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Non-binary turbo codes: design, simplified decoding and comparison with SoA codes

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EPIC Grant Agreement No 760150

OUTLINE

- 1. Context and state of the art
- 2. Design of NB-TCs
- 3. Union bound evaluation for NB-TCs
- 4. Low-complexity decoding algorithm
- 5. Performance comparison
- 6. Conclusion and future work



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OUTLINE

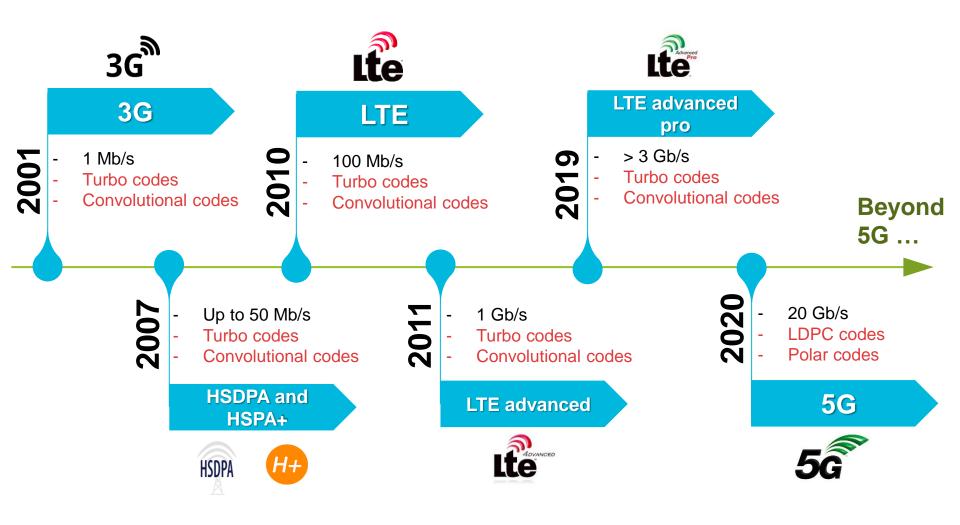
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Evolution of communication standards



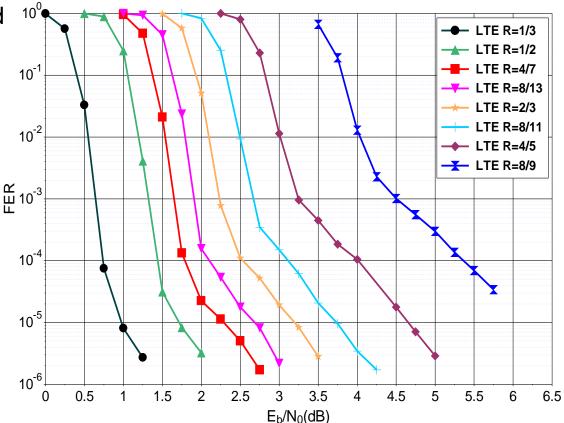




DESIGN OF NON-BINARY TURBO CODES

Why new families of error correcting codes should be considered?

- The performance of standardized codes is to be improved:
 - at high SNRs for high coding rates,
 - for their association with high order modulations,
 - for short block sizes targeting low latency applications.



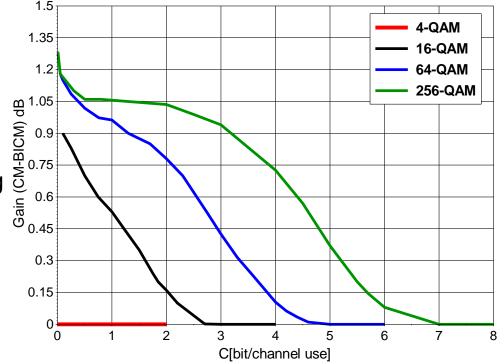
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Motivations to study non-binary (NB) codes

Advantages of NB codes

- Lower correlation in the decoding process [1]
- Theoretical performance gains for high modulation orders [2]
 - Gap closed by Probabilistic Shaping (PS) for optical channels [3]
 - PS + NB codes better than PS + binary codes [4]
- More freedom degrees can be investigated Existing NB codes
- Reed-Solomon codes
- ► NB-LDPC codes
- Some studies targeted NB-CC and NB-TC



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[1] C. Berrou, M. Jezequel, C. Douillard, and S. Kerouedan, "The advantages of non binary turbo codes," in IEEE Inform. Theory Workshop, Cairns, Queensland, Australia, 2001, pp. 61–63.

[2] G. Caire, G. Taricco, and E. Biglieri, "Bit-interleaved coded modulation," IEEE Trans. Inform. Theory, vol. 44, no. 3, pp. 927–946, 1998

[3] Böcherer, G., Steiner, F., & Schulte, P. (2015). Bandwidth efficient and ratematched low-density parity-check coded modulation. *IEEE Transactions on Communications*, 63(12), 4651-4665.

[4] Steiner, F., Liva, G., & Böcherer, G. (2017, December). Ultra-sparse non-binary LDPC codes for probabilistic amplitude shaping. IEEE *Global Commun. Conf* (pp. 1-5).



State of the art in NB-CCs and NB-TCs (1/2)

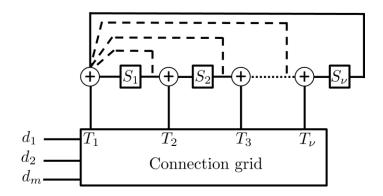
m-binary codes [5]

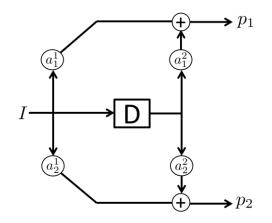
- Better convergence.
- Larger minimum distance than binary codes.
- Less sensitivity to puncturing.
- Reduced latency.
- Robustness of the decoder.
 NB-CC over rings [6]
- Good performance with phase modulations.
- Adapted to perform with m-QAM constellations.
- Not suitable for NB-TCs: non-systematic and non-recursive.

[5] C. Douillard and C. Berrou, "Turbo codes with rate-m/(m+ 1) constituent convolutional codes," IEEE Trans. Commun., vol. 53, no. 10, pp. 1630–1638, 2005.

[6] J. L. Massey and T. Mittelholzer, "Convolutional codes over rings," in 4th Joint Swedish Soviet Int. Workshop Inform. Theory, Gotland, Sweden, 1989, pp. 14–18.







NB-TC based on time-variant NB-CCs [7][8]

Parallel concatenation

A protograph sub-ensemble of a regular NB-LDPC code is derived to define the NB-TCs.

$H = \begin{bmatrix} \\ \end{bmatrix}$		$\tilde{P}^{(1)}$	0]
$H = \begin{bmatrix} \\ \\ \end{bmatrix}$	Π	0	$\tilde{P}^{(2)}$	

Serial concatenation

The protograph is stretched to form a serial structure:

$$H = \begin{bmatrix} \tilde{\Pi}^{-1} & \tilde{P}^{(2)} & \tilde{I}^{-1} & \tilde{P}^{(1)} \end{bmatrix}$$



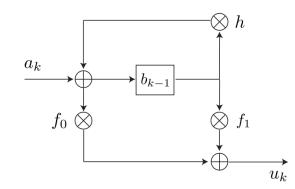
DESIGN OF NON-BINARY TURBO CODES

 p_i

NB-TC over GF(q), q > 2 [9]

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▶ Based on NB-CCs over GF(q)

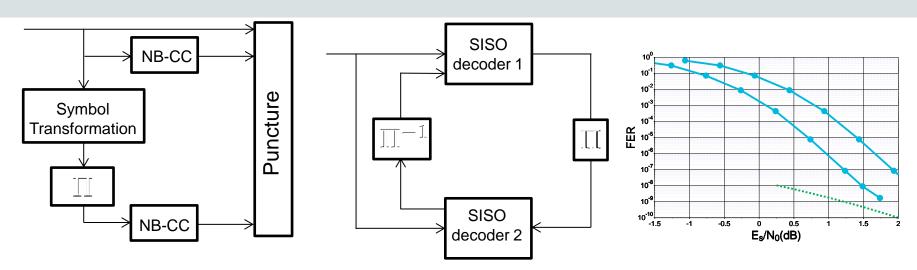


- ► Good NB-TCs using:
 - union bounds,
 - EXIT charts.

[7] G. Liva, S. Scalise, E. Paolini, and M. Chiani, "Turbo codes based on time- variant memory-1 convolutional codes over Fq," in IEEE Int. Conf. Commun., Kyoto, Japan, June 2011, pp. 1–6.
[8] G. Liva, E. Paolini, B. Matuz, S. Scalise, and M. Chiani, "Short turbo codes over high order fields," IEEE Trans. Commun., vol. 61, no. 6, pp. 2201–2211, June 2013.

[9] Matsumine, T., & Ochiai, H. (2018). Capacity-Approaching Non-Binary Turbo Codes: A Hybrid Design Based on EXIT Charts and Union Bounds. *IEEE Access*, *6*, 70952-70963.

Goals of the performed work



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Main contributions of the work:

- New structure of NB-CCs
- Interleaving constraints
- Two puncturing techniques
- Bijective symbol transformation block
- Estimation of the distance spectra and evaluation of union bounds
- A low-complexity decoder

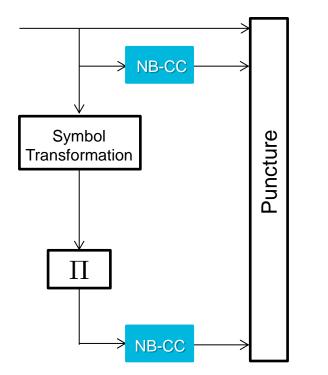


Performance comparison with existing codes

DESIGN OF NON-BINARY TURBO CODES

Outline

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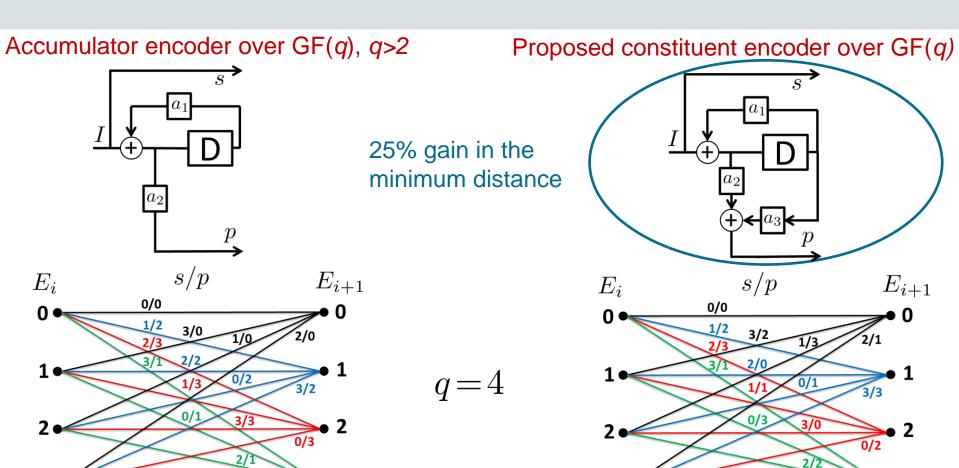


1. Context and state of the art

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Proposed convolutional code design



Same Parity when arriving to a state \rightarrow systematic value responsible for the distance



 E_i

DESIGN OF NON-BINARY TURBO CODES

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Distinct parity when arriving to a state

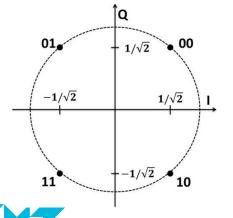
 \rightarrow systematic and parity symbols

contribute to the distance

Distance spectrum calculation

Error-prone sequences: Diverging-Converging (DC)

4-QAM Constellation



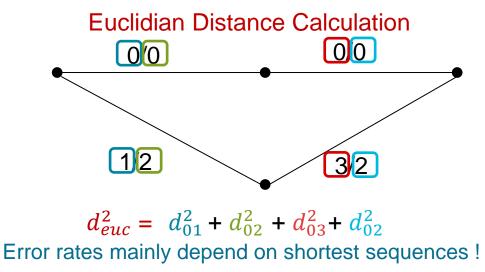


DESIGN OF NON-BINARY TURBO CODES

Non-linearity of the nonbinary coded modulation

For high constellation orders:

- Non-binary coded modulations are nonlinear in signal space
- Different protection levels for constellation symbols
- All-zero sequence cannot be used as reference



Proposed algorithm to find the best NB-CC over GF(q) 13/54

Code enumeration over GF(q) for the proposed structure When the constellation mapping varies:

- $q! * (q-1)^3$ possible codes to test
- ⇒ Prohibitive search complexity for high values of *q*.
- ⇒ Question: Do we really need to consider all these combinations ?

Answer: No, for the proposed structure.

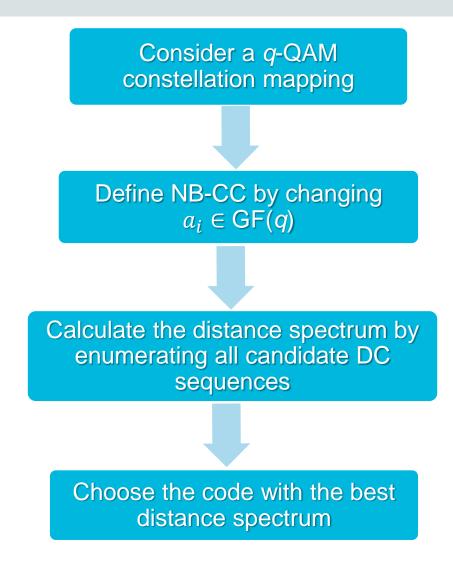
When the constellation mapping is fixed:

• $(q-1)^3$ possible codes to test

Proposal: Limitation of the enumerated DC sequences to the short ones

- Acceptable complexity
- Accurate enumeration of the two lowest distances of the code

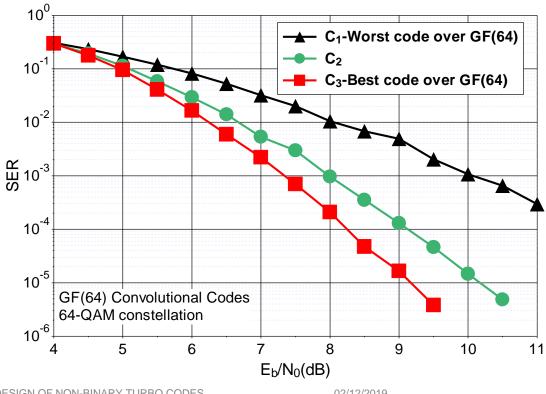




Examples of codes over GF(64)

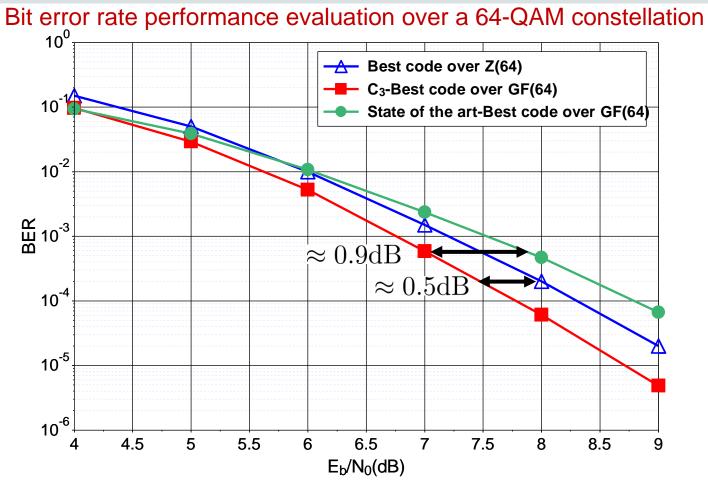
Results over GF(64)

Code	C_1	C_2	C_3
(a_1, a_2, a_3)	(41, 2, 0)	(41, 1, 24)	(31,5,18)
$d_1^2 (= d_{min}^2)$	0.38	1.14	1.52
$n(d_1)$	238422	1542390	652698
d_{2}^{2}	0.57	1.23	1.61
$n(d_2)$	230886	4111444	1084014





Comparison with NB-CC over rings



[10] T. Konishi, "A coded modulation scheme for 64-QAM with a matched mapping," in IEEE Int. Symp. Inform. Theory and its Applications (ISITA), Oct. 2014, pp. 191–195.



DESIGN OF NON-BINARY TURBO CODES

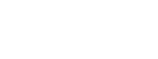
[11] Matsumine, T., & Ochiai, H. (2018). Capacity-Approaching Non-Binary Turbo Codes: A Hybrid Design Based on EXIT Charts and Union Bounds. *IEEE Access*, 6, 70952-70963.

NB-CCs for low rate codes



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Results over GF(64) A general structure of NB-CCs 10⁰ sA..... Best CSED Worst CSED 10⁻¹ R=1/510⁻² R=1/3Ж Ш 0⁻³ Summary 10⁻⁴ p^n 10⁻⁵ GF(64) Convolutional Codes ì 64-QAM Symbols 10^{-6⊥} 12 2 6 8 10 0 4



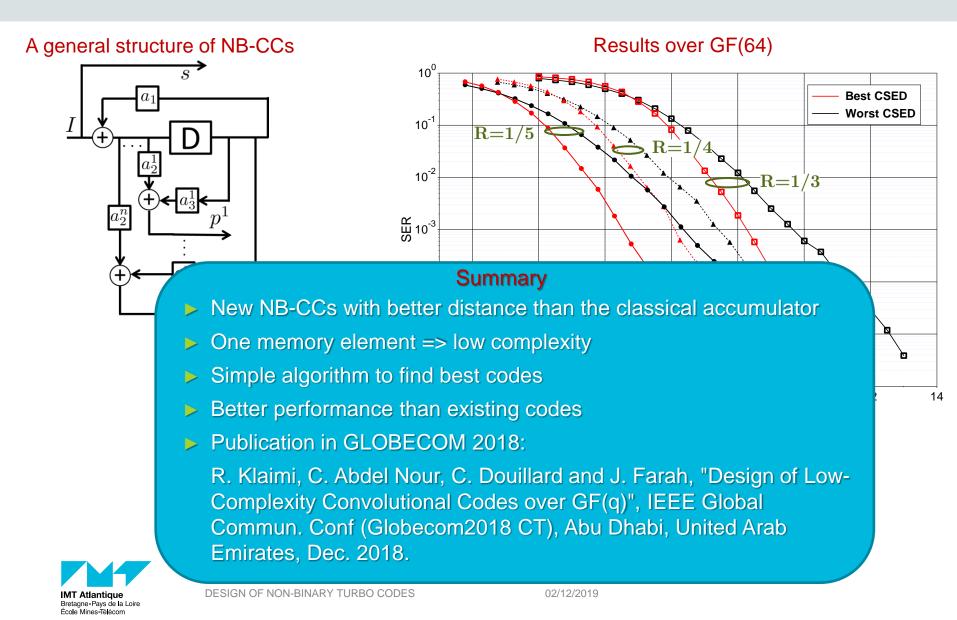
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DESIGN OF NON-BINARY TURBO CODES

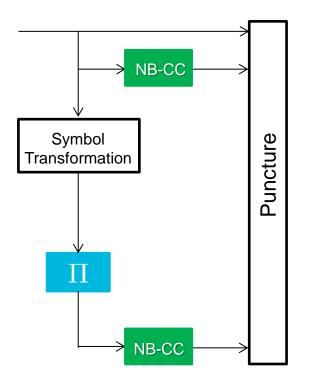
 $E_s/N_0(dB)$

NB-CCs for low rate codes



Outline

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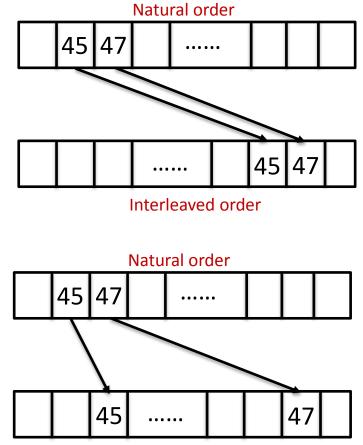


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Interleaving design: criteria to choose the best interleaver for NB-TCs

Spreading constraint: avoid same short-DC sequence at both decoders.



Interleaved order



Interleaving design: criteria to choose the best interleaver for NB-TCs

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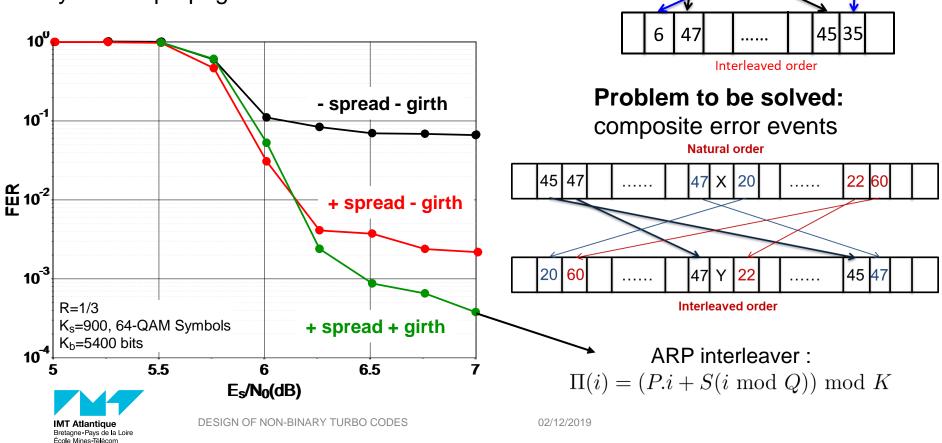
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Natural order

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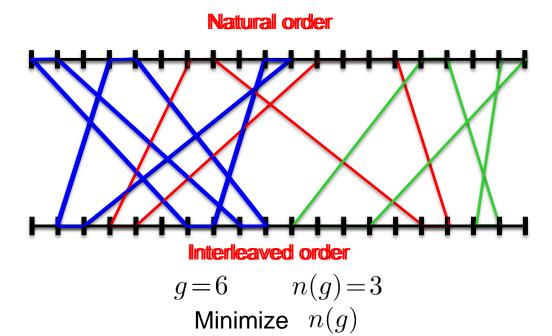
- Spreading constraint: avoid same short-DC sequence at both decoders.
- Girth constraint: short correlation cycles => propagated errors.



Interleaving design: reducing correlation effect for short frame sizes

Correlation cycle possible problems:

► High minimum cycle occurrence n(g)



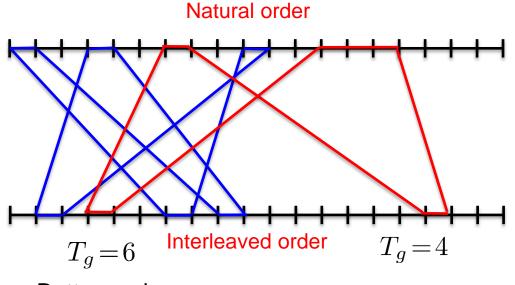


Interleaving design: reducing correlation effect for short frame sizes

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Correlation cycle possible problems:

- High minimum cycle occurrence n(g)
- Low information exchange between component decoders



Better cycles

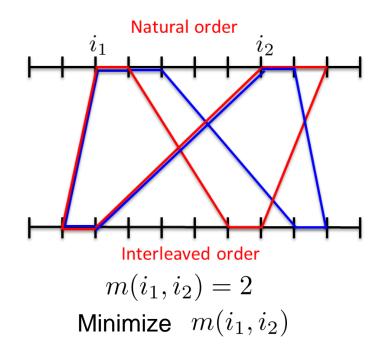


Interleaving design: reducing correlation effect for short frame sizes

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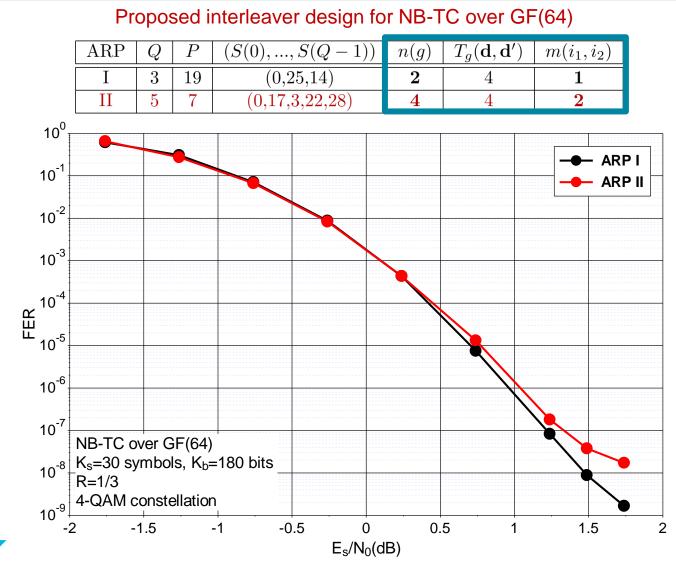
Correlation cycle possible problems:

- High minimum cycle occurrence n(g)
- Low information exchange between component decoders
- Couples of symbols occurrence in minimum cycles $m(i_1, i_2)$.





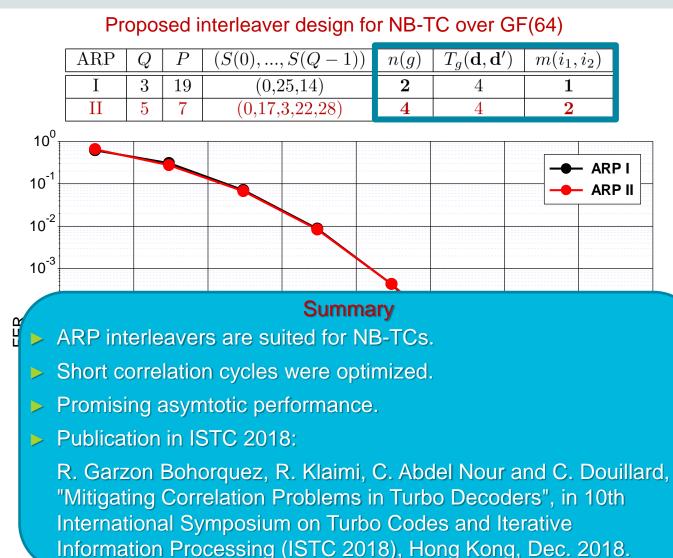
Interleaving design: obtained results





DESIGN OF NON-BINARY TURBO CODES

Interleaving design: obtained results

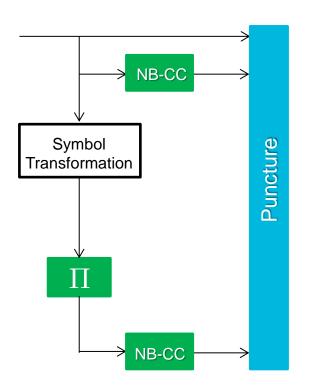




DESIGN OF NON-BINARY TURBO CODES

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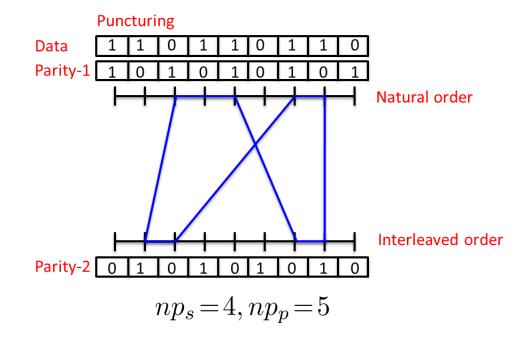
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Proposed puncturing: symbol puncturing (1/2)

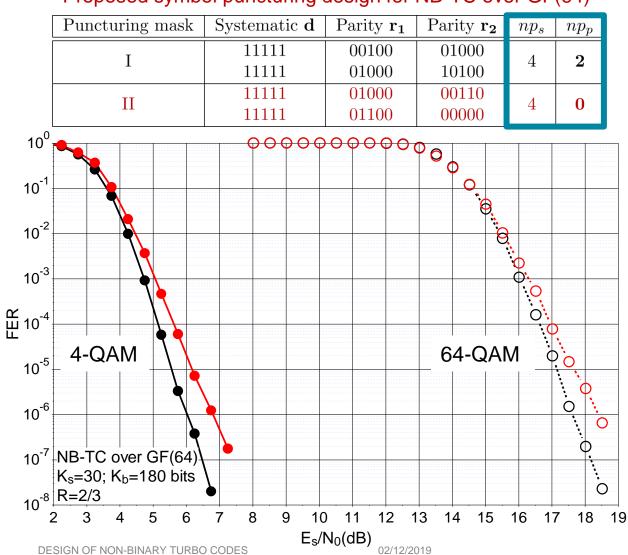
- Low number of non-punctured symbols in correlation cycles
 => possible error event.
- Avoid to puncture symbols from short correlation cycles.
- Higher mutual information propagated in short correlation cycles.





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Proposed puncturing: symbol puncturing (2/2)



Proposed symbol puncturing design for NB-TC over GF(64)

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Proposed puncturing: bit puncturing (1/2)

Applied method:

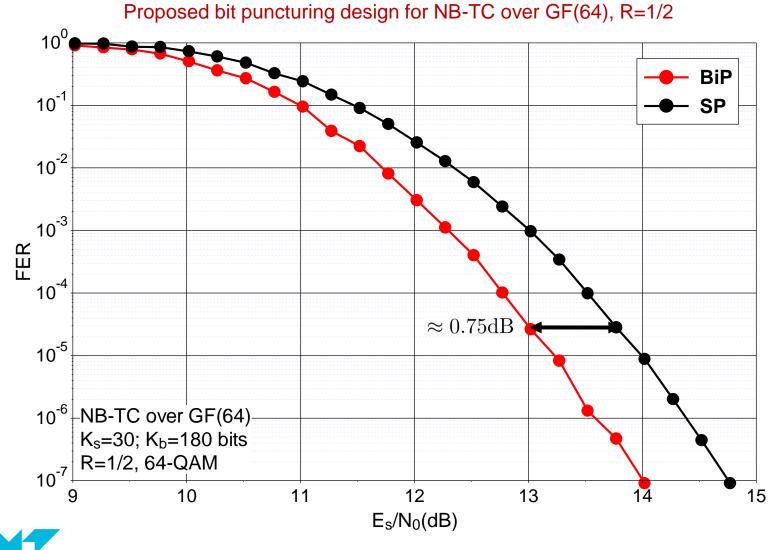
- Bits are protected differently in high order constellations.
- Several symbol protection levels.

 Under consideration for possible patent filing.



Proposed puncturing: bit puncturing (2/2)

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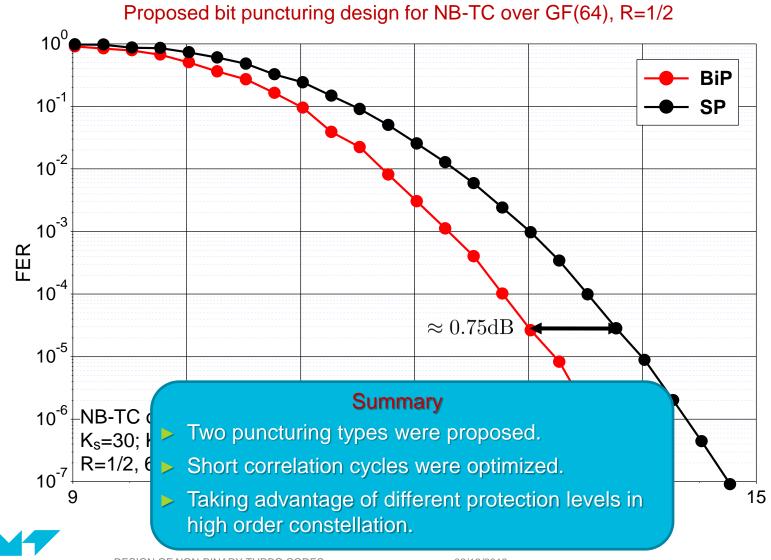


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DESIGN OF NON-BINARY TURBO CODES

Proposed puncturing: bit puncturing (2/2)

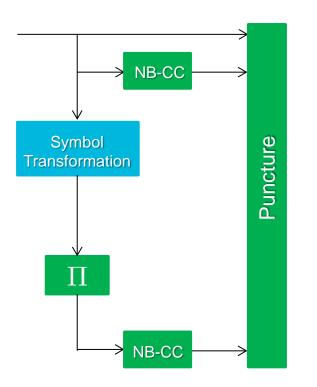
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IMT Atlantique Bretagne-Pays de la Loire École Mines-Télécom DESIGN OF NON-BINARY TURBO CODES

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1. Context and state of the art

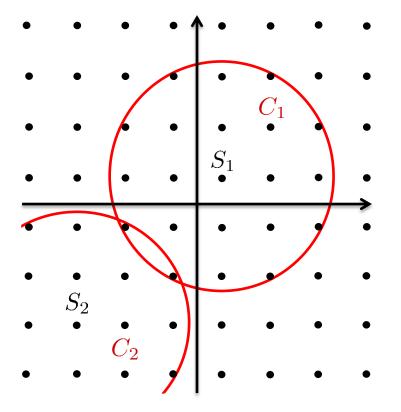
2. Design of NB-TCs:

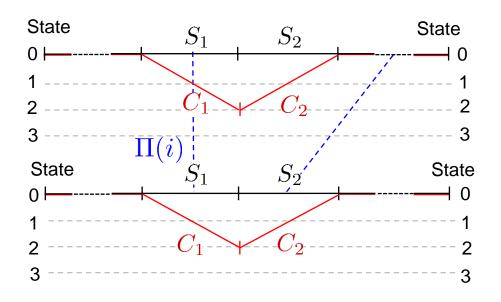
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DESIGN OF NON-BINARY TURBO CODES

Error prone sequences of NB-TCs: neighboring systematic symbols





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Repeated error pattern at both decoders => same false decoded sequence at both SISO decoders.

Neighboring constellation symbols => low CSED. How to avoid them?

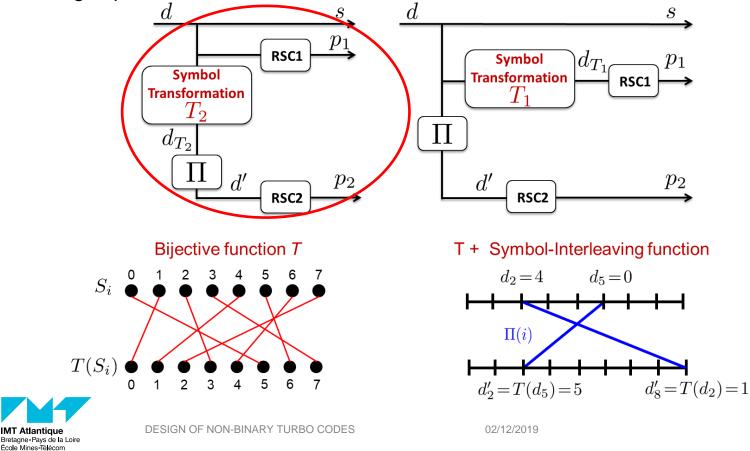


DESIGN OF NON-BINARY TURBO CODES

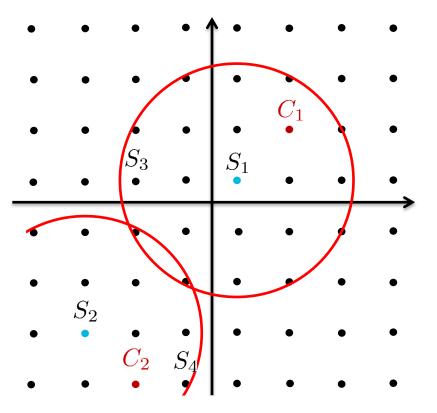
Symbol transformation: an additional freedom degree for NB-TCs

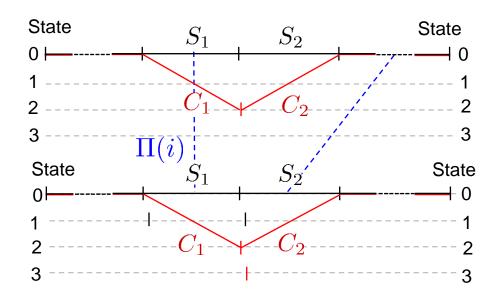
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- ▶ New freedom degree can be investigated in the case of NB-TCs.
- ► A bijective symbol transformation can be added before one of the constituent encoders.
- The repetition structure is replaced by the transformation block to enhance the error correcting capabilities.



Why is it beneficial?





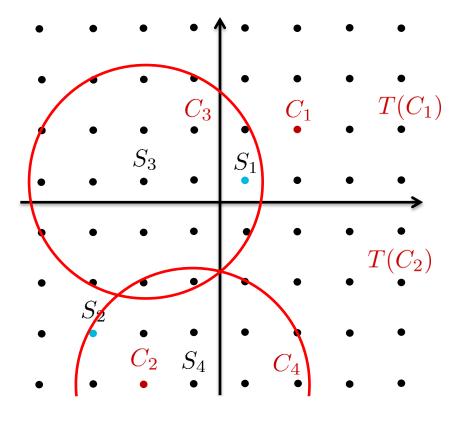
Repeated error pattern at both decoders => same false decoded sequence at both SISO decoders.

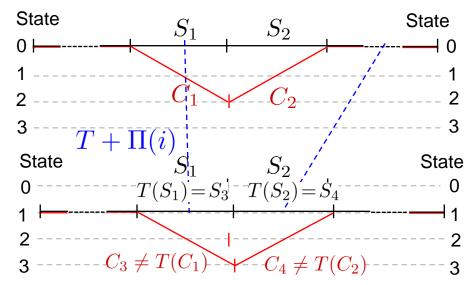
Neighboring constellation symbols => low CSED. How to avoid them?



DESIGN OF NON-BINARY TURBO CODES

Why is it beneficial?





Bijective transformation results in different sequences at both decoders => different candidates at decoders => better information exchange.

Initial design constraint: maximize $\delta(S_i, S_j) = d^2(S_i, S_j) + d^2(T(S_i), T(S_j))$



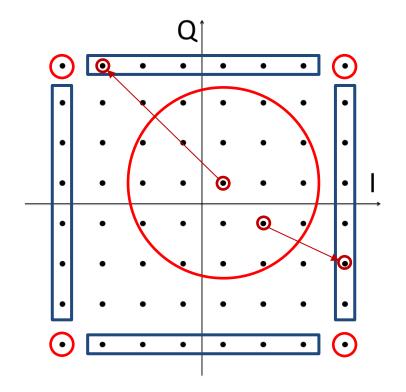
DESIGN OF NON-BINARY TURBO CODES

GF(64) code mapped to a 64-QAM constellation

- Different protection levels for symbols in a 64-QAM constellation
- Neighboring symbols lead to short DC sequences.

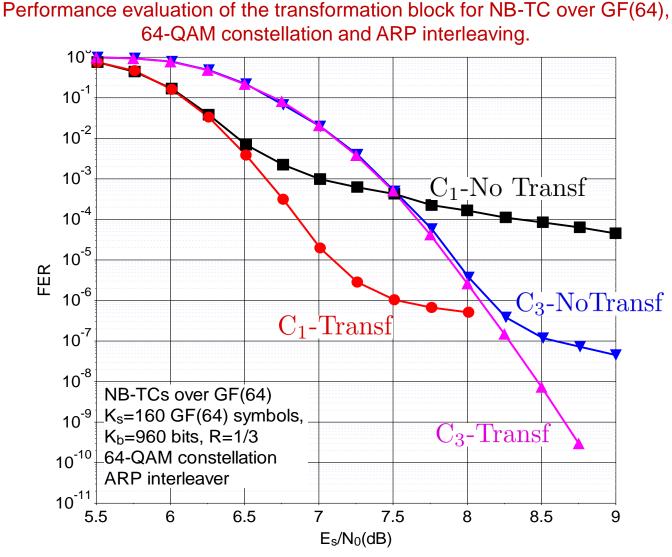
Proposed constraints to design a good transformation function

- Highly protected symbols => low protected symbols.
- Neighboring symbols => nonneighboring ones.





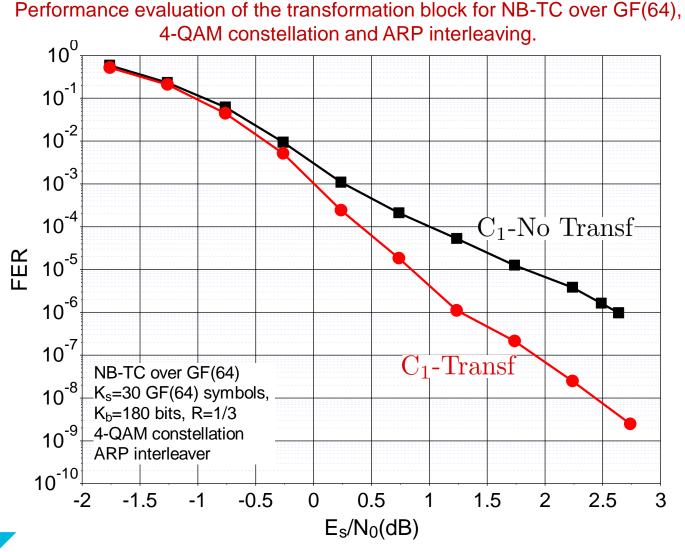
Results over 64-QAM constellation: ARP interleaving 38/54





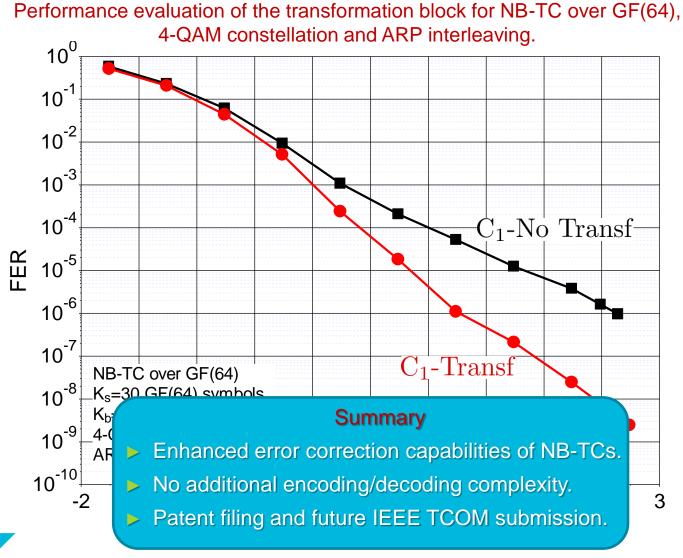
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Results over 4-QAM constellation: ARP interleaving 39/54



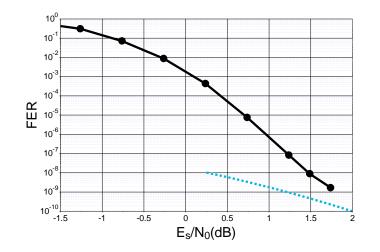
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Results over 4-QAM constellation: ARP interleaving 40/54





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Union bounds evaluation: problem description and equations

Union bounds are used to predict the asymptotic behavior of a FEC.

	B-TC	NB-TC
B modulation	~	 ✓
NB modulation	~	X

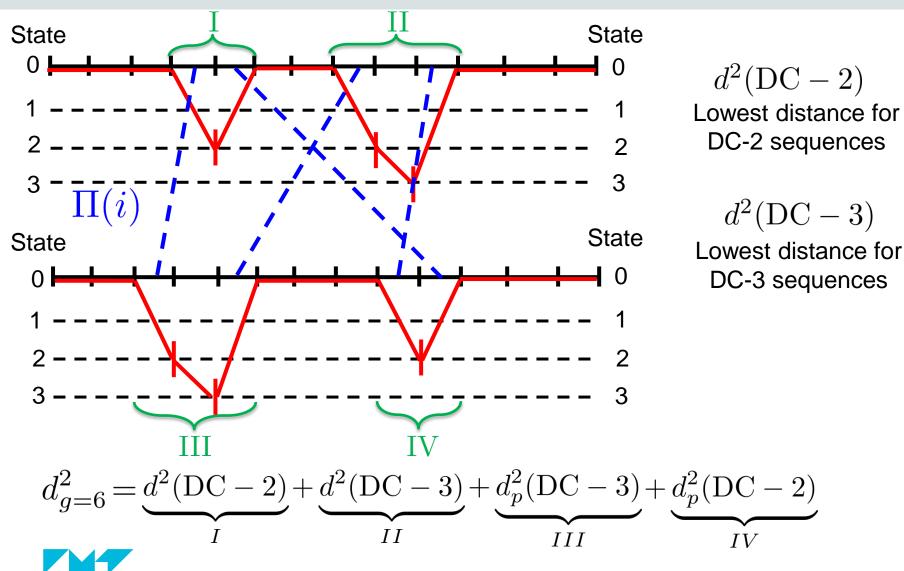
Union bound in the case of NB-TCs:

$$P_f \leq 2 \sum_{i=1}^{i_{max}} r(d_i) Q\left(\frac{d_i}{2\sigma}\right)$$

- **Problem:** identification of the cumulated Euclidean distance spectrum of NB-TCs.
- Two types of error-prone sequences of NB-TCs are observed:
 - error events caused by short correlation cycles,
 - error events caused by low interleaving spread.



Error-prone sequences: short correlation cycles

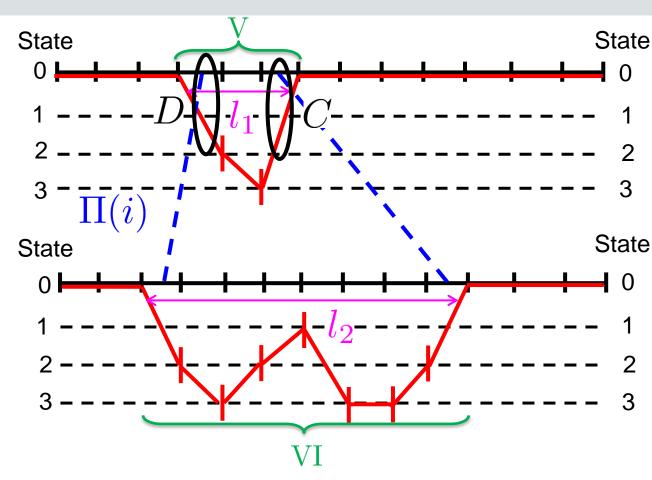




DESIGN OF NON-BINARY TURBO CODES

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Error-prone sequences: low spreading values



 $d^2(D)$

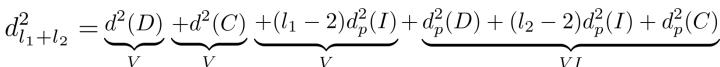
Lowest distance between symbols in the diverging transitions.

$$d^2(C)$$

Lowest distance between symbols in the converging transitions.

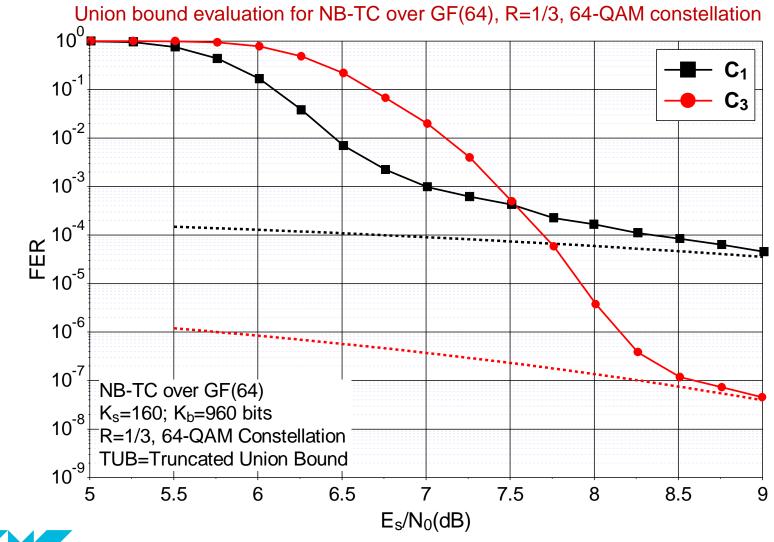
$$d_p^2(I)$$

Lowest distance between parity symbols in the intermediate transitions.



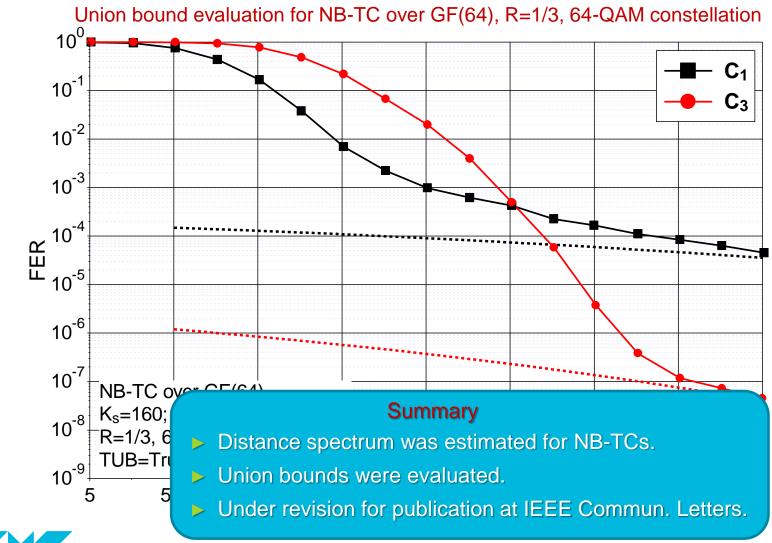


Union bounds evaluation: results (2/2)





Union bounds evaluation: results (2/2)

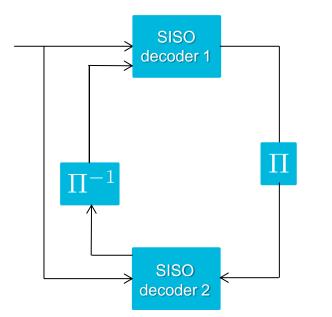




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Min-Log-MAP decoding algorithm for NB-TCs: complexity issue

$$\alpha_i(j) = \min_{j' \in \{0 \cdots q-1\}} (\alpha_{i-1}(j') + \gamma_{s,i-1}(j',j) + \gamma_{p,i-1}(j',j))$$

Backward state metric

$$\beta_i(j) = \min_{j' \in \{0 \cdots q-1\}} (\beta_{i+1}(j') + \gamma_{s,i}(j,j') + \gamma_{p,i}(j,j'))$$

q^2 ACS operations

Complexity

 q^2 ACS operations

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Extrinsic LLR

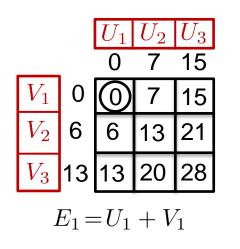
 $L_{i}^{e}(a) = \min_{(j,j') \in \{0 \cdots q-1\}^{2} \mid s(j,j') = a} (\alpha_{i}(j) + \beta_{i+1}(j') + \gamma_{p,i}(j,j')) \qquad q^{2} \text{ ACS operations}$

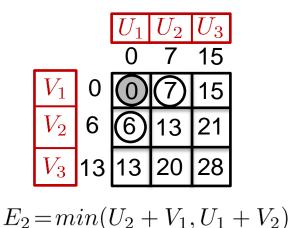
 $3q^2$ ACS operations at each trellis stage



Complexity reduction for NB-LDPC decoders

- Extended Min-Sum (EMS) decoding algorithm
- Limiting the number of considered LLRs to the (n_m) most reliable ones ^[12]
- Limiting the number of Add-Compare-Select (ACS) operations through the use of bubble check algorithm.^[13]





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[12] A. Voicila, D. Declercq, F. Verdier, M. Fossorier, and P. Urard, "Low-complexity, low-memory EMS algorithm for non-binary LDPC codes" in *IEEE International Conference on Communications. IEEE, 2007, pp. 671–676.*

[13] E. Boutillon and L. Conde-Canencia, "Bubble check: a simplified algorithm for elementary check node processing in extended minsum non-binary LDPC decoders" *Electronics Letters, vol. 46,* no. 9, pp. 633–634, 2010.



Proposed low-complexity decoding algorithm

Bubble check: not directly applicable for Min-Log-MAP

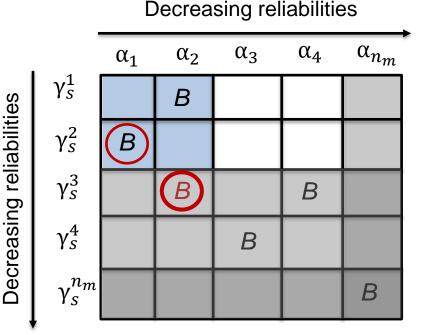
- Output sorting is not required.
- Three terms in each calculation => 3D bubble check.
- Not all transition/state metric combinations are possible: some cells in the table remain empty.

Proposal

- 2D-bubble proposed => compute forward, backward recursions and extrinsic info.
- n_m LLRs are used.
- No connections between highly reliable values => exit algorithm.
- Bubble identified => introduction of a virtual bubble => possible early stopping.

$$\alpha_i(j) = \min_{j' \in \{0 \cdots q-1\}} (\alpha_{i-1}(j') + \gamma_{s,i-1}(j',j) + \gamma_{p,i-1}(j',j))$$

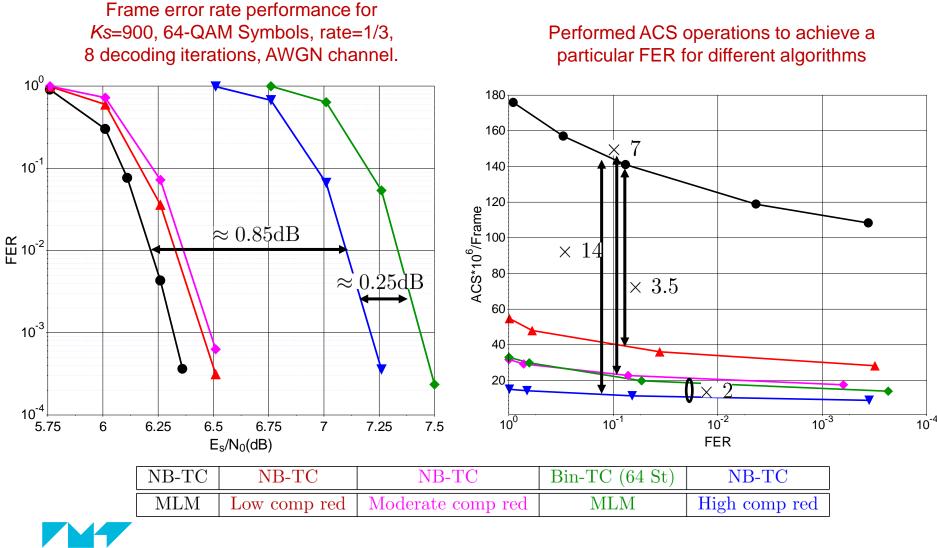
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$$\alpha_i(j) = \min(\alpha_i(j), B)$$



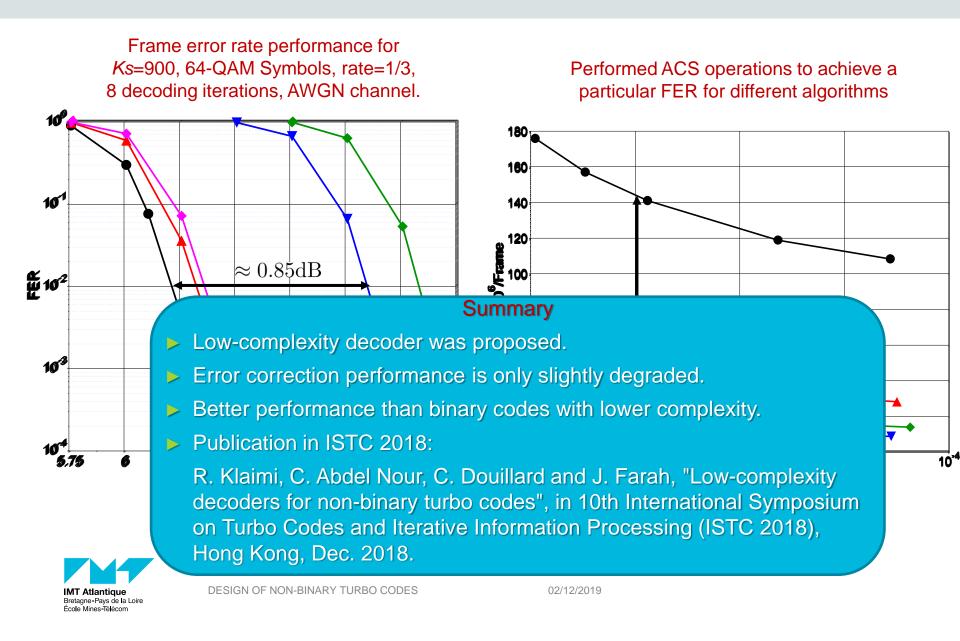
Error correction performance and decoding complexity 51/54



DESIGN OF NON-BINARY TURBO CODES

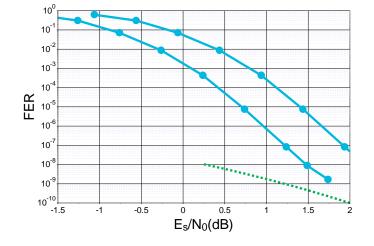
IMT Atlantique Bretagne-Pays de la Loire École Mines-Télécom 02/12/2019

Error correction performance and decoding complexity 52/54



Outline





- 1. Context and state of the art
- 2. Design of NB-TCs
- 3. Union bound evaluation for NB-TCs
- 4. Low-complexity decoding algorithm
- 5. Performance comparison
- 6. Conclusion and future work



Performance comparison with existing codes: comparison with LTE code



LTE

MLM

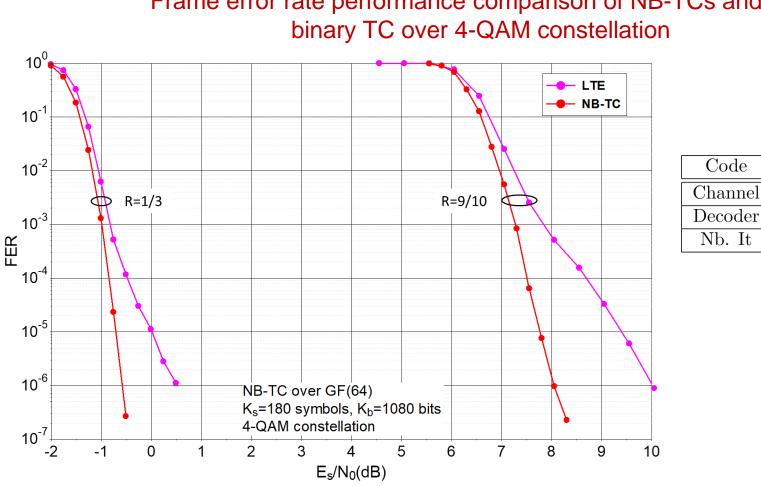
8

NB-TC

MLM

8

AWGN

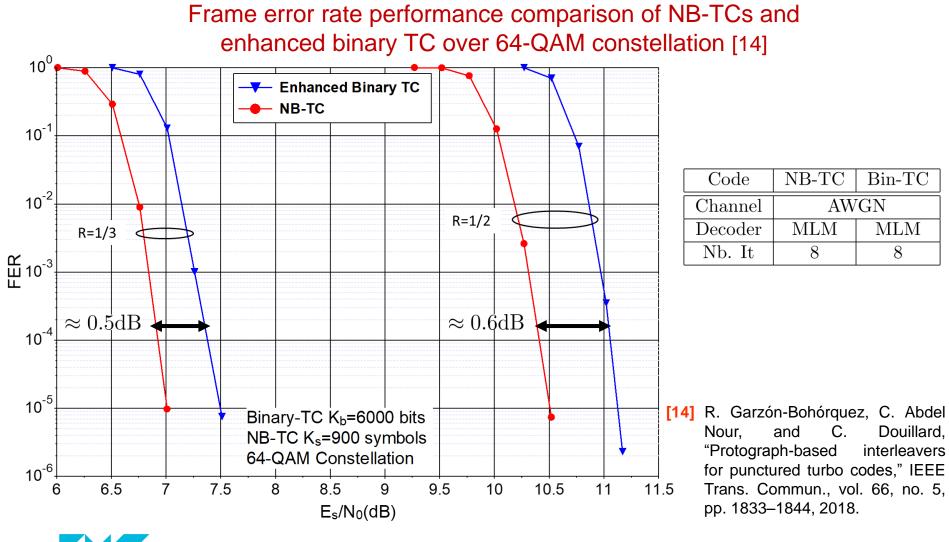


Frame error rate performance comparison of NB-TCs and LTE



Performance comparison with existing codes: comparison with enhanced binary turbo code

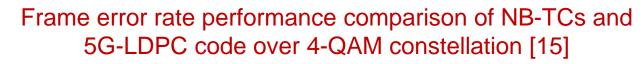


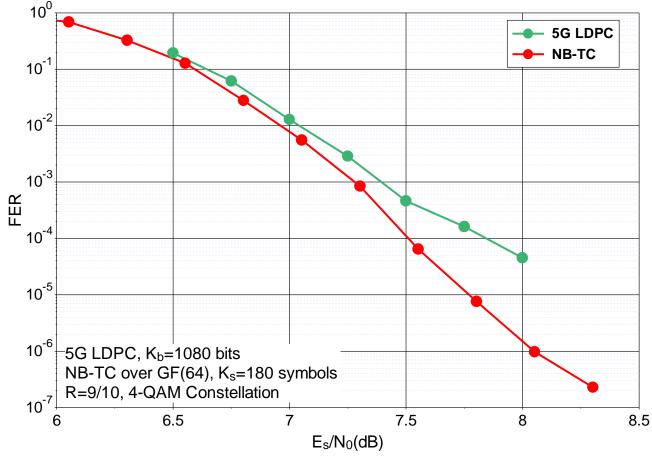




Performance comparison with existing codes: comparison with 5G-LDPC code

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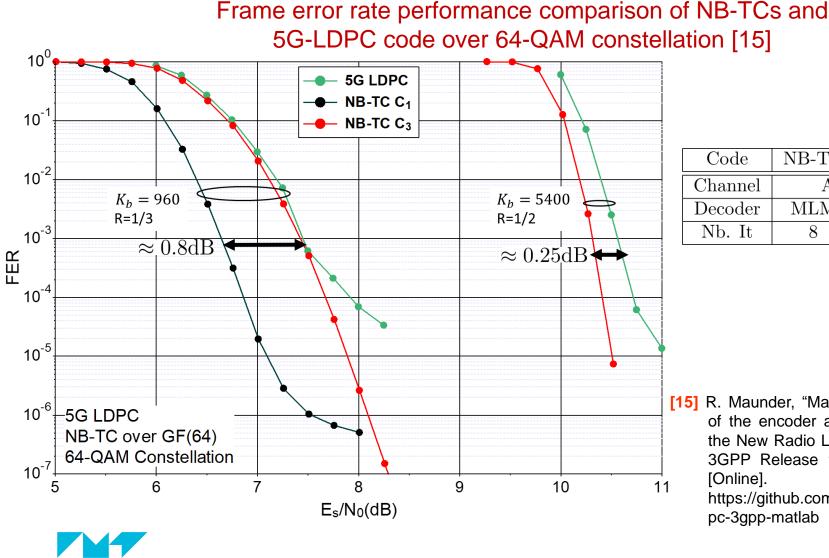
Code	NB-TC	LDPC
Channel	AW	GN
Decoder	MLM	BP
Nb. It	8	50

[15] R. Maunder, "Matlab simulations of the encoder and decoder for the New Radio LDPC code from 3GPP Release 15," Nov 2018.
[Online]. Available: https://github.com/robmaunder/ld pc-3gpp-matlab



Performance comparison with existing codes: comparison with 5G-LDPC code

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Code	NB-TC	LDPC
Channel	AWGN	
Decoder	MLM	BP
Nb. It	8	50

[15] R. Maunder, "Matlab simulations of the encoder and decoder for the New Radio LDPC code from 3GPP Release 15," Nov 2018. Available: [Online]. https://github.com/robmaunder/ld pc-3gpp-matlab

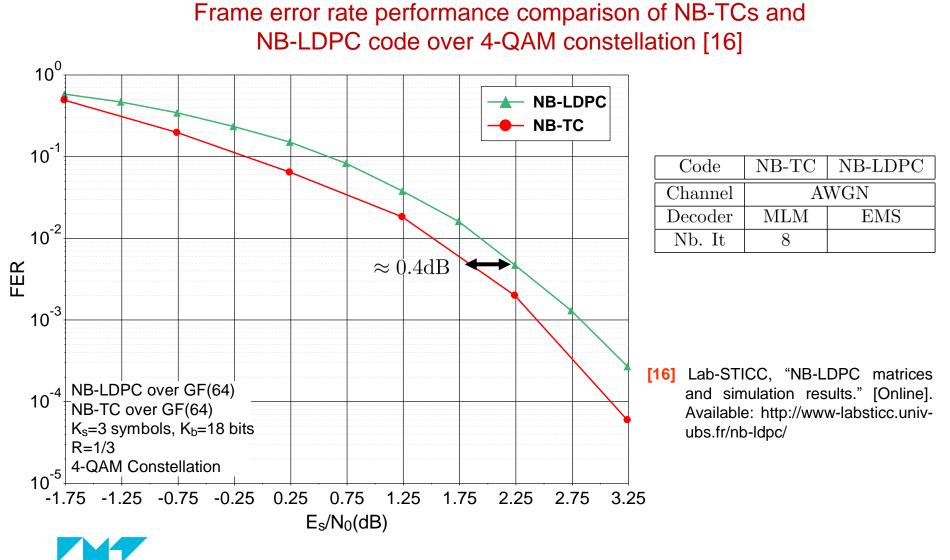
DESIGN OF NON-BINARY TURBO CODES

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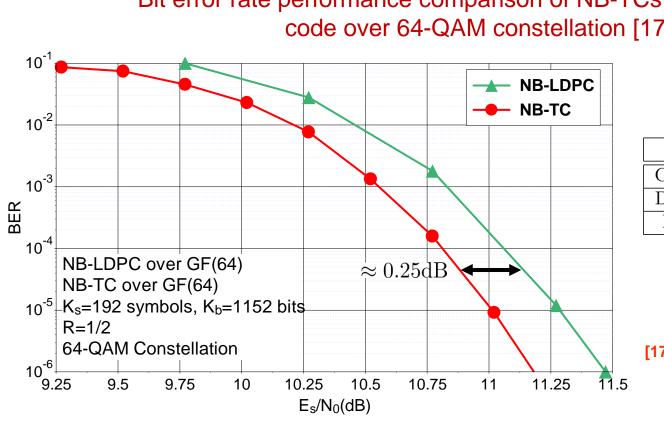
Performance comparison with existing codes: comparison with NB-LDPC code

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Performance comparison with existing codes: comparison with NB-LDPC code

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59/54
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Bit error rate performance comparison of NB-TCs and NB-LDPC code over 64-QAM constellation [17]

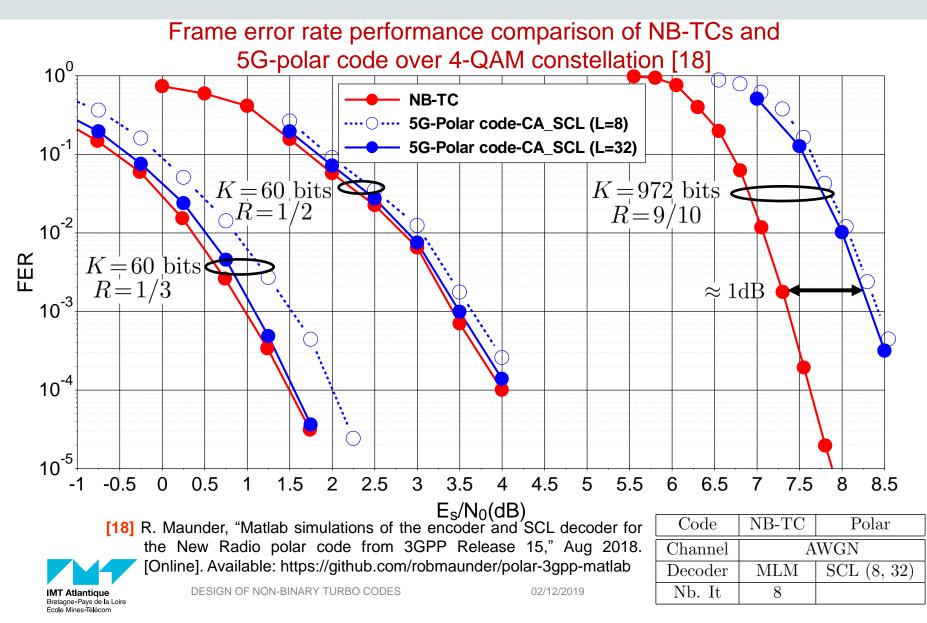
Code	NB-TC	NB-LDPC
Channel	AWGN	
Decoder	MLM	FFT-QSPA
Nb. It	8	50

[17] D. Feng, H. Xu, Q. Zhang, Q. Li, Y. Qu, and B. Bai, "Nonbinary LDPC-Coded Modulation System **High-Speed** Mobile in Communications," IEEE Access, vol. 6, pp. 50994–51001, 2018.



Performance comparison with existing codes: comparison with 5G-polar code





OUTLINE

- **1. Context and state of the art**
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Conclusion

- Proposal of a new low-complexity structure of NB-CC with largely improved CSED (> 25%).
- Error patterns analyzed => design of suitable ARP interleavers.
- Study of puncturing schemes => bit puncturing with largely improved performance (≈0.75 dB).
- Introduction of a transformation block => very low error floors (< 10^{-10} of FER).
- ► Union bounds for NB-TC+high order modulations can now be evaluated.
- First low-complexity decoding algorithm for NB-TCs
 - E.g. 7x reduction in nb of ACS vs MLM for 0.15dB perf penalty.
 - Better perf than 64-state binary codes with lower complexity.
 - Several perf/complexity tradeoffs.
- Proposed NB-TCs are shown to outperform existing binary and non-binary FEC codes under different simulation conditions.



Future work

- Towards a complete FEC solution
 - Rate adaptability mechanism.
 - Optimization of the proposed blocks depending on the application.
- Further design exploration
 - Time variant NB-CCs => break short DC sequences.
 - Enhanced design of the transformation block.
 - NB-TCs with probabilistic shaping.
- Hardware implementation
 - Investigation of different perf/complexity tradeoffs.
 - Design space exploration of HW architectures.



List of publications

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- ► Journal papers:
 - R. Klaimi, C. Abdel Nour, C. Douillard and J.Farah "Union bound evaluation for Non-Binary Turbo Codes", submitted to IEEE Commun. Letters.
 - R. Klaimi, C. Abdel Nour, C. Douillard and J.Farah "Enhancing performance of non-binary turbo codes via symbol transformation", to be submitted to IEEE Trans. on Commun.
- Conference papers:
 - R. Klaimi, C. Abdel Nour, C. Douillard and J. Farah, "Design of Low-Complexity Convolutional Codes over GF(q)", IEEE Global Commun. Conf.: Communication Theory (Globecom2018 CT), Abu Dhabi, United Arab Emirates, Dec. 2018.
 - R. Garzon Bohorquez, R. Klaimi, C. Abdel Nour and C. Douillard, "Mitigating Correlation Problems in Turbo Decoders", in 10th International Symposium on Turbo Codes and Iterative Information Processing (ISTC 2018), Hong Kong, Dec. 2018.
 - R. Klaimi, C. Abdel Nour, C. Douillard and J. Farah, "Low-complexity decoders for non-binary turbo codes", in 10th International Symposium on Turbo Codes and Iterative Information Processing (ISTC 2018), Hong Kong, Dec. 2018.
- Patent filing:
 - R. Klaimi, C. Abdel Nour and C. Douillard "Procédé de génération d'un signal mettant en œuvre un turbo-codeur, dispositif et programme d'ordinateur correspondants (in French)", Patent application FR 1873615, Dec. 2018.
 - Patent filing under consideration for the bit puncturing technique.

