Adaptive Quantization of the Polar Successive Cancellation Decoder

Yiğit Ertuğrul and Erdal Arıkan

Department of Electrical and Electronics Engineering, Bilkent University

{ertugrul, arikan}@ee.bilkent.edu.tr

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Introduction

Polar codes and its implementation challenges are an active study field since [1]. SC decoding algorithm is the well known polar decoding algortihm described in [1]. The chip area of the SC decoder is dominated by memory and register chains in the deeply-pipelined architecture [2]. An adaptive quantization technique is proposed to reduce the memory complexity of the polar SC decoder. Due to polarization of synthesized bit channels, some portion of the bits could be decoded by diminishing the log-likelihood ratios' (LLRs) bit precision with a negligible communications performance loss. Unlike standard SC decoding, which operates with constant bit precision throughout decoding, this method utilizes varying number of bits to represent LLRs among the levels of the decoder. Bit precision of each decoding level could be determined by a heuristic method. Another method is motivated by maximizing the mutual information between the levels of the decoder [3]. Adaptive quantization method reduces the memory complexity of the decoder proportional to coding rate.

Algorithm

In order to find a systematic way of LLR quantization, we propose reducing precision bits based on reliability order of the bit channels. LLR precision could be changed either at the input of the well known *f* & *g* processing functions or at the output. Due to sequential nature of the polar SC decoder, some portion of the LLRs must be stored and reused for another operation. The amount of storage could be dropped even we reduce the bit precision at the output of the *f* & *g* functions. After the number of quantization bits are calculated, we use the following formula to reduce the number of quantization bits. This formula reduces the LLR bit precision starting from the least significant bit.

$$\ell' = \text{sgn}(\ell)\left\lfloor\frac{|\ell|}{2^{Q-Q'}}\right\rfloor$$
$$= \min\left(2^{Q'-1}-1, \ell'\right)$$
$$= max\left(-(2^{Q'-1}-1), \ell'\right)$$

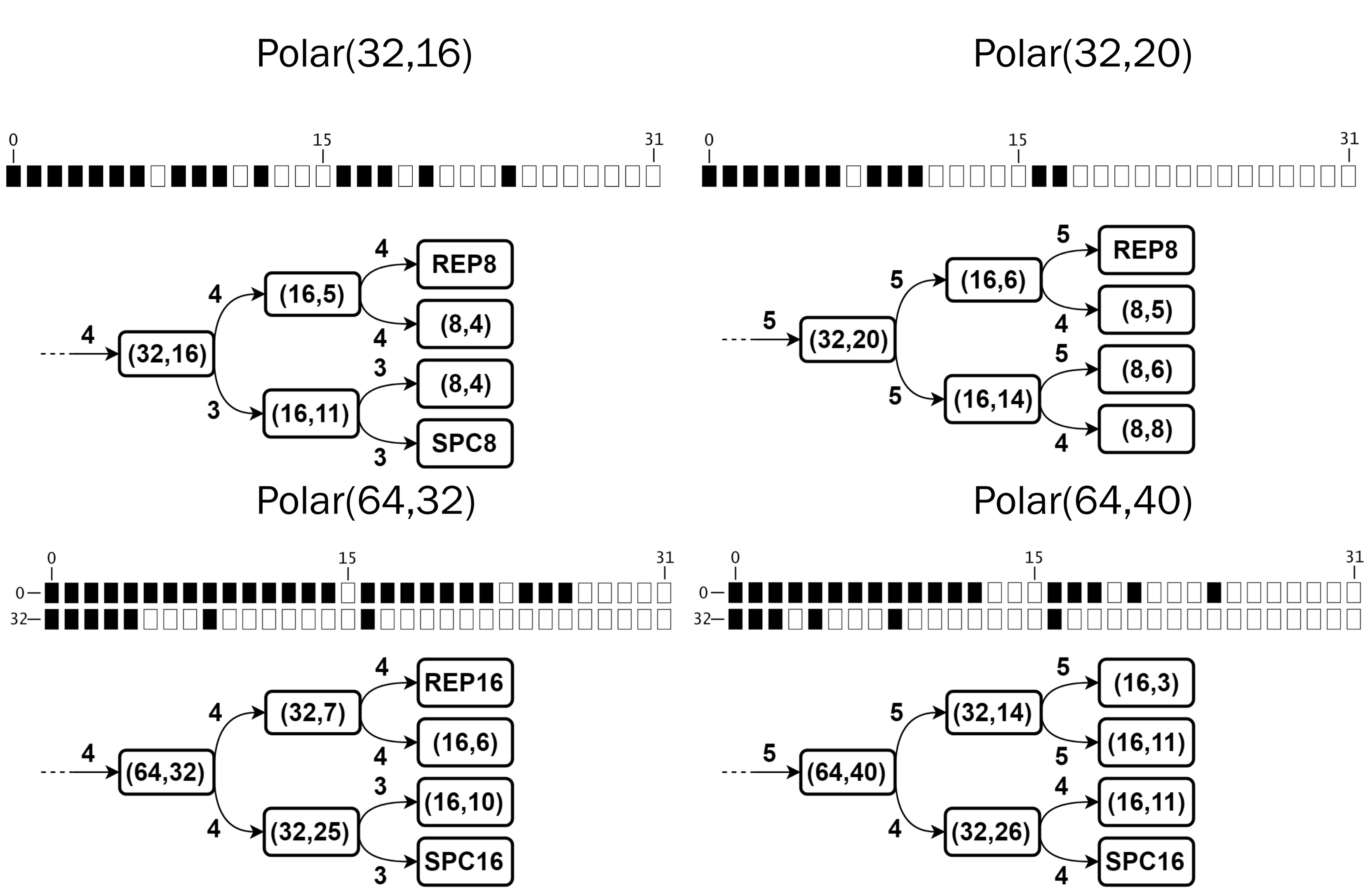
where ℓ is the LLR value, ℓ' is the LLR value after quantization operation, Q is the input LLR bit precision and Q' is the target LLR bit precision. Memory reduction of this method is measured by defining

$$\bar{Q} = \frac{\sum_{i}^{\lg(N)}\sum_{j=1}^{2^i} N_{ij}Q_{ij}t_{ij}}{\sum_{i}^{\lg(N)}\sum_{j=1}^{2^i} N_{ij}t_{ij}}$$

where N_{ij} is the block length of the sub-tree, Q_{ij} is the LLR bit precision of the cornensponding leaf node and t_{ij} is the number of clock cycles that the LLR vector is stored. $\bar{Q}$ could be thought of like average LLR bit precision. Without using adaptive quantization $\bar{Q} = Q$. Example (1024,854) polar decoder has $\bar{Q} = 4.24$ which is approximately 15% memory reduction compared to fixed 5 bit precision implementation.

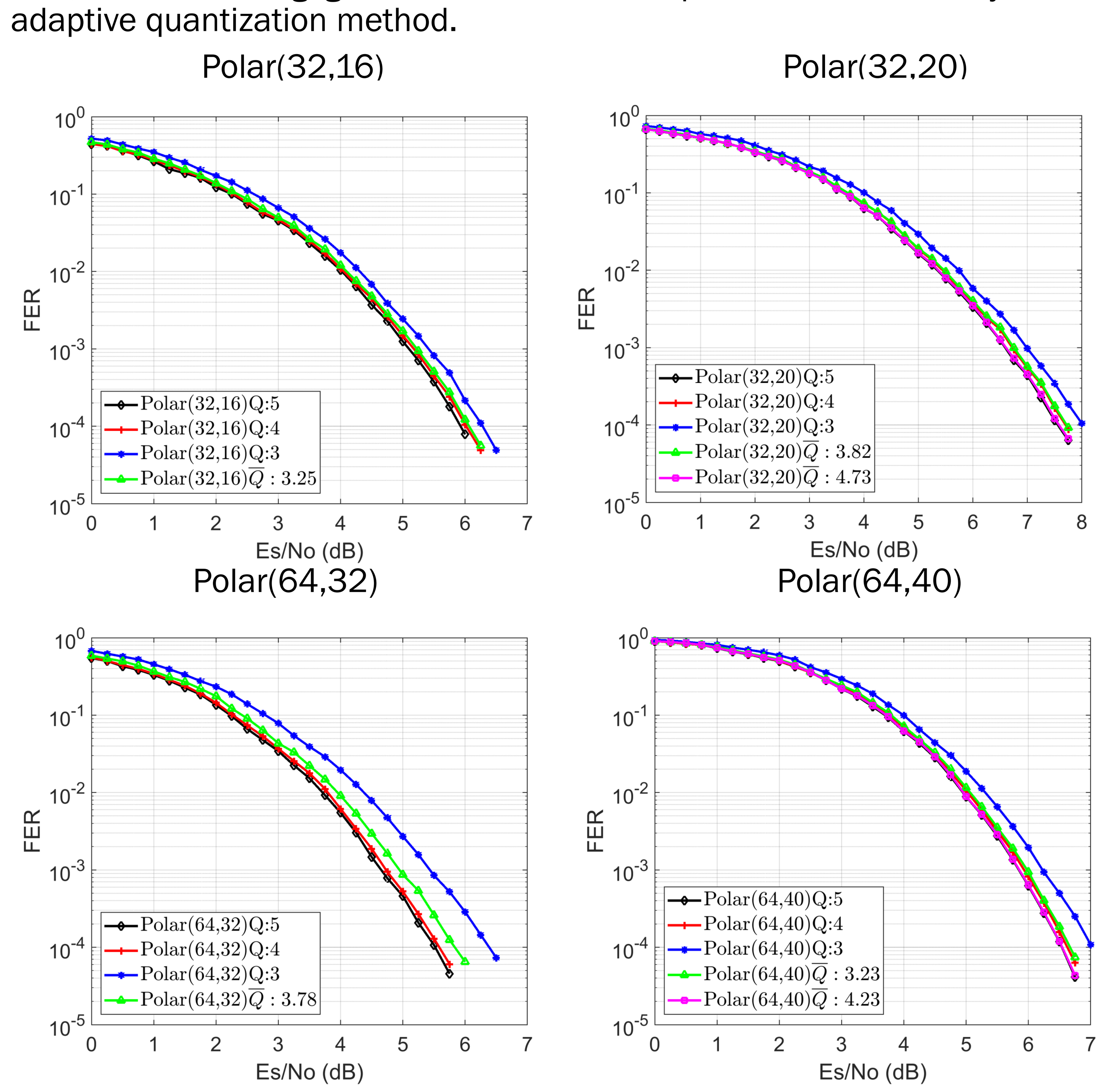
Code Parameters

Polar code design and adaptively quantized polar SC decoder parameters are described in this section. Block lengths 32 and 64, coding rates 0.50 and 0.62 are selected in order to demonstrate the effect of adaptive quantization method. SC decoding tree for each code is illustrated below where LLR quantization of each leaf Q_{ij} is indicated on arrows. For each decoding tree the frozen index vector is indicated where black colour represents a frozen bit and white colour represents a free bit.



Simulation Results

Simulation results of polar SC decoder under AWGN channel and BPSK modulation is illustrated in this section. Input LLR vector has 3, 4 and 5 bits precision quantized from floating point values. Target frame error rate is selected as 10^{-4}. Maximum number of trials for each SNR point is 10^7. $\bar{Q}$ is calculated by recording t_{ij}, Q_{ij} and N_{ij} on MATLAB environment. Frame error rate (FER) versus SNR performance of polar codes are depicted below. Note that, we have negligible communications perfromance loss by utilizing adaptive quantization method.



Memory reduction after utilizing adaptive quantization method is shown in table below where memory reduction is measured with respect to input bit precision Q_{in}.

N	K	R	Q_{in}	$\bar{Q}$	Memory Reduction
32	16	0.50	4	3.25	18%
32	20	0.62	4	3.82	4%
32	20	0.62	5	4.73	5%
64	32	0.50	4	3.78	5%
64	40	0.62	4	3.23	19%
64	40	0.62	5	4.23	15%

Conclusions

In this work, memory complexity of the unrolled and pipelined polar SC decoder is reduced by using an adaptive quantization method with negligible communications performance loss. We have demonstrated memory reduction upto 19% where this number is highly dependent on block length, coding rate and code design parameters. This work was restricted to a given polar code construction. In other words, adaptive quantization technique is used for a given fixed polar code design. Future work will be focused on polar code constructions that are aware of the quantization of the corresponding sub-channel. Furthermore, nonuniform quantization will be studied and could be utilized by transforming LLR calculations into LUTs in hardware.

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