NEUTRON NOISE SIMULATIONS IN A HETEROGENEOUS SYSTEM: A COMPARISON BETWEEN A DIFFUSION-BASED AND A DISCRETE ORDINATES SOLVER

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ABSTRACT

A comparative study on two neutron noise solvers is presented. The first solver is the simulator CORE SIM which is based on neutron diffusion theory, while the second one makes use of a discrete ordinates method. For the comparison, a two-dimensional, heterogeneous critical system with a localized perturbation, is considered. The perturbation is defined as a fluctuation of the macroscopic neutron capture cross-section in one point of the system. Differences can be found between the two solvers because of the heterogeneities of the system and the perturbation.

KEYWORDS: neutron noise, diffusion theory, discrete ordinates

1. INTRODUCTION

Reactor neutron noise is an example of a transport problem in a stochastic medium. In fact, processes such as flow-induced vibrations in nuclear power reactors, lead to core properties varying in time and space around expected mean values. The resulting fluctuations in the neutron flux, denoted as neutron noise, are helpful for core monitoring and diagnostics. When simulating reactor neutron noise, neutron diffusion theory is widely applied because of less demanding computational constraints. More accurate solvers based on higher-order approximations of the neutron transport equation, allow for a better modelling of neutron noise problems and can provide reference solutions, from which the validity of the diffusion approach can be assessed (e.g., see [1] and [2]). The current paper presents a comparative study between the diffusion-based neutron noise simulator CORE SIM [3] and a discrete ordinates solver [4]. The case of a 2-dimensional heterogeneous system with a neutron noise source defined as a fluctuation of the macroscopic neutron capture cross-section in one point, is analyzed.

2. DESCRIPTION OF CORE SIM AND THE DISCRETE ORDINATES SOLVER

The simulator CORE SIM and the discrete ordinates solver perform neutron noise calculations in the frequency domain, so that computationally expensive time-dependent simulations are avoided.

The neutron noise equation in the frequency domain is derived from the following procedure. A critical nuclear system is considered. The perturbations that induce the neutron noise, are modelled as stationary fluctuations of the macroscopic neutron cross-sections, with a prescribed amplitude (much smaller than the mean values of the cross-sections) and angular frequency $\omega = 2\pi f$. In the time-dependent neutron transport equation with a given number of groups of delayed neutrons, the neutron flux, the concentrations of precursors of delayed neutrons and the macroscopic neutron cross-sections are expressed as sums of static mean values (generically denoted as X_0) and time-dependent fluctuating values (generically denoted as $\delta X(t)$). The other quantities are assumed to be constant in time. The static balance equations are then removed from the dynamic equations and a Fourier transformation is applied with respect to time. Assuming linear theory to be valid, the second-order terms associated with the perturbations are neglected. Thus, the final multi-energy group relationships for the evaluation of the angular neutron noise $\delta\psi_g(\vec{r},\widehat{\Omega},\omega)$ in the frequency domain, are:

$$\left[\widehat{\Omega}\cdot\nabla + \Sigma_{t,g,0}(\vec{r}) + \frac{i\omega}{v_g}\right]\delta\psi_g(\vec{r},\widehat{\Omega},\omega) = \frac{1}{4\pi}\sum_{g'}\Sigma_{s,g'\to g,0}(\vec{r})\delta\phi_{g'}(\vec{r},\omega) + \frac{1}{4\pi k}\left[\chi_{p,g}(\vec{r})\left(1 - \sum_q \beta_q(\vec{r})\right) + \sum_q \chi_{q,g}(\vec{r})\frac{\lambda_q\beta_q(\vec{r})}{i\omega+\lambda_q}\right]\sum_{g'}\nu\Sigma_{f,g',0}(\vec{r})\delta\phi_{g'}(\vec{r},\omega) + S_g(\vec{r},\widehat{\Omega},\omega)\ , \quad (1)$$

The term $S_g(\vec{r},\widehat{\Omega},\omega)$ represents the neutron noise source and it reads as:

$$S_g(\vec{r},\widehat{\Omega},\omega) = -\delta\Sigma_{t,g}(\vec{r},\omega)\psi_{g,0}(\vec{r},\widehat{\Omega}) + \frac{1}{4\pi}\sum_{g'}\delta\Sigma_{s,g'\to g}(\vec{r},\omega)\phi_{g',0}(\vec{r}) + \frac{1}{4\pi k}\left[\chi_{p,g}(\vec{r})\left(1 - \sum_q \beta_q(\vec{r})\right) + \sum_q \chi_{q,g}(\vec{r})\frac{\lambda_q\beta_q(\vec{r})}{i\omega+\lambda_q}\right]\sum_{g'}\nu\delta\Sigma_{f,g'}(\vec{r},\omega)\phi_{g',0}(\vec{r})\ , \quad (2)$$

As shown in Eq. (2), the static angular neutron flux $\psi_{g,0}(\vec{r},\widehat{\Omega})$ and the static scalar neutron flux $\phi_{g',0}(\vec{r})$ are needed. Therefore, the criticality problem has to be solved before the neutron noise can be determined. The solution of the neutron noise problem in the frequency domain leads to complex quantities, from which amplitude and phase are estimated.

Different approximations of Eqs. (1)-(2) are implemented in the two solvers. The tool CORE SIM relies on neutron diffusion theory, while the second solver evaluates the neutron flux and relative noise via a discrete ordinate method (with isotropic scattering). In both algorithms the spatial discretization is obtained from a finite difference approach, and the neutron energy discretization is limited to the fast and thermal groups.

3. A HETEROGENEOUS SYSTEM WITH A LOCALIZED NEUTRON NOISE SOURCE

A neutron noise problem in a 2-D heterogeneous system is considered. The system is based on the C3 benchmark on deterministic transport calculations [5] and it is perturbed by introducing a localized neutron noise source. The system configuration is given in Figure 1. It consists of two UO2 fuel assemblies (at North-West and South-East positions) and two MOX fuel assemblies (at North-East and South-West positions). The size of each fuel assembly is 21.42 cm x 21.42 cm. The dark blue squares in the illustration are guide tubes; the ones in the center of the fuel assemblies contain fission chambers. Reflective boundary conditions are imposed. The perturbation is a fluctuation of 5% of the fast and thermal neutron capture cross-sections in the fuel cell (16,19) identified with a red square in Figure 1.

Different computational spatial grids are tested for the calculation. The coarser mesh is such that each fuel cell is matched with a node (of size of 1.26 cm x 1.26 cm). Then the resolution is progressively refined from 2x2 to 8x8 mesh nodes per fuel cell. For the discrete ordinates solver, the angular variable is discretized according to the S_8 approximation. Figure 2 shows the predicted amplitude of the neutron noise at the location of the perturbation, with respect to the resolution of the grid. In this case the frequency of the neutron noise source is 1 Hz. Both the simulations with CORE SIM (blue solid line with circles) and the discrete ordinates solver (blue solid line with asterisks) need at least 5x5 nodes per fuel cell/guide tube to be mesh-independent, i.e. the solution of each of the solvers is approximately invariant for any mesh with 5x5 nodes per fuel cell/guide tube or more and so is the relative differences between the two (red dashed line).

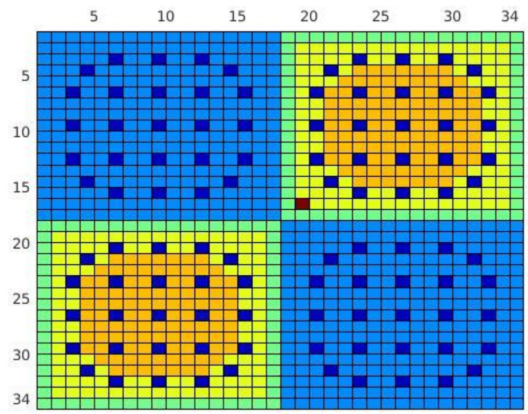

Figure 1. C3 configuration and location of the neutron noise source (in red)

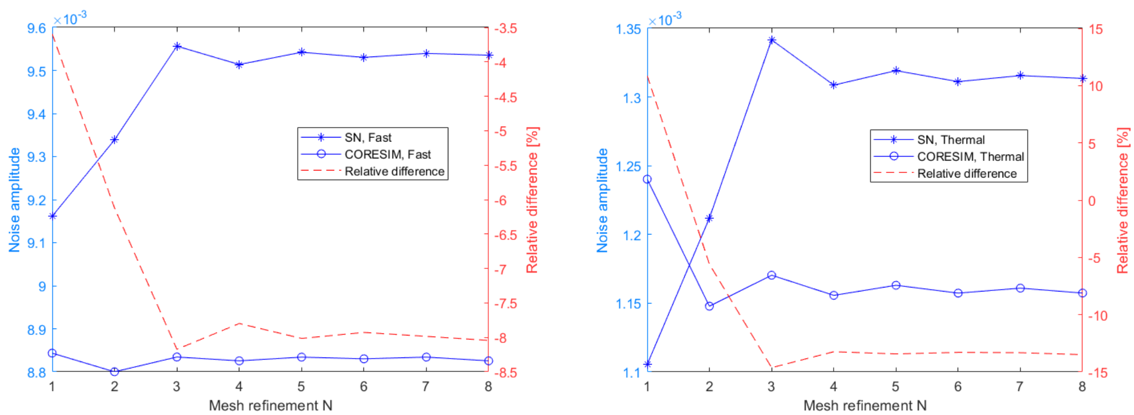

Figure 2. Amplitude of the fast (left) and thermal (right) neutron noise, at the location of the neutron noise source: predictions of the two solvers and relative differences, with respect to the resolution of the spatial grid

Considering the mesh with 5x5 nodes per fuel cell/guide tube, the relative differences between CORE SIM and the discrete ordinates solver are estimated for both the static scalar neutron flux and the neutron noise. The results are shown in Figures 3 and 4.

In the static calculation, the largest differences are related to the guide tubes, i.e. ~2.5% for the fast neutron flux and ~ -14% for the thermal flux (see plots at the top of Figure 3). The guide tubes introduce abrupt variations of the material properties of the system. Higher-order transport methods are considered to be more adequate than a diffusion approximation, to reproduce these kinds of effects associated with heterogeneities. The discrepancies of the static case are reflected on the neutron noise calculations, too.

Such an outcome is expected because of two reasons. First, the static neutron flux is used in the solution scheme of the neutron noise problem (see Eqs. (1)-(2)). Second, the system under study is small and its response is characterized by a dominant point-kinetic component, thus the neutron noise tends to follow the static flux. Nevertheless, additional large discrepancies are found at the location of the neutron noise source and in its close surroundings: ~ -8% for the fast neutron noise amplitude and ~ -14% for the thermal neutron noise amplitude (see plots at the bottom of Figure 3). As shown in Figure 4, the differences between the phase of the neutron noise calculated with the two solvers are negligible (~ -0.07 and ~ -0.09% at the location of the perturbation).

In Figures 3 and 4, 'ray effect' due to the S_8 discretization of the angular variable can be seen, although they are not severe. The use of the S_{16} approximation leads to some improvements, but without any relevant change (compare Figures 3 and 4 with Figure 5 and 6).

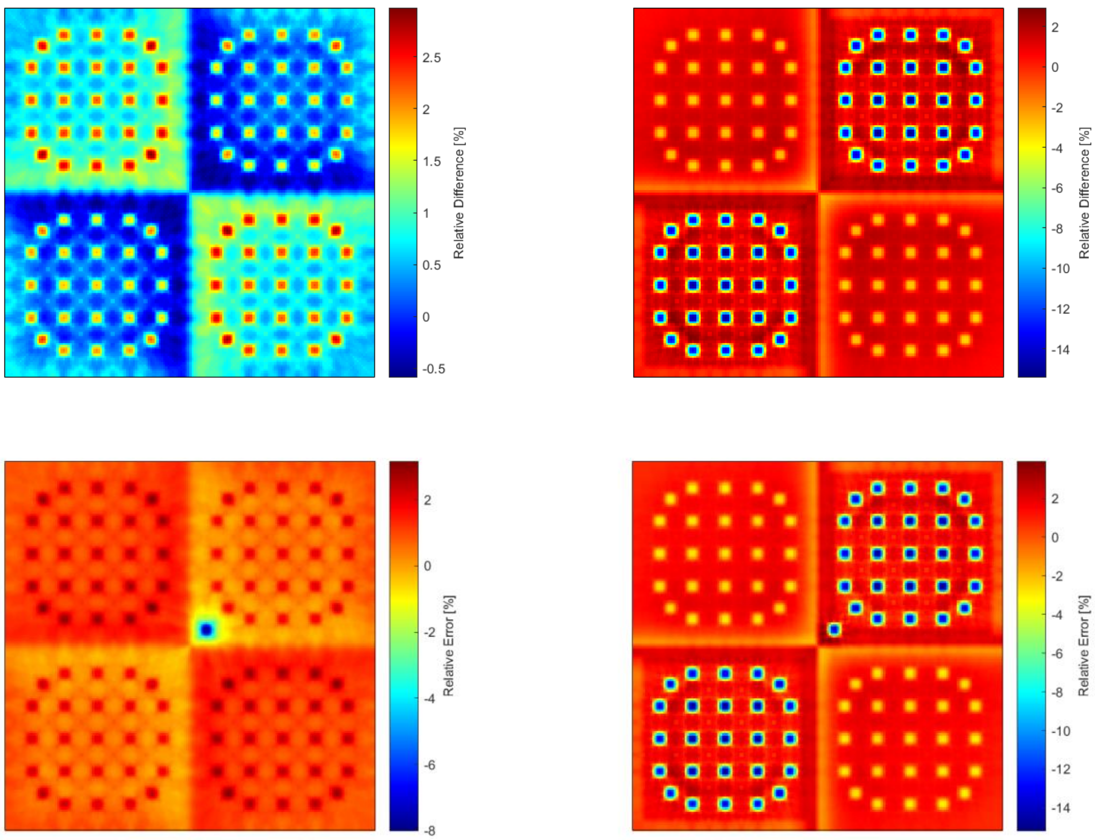

Figure 3. Spatial distribution of the relative differences in the fast (top-left) and thermal (top-right) static neutron flux, and in the amplitude of the fast (bottom-left) and thermal (bottom-right) neutron noise, between CORE SIM and the discrete ordinates solver with S_8

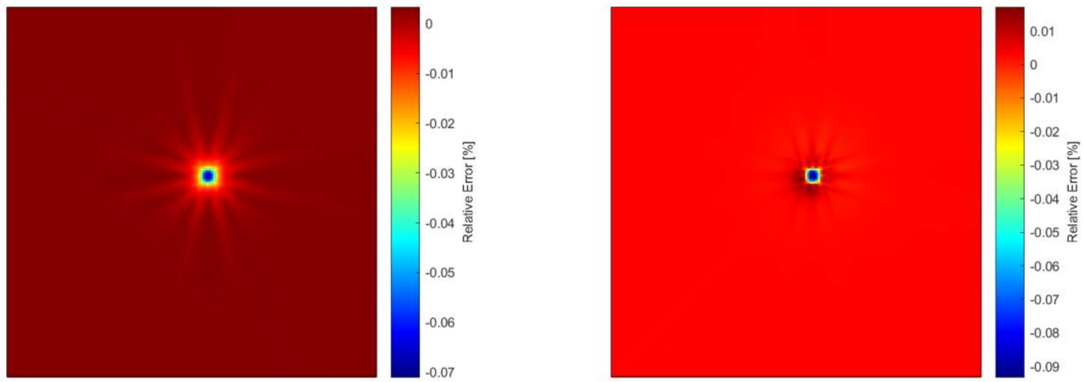

Figure 4. Spatial distribution of the relative differences in the fast (left) and thermal (right) neutron noise phase, between CORE SIM and the discrete ordinates solver with S_8

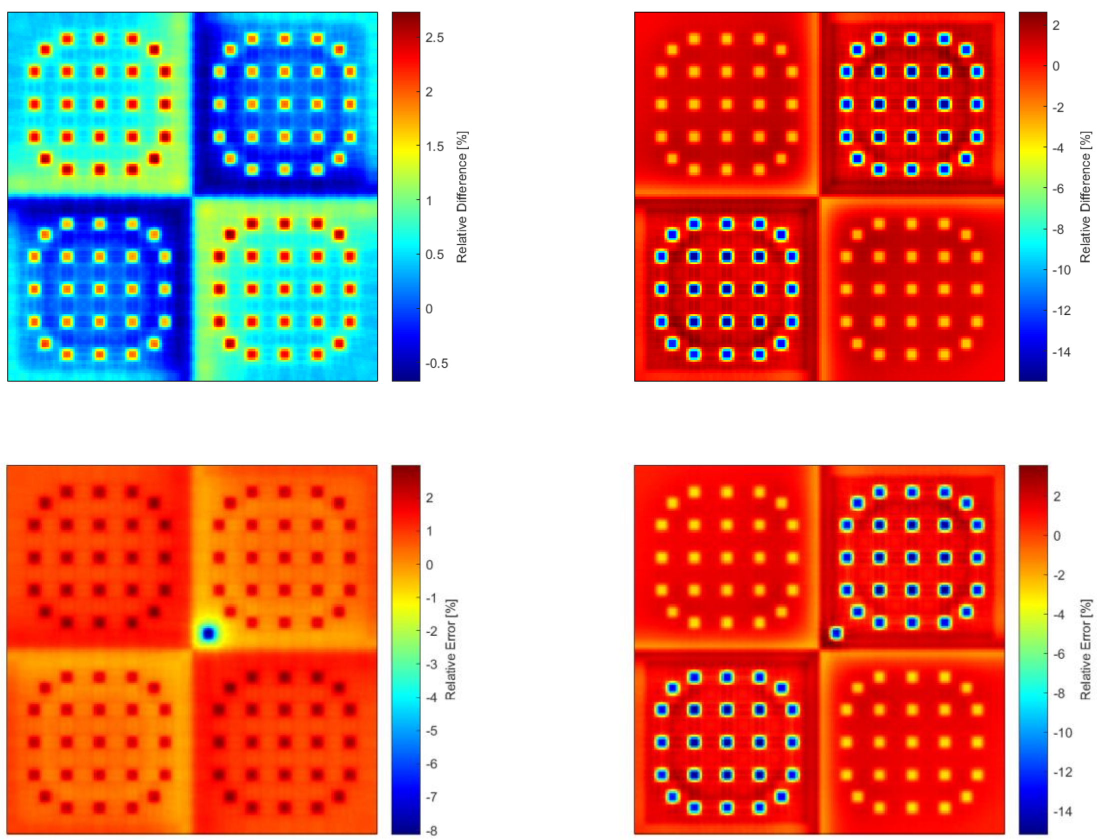

Figure 5. Spatial distribution of the differences in the fast (top-left) and thermal (top-right) static neutron flux, and in the amplitude of the fast (bottom-left) and thermal (bottom-right) neutron noise, between CORE SIM and the discrete ordinates solver with S_{16}

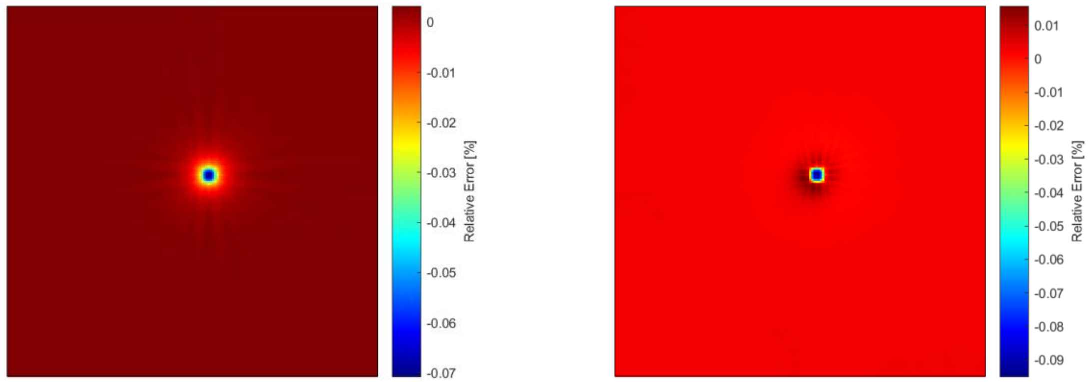

Figure 6. Spatial distribution of the relative differences in the fast (left) and thermal (right) neutron noise phase, between CORE SIM and the discrete ordinates solver with S_{16}

The effect of the frequency of the neutron noise source, is investigated with respect to the range between 0.01 Hz and 100 Hz. The resolution of the spatial mesh is chosen to be 5x5 nodes per fuel cell/guide tube, and the S_8 approximation is used for the discrete ordinate solver. The results for the neutron noise amplitude are presented in Figures 7 to 10 and they are respectively related to the following locations: (16,19) where the perturbation is placed; (17,18) as representative of fuel cells close to the perturbation; (25,10) as representative of the fuel cells far away from the perturbation; (31,4) as representative of the guide tubes in the MOX fuel assemblies. In addition, Figures 11 and 12 provide an example of the behavior of the phase of the neutron noise, taken respectively at the location of the perturbation (16,19) and at the location (17,18) close to the perturbation.

In general, the behavior of the predicted neutron noise with respect to the frequency is consistent with the theoretical zero-power reactor transfer function. Accordingly, at low and high frequencies the neutron noise amplitude decreases with the increase of frequency, while at intermediate frequencies a plateau region can be identified where the neutron noise amplitude is approximately constant. The neutron noise phase resembles a bell-shaped curve, and it is also approximately constant at intermediate frequencies. The discrepancies between CORE SIM and the discrete ordinates solver change over the frequency range, although they are nearly insensitive in the plateau region.

The discrepancies of the neutron noise amplitudes calculated with the 2 solvers at the location of the perturbation and at locations close to the perturbation, are relatively large and increase with frequency (see Figures 7 and 8). In particular, at the location of the neutron noise source, the relative differences for the fast amplitude vary from ~ -4% to ~ -11% over the frequency range, and the ones for the thermal amplitude vary from ~ -8% to ~ -24%. Since a strong gradient of the neutron flux occurs near the perturbation, a higher-order transport method may reproduce the phenomenon better than a diffusion-based model. In addition, the differences may be more notable at higher frequencies for which the propagating effect of the disturbance needs to follow faster fluctuations of the system properties.

The discrepancies between CORE SIM and the discrete ordinates solver at locations of fuel cells far away from the perturbation, are relatively small and quite insensitive to frequency. For position (25,10), the relative differences of the amplitude are below 1% for the fast group, and around 1.2 – 1.6% for the thermal group (see Figure 9). Considering the highly localized effect of the perturbation, these discrepancies are mainly due to the static calculations which are not dependent on the frequency of the neutron noise source (see Figure 3).

When considering the neutron noise in the guide tubes, discrepancies between CORE SIM and the discrete ordinates solver are significant, but they are only weakly affected by the frequency of the neutron noise source. As shown in Figure 10, the relative differences in the guide tube (31,4) reach ~12%. These large values are mainly related to the large discrepancies already existing in the static neutron flux because of the sharp heterogeneity introduced with the guide tubes (see Figure 3), and thus are nearly independent from the frequency.

The discrepancies between the neutron noise phases evaluated with CORE SIM and with the discrete ordinates solver are relatively small and constant in the plateau region. For this interval of frequencies, a phase close to 180 degrees is expected because the perturbation of the macroscopic neutron capture cross-section induces an out-of-phase response of the neutron flux. Outside the plateau region, the relative differences may be somewhat larger. At the location of the perturbation, they are found to be (see Figure 11): ~ -1% for the fast neutron noise and ~ -3% for the thermal neutron noise at the frequency of 0.01 Hz; and ~ -6% for the fast neutron noise and ~ -3.5% for the thermal neutron noise at the frequency of 100 Hz. When taking other locations, CORE SIM and the discrete ordinates solver provide very similar results. For example, the relative differences are approximately below 1.2% already in the fuel cell (17,18) which is just next to the perturbed one (see Figure 12).

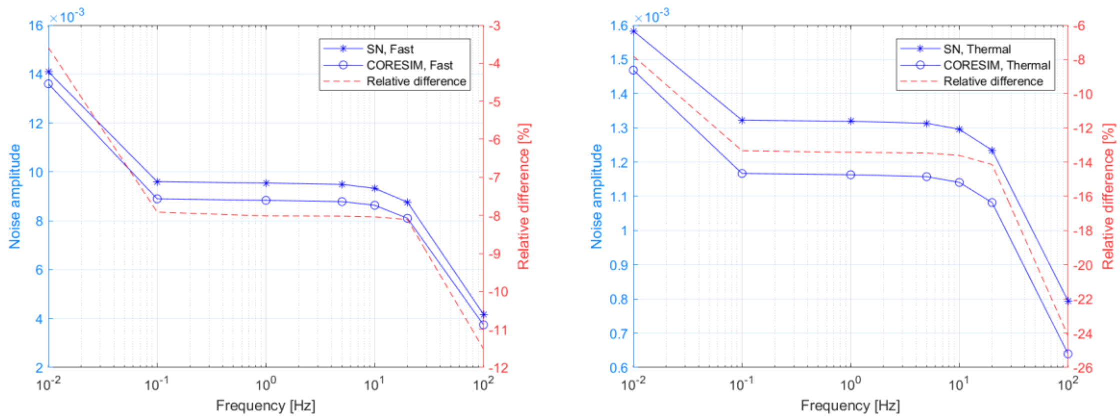

Figure 7. Relative differences between the 2 solvers at the location of the neutron noise source, for the amplitude of the fast (left) and thermal (right) neutron noise

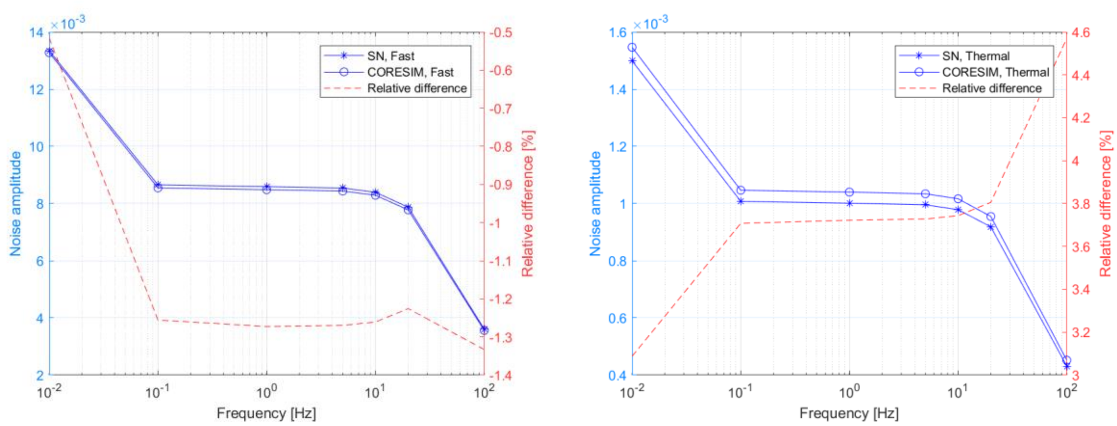

Figure 8. Relative differences between the 2 solvers at the location (17,18), for the amplitude of the fast (left) and thermal (right) neutron noise

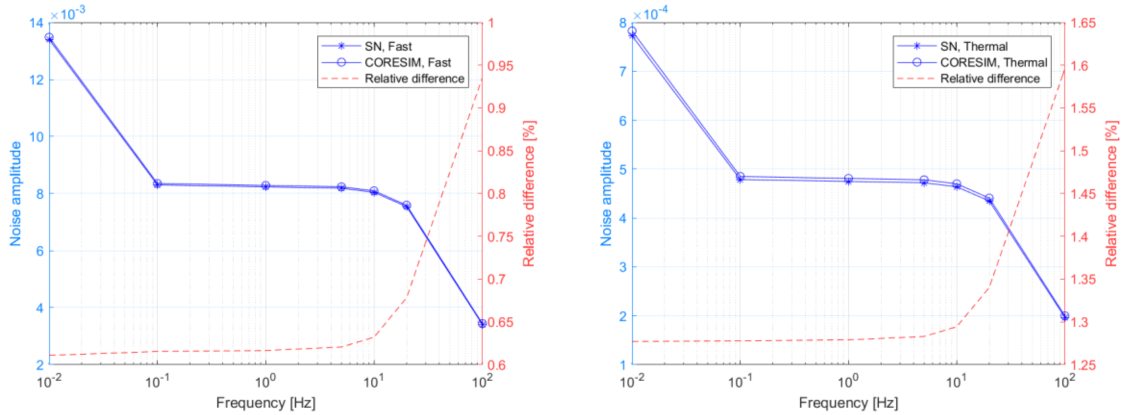

Figure 9. Relative differences between the 2 solvers at the location (25,10), for the amplitude of the fast (left) and thermal (right) neutron noise

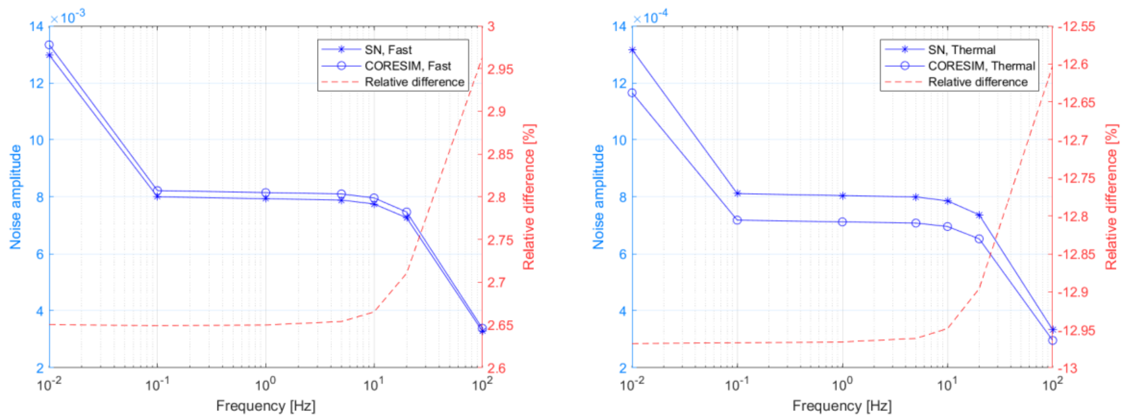

Figure 10. Relative differences between the 2 solvers at the location (31,4), for the amplitude of the fast (left) and thermal (right) neutron noise

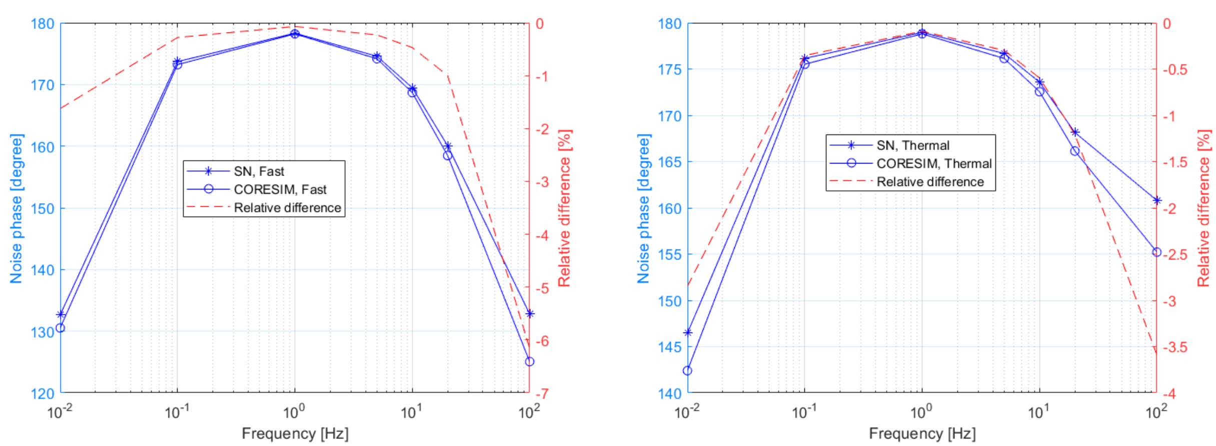

Figure 11. Relative differences between the 2 solvers at the location of the neutron noise source, for the phase of the fast (left) and thermal (right) neutron noise

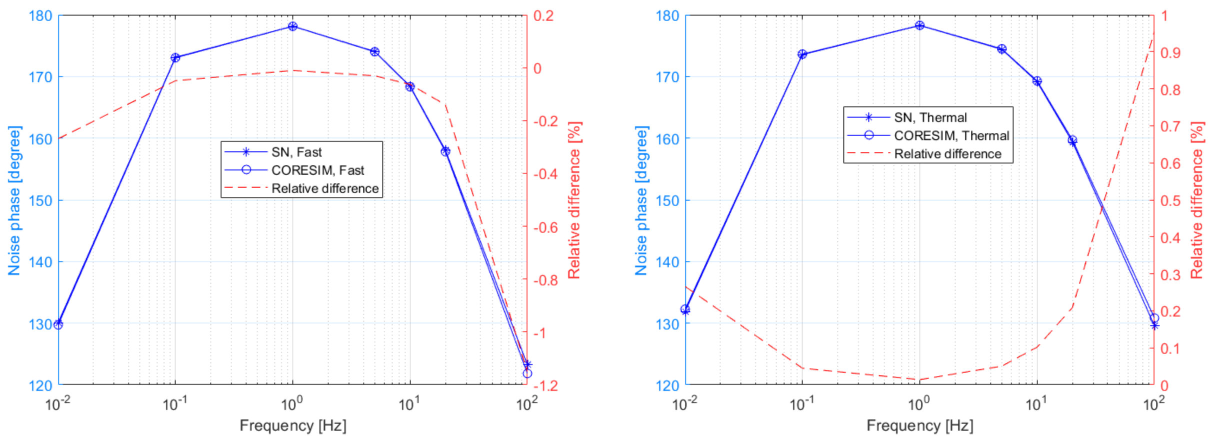

Figure 12. Relative differences between the 2 solvers at the location (17,18), for the phase of the fast (left) and thermal (right) neutron noise

4. CONCLUSIONS

The problem of a localized neutron noise source in a 2-dimensional, heterogeneous critical system, was used to compare the diffusion-based tool CORE SIM and a discrete ordinates solver. The critical system configuration was taken from the C3 benchmark calculations of power distribution within fuel assemblies, and a perturbation was introduced into the system as the fluctuation of the two-energy macroscopic neutron capture cross-sections associated with one point.

Since the system is small, its response to perturbations is dominated by the point-kinetic component and the induced neutron noise mainly follows the static neutron flux. The study showed that the neutron noise predicted with CORE SIM and with the discrete ordinates solver is consistent with the zero-power reactor transfer function. The agreement between the two solvers was found to be relatively good, even though discrepancies may arise from the heterogeneities of the system and the perturbation.

In the close surroundings of the perturbation the discrepancies in the neutron noise amplitude are large and affected by the frequency of the neutron noise source. This is due to the fact that the neutron noise source has a strong local impact. Then a higher-order transport method may reproduce the phenomenon better than a diffusion-based model. In addition, the differences may be more notable at higher frequencies for which the propagating effect of the disturbance follows faster fluctuations of the system properties.

When considering locations that are far away from the perturbation, two main trends were identified. First, the discrepancies in the neutron noise amplitude were large and insensitive to the frequency for the locations of the guide tubes. In this case the abrupt variations of material properties such as the ones introduced with the guide tubes, cause large differences between diffusion and higher-order transport methods already in the static calculations (which are needed for the neutron noise calculations and are independent from frequency). Second, the discrepancies between the amplitudes were smaller and again insensitive to the frequency for the locations of the fuel pins that are sufficiently distant from the perturbation and that characterize the system heterogeneities to a less remarkable extent (and thus the diffusion approximation agrees better with higher-order transport).

The deviation between the neutron noise phases evaluated with CORE SIM and with the discrete ordinates are small, but they may be more important at the location of the perturbation when the frequency of the neutron noise source is very low or very high.

Future work should investigate diffusion and higher-order transport methods with respect to other type of neutron noise source (e.g., perturbation of two-energy macroscopic neutron fission cross-sections). Moreover, the effect of the anisotropy of the scattering should be assessed.

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