Task-Space Impedance Controller Using Dual Quaternion Logarithm

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***Abstract*— This paper proposes a task-space impedance controller using the dual quaternion logarithm, combining the translation and rotation impedance in a single mathematical structure. The controller is composed of an outer loop impedance controller to impose a desired apparent impedance to the robot and an inner motion control loop to ensure trajectory tracking of the end-effector pose. Experiments showing the effectiveness of the controller were run on a KUKA LWR4+ robot with a force/torque sensor in the end-effector.**

I. Introduction

The research on physical human-robot interaction (pHRI) has been growing in the last years, in which robots take advantage of their strength and precision capability to assist humans in performing tasks in different environments [1]. For a safe pHRI, it is crucial to ensure a suitable compliant robot behavior, which can be imposed by controlling its apparent impedance [2].

Considering the execution of six degree of freedom (DOF) tasks, both end-effector position and orientation must be handled. In classic approaches, these parts are uncoupled in the control law and the orientation is usually based on minimal representations, such as the Euler angles, which have representation singularities [2]. Caccavale et al. propose to use a unit quaternion for the orientation displacement, but they use two different control laws for the position and orientation [2]. Furthermore, their formulation presents one unstable equilibrium point. That work was later extended to propose a coupled controller [3]. However, although they use unit quaternions to represent the orientation displacement in the impedance law, they still use rotation matrices to perform transformations, making use of different representations in the same framework.

We propose a new coupled six-DOF impedance controller based on the dual quaternion (DQ) logarithm of the task-space displacement, which is simple but effective, as shown by experiments on a real 7-DOF manipulator robot.

II. Mathematical background

Considering a unit DQ representing a six-dimensional pose $\underline{\boldsymbol{x}} = \boldsymbol{r} + (1/2)\varepsilon \boldsymbol{p}\boldsymbol{r}$, where $\boldsymbol{p} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ is the three-dimensional position, and $\boldsymbol{r} = \cos(\phi/2) + \boldsymbol{n}\sin(\phi/2)$ is the rotation of the angle ϕ around the rotation axis $\boldsymbol{n} = n_x\hat{\imath} + n_y\hat{\jmath} + n_z\hat{k}$, its logarithm is $\log \underline{\boldsymbol{x}} = (\phi \boldsymbol{n} + \varepsilon \boldsymbol{p})/2$ [4].

Letting $\underline{\boldsymbol{y}} = \log \underline{\boldsymbol{x}}$, there exists a matrix $\boldsymbol{Q}_8(\underline{\boldsymbol{x}}) \in \mathbb{R}^{8\times 6}$ such that the following relation is true [5]

$$\operatorname{vec}_8(\dot{\underline{\boldsymbol{x}}}) = \boldsymbol{Q}_8(\underline{\boldsymbol{x}}) \operatorname{vec}_6(\dot{\underline{\boldsymbol{y}}}), \tag{1}$$

where the operators vec_8 and vec_6 map the (pure) DQ in a vector, that is, $\operatorname{vec}_8 : \mathcal{H} \to \mathbb{R}^8$, and $\operatorname{vec}_6 : \mathcal{H}_p \to \mathbb{R}^6$.

A DQ error $\tilde{\underline{\boldsymbol{x}}}$ can be defined as the spacial difference in $\mathrm{Spin}(3)\ltimes\mathbb{R}^3$ [6], that is, $\tilde{\underline{\boldsymbol{x}}} \triangleq \underline{\boldsymbol{x}}^* \underline{\boldsymbol{x}}_d$, where $\underline{\boldsymbol{x}}$ and $\underline{\boldsymbol{x}}_d$ are respectively the current and the desired pose such that $\underline{\boldsymbol{x}} = \underline{\boldsymbol{x}}_d$ implies $\tilde{\underline{\boldsymbol{x}}} = 1$.

The logarithm $\tilde{\underline{\boldsymbol{y}}} \triangleq \log \tilde{\underline{\boldsymbol{x}}}$ can be used to translate the error to the origin as $\tilde{\underline{\boldsymbol{x}}} \to 1$ implies $\tilde{\underline{\boldsymbol{y}}} \to 0$.

The force $\boldsymbol{f}_{\log} \in \mathbb{R}^6$ related to the logarithm is given by

$$\boldsymbol{f}_{\log} \triangleq \boldsymbol{G}_{\log}(\underline{\boldsymbol{x}})^T \operatorname{vec}_8(\underline{\varsigma}), \tag{2}$$

where $\underline{\varsigma} \in \mathcal{H}_p$ is the wrench and $\boldsymbol{G}_{\log}(\underline{\boldsymbol{x}}) \triangleq \boldsymbol{G}(\underline{\boldsymbol{x}})\boldsymbol{Q}_8(\underline{\boldsymbol{x}})$ is the generalized Jacobian matrix related to the logarithm, with $\boldsymbol{G}(\underline{\boldsymbol{x}}) \in \mathbb{R}^{8\times 8}$ being the generalized Jacobian matrix [4].

III. Control strategy

Given a desired pose $\underline{\boldsymbol{x}}_d$ of an end-effector that interacts with the environment, we consider another (reference) frame specified by $\underline{\boldsymbol{x}}_r$ such that a desired apparent impedance can be imposed on the pose displacements between the two frames [2].

Considering the displacement between $\underline{\boldsymbol{x}}_d$ and $\underline{\boldsymbol{x}}_r$, the impedance equation is given by

$$\boldsymbol{M}_d \ddot{\tilde{\boldsymbol{y}}}_{\mathrm{rd}} + \boldsymbol{B}_d \dot{\tilde{\boldsymbol{y}}}_{\mathrm{rd}} + \boldsymbol{K}_d \tilde{\boldsymbol{y}}_{\mathrm{rd}} = -\boldsymbol{f}^r_{\log}, \tag{3}$$

where $\boldsymbol{M}_d, \boldsymbol{B}_d, \boldsymbol{K}_d \in \mathbb{R}^{6\times 6}$ are the apparent desired inertia, damping, and stiffness matrices, with $\boldsymbol{M}_d$ and $\boldsymbol{K}_d$ positive definite matrices, $\tilde{\boldsymbol{y}}_{\mathrm{rd}} \triangleq \operatorname{vec}_6(\log \tilde{\underline{\boldsymbol{x}}}_{\mathrm{rd}})$, with $\tilde{\underline{\boldsymbol{x}}}_{\mathrm{rd}} \triangleq \underline{\boldsymbol{x}}_r^* \underline{\boldsymbol{x}}_d$, and $\boldsymbol{f}^r_{\log} = \boldsymbol{G}_{\log}(\tilde{\underline{\boldsymbol{x}}}_{\mathrm{rd}})^T \operatorname{vec}_8(\underline{\varsigma}^r)$ is the external wrench transformed to be consistent with the logarithm mapping and referenced to $\underline{\boldsymbol{x}}_r$. Hence, the impedance control law is given by

$$\ddot{\tilde{\boldsymbol{y}}}_{\mathrm{rd}} = \boldsymbol{M}_d^{-1}\left(-\boldsymbol{B}_d \dot{\tilde{\boldsymbol{y}}}_{\mathrm{rd}} - \boldsymbol{K}_d \tilde{\boldsymbol{y}}_{\mathrm{rd}} - \boldsymbol{f}_{\log}\right). \tag{4}$$

From $\ddot{\tilde{\boldsymbol{y}}}_{\mathrm{rd}}$, we use (1) and its derivative to find the displacement $\tilde{\underline{\boldsymbol{x}}}_{\mathrm{rd}}$ and its first and second derivatives and then $\underline{\boldsymbol{x}}_r$ and its derivatives are retrieved through the definition of $\tilde{\underline{\boldsymbol{x}}}_{\mathrm{rd}}$.

The closed loop system is composed of the impedance law (4) in a outer loop, and a motion controller in an inner loop to control the end-effector pose according to the reference pose $\underline{\boldsymbol{x}}_r$, as illustrated in Fig. 1.

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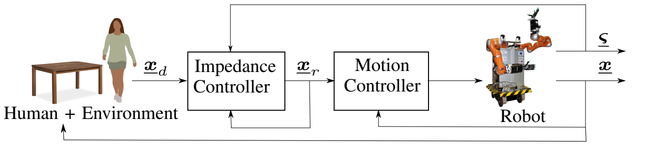

Fig. 1: Scheme illustrating the control law composed of an outer loop with an impedance law and an inner loop with a motion controller.

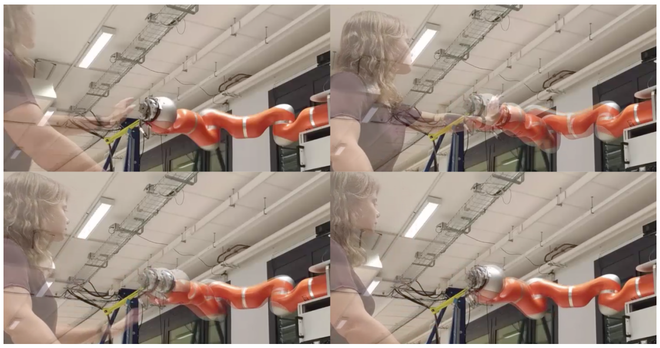

Fig. 2: Behavior of the KUKA LWR4+ when a human pushes its end-effector.

IV. Experiments and Discussions

Experiments were run on a KUKA LWR4+ robot manipulator with a force/torque sensor located at its end-effector, with a sample time of $5\,\mathrm{ms}$. The desired impedance matrices were $\boldsymbol{M}_d = 1.5\boldsymbol{I}$, $\boldsymbol{B}_d = 300\boldsymbol{I}$, and $\boldsymbol{K}_d = 1000\boldsymbol{I}$, where $\boldsymbol{I} \in \mathbb{R}^{6\times 6}$ is the identity matrix. A second-order kinematic controller was used in the inner loop to track the pose $\underline{\boldsymbol{x}}_r$.

The desired end-effector pose was its initial one, so in the absence of external wrenches it should remain at this initial pose. However, when an external wrench is applied to it (in this case by pushing/pulling the end-effector in different directions), the robot should move complacently, according to the desired imposed impedance. Fig. 2 presents some snapshots of the robot movement when a person pushes its end-effector. The end-effector goes back and after the person releases it, it returns close to its initial pose. The initial KUKA's configuration is also shown transparently in the image.

Fig. 3a shows the wrench coefficients read from the force/torque sensor, and the transformed wrench $\boldsymbol{f}^r_{\log}$, while Fig. 3b shows the coefficients of the logarithm of the poses $\underline{\boldsymbol{x}}_d$, $\underline{\boldsymbol{x}}_r$, and $\underline{\boldsymbol{x}}$. We can see by Fig. 3 that when there is a force, the reference pose $\underline{\boldsymbol{x}}_r$ is different from the desired pose $\underline{\boldsymbol{x}}_d$, and the end-effector $\underline{\boldsymbol{x}}$ follows $\underline{\boldsymbol{x}}_r$.

V. Conclusion

The proposed impedance controller consists of a coupled control law for six-DOF tasks by using the DQ logarithm mapping. In the presence of external forces/torques acting on the end-effector, the resultant reference pose is different from the desired one, which ensures a compliant behavior because of the desired apparent impedance imposed to the robot. In the absence of external wrenches, the desired and reference poses are the same and an inner motion control loop ensures the trajectory tracking of the robot end-effector, according to the reference pose.

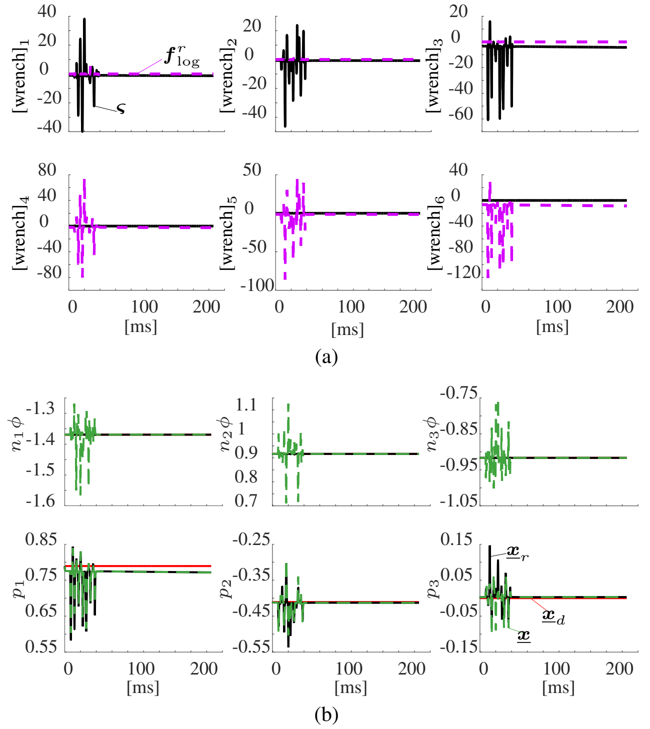

Fig. 3: (a) Coefficients of the external wrench exerted on the robot's end-effector for the value read from the sensor ς, and the modified $\boldsymbol{f}^r_{\log}$. (b) Logarithm coefficients of $\underline{\boldsymbol{x}}_d$, $\underline{\boldsymbol{x}}_r$, and $\underline{\boldsymbol{x}}$.

Future works include the improvement of the proposed control law in order to have impedance matrices with a clear physical meaning, and also a meaningful relation between the desired stiffness and the task geometry.

References

[1] P. Hinds, T. Roberts, and H. Jones, "Whose Job Is It Anyway? A Study of Human-Robot Interaction in a Collaborative Task," *Human-Computer Interact.*, vol. 19, no. 1, pp. 151–181, jun 2004.

[2] F. Caccavale, C. Natale, B. Siciliano, and L. Villani, "Six-DOF impedance control based on angle/axis representations," *IEEE Trans. Robot. Autom.*, vol. 15, no. 2, pp. 289–300, 1999.

[3] F. Caccavale, P. Chiacchio, A. Marino, and L. Villani, "Six-DOF Impedance Control of Dual-Arm Cooperative Manipulators," *IEEE/ASME Trans. Mechatronics*, vol. 13, no. 5, pp. 576–586, oct 2008.

[4] B. V. Adorno, "Two-arm Manipulation: From Manipulators to Enhanced Human-Robot Collaboration [Contribution à la manipulation à deux bras : des manipulateurs à la collaboration homme-robot]," Ph.D. dissertation, Université Montpellier II, 2011.

[5] H. J. Savino, L. C. Pimenta, J. A. Shah, and B. V. Adorno, "Pose consensus based on dual quaternion algebra with application to decentralized formation control of mobile manipulators," *Journal of the Franklin Institute*, pp. 1–37, oct 2019.

[6] L. F. C. Figueredo, B. V. Adorno, J. Y. Ishihara, and G. A. Borges, "Robust kinematic control of manipulator robots using dual quaternion representation," in *Proc. - IEEE Int. Conf. Robot. Autom.* Karlsruhe: IEEE, 2013, pp. 1949–1955.