NEUTRON KINETICS EQUATIONS IN APOLLO3® CODE FOR APPLICATION TO NOISE PROBLEMS

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ABSTRACT

A 2-D noise model is implemented in the deterministic reactor code APOLLO3® to simulate a periodic oscillation of a structural component. The Two/Three Dimensional Transport (TDT) solver, using the Method of Characteristics, is adopted for the calculation of the case studies, constituted by a moving detector and control-rod bundle. The period is constructed by properly linking the geometries corresponding to the temporal positions. The calculation is entirely performed in the real time domain, without resorting to the traditional frequency approach. A dynamic eigenvalue is defined that takes into account the system average reactivity over a period. The algorithm is accelerated by the DP_N synthetic method. For each cell of the domain, the time values of fission rates are analysed to determine the noise extent.

KEYWORDS: Neutron noise, Temporal domain, 2D MOC

1. INTRODUCTION

In reactor physics the so-called "power reactor noise" [1] is the oscillatory effect on the neutron flux of the perturbation of macroscopic cross sections, due either to vibrations of the structural components or to fluctuations of the coolant density. Whenever these excursions exceed the safety limits, the noise signal can be analysed to identify and localize malfunctions without the need of destructive diagnostics, hence the interest in improving the detection and the elaboration of this signal.

Traditionally, neutron noise can be simulated by a stochastic model generating the small deviations randomly or by considering a fixed, periodic modification of cross sections [2]. In both cases the common starting point is a critical condition and the cross-section variation, acting as noise source, produces fluctuations in the neutron flux which can be either random or deterministic, depending on the nature of the source [1]. The classical way to study oscillations of nuclear properties adopts a frequency-based approach: a system of linearized equations in the frequency domain, derived from Boltzmann's through Fourier transform, is solved to obtain the coefficients needed to reconstruct the flux temporal behaviour. A drawback of this procedure is the appearance of a frequency dependence in cross sections, with consequent complications in the numerical discretizations. Moreover, this approach is perturbative: it is based on the assumption that properties undergo small variations. These problematics push for the development of alternative strategies, as the one discussed in the present paper. Another example is given by [3], where neutronics and thermal-hydraulics are coupled to investigate, in the time-domain, the effect of the oscillation of homogenized fuel assemblies.

A deterministic method is here presented, aiming to simulate the periodic oscillation of a system component. The period is discretized into a set of sub-intervals, and each of them is associated to a geometric configuration, that is, to a position of the component. In doing so, noise is studied within the real temporal domain, without the need of the "small-perturbation hypothesis". In fact, starting from a critical, unperturbed condition, any deviation from the average criticality is taken into account by a "dynamic eigenvalue" defined ad hoc. The computational scheme is realized inside the TDT (Two/Three Dimensional Transport) solver [4,5] and implemented in the APOLLO3® system.

The paper describes first the physical aspects of the problem, the equations involved as well as their declination to the case in question; this is done in Section 2. Section 3 deals with the resolution strategy, which is basically an adaptation of the classical power method. In Section 4 the characteristics of the considered geometries are addressed, showing the component displacements and the obtained results in terms of dynamic eigenvalues and fission-rate oscillation amplitude. Lastly, Section 5 reports conclusions and future perspectives of the presented noise approach.

2. THE KINETIC PROBLEM

Starting from a static situation, noise introduces into the system a temporal behaviour of non-negligible extent. Thus, a kinetic system of equations has to be considered, coupling the time-dependent Boltzmann equation with the equations for the concentrations of delayed-neutron precursors:

$$
\begin{cases}
\left(\frac{1}{v}\partial_t + \mathcal{L} - \mathcal{H}\right)\psi(\vec{r},\vec{\Omega},E,t) = \\
\qquad = \sum_{j=1}^{N_f} \frac{\chi_j^P(E)}{4\pi} \int_E dE' \Big(1 - \sum_i \beta_{i,j}(E')\Big) \nu\Sigma_{f,j}(\vec{r},E',t)\phi(\vec{r},E',t) + \\
\qquad + \sum_{j=1}^{N_f} \sum_{i=1}^{N_i} \frac{\chi_{i,j}^D(E)}{4\pi} \lambda_i C_{i,j}(\vec{r},t) \\
\\
\partial_t C_{i,j}(\vec{r},t) = -\lambda_i C_{i,j}(\vec{r},t) + \int_E dE' \beta_{i,j}(E') \nu\Sigma_{f,j}(\vec{r},E',t)\phi(\vec{r},E',t) \\
\qquad \qquad i = 1, N_i \;\; j = 1, N_f
\end{cases}
\tag{1}
$$

Here, $\mathcal{L}\psi$ and $\mathcal{H}\psi$ represent the transport term (streaming and collision) and the transfer one, respectively; in the fission term on the right-hand side the prompt and delayed contributions are expressed separately: i is the index of the delayed-neutron family, j the one of the fissile isotope, N_i is the number of families and N_f the amount of fissile isotopes considered, χ^P and χ^D are repectively the prompt and delayed-neutron emission spectra, λ is the decay constant, β the delayed-neutron fraction and $\nu\Sigma_f$ the average number of neutrons released by fission multiplied by the macroscopic fission cross section. In order to deal only with isotope-independent quantities, for each family i its set of N_f equations is substituted by

$$\partial_t C_i(\vec{r},t) = -\lambda_i C_i(\vec{r},t) + \sum_{j=1}^{N_f} \int_E dE'\, \beta_{i,j}(E')\nu\Sigma_{f,j}(\vec{r},E',t)\phi(\vec{r},E',t), \tag{2}$$

where $C_i(\vec{r},t) = \sum_{j=1}^{N_f} C_{i,j}(\vec{r},t)$. The delayed source is then rewritten as $\sum_i \bar{\chi}_i^D(\vec{r},E)\lambda_i C_i(\vec{r},t)$, where the delayed-neutron per-family emission spectra $\bar{\chi}_i^D$ are computed as

$$\bar{\chi}_i^D(\vec{r},E) = \frac{\sum_{j=1}^{N_f} \chi_{i,j}^D(E) C_{i,j}(\vec{r})}{\sum_{j=1}^{N_f} C_{i,j}(\vec{r})}. \tag{3}$$

To provide a more accurate description of the system, the previous operation is done using the precursor concentrations obtained in the unperturbed condition (when the component is in its original position), instead of reference values.

The solution sought is an asymptotic dynamic equilibrium, wherein the periodic oscillation of structures implies the periodic variation of physical quantities: this property can be exploited to manipulate the delayed fission source, in order to have only the flux as unknown. This aspect and the definition of the dynamic eigenvalue, characterizing the deviation of the system from the criticality over the period, are illustrated in the following, together with the adiabatic approximation adopted.

2.1. Delayed Fission Source Treatment

In order to express the delayed contribution to the fission source in terms of the variable flux, the precursor equations are firstly integrated over time: thanks to periodicity, this step leads to write for each family i

$$C_i(\vec{r},t) = \int_0^T dt' \left(\frac{e^{-\lambda_i T}}{1-e^{-\lambda_i T}} + \theta_{[0,t]} \right) e^{-\lambda_i (t-t')} \sum_{j=1}^{N_f} \int_E dE'\, \beta_{i,j}(E')\, \nu\Sigma_{f,j}(\vec{r},E',t')\, \phi(\vec{r},E',t'), \tag{4}$$

where T is the period of oscillation and $\theta_{[0,t]}$ is equal to 1 if $t' \in [0,t]$, to 0 otherwise. This result is then substituted in the last term of Boltzmann equation, which, due to the time integral of Eq. (4), at each time t is now dependent on the flux behaviour over the entire period. However, the constraints of the numerical discretization limit the number of available flux solutions to the number N of period sub-intervals: to approximate the time integral a quadrature formula is then developed, for which the condition is imposed that it solves exactly the integral of any T-periodic function containing terms up to a certain frequency, based on the chosen number of sub-intervals.

Therefore, the delayed source in the sub-interval centred around the instant t_k reads

$$\sum_i \bar{\chi}_i^D(\vec{r},E)\lambda_i C_i(\vec{r},t_k) =$$
$$\sum_i \bar{\chi}_i^D(\vec{r},E)\lambda_i \sum_{k'=1}^{N} w_{i,k}(t_{k'}) \sum_{j=1}^{N_f} \int_E dE'\, \beta_{i,j}(E')\, \nu\Sigma_{f,j}(\vec{r},E',t_{k'})\phi(\vec{r},E',t_{k'}), \tag{5}$$

$w_{i,k}(t_{k'})$ being the k'-th weighting coefficient for delayed family i at time t_k.

2.2. Adiabatic Approximation

A priori, one could deal with the time-derivative term present in Boltzmann equation approximating the derivative of the angular flux moments, which derive from the expansion over spherical harmonics, through a Finite-Difference approach:

$$\frac{1}{v}\,\partial_t\psi(\vec{r},\vec{\Omega},E,t_k) \approx \sum_n A_n(\vec{\Omega})\frac{\partial_t\Phi_n(\vec{r},E,t_k)}{v} \approx \sum_n A_n \frac{\Phi_n(\vec{r},E,t_{k+1}) - \Phi_n(\vec{r},E,t_{k-1})}{\frac{2T}{N}v}. \tag{6}$$

The relevance of this term clearly depends on the chosen period of oscillation: to be consistent with the typical order of magnitude of mechanical vibrations [6], a frequency of 1 Hz has been considered within the simulation (T = 1 s). It has been tested that, for such a value, the flux derivative with respect to time produces no significant effect on the global source and therefore has been neglected.

2.3. Dynamic Eigenvalue

Since the model in question aims to simulate a periodic behaviour, in order for the physical quantities to be actually periodic the system cannot but be critical over the period, that is, it has to keep the net neutron balance constant from a period of oscillation to another. For this purpose a dynamic eigenvalue is introduced in the time-dependent Boltzmann equation:

$$\big(\mathcal{L} + DB^2 - \mathcal{H}\big)\psi(t) = \frac{1}{k_D}\mathcal{F}\phi(t), \tag{7}$$

$\mathcal{F}\phi(t)$ expressing in a compact way the right-hand side of the transport equation (the fission source). The term DB^2, needed to achieve the initial criticality, is computed when the system is in the unperturbed condition by means of a homogeneous leakage model [7] available in APOLLO3®, and then employed for the rest of the simulation. The dynamic eigenvalue k_D is searched based on the condition that

$$\int_T dt < \mathcal{F}\phi(t) > \; = const, \tag{8}$$

where '$< \cdots >$' represents the integration over the phase space. Similarly to the classical power-iteration method, the dynamic eigenvalue is updated as the ratio between the current fission source and the normalized previous one; fulfilling condition (8) requires however to consider time integrals of the source:

$$k_D^n = \frac{\int_T dt < \big(\mathcal{F}_\mathcal{D}\vec{\phi}^n\big)(t) >}{1/k_D^{n-1}\int_T dt < \big(\mathcal{F}_\mathcal{D}\vec{\phi}^{n-1}\big)(t) >} \approx \frac{\sum_{k=1}^N < \big(\mathcal{F}_\mathcal{D}\vec{\phi}^n\big)(t_k) >}{1/k_D^{n-1}\sum_{k=1}^N < \big(\mathcal{F}_\mathcal{D}\vec{\phi}^{n-1}\big)(t_k) >}. \tag{9}$$

In the expression above $\left(\mathcal{F}_{\mathcal{D}}\vec{\phi}\right)(t)$ is the delayed source evaluated at time t and the vector notation for the flux suggests the dependence of this term on the whole period. The time integrals are this time approximated by constant-weight sums. The use of the delayed source instead of the total one is related to the iteration strategy adopted, as will be discussed in the next section. The dynamic eigenvalue here defined plays the role of a fictitious feedback: periodicity could be guaranteed by an external feedback as, for instance, a certain boron concentration to be determined iteratively; in that case, the dynamic eigenvalue would simply be equal to 1.

3. ITERATIVE ALGORITHM AND ACCELERATION

The implemented noise simulation consists in an iterative cycle external to the power method outer iterations. At the beginning of each "noise" iteration (index n) the delayed fission source is fixed along the entire period; then, for each time sub-interval k, the following fixed-source problem is solved:

$$\left(\hat{\mathcal{L}} - \mathcal{H}\right)\psi^{n,o}(t_k) - \frac{1}{k_D^{n-1}}\mathcal{F}_{\mathcal{P}}\phi^{n,o-1}(t_k) = S^{n-1}(t_k), \tag{10}$$

where $S^{n-1}(t_k) = \frac{1}{k_D^{n-1}}\mathcal{F}_{\mathcal{D}}\vec{\phi}^{n-1}(t_k)$ and $\hat{\mathcal{L}} = \mathcal{L} + DB^2$. If k_D is not too far from 1, in particular if this difference is smaller than the delayed-neutron fraction of the set of fissile isotopes considered, problem (10) is expected to converge. The algorithm requires to find the flux solutions of the N sub-intervals in order to update the delayed source and the dynamic eigenvalue (Eq. (9)) for the next noise iteration.

The inner cycles corresponding index o are accelerated by means of the DP_N synthetic method [5] already implemented in the APOLLO3® code: basically, the last transport solution, referred to as

Table 1: Geometry and oscillation data

Dimensions	**[cm]**
Cell side	1.26502
Guide tube inner radius	$5.72379\ 10^{-1}$
Detector anode radius	$1.50000\ 10^{-1}$
Detector cathode outer radius	$2.00000\ 10^{-1}$
Detector envelope outer radius	$2.80000\ 10^{-1}$
Detector shift amplitude	$\pm 2.60000\ 10^{-1}$
Control-rod cladding radius	$4.86125\ 10^{-1}$
Rod bundle shift amplitude	$\pm 8.00000\ 10^{-2}$

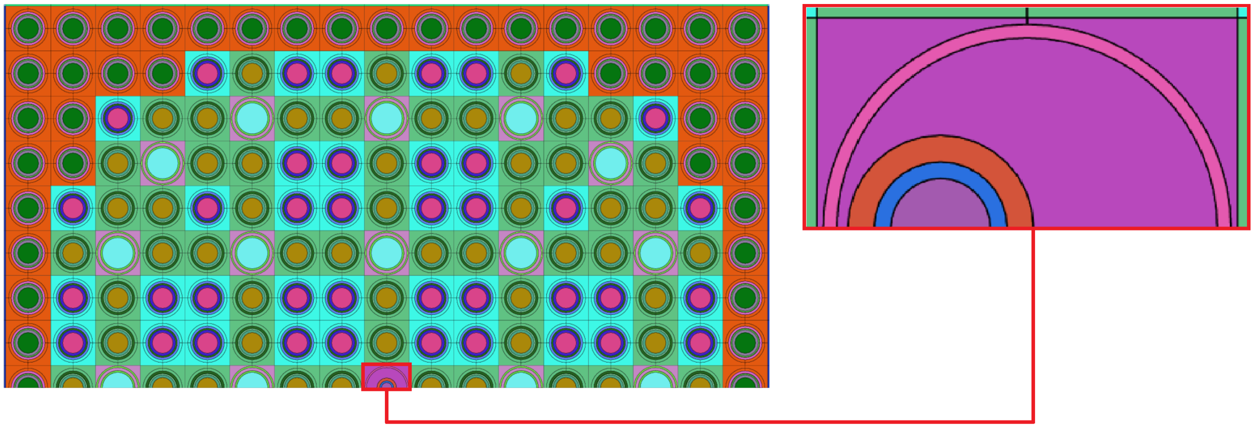


Figure 1: PWR fuel assembly. The halved domain is shown. The fission-chamber detector is inserted in the central cell (the zoom shows its leftmost position).

$\psi^{o-\frac{1}{2}}$, is "corrected" by a term $\delta\psi^{o-\frac{1}{2}}$ which is solution of the simplified problem

$$\big(\mathcal{L}_{DP_N} - \mathcal{H}\big)\delta\psi^{o-\frac{1}{2}}(t_k) - \frac{1}{k_D}\mathcal{F}_{\mathcal{P}}\delta\phi^{o-\frac{1}{2}}(t_k) = \frac{1}{k_D}\mathcal{F}_{\mathcal{P}}\Delta\phi(t_k), \tag{11}$$

where the transport operator has been substituted by its DP_N version and the source is now given by the difference between the two latest transport iterates ($\Delta\phi = \phi^{o-\frac{1}{2}} - \phi^{o-1}$). The correction term is then added to retrieve the next iterate:

$$\psi^{o}(t_k) = \psi^{o-\frac{1}{2}}(t_k) + \delta\psi^{o-\frac{1}{2}}(t_k). \tag{12}$$

The convergence requirements involve both the dynamic eigenvalue and the delayed-fission sources at every time point k.

4. CASE STUDIES AND RESULTS

The described method has been implemented for two different 2-D geometries, both involving one or more components oscillating within guide tubes. The movement is supposed to take place horizontally, along the tube diameter, and at constant speed: if N period sub-intervals are considered, this leads to have $N/2+1$ equidistant positions, including the initial one. It is worth noting that this is quite an ideal case: a more realistic simulation would require a library of frequency modes to reconstruct the oscillation. The use of Silène software [8] simplified the geometries construction, making it possible to easily obtain non-concentric components (which is essential for simulating the movement of one structure within another). For both cases reflective boundary conditions are assumed, with temperatures equal to 841.00 K in the fuel and to 579.55 K in all other materials. The most relevant geometric data are reported in Table 1.

The first system (Fig. 1) is a 1.8% enriched PWR assembly (17×17) with a fission-chamber detector inside the water tube of the central cell; such an instrument is tipically inserted from above, where its unique constraint is, and given the considerable length of the insertion arm the detector is likely to swing due to the coolant turbulence. It should be noted that a 2-D representation

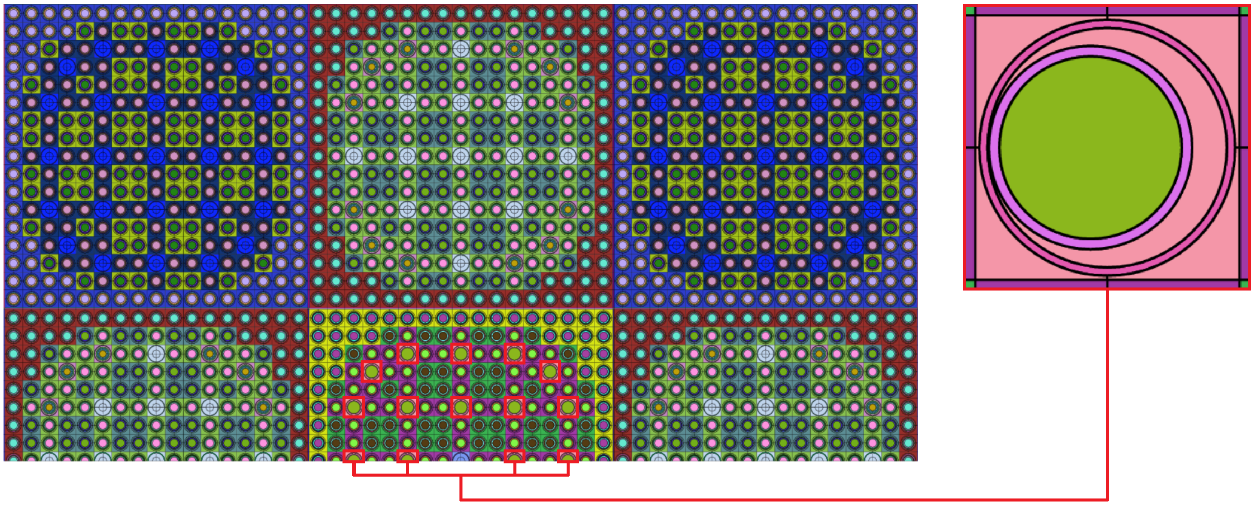


Figure 2: Cluster of 9 PWR fuel assemblies. The halved domain is shown. The moving control rods are in the central assembly (the zoom shows their leftmost position).

hardly resembles reality, since the detector length is at least an order of magnitude lower than the fuel assembly; as a consequence, the measured noise effect is probably higher than what would actually be.

The other system (Fig. 2) is a cluster of 9 PWR fuel assemblies, arranged on a 3×3 grid, with control rods inserted in the central one. In this case the 2-D model describes quite faithfully the behaviour of a mid-height core section, provided that rods be fully inserted, thanks to the weak axial heterogeneity of PWRs. Again, the coolant turbulent motion may induce vibrations of structures and in particular of control rods, as they are contrained only on one side. For simplicity, a coherent horizontal oscillation of the whole rod bundle is assumed to take place. For the sake of completeness, each of the assemblies adjacent to the central one contains 12 pyrex rods and 2.4% enriched fuel, while the enrichment is 1.8% in the others.

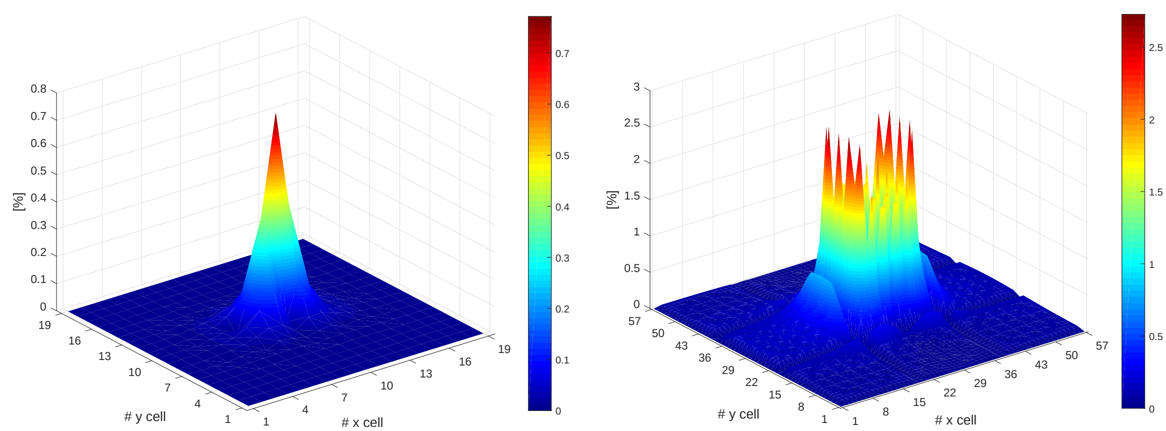
For the two cases Tab. 2 shows the dynamic-eigenvalue results together with the "static" values, which are obtained considering each position along the period independently. The calculations simulate oscillations of 1 Hz using $N = 8$ time sub-intervals and as many positions, with the component(s) starting from the non-oscillation position and moving first up to the leftmost position and then to the rightmost one, before returning to the centre. In view of the weak eigenvalue variation among static values, in particular with respect to the delayed-neutron fraction, the algorithm described in Sec. 3 was expected to converge with a dynamic eigenvalue comprised between the maximum and minimum k_{eff}, as it actually did.

More interesting results can be found analysing the variations of fission rates, especially from an industrial perspective, since they directly impact the power output. Denoting by V_i the volume of the general fuel cell i, the fission rate τ_i relative to the same cell can be expressed, as a function of time, as

$$\tau_i(t) = \int_{V_i} d\vec{r} \int_E dE\, \Sigma_f(\vec{r}, E, t)\phi(\vec{r}, E, t). \tag{13}$$

Table 2: Eigenvalue results

Moving component		Detector	Rod bundle
Static eigenvalues	1	1.000006	1.000003
	2	1.000001	0.999970
	3	0.999998	0.999876
	4	1.000001	0.999970
	5	1.000008	1.000003
	6	1.000001	0.999971
	7	0.999998	0.999878
	8	1.000001	0.999971
Dynamic eigenvalue $(f = 1\ Hz)$		1.000004	0.999946



(a) (b)

Figure 3: Fission-rate variation amplitude relative to the period average, plotted over the whole domains for the two case studies (a: detector, b: control rods).

As a consequence of the periodic structural oscillation, fission rates acquire a periodic behaviour with the same period, attributable to local variations in the moderating ratio. To measure the extent of this effect one can consider, for each cell, the deviations from the average value over the period: identifying this latter as $\bar{\tau}_i$ and as τ_i^{max} and τ_i^{min} the maximum and minimum values, respectively, the maximum relative deviation reads

$$\left(\frac{\delta\tau}{\tau}\right)_i = \frac{1}{\bar{\tau}_i}\ \max\{(\tau_i^{\max} - \bar{\tau}_i); (\bar{\tau}_i - \tau_i^{\min})\}. \tag{14}$$

This quantity has been chosen to quantify the noise effect, and has therefore been computed and plotted over the whole domain for each case study (Fig. 3).

It can be seen that the fission-rate oscillation due to the detector has a quite small amplitude: it peaks at 0.77% in the cell containing the fission chamber and does not go over 0.42% in the two most perturbed fuel cells, to the left and to the right of the previous one. By going towards the boundary, parallel to the oscillation direction, the amplitude decreases up to a value two orders of magnitude lower than the one above. The cells along the perpendicular direction are less affected (about one order of magnitude less) than the previous ones and, clearly, the amplitude of the cells in between assumes intermediate values. These results are nevertheless quite high in comparison to the requirements of an accurate in-core measurement, probably due to the limitations of a 2-D description of the system.

In the case of the rod bundle displacement the measured amplitude is much higher, as it reaches 2.64% in the cells adjacent, along the oscillation direction, to the most external rods (not along the horizontal symmetry axis, though, but along the two successive horizontal rows of control rods). As in the previous case, the effect along the perpendicular direction is about an order of magnitude lower. However, the noise effect appears to be limited to the central assembly: the perturbation in the left and right assemblies peaks at 0.80%, and in the six assemblies above and below the moving rods the maximum amplitude is just 0.30%. This leads to present the main current limitation of the cases analysed: considering the local temporal fluctuation of fission rates no out-of-phase behaviour has been observed. This may be due to the limited size of the system, so that one may be induced to study a larger cluster. However, aside for the computational cost that would at least triple, if for the added assemblies the attenuation behaved as between the central assembly and its neighbours there would be no less than four orders of magnitude between the most perturbed cell and the one furthest from the centre (for a 5×5 cluster): the physical noise would risk to get confused with the numeric one. Future developments should better investigate this aspect.

In any case, the computed oscillation is interesting twofold: on one side, the absence of macroscopic effects on reactivity is consistent with the observed behaviour in real systems; on the other side the calculation shows the presence of measurable and important local flux fluctuations that can impact on the thermo-mechanical system response.

5. CONCLUSIONS

The noise model presented in this paper simulates periodic oscillations of structural components and analyses their effects on reactivity and fission rates. The positions over time of the detector and of the control-rod bundle are identified by different geometries, which are linked in the proper order to construct the oscillation period. Following this procedure, noise is studied within the real temporal domain, without the need of Fourier transforming and of the small-perturbation hypothesis used in the traditional frequency-based approach.

The fission source of delayed neutrons required a specific treatment, leading to express it as a function of the flux values over all the period and, by discretizing the latter in a finite number of sub-intervals, to evaluate the time integral by a quadrature formula suitable for periodic functions.

Each noise iteration is made by a set of power-method outer iterations (one per temporal sub-interval) and updates the here defined dynamic eigenvalue by means of delayed sources until the convergence of this value and of each fission source over the period. The DP_N synthetic acceleration is applied to the outer iterations relative to each temporal point to achieve a faster convergence.

Looking at our results, the oscillations produce a relevant noise with regard to fission rates: in particular, for the control-rod case the amplitude reaches 2.64% of the period average value. The reactivity is much less affected, as the deviation of the dynamic eigenvalue from the static situation is practically insignificant, at least to nuclear engineering. This is somehow a reassuring result, since practical experience in nominal operating has never shown evident effects on reactivity due to vibrating rods. On the other hand, the computational resources limited the analysis to cases that are not very significant: regarding the detector assembly, because a 2-D domain cannot accurately describe reality, since an object measured in centimetres is inserted in a system several meters high; as for the rod bundle because, due to the combination of a relatively weak noise source and a relatively small domain, no phase shift has been detected.

In conclusion, the temporal noise model shows significative fluctuations of per-cell fission rates, suggesting the possibility of detecting structure vibrations by analyzing variations of the local flux. In the future, one may be interested in studying larger and even 3-D geometries, which would permit to measure the noise effect on a relevant portion of a reactor and, potentially, on its totality. In particular, a whole oscillating assembly is a promising candidate as noise source: the hope is to be able to detect some out-of-phase behaviour, that would open the doors to frequency analysis.

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