Application of AK-SYS method for time-dependent reliability analysis

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Abstract :

The aim of this paper is to propose an approach to address time-variant reliability problems using system reliability methods. The remaining challenge in time-variant reliability is related to problems with low failure probabilities, high dimensionality, and including costly-to-evaluate performance functions. One of the main steps towards resolving time-dependent problems is to descretize the desired time interval. This step creates a similarity between time-variant and system reliability problems. Therefore efficient system reliability methods can be used to address time-variant reliability problems. AK-SYS is an efficient reliability method for systems that can be employed for performance functions that are expensive to evaluate. In this paper, application of this method for time-variant reliability problems is investigated.

Mots clefs : time-variant reliability, Kriging, system reliability

1 Introduction

Performance in many of engineering systems is a function of time. Time-variant reliability shows how properly a system fulfill its duties under given conditions during its lifetime. This can provide an indicator for maintenance and inspection planning of structures and life cycle cost analysis [1]. Involving time makes reliability calculations more complicated since it adds an extra dimension to the problem. The challenge in time-variant reliability methods is to have a reasonable trade-off between the accuracy and efficiency especially for problems with low failure probabilities and/or high dimension. Hence, the aim of this study is to introduce a new approach for time-variant reliability analysis with a reasonable level of efficiency and accuracy.

Different methods have already been developed for time-variant reliability analysis. They are generally categorized into two groups namely first passage-based and extreme value-based methods. Methods in the first category use the out-crossing rate to estimate the failure probabilities. Rice formula [2], PHI2 [3] and PHI2+ [4] are among the popular methods in the first group. Methods in the second group are based on the estimation of the extreme response of a time-variant performance function. Estimation

of the extreme response is not so simple for costly-to-evaluate performance functions. Hence, meta-models such as Kriging and Polynomial Chaos Expansion (PCE) are used to approximate the extreme response. NERS [5], Mixed-EGO [6], and SILK [7] are among the Kriging based methods and t-PCE is a PCE-based method [8].

Discretizing the time is one way to overcome the difficulty of dealing with continues time in most of the time-variant reliability methods. The desired time interval is divided in a finite number of time nodes and one can consider a performance function for each time node. This makes the problem very similar to the reliability analysis of a serially connected system. Therefore, the goal here is to employ an efficient system reliability method for time-variant reliability problems. AK-SYS is a Kriging-based reliability method for systems that is going to be used for this reason [9].

The reminder of this paper is organized as follows: Section 2 reviews time-variant reliability. System reliability and AK-SYS is then reviewed in Section 3. Section 4 is related to the proposed methodology. Two case studies are used in Section 5 to show the accuracy and efficiency of the new method. A short conclusion is finally provided in Section 6.

2 Time-variant reliability analysis

In most of the reliability problems, time-variant analysis is inevitable due to the temporal nature of material properties, loading, and geometrical parameters. Time-variant reliability analysis is more complicated than time-invariant analysis by introducing the time into the problem. The general time-dependent performance function $G(\mathbf{X}, \mathbf{Y}(t), t)$ involves input random variables $\mathbf{X}$, random processes $\mathbf{Y}(t)$ and time t. However, by using some process representations such as Karhunen-Loeve (KL) transformation or spectral representation [10] the general problem can be reduced to an explicit performance function ($G(\mathbf{X}, t)$).

The main objective of a time-variant reliability analysis is to calculate the cumulative probability of failure for a system. It tries to measure the probability of having at least one failure over a given period of time $[t_0, t_l]$, see Equation 1. In the other hand, an instantaneous failure probability can be defined for each time instant t as in Equation 2. Figure 1 shows the difference between these two failure probabilities [11].

$$P_{f,c}(t_0, t_l) = \text{Prob}(\exists \tau \in [t_0, t_l], G(\mathbf{X}, \tau) \leq 0) \tag{1}$$

$$P_{f,i}(t) = \text{Prob}(G(\mathbf{X}, t) \leq 0) \tag{2}$$

Descretizing the lifetime of a system allows an equivalence between time-variant and system reliability analyses. This similarity let us to employ system reliability methods to solve time-variant reliability problems. Among recently developed methods for system reliability, AK-SYS has a promising efficiency. Consequently, application of this method for time-variant reliability analysis will be explained.

3 System reliability and AK-SYS

Reliability analysis at the system level is different than for components since a system can have several failure modes. This study considers the reliability of serially connected systems only. In series systems

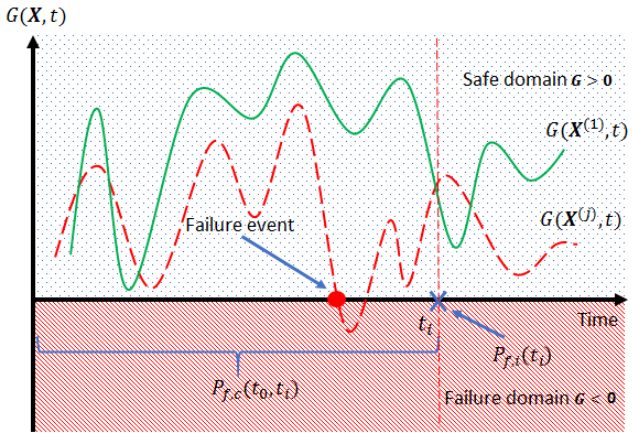

Figure 1: An illustration of cumulative and instantaneous failure probabilities in time-variant problems

with p components, failure happens if one component fails. The jth failure event of the system can be formulated as

$$E_j = \{G_j(\mathbf{X}) \leq 0\} \tag{3}$$

where $G_j(.),\ j = 1, ..., p$ denotes the performance function defined for jth component. The system's failure probability is defined as the probability of the union of all events.

$$P_f = \text{Prob}(\cup_{j=1}^{p} E_j) = \text{Prob}(\cup_{j=1}^{p} G_j(\mathbf{X}) \leq 0) \tag{4}$$

AK-SYS is one of the efficient methods to calculate this failure probability especially when computationally expensive performance functions are involved. It works in the same way as Monte Carlo simulation (MCS). However, in AK-SYS each performance function of the system is replaced with Kriging meta-model. One important step in using meta-models is the enrichment process to reach a reasonable level of accuracy. For this reason, AK-SYS uses an active learning process using the learning function U that is introduced in [12]. A modification has been done on this learning function to make it suitable for systems. The new learning function is called composite criterion learning function and it is formulated in Equation 5 by:

$$U_s(\mathbf{x}^{(i)}) = \frac{|\hat{G}_s(\mathbf{x}^{(i)})|}{\sigma_{\hat{G}_s(\mathbf{x}^{(i)})}} \tag{5}$$

This learning process is applied on N Monte Carlo generated input samples. For each point $\mathbf{x}^{(i)},\ i = 1, ..., N$ the minimum performance function $\hat{G}_s$ among $\hat{G}_j(\mathbf{X}),\ j = 1, ..., p$ is found first. The learning process is then performed on this function to find the best training point. The point $\mathbf{x}^{(i)}$ that minimizes U_s is the best training point. The Kriging meta-model $\hat{G}_s$ is subsequently updated. The learning process stops when for all sample points $\min U_s \geq 2$. Using this learning process for the enrichment process makes AK-SYS very efficient because it only updates the performance functions which significantly impact the system's failure. Besides, it is a general method meaning that it can be applied on different kinds of limit states since it does not make any assumptions on them.

4 Proposed methodology

4.1 From time-variant to system reliability

Discretizing the time is one step to tackle the difficulty of dealing with time-variant problems. In this manner, the lifetime of a system will be divided into N_t nodes. An instantaneous performance function $G_n(\mathbf{X})$ is defined for each node $t_n,\ n = 0, ..., N_t$. Also, a failure event can be defined for each time node as expressed in Equation 6. Figure 2 shows an illustration of the time descretization.

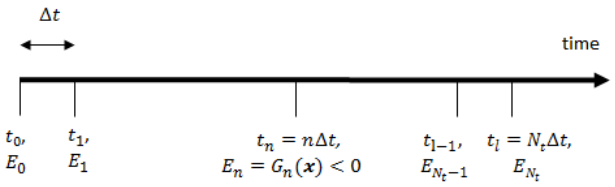

Figure 2: A representation of time discretization

$$E_n = \{\mathbf{x} : \ G_n(\mathbf{x}) < 0)\} \tag{6}$$

Therefore, the cumulative failure probability in time-variant reliability problems can be approximated by Equation 7.

$$P_{f,c}(t_0, t_l) \approx \text{Prob}\{\cup_{n=0}^{N_t} E_n\} = \text{Prob}\{\cup_{n=0}^{N_t} (G_n(\mathbf{X}) < 0)\} \tag{7}$$

Comparing Equations 7 and 4 helps us to highlight the similarity between time-variant and system reliability analyses. In the next part, application of AK-SYS for time-variant reliability assessment is introduced.

4.2 AK-SYS for time-variant reliability analysis

The main reason of introducing the new approach using AK-SYS is to exploit the efficiency and generality of this method for time-variant reliability assessment. Different steps of the proposed methodology is depicted in Figure 3. The algorithm starts by discretizing the desired lifetime into N_t time nodes, generating the initial Monte Carlo population of size N_{MCS} from the joint distribution of $\mathbf{X}$, and preparing the initial DOE of size N_{DOE}. The DOE is subsequently used to calibrate N_t Kriging meta-models related to N_t time nodes. The composite criterion learning function U_s is employed in the next step. The sample point that minimizes the learning function is used to enrich the DOE and to train the meta-models. The learning process continues until the minimum value of learning function is greater than 2 over the Monte Carlo population.

The cumulative probability of failure can be calculated in the next step. This can be done in the same manner as MCS where the original performance functions are replaced by Kriging meta-models. The ratio between the failed realizations of the time-variant performance function over the total number of generated realizations N_{MCS} realizations calculates the cumulative failure probability. The coefficient of variation of $\hat{P}_{f,c}$ computed by Equation 9 is checked. This helps to make sure that the resulting failure probability is close enought to the result of MCS.

$$\hat{P}_{f,c}(t_0, t_l) = \frac{\hat{N}_{fail}(t_0, t_l)}{N_{MCS}} \tag{8}$$

$$C.O.V_{\hat{P}_{f,c}} = \sqrt{\frac{1 - \hat{P}_{f,c}(t_0, t_l)}{N_{MCS} \times \hat{P}_{f,c}(t_0, t_l)}} \tag{9}$$

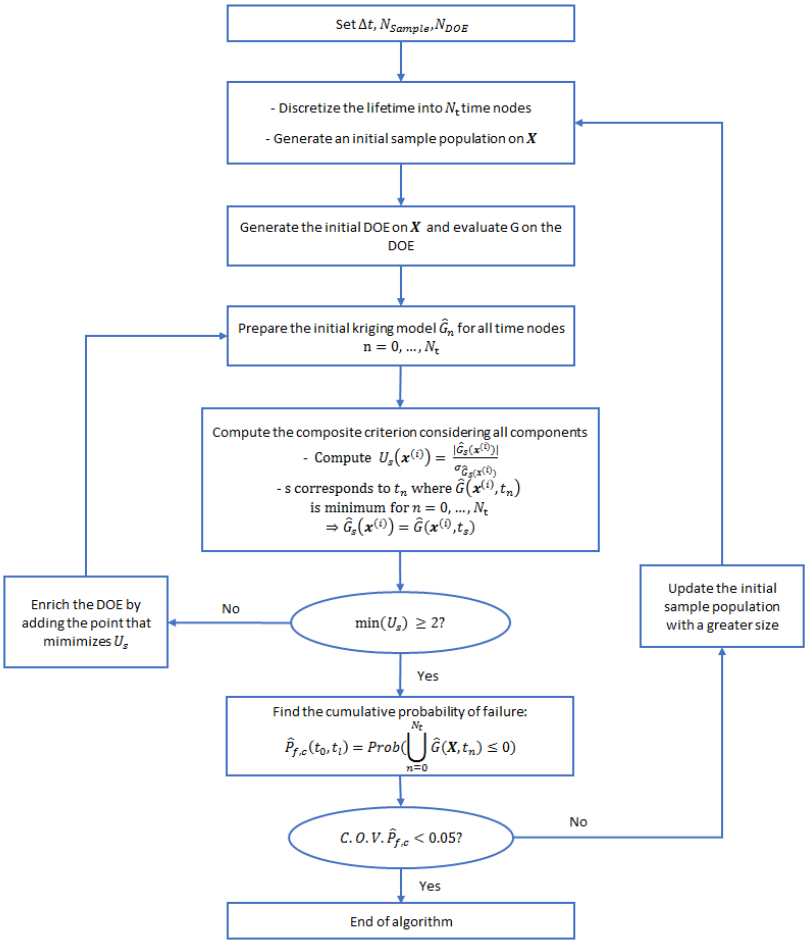

Figure 3: General algorithm for proposed methodology

5 Case studies

The new method is tested on two numerical cases in this section and the results are compared with MCS. To demonstrate the validity of the model, the number of realizations that are misclassified is calculated. This can be calculated by Equation 10. The relative percentage error ,see Equation 12, can also be used for the sake of comparison if there is any misclassification. It should be noted that for comparison purpose, the same time discretization strategy is used for both methods.

$$N_{misclass} = \sum_{n=1}^{N_t} \sum_{i=1}^{N_{MCS}} I(\mathbf{x}^{(i)}, t_n) \tag{10}$$

where

$$I(\mathbf{x}^{(i)}, t_n) = \begin{cases} 1 & if \quad \hat{G}(\mathbf{x}^{(i)}, t_n) \times G(\mathbf{x}^{(i)}, t_n) < 0 \\ 0 & \text{Otherwise} \end{cases} \tag{11}$$

$$Error(\%) = \frac{|P_{MCS} - \hat{P}_{f,c}|}{P_{MCS}} \times 100 \tag{12}$$

5.1 Example 1: A nonlinear model

A nonlinear performance function with only one random variable has been chosen for illustration purpose. This performance function is formulated in Equation 13. It includes one random variable X where $X \sim N(10, 1)$ [6].

$$G(X, t) = 0.014 - \frac{1}{X^2 + 4}\text{sin}(2.5X)\text{cos}(t + 0.4)^2 \tag{13}$$

The cumulative probability of failure over $[1, 2.5]$ is given by:

$$P_{f,c}(1, 2.5) = \text{Prob}(\exists \tau \in [1, 2.5],\ G(X, \tau) \leq 0) \tag{14}$$

A Monte Carlo population of size 5×10^5 and DOE of size 10 is used. Table 1 shows the results for different descritization strategies where the probabilities calculated by MCS and the new methodology are exactly the same. It can also be noted that the cumulative failure probability increases for cases with higher number of time nodes. This shows that the descritization is crucial for estimation of failure probabilities. Also, it should be mentioned that the COV is less than 0.05 for all cases.

Table 1: Results for example 1

Time Nodes	P_MCS	P_AK-SYS-based	N_calls	Misclass	Error(%)
5	0.002048	0.002048	19	0	0
10	0.002046	0.002046	20	0	0
20	0.010068	0.010068	23	0	0
50	0.011588	0.011588	21	0	0

5.2 Example 2: A general model

A more complicated performance function with dimension 10 is considered in this section. The performance function is defined in Equation 15 where $L = 5m$, $k = 5 \times 10^{-5}$, and $\rho_{st} = 7.85 \times 10^4 N$. σ_u, a_0, b_0 are input random variables and $F(t)$ is an input stochastic process. $F(t)$ is decomposed into seven random variables using a spectral representation. Table 2 provides distributions of involved random variables and their parameters for the input random variables.

$$G(\mathbf{X}, Y(t), t) = -(F(t)L/4 + \rho_{st} a_0 b_0 L^2/8) + (a_0 - 2kt)(b_0 - 2kt)^2 \sigma_u/4 \tag{15}$$

$$F(t) = 6500 + \sum_{i=1}^{7} \xi_i (\sum_{j=1}^{7} (a_{ij} sin(b_{ij} t + c_{ij}))) \tag{16}$$

The matrices for coefficients a_{ij}, b_{ij}, and c_{ij} can be found in [6]. The cumulative probability of failure for 35 years lifetime for this performance function is defined as:

$$P_{f,c}(0,35) = \text{Prob}(\exists\tau \in [0,35],\ G(\mathbf{x}, Y(t), \tau) \leq 0) \tag{17}$$

Table 2: Distribution of random variables for example 2

variable	Mean	Standard deviation	Distribution
σ_u(Pa)	2.4×10^8	2×10^7	Normal
a_0(m)	0.2	0.01	Normal
b_0(m)	0.04	4×10^{-3}	Normal
ξ_1	0	100	Normal
ξ_2	0	50	Normal
ξ_3	0	98	Normal
ξ_4	0	121	Normal
ξ_5	0	227	Normal
ξ_6	0	98	Normal
ξ_7	0	121	Normal

A Monte Carlo population of size 5×10^4 and an initial DOE of size 50 is used for this case. The results for this example are provided in Table 3. Some misclassification appears for different discretization strategies. However, comparing the relative percentage error still shows a promising accuracy for this approach. Finally, comparing the results with the results of first example shows that by increasing the dimension, the number of calls to the performance function increases as well as the number of misclassification.

Table 3: Results for example 2

Time Nodes	P_MCS	P_AK-SYS-based	N_calls	Misclass	Error(%)
5	0.04416	0.04418	73	7	0.045
10	0.044065	0.044025	79	12	0.091
20	0.044163	0.044063	77	17	0.226
50	0.044203	0.044243	77	14	0.090

6 Conclusion

This paper proposes a new approach for time-variant reliability analysis. This method takes advantage of AK-SYS method which is a system reliability method. In fact time descritization makes the similarity between time-variant and system reliability analyses. Two illustrative examples show the efficiency and accuracy of the proposed methodology. It can be seen that for problems with low dimensionality the results are exactly the same as MCS, but by increasing the dimension, the error slightly increases. However, the results remain very accurate with only few calls to the original performance functions comparing to MCS.

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