suspension system tailored in such a way to have low enough thermal noise associated to the suspensions and with the introduction of low dissipating mirror coatings. In particular, the use of high reflective mirrors with Advanced Virgo-like coatings and $10g$ mass suspended with monolithic Virgo-like suspension system optimises both mirror and suspension thermal noise below the radiation pressure fluctuations, fundamental condition to have the optical spring effect observable and exploitable.

3 Noise sources

Detailed analysis has been carried out before validating the suspension system which better satisfies the thermal noise requirements, as well as the feasibility constraints. We assume that the system is kept under vacuum and at room temperature. In our model, the fused silica beam splitter and the fused silica mirrors, forming the two Fabry-Pérot cavities, are suspended by two wires from an intermediate stage made of steel (called marionette) which in turn is hung, from its suspension point, to a bigger plate through one steel wire. To increase the suspension mechanical quality factor (i.e. to reduce the associated thermal noise [21]) each one of the interferometer main optics is suspended with two thin silica (SiO_2) fibers, silicate-bonded at the two flat sides of the bulks. At the top level, the fiber anchors are silicate-bonded to super-polished steel interfaces. For all the parameters (length l and diameter d of the fiber, suspended mass m) of the suspension that we have investigated, we evaluated the thermal noise of the mirror suspension and the thermal noise related to the high-finesse coating of the mirror. Then, we compared them to the quantum radiation pressure noise evaluated considering an input laser power $P_{in} = 2.5W$ and a cavity finesse $\mathcal{F}$ of the order of few tens of thousands [22]. This preliminary analysis allowed us to select those parameter combinations which appear to be a good trade-off for the feasibility of a quantum radiation pressure noise-limited interferometer with suspended mirror. Then, we decided to analyse a double pendulum configuration, instead of a simple pendulum one. Indeed, a double stage suspension system, since it allows to both attenuate the seismic noise at the level of the suspended mirror and to reduce the suspension thermal noise, turns out to be more suitable for our purpose.

3.1 Suspension Thermal Noise

The thermal noise for the double pendulum system has been estimated by using the approach of the analytic model of the sequential pendula [23]. The double pendulum is composed of a first steering stage (Marionette) plus the mirror. The equations of motion of the system give the impedance matrix $Z_{ij}(\omega)$ which takes into account all the dissipation processes present in the system [24, 25] [26, 27]. Then, in order to calculate the thermal noise of the mirror suspension, which in our case corresponds to the element 2,2 of the impedance matrix, we have to apply the fluctuation dissipation theorem (FDT) [28] to the element 2,2 of the inverse of the impedance matrix:

$$X_{therm,2}(\omega) = \sqrt{\frac{4k_B T}{\omega^2} \Re\{[Z^{-1}(\omega)]_{22}\}}, \tag{1}$$

where k_B is the Boltzmann constant, and T the temperature. With a similar approach, we estimate the transverse vibration modes of the suspension wire, which are called violin modes, in analogy with vibrating strings. Indeed, in suspension wires, when the tension due to the applied load dominates over their internal elasticity, they behave like vibrating strings. This effect, in terms of the power spectral density of the mirror displacement, can be modeled according to the approach made by Gonzáles and Saulson ($X_{vio}(\omega)$) [29]. The total thermal noise of the double suspension system for the considered mirror is given by the sum of the two contributions as follows:

$$X_{ThNS}(\omega) = \sqrt{X^2_{therm,2}(\omega) + X^2_{vio}(\omega)}. \tag{2}$$

3.2 Mirror Thermal Noise

The mirror thermal noise (ThNM) due to the reflective coating is calculated according to Levin's approach [30] and using a finite element analysis (FEA) with ANSYS® software. The mirror thermal noise is then given by:

$$X_{ThNM}(\omega) = \sqrt{\frac{8k_B T}{\omega F_0^2} U_{mir}\phi_{tot}}, \tag{3}$$

where U_{mir} is the total strain energy stored by the suspended mirror under an impinging Gaussian pressure:

$$P = \frac{2F_0}{\pi w^2}\, e^{-\frac{2r^2}{w^2}} \tag{4}$$

given the integrated unitary force F_0, the beam waist w of the laser on the mirror surface, and the radial coordinate r on the mirror surface. All the dissipation processes are taken into account by the total loss angle ϕ_{tot}, which is the sum of all the dissipative contributions calculated with the FEA and according to this formula:

$$\phi_{tot} = \phi_B + \sum_{lay} \frac{U_{lay}}{U_{tot}} \phi^{lay}_{mat} \tag{5}$$

where we considered the loss angles related to the bulk ϕ_B and to the layer material ϕ^{lay}_{mat} multiplied by the fraction of strain energy of the layer U_{lay}, over the total strain energy U_{tot}. U_{lay} and U_{tot} are evaluated through finite element analysis. In the double pendulum model for SIPS, the dissipative layers considered are those related to the coating on the mirror intra-cavity surface and to the silicate bonding layers used to monolithically attach the suspension wires to the mirror. In order to reach a finesse in the range of some tens of thousands, we considered low refractive index (LR) and high refractive index (HR) coating layers in an Advanced Virgo-like configuration (see table 1) [31]. For the silicate bonding layer related to the mirror suspension we considered a thickness of $315nm$, which is easily achievable applying the silicate bonding between the mirror lateral face and the SiO_2 fiber anchor. The intrinsic losses of the material composing the silicate bonding can be considered equal to $\phi_{sibon} = 0.1$, while the density is the same of the bulk material 2200 kg/m^3 and the Young modulus is 73 GPa. Therefore, we obtain:

$$\phi_{tot} = \phi_B + \frac{U^{HR}_{coat}}{U_{tot}}\phi^{HR}_{mat} + \frac{U^{LR}_{coat}}{U_{tot}}\phi^{LR}_{mat} + \frac{U_{SB}}{U_{tot}}\phi_{SB}, \tag{6}$$

where the strain energy ratios for the coatings (U^{LR}_{coat}, U^{HR}_{coat}) and for the silicate bonding (U_{SB}) layers are calculated from FEA simulations.

3.3 Seismic noise

Another noise source, which can prevent SIPS experiment from being a quantum radiation pressure noise limited interferometer, in the audio band, comes from the seismic background. To a first approximation, the power spectral density of the position fluctuations due to the seismic noise are proportional to ω^{-2}. A representative model of the seismic background on Earth is given by the Peterson's models [32]. Gravitational wave detectors, like Virgo and LIGO, demonstrated that it is possible to filter out this noise, even at low frequency, using passive and active systems. In particular, in Virgo the Superattenuator (SA) is capable of 180dB of attenuation at 10Hz [13, 14]. To exploit this seismic attenuation, SIPS has been initially designed to be suspended from the last steering filter of the SA chain. Indeed, a full scale prototype of a Superattenuator is currently available for test in the facility located at EGO (Virgo site) and plans were made to use it for the SIPS experiment. Before this integration, we decided to use the SIPS interferometer as a tabletop demonstrator of the EPR experiment at 1500W R&D laboratory at EGO (Virgo site). Hence, in the noise budget of SIPS we take into account the seismic noise measured at this location [33]. Simple passive seismic dampers can be installed below the vacuum chamber hosting the experiment to further reduce the seismic noise level in the band of interest.

3.4 Quantum radiation pressure noise

All the noise sources described in the previous subsections, should be compared to the quantum radiation pressure noise (RPN). For a pendulum suspension in a Fabry-Pérot cavity, RPN mirror fluctuations can be modeled following a classical approach [22]:

$$X_{RP}(\omega) = \frac{2\mathcal{F}}{\pi m}\sqrt{\frac{8h_P P_{in}/(\lambda c)}{(\omega_p^2 - \omega^2)^2 + (\omega_p^2\phi_p(\omega))^2}} \tag{7}$$

where h_P is the Planck constant, c the speed of light, ω_p the pendulum resonance of the suspended mirror of mass $m = 10g$, $\phi_p(\omega)$ the pendulum loss angle which takes into account all the dissipative process in the suspension (see sec. 3.1) [24, 25, 26, 27]. RPN has been estimated by assuming a finesse of $\mathcal{F} = 3\cdot 10^4$, a wavelength $\lambda = 1064nm$ and an input laser power of $P_{in} = 2.5W$ (red curve in figure 1). We also evaluated the possibility to double the power to $P_{in} = 5W$ (green curve in figure 1), but this seems to not increase significantly RPN with respect to the thermal noises.

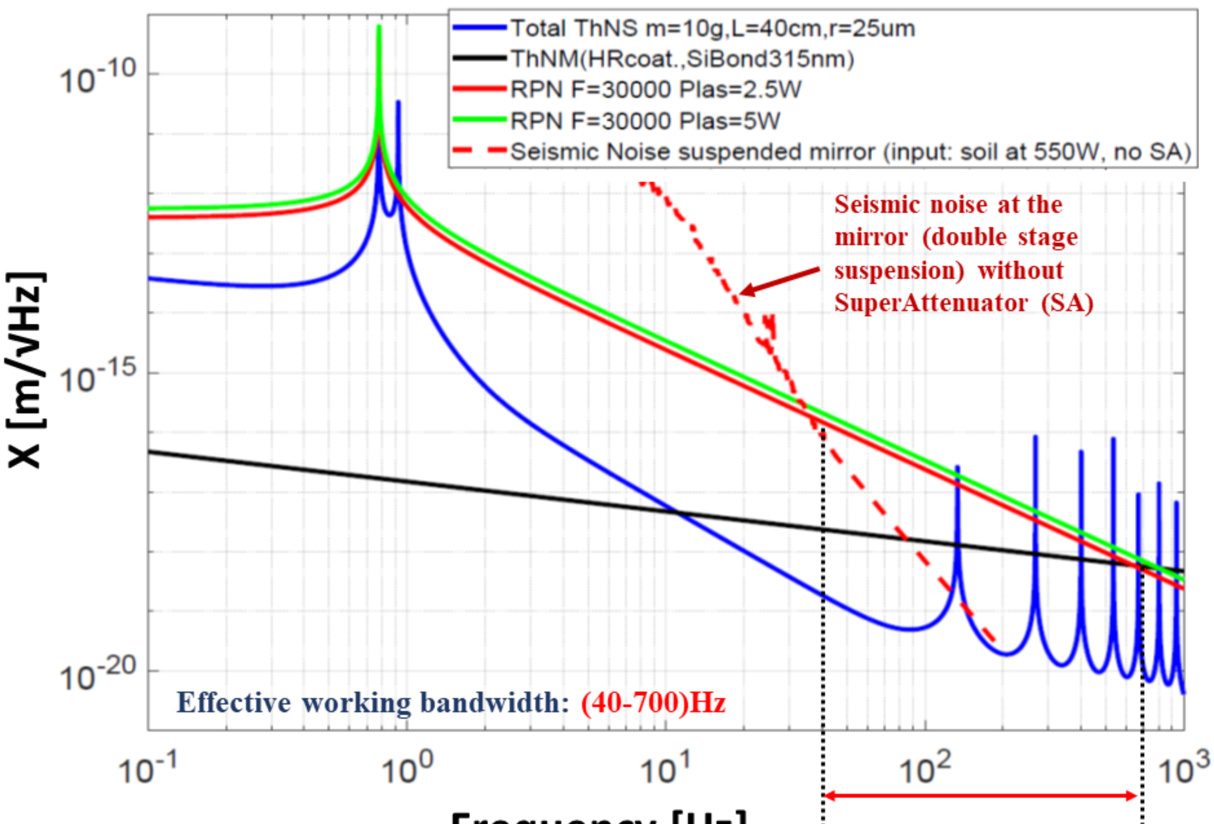

Figure 1: Expected noise budget of SIPS compared with the seismic noise at Virgo site (red dashed curve). Thermal noise of the suspension (blue curve) and of the mirror (dark curve) estimated for the end mirror of SIPS is almost three order of magnitude below the radiation pressure noise at 10Hz (red and green curve). This assures a broadband frequency range of working for SIPS. Nevertheless, for the integration with the EPR experiment without the use of a superattenuator, we should consider the seismic noise at Virgo site, which still leaves us working frequency range of 40 - 700Hz.

3.5 Other sources of noise

SIPS will take advantage of an interferometric configuration working in dark fringe condition, which, as well known, cancels out all the noise sources which are common to the two arms, like those related to the laser instabilities (RIN, etc.). Therefore, in this first noise budget estimation for SIPS integration with EPR setup, we can neglect all the other common mode noise sources.

4 SIPS status and integration with EPR

From the suspension thermal noise analysis done, and also according to technical feasibility of thin silica fibers, the best combination of parameters for the end mirror of the SIPS interferometer is: suspended mirror mass $m = 10g$, fiber diameter $d = 50\mu m$ and length $l = 40cm$. Even increasing the mirror mass to $m \sim 100g$, the same values for the diameter and length of the silica fiber are a good trade-off. The mirror mass of $10g$ is compliant with a cylindrical bulk made of fused silica with diameter $1''$ and thickness $t = 1cm$, values that we choose for the end mirrors of the SIPS interferometer. For the beam splitter and input mirrors we can use a slightly heavier mass, but still within the order of magnitude of $100g$. Therefore, we choose a value of $300g$ which corresponds to a fused silica bulk having diameter $3''$ and thickness $3cm$. In order to reach the suitable value for the $finesse$, a high reflective coating on its intra-cavity surface, providing a transmissivity of T=1ppm at $1064nm$ is required. Then, in the finite element analysis described in sec.3.2, we have considered Advanced Virgo-like coatings for the cavity end mirror (see table 1). Moreover, in the analysis we set a beam waist on the mirror surface equal to $w = 254\mu m$, value which is 1/100 of the end mirror diameter, and satisfies the stability condition for the optical cavity.

Table 1: Parameters used in our analysis for the coating layers of the Fabry-Pérot cavity end mirror. We considered Advanced Virgo-like coatings [31].

Coating	**High Refractive**	**Low Refractive**
Material	Ta_2O_5	SiO_2
Density	8200 kg/m^3	2200 kg/m^3
Young's modulus	140 GPa	72.2 GPa
Loss angle	$2 \cdot 10^{-4}$	$5 \cdot 10^{-5}$
Poisson ratio	0.26	0.17
Thickness	2.393 μm	4.1086 μm

The experiment setup of SIPS is currently under development, with the majority of the mechanical parts already produced and assembled. All the main optics substrates have been procured, and the coatings have been done on the substrates. The monolithic suspension system has been designed, and the production of thin silica fiber has been successfully tested. The local control system of the double mirror suspension has been developed and tested on a dummy aluminium end mirror suspended to the steel marionette trough a niobium wire of $60\mu m$ section. This local control system is based on optical levers, which are used for the readout, and a system of coils and magnets for the actuation. The local control system will be implemented to all the main optics suspended. The logic of the global control system is under study, with particular regards to the compatibility with the EPR setup. The EPR experiment is also under development [15], and it will be an upgrade of the automated squeezing bench that we have in the R&D squeezing laboratory at 1500W of Virgo site [34]. The general design for the integration of SIPS with EPR experiment at 1500W has been developed, whereas the detailed design still needs to be finalised in the very next future. The expected noise budget of SIPS is shown in figure 1. Thermal noise of the suspension (blue curve) and of the mirror (dark curve), estimated for the end mirror of SIPS, is almost three order of magnitude below the quantum radiation pressure noise at 10Hz (red and green curve). This assures a broad frequency range of working for SIPS. Nevertheless, since the SA research facility at EGO cannot be ready before the planned integration with EPR setup, in the noise budget of SIPS we adopt a conservative approach considering the seismic noise at Virgo site as seen by the double-stage suspended mirror (red curve). The plot of figure 1, tells us that, even without the use of a superattenuator, SIPS interferometer can be used as a test bench for the EPR experiment. When integrated, it can assure a quantum radiation pressure limited bandwidth of $(40 - 700)Hz$.

5 Conclusions

The study presented in this paper demonstrates that it is possible to realise a small scale suspended interferometer where the test mirrors which are $10g$-scale, not micro-scale, are in quantum radiation pressure noise regime. Therefore, it can be used to generate ponderomotive squeezing. Common mode noises, such as RIN, are indeed a potential limit in an optical cavity. For this reason, we chose a design based on an interferometer with symmetric arms, and optics in the two Fabry-Pérot cavities with the same specifics. Then, choosing a working point close to the dark fringe, we can exploit the self-subtraction of fluctuations due to the laser. Working at the dark fringe is also motivated by the need to produce a squeezed vacuum field at the output port of the system. Thanks to a suitable suspension system composed of a double pendulum with a monolithic suspension, suspension thermal noise is lower (of a factor 600 at 10Hz) than the quantum radiation pressure noise. The use of Advanced Virgo-like coatings allows to keep the mirror thermal noise low enough (of a factor 600 at 10Hz) to make radiation pressure noise dominating. The seismic noise can be attenuated by attaching the interferometer bench to the last suspension stage of a superattenuator facility (a chain of mechanical filter plus an inverted pendulum) of the same kind of those used to suspend the Virgo test masses [14],[13]. Active filters, such as those used in LIGO, are a possible alternative to be taken into account to extend the operation bandwidth. Once completed, SIPS will be capable to produce frequency-dependent squeezing ranging in the detection band of ground based gravitational wave interferometers. A possible spin-off of SIPS experiment consists in interfacing it with the EPR experiment for frequency-dependent squeezing generation. Indeed, the EPR experiment that we are developing at Virgo site [15] needs to be tested in a quantum radiation pressure noise limited interferometer before a possible implementation in large scale GW interferometers like Virgo. Even without using a superattenuator, the tabletop setup of SIPS will ensure a quantum radiation pressure noise-limited bandwidth of $(40 - 700)Hz$, in which the EPR squeezing effect can be observed.

Acknowledgements

We want to acknowledge all the groups which funded this study from the very beginning: in particular INFN CSN5, the PRIN PPPS and the POLIS collaboration for the initial R&D of this experiment. In particular, we want to thank all the people who support us from the Pisa INFN and EGO: A. Basti, F. Frasconi, G. Cella and A. Pasqualetti. A special thank goes to F. Piergiovanni and M. Montani for the feasibility study of the thin silica fibers.

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