

Dualism QM and GR

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Hawking and Beckenstein's theory of the thermodynamics of black holes indicates that there is a connection between quantum information and gravity. In general, their result is called the holographic principle. According to it, the entropy of a black hole is proportional to the area of the sphere of the event horizon. In this paper, noncommutative geometry is generalized using the holographic principle. Under certain assumptions, it is possible to obtain results from this synthesis regarding the geometry of the Minkowski space-time. To do this, we consider two main provisions for the generalization of noncommutative geometry.

1. The operators of noncommutative geometry are generalized, in which the coordinates of space and the canonical coordinates do not commute with each other.

$$x_k = (x, y, z)$$

$$\{x_k; y_k\} = i \frac{l_p^2}{\pi}$$

$$l_p^2 = \frac{Gh}{c^3}$$

2. Coordinates of space become operators.

$$x_k \Psi = i \frac{l_p^2}{\pi} \frac{\partial \Psi}{\partial y_k}$$

So the derivative takes the canonical coordinates of noncommutative geometry.

From this synthesis of the holographic principle with the Hilbert space of states of noncommutative geometry, we obtain the concept of a complex vector on a plane with coordinates.

$$z_k = x_k + i y_k$$

This allows you to enter a direct distance in the phase space of non-commutative geometry

$$s^2 = \tilde{z}^k z_k = (x^k - i y^k)(x_k + i y_k) = x^k x_k + y^k y_k$$

Thus, we will assume that the distance in the phase space of non-commutative geometry will be a measure of the definition of a new type of geometry, in which the space coordinates and the canonical coordinates have the same physical dimension.

1. Quantum tunneling of noncommutative geometry

In the wave function, Euclidean gravity is considered as the complete quantum state of the Universe in the Euclidean form (Hawking) as a form of tunneling.

$$\Psi_H = \int e^{-I\{dg\}}$$

In this case, the coordinates of space are not just numbers, but the internal characteristics of the quantum state of the Universe, that is, operators.

$$x_k \Psi = i \frac{l_p^2}{\pi} \frac{\partial \Psi}{\partial y_k}$$

If the wave function is a real number, then tunneling takes place. Let the canonical coordinates have imaginary values (spatial coordinates are real numbers).

$$y_k = i q_k$$

We introduce the entropy of a tunnel junction as the natural logarithm of the probability of the wave function

$$S = \ln|\Psi|^2$$

Let the tunneling be isotropic in all directions of space

$$r = \sqrt{x^2 + y^2 + z^2}$$

$$r dq_r = \frac{l_p^2}{\pi} \frac{d\Psi}{\Psi}$$

Hence the tunneling parameter

$$q_r = \frac{l_p^2}{2\pi} \oint \frac{dS}{r}$$

Such a definition makes it possible to understand the occurrence of a pseudo-Euclidean distance of noncommutative geometry.

$$s^2 = x^k x_k + y^k y_k = r^2 - q_r^2 = r^2 - \left(\frac{l_p^2}{2\pi} \oint \frac{dS}{r} \right)^2$$

This structure already differs sharply from the Euclidean definition. It contains special structures of cones, which are divided into specific areas for the phase space of noncommutative geometry. This definition of the pseudo-Euclidean interval, if the entropy of tunneling is isotropic non-commutative geometry, is determined on the surface boundary.

2. Pseudo-Euclidean interval of noncommutative geometry and the structure of space-time.

In the last chapter, a definition of the pseudo-Euclidean interval in the pseudo-Euclidean phase space of non-commutative geometry is obtained, in which quantum tunneling passes through the boundary. And there are special structures, cones. Minkowski spacetime has the same structure.

$$s^2 = r^2 - (c t)^2$$

Here we will consider this as not a coincidence, but a fundamental consequence of the geometry of the Minkowski space-time from the definition of a holographic interval.

$$s^2 = r^2 - \left(\frac{l_p^2}{2\pi} \oint \frac{dS}{r} \right)^2 = r^2 - (c t)^2$$

From here, the definition of time is obtained, as the ratio of the entropy at the boundary of the sphere to its radius. This is the definition of arising time. Where the entropy at the boundary of the sphere should be considered as entropy of entanglement between the boundary of the sphere and the point inside, where the moment of time is determined. In general, the resulting time will be as a closed surface integral

$$t = \frac{Gh}{2\pi c^4} \oint \frac{dS}{r}$$

In this form, you can come to the general formula for any closed arbitrary surface. In this formula, time is determined at a certain point, where a closed surface is taken around through the integral of entropy of entanglement on a given surface. In this case, the differential form of the space-time interval will be

$$ds^2 = dr^2 - \left(\frac{l_p^2}{2\pi} d \oint \frac{dS}{r} \right)^2$$

This is the general definition of the new differential interval of space-time. If the time counts equally uniformly in all points of space. The definition of arising time can be given as the ratio of the sphere entropy to the radius.

$$t = \frac{Gh}{2\pi c^4} \frac{S(r)}{r}$$

3. Invariance of the definition of arising time.

Consider a simplified version for the sphere. In a moving system, the radius of the sphere intersects the coordinates and is determined through the Lorentz transformations. If we assume that the entropy of a given spherical surface is the same everywhere in any reference system, that is, the invariant

$$\frac{Gh}{2\pi c^4} dS = (r - v t) dt$$

Then time in the moving frame will have the following definition

$$t^l = \frac{Gh}{2\pi c^4} \int_A \frac{dS}{r^l} = \int \frac{(r - v t)}{r^l} dt = \int \sqrt{1 - \frac{v^2}{c^2}} dt$$

In general, a time dilation formula is obtained for a moving frame of reference. From this it follows that the definition of “surface” time is invariant with respect to the Lorentz transformations. This proof was for a spherical surface, but it is possible to show the validity of arbitrary closed surfaces with the help of special mathematical methods.

4. Holographic time focusing and relativistic correction.

Holographic definition of time assumes that time is focused inside a closed surface at any point of the internal volume. Accordingly, in this approach, time is considered as a closed surface integral along the entropy area of entanglement.

$$t = \frac{Gh}{2\pi c^4} \oint \frac{dS}{r}$$

This definition allows you to introduce the concept of focusing time. The focus value can be defined as the inverse relation to the radius from the focal point to the point of the closed surface.

$$F = \frac{1}{r}$$

In this case, the holographic time focusing is determined by the formula

$$t = \frac{Gh}{2\pi c^4} \oint F dS$$

In addition, it must be borne in mind that an arbitrary closed surface can be not only a static surface, but a kinematic one. Each point of the surface can have its own speed of movement. In this case, the speeds that are perpendicular to the radius affect the geometric dimensions of the surface. This is due to the reduction in the length of Lorenz. Therefore, entropy of entanglement of a surface has a relativistic modification due to the Lorentz contraction.

$$dS \rightarrow \frac{dS}{\sqrt{1 - \frac{v^2}{c^2} \sin^2 \varphi}}$$

$$\varphi = (\vec{v}, \vec{r})$$

Here, the flat angle between the velocity vector of a surface point and the radius-vector.

Hence, the relativistic modification of time holography has the following form.

$$t = \frac{Gh}{2\pi c^4} \oint \frac{F dS}{\sqrt{1 - \frac{v^2}{c^2} \sin^2 \varphi}}$$

5. General relativity and holography time

Consider the holographic definition of time in a spherically symmetric form. The course of time is defined as the ratio of the entropy of entangling a sphere to its radius. Where the entropy of the entanglement of a sphere is the entropy of entanglement between the space inside the sphere and the space outside the sphere.

$$t = \frac{Gh}{2\pi c^4} \frac{S(r)}{r}$$

Now we will place the point mass M in the center of the sphere. If the point mass processes information according to the Margolis-Levitin theorem at the quantum level, then by entanglement, the total holography on the sphere will change.

$$S^{\downarrow} - S = I$$

As a result of changes in entanglement on the surface of a sphere, the course of time also changes at different points in space.

$$I = \frac{2\pi M c^2}{h} t$$

$$t^l - t = \frac{Gh}{2\pi c^4} \frac{S^l - S}{r} = \frac{G M}{c^2 r} t$$

This effect is called gravitational time dilation. The key concept of general relativity, which with the use of geometry of Minkowski leads to the idea of gravity as the curvature of space-time, that is, to the Einstein equations.

$$R_{ik} - \frac{1}{2} g_{ik} R = \frac{8\pi G}{c^4} T_{ik}$$

6. Quantum mechanics and holography of time.

In the last chapter it was shown that general relativity occurs through the confusion of boundaries, if we consider the application of quantum information to the holography of time. On the other hand, the symmetry of time holography can be shown if the general theory of relativity is dual to quantum mechanics. In other words, one can obtain quantum mechanics using time holography and the special geometry of the general relativity.

One promising discovery is that quantum entanglement and the wormhole are equivalent to each other. Juan Maldensana gave a short name ER = EPR. Let us try to apply this idea to the emergence of quantum mechanics from the geometry of general relativity.

Consider the special geometry of a Euclidean wormhole with a metric

$$ds^2 = (c dt)^2 + \gamma_{ik} dx^i dx^k$$

If we enter imaginary time

$$d\tau = i dt$$

It turns out the wormhole with the usual metric

$$ds^2 = -(c d\tau)^2 + \gamma_{ik} dx^i dx^k$$

We assume that inside the Euclidean wormhole time is an imaginary value relative to the external time of the observer. We assume that inside the Euclidean wormhole time is an imaginary value relative to the external time of the observer.

We give a holographic definition of imaginary time.

$$\tau = \frac{Gh}{2\pi c^4} \oint \frac{dS}{r}$$

Here the radius corresponds to the Schwarzschild radius.

$$r = r_g = \frac{2G}{c^4} E$$

Hence the definition of imaginary time through holographic entanglement.

$$d\tau = \frac{Gh}{2\pi c^4} \frac{dS}{r} = \frac{h}{4\pi} \frac{dS}{E}$$

We define holographic entropy in terms of the logarithm of a certain function.

$$dS = -2 \frac{d\Psi}{\Psi}$$

The result is the Schrödinger equation for a particle with a total energy.

$$i \frac{\hbar}{2\pi} \frac{\partial \Psi}{\partial t} = E \Psi$$

In addition, the action for a quantum particle is equivalent to the description in terms of holography and holographic information.

$$S_{action} \rightarrow I$$

Thus, the holography of time is symmetrical about the dualism between quantum mechanics and the general relativity.

Conclusion

In general, the main new position of noncommutative geometry in the synthesis with the holographic principle gives a promising start. First, a new concept of distance in the phase space of non-commutative geometry is obtained. In the second, we can consider noncommutative geometry as a pre-geometry for the resulting space-time when considering the quantum tunneling approach. Thirdly, the arising time takes place in noncommutative geometry and this largely explains the necessity of introducing time as an imaginary coordinate for Minkowski space-time. Quantum tunneling of noncommutative geometry gives the definition of time in the form of holography, that is, in the form of a closed surface integral. Ultimately, the holography of time shows the dualism between quantum mechanics and the general theory of relativity.

- [1] R. Penrose, The central programme of twistor theory, *Chaos, Solitons & Fractals*, 10 (1999) 581-611.
- [2] J. Baez, Spin foam models, *Class. Quant. Grav.* 15 (1998) 1827-1858.
- [3] D. Barbasch and D. Vogan, Jr., Unipotent representations of the complex semisimple groups, *Ann. Math.* 121 41-110.
- [4] S. Gindikin, SO(1;n)-twistors, *J. Geom. & Phys.* 26 (1998) 26-36.
- [5] Maldacena J, Susskind L. Cool horizons for entangled black holes. *Fortsch. Phys.* 2013; 61: 781-811. doi: 10.1002/prop.201300020
- [6] D. H. T. Franco and C. M. M. Polito, "CPT, spin-statistics and analytic wavefront set in non-commutative field theories," arXiv:hep-th/0403028.
- [7] J. Gomis and T. Mehen, "Space-time noncommutative field theories and unitarity," *Nucl. Phys. B* 591 (2000) 265 [arXiv:hep-th/0005129].
- [8] S. Hawking and G. Ellis, "The large scale structure of space-time," Cambridge University Press, 1973.