

# The Papers of Independent Authors 

Volume 17

Mathematics \5<br>Meteorology $\backslash 61$<br>Physics and Astronomy $\backslash 69$<br>Authors \167

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Site with information for the author - http://izdatelstwo.com
The contact information - publisher-dna@hotmail.com
Fax: ++972-8-8691348
Address: POB 15302, Bene-Ayish, Israel, 60860
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$$
\begin{aligned}
& \| \begin{array}{r}
\text { Truth - the daughter of time, instead of authority } \\
\text { Frensis Bacon }
\end{array} \\
& \| \begin{array}{r}
\text { Everyone has the right to freedom of opinion and expression; } \\
\text { this right includes freedom to hold opinions without } \\
\text { interference and to seek, receive and impart information and } \\
\text { ideas through any media and regardless of frontiers. } \\
\text { United Nations OrganizationUniversal. }
\end{array} \\
& \text { Declaration of Human Rights. Article } 19 .
\end{aligned}
$$

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Solomon I. Khmelnik.

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## Yarosh V.S.

# Non-modular elliptic curves and congruents numbers of pythagorean 

## Annotation

The article «Non-modular elliptic curves and congruents numbers of Pythagorean» - the play-off to dramatic history of the determination of real existence of the simple proof Fermat's Last Theorem. Known italian mathematician Paulo Ribenboym wrote in its book Fermat's Last Theorem: «Some mathematicians not satisfied method proof, using elliptical curves and modular of the form, as alien this problem. Wholly reasonable problem to try to find another, more idle time proof Fermat's Last Theorem». Known american mathematician Harold M Edwards wrote also about this in its book Fermat's Last Theorem: "Certainly, wholly possible that best european mathematicians approached to problem false fetter and that exists some simple idea - possible openning Fermat, acceptable to all events." Such idea was offered by author of this article Her have blocked the known mathematicians to Russia, which approached to problem false fetter. The Copies blocking documents reader will find on put http://yvsevolod-28.narod.ru/index.html. In spite of official blocking, the idea was published in book, published in russian and english. The Role of the key to asked a decision of the problem execute primitiv and congruents of the numbers of Pythagorean. :In proposed article is described proof of real existence endless ensemble non-modular elliptical curves. The mail goal of the article is to consider to common solution the system equations of P.Fermat, G.Frey and its application to the non-modular elliptic curves. It is exact proof for the facts: Hypothesis of G.Shimura-Y. Taniyama: All elliptic curves is modular curves - it is wrong. Proof of A.Wiles for Last Theorem of P.Fermat is doubtful.

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1. Introduction.
2.Non-modular elliptic curves and The Primitive three-tuples of Pythagoras
2. Unique characteristics of congruous numbers of Pythagoras
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## 1. Introduction

The main goal of the article is to consider to common solution the system equations of P. Fermat, G. Frey and its application to the nonmodular elliptic curves. It is exact proof for the facts:

1. Hypothesis of G. Shimura - Y. Taniyama: All elliptic curves is modular curves - it is wrong.
2. Proof of A. Wiles for Last Theorem of P. Fermat is doubtful. The Known that proof of the Last theorem of P. Fermat is founded on hypothesis Shimura-Taniyama, which confirms: All are an elliptical curves is modular curves [1].
The First general phrases from article [2] doctor A. Willes are indicative of this. Here is these first general phrases:

An elliptic curve over $\mathbf{Q}$ is said to be modular if it has a finite covering by a modular curve of the form $X_{0}(N)$. Any such elliptic curve has the property that its Hasse-Weil zeta function has an analytic continuation and satisfies a functional equation of the standard type. If an elliptic curve over $\mathbf{Q}$ with a given $j$-invariant is modular then it is easy to see that all elliptic curves with the same $j$-invariant are modular (in which case we say that the $j$-invariant is modular). A well-known conjecture which grew out of the work of Shimura and Taniyama in the 1950's and 1960's asserts that every elliptic curve over Q is modular. However, it only became widely known through its publication in a paper of Weil in 1967 [We] (as an exercise for the interested reader!), in which, moreover, Weil gave conceptual evidence for the conjecture. Although it had been numerically verified in many cases, prior to the results described in this paper it had only been known that finitely many $j$-invariants were modular.

In 1985 Frey made the remarkable observation that this conjecture should imply Fermat's Last Theorem. The precise mechanism relating the two was formulated by Serre as the $\varepsilon$-conjecture and this was then proved by Ribet in the summer of 1986. Ribet's result only requires one to prove the conjecture for semistable elliptic curves in order to deduce Fermat's Last Theorem.

In proposed article is described proof of real existence endless ensemble non-modular elliptical curves.
There of proof follows the conclusion about that, that proof fLast Theorem, offered by doctor A. Willis [2], is wrongly.
Proof of this phenomenon to numbers theories is based on substitution, which author of the article construct from primitive three-tuple of Pythagoras [3]:

$$
\begin{gathered}
a_{0}=v^{2}-u^{2} \\
b_{0}=2 \mathbf{v u} \\
\mathbf{c}_{0}=v^{2}+u^{2}
\end{gathered}
$$

## 2.Non-modular elliptic curves and The Primitive three-tuples of Pythagoras

The Primitive three-tuples of Pythagoras serves the connecting-links between descriptions characteristic natural numbers, characteristic threedimensional space Euklid's, characteristic of the Fermat's equation, characteristic non-modular elliptical crooked and characteristic heavy atoms material - aktnoids. Information on non-modular elliptical crooked contains known equation of Frey G. All mentionned characteristic of numbers has a feedforward with structure heavy atoms material - an actinoids. Such way in given article uniquely dares ontological problem about unity mathematicians and physicists. This problem in vain solve in Internet participants of the forum " Physics and mathematics- that primary?" On put "Chislonavtika" this problem is long ago solved on base of the ancient science -a Theosophies, which creed the ancient civilization to India, Egypt and Greeces [4].
In mathematic (that is in nature well) everything is in interrelationship, all the lotis embraced by direct and indirect this both visible and invisible, strong and weak.
The mail goal of the article is to consider to common solution the system equations of P. Fermat, G. Frey and its application to the non-modular elliptic curves.
It is exact proof for the facts: Hypothesis of G. Shimura-Y. Taniyama: All elliptic curves is modular curves - it is wrong Proof of A. Wiles for Last Theorem of P. Fermat is doubtful.

It is known: David Hilbert, while solving the problem of Gordan's invariants, presented a universal formulation of this problem:
«Is given an endless system of forms of a finite number of variables. Under what circumstances exists a finite system of forms through which all others are expressed in the form of linear combinations as rational functions of the variables» [5].
Universality of the given formulation lies in the fact that it contain in a generalized form the description of a final solution of the Last Theorem Fermat's.
In our cause -this infinitely multitude equations:

$$
\mathbf{a}^{\mathbf{n}}+\mathbf{b}^{\mathbf{n}}=\mathbf{c}^{\mathbf{n}}
$$

each of which is realized at a concrete exponent of power n . The number of the generalized variable is finite: $\mathrm{n}, \mathrm{c}, \mathrm{b}$, a. In our case, use is made of three forms:

$$
\begin{gather*}
\left(\mathrm{a}_{o}^{2} \times \mathrm{a}_{o}^{\mathrm{n}-2}\right)+\left(b_{o}^{2} \times \mathrm{a}_{o}^{\mathrm{n}-2}\right)=\left(c_{o}^{2} \times \mathrm{a}_{o}^{\mathrm{n}-2}\right) \\
\left(\mathrm{a}_{0}^{2} \times \mathbf{b}_{o}^{\mathrm{n}-2}\right)+\left(b_{o}^{2} \times b_{o}^{\mathrm{n}-2}\right)=\left(c_{o}^{2} \times b_{o}^{\mathrm{n}-2}\right)  \tag{E}\\
\left(\mathrm{a}_{0}^{2} \times \mathbf{c}_{0}^{\mathrm{n}-2}\right)+\left(b_{0}^{2} \times \mathbf{c}_{o}^{\mathrm{n}-2}\right)=\left(c_{o}^{2} \times \mathbf{c}_{0}^{\mathrm{n}-2}\right)
\end{gather*}
$$

based on the Pythagorean equation:

$$
a_{o}^{2}+b_{o}^{2}=c_{o}^{2}
$$

The integral rational functions of the variables appeared to be the proportionality coefficients:

$$
\begin{aligned}
S_{a} & =\mathbf{a}_{o}^{\mathrm{n}-2} \\
\mathbf{S}_{\mathrm{b}} & =\mathbf{b}_{o}^{\mathrm{n}-2} \\
\mathbf{S}_{\mathrm{c}} & =\mathbf{c}_{\mathrm{o}}^{\mathrm{n}-2}
\end{aligned}
$$

Further on, let's add term by term the obtained equations (E) and arithmetically average these sums.
As a result, we will obtain one combined equation

$$
\begin{equation*}
\left(\mathbf{a}_{o}^{2} \times \mathbf{D}_{\mathrm{n}}\right)+\left(\mathbf{b}_{o}^{2} \times \mathbf{D}_{\mathrm{n}}\right)=\left(\mathbf{c}_{o}^{2} \times \mathbf{D}_{\mathrm{n}}\right) \tag{F}
\end{equation*}
$$

Here

$$
\mathrm{D}_{\mathrm{n}}=\left(\mathbf{a}_{0}^{\mathrm{n}-2}+\mathbf{b}_{0}^{\mathrm{n}-2}+\mathrm{c}_{\mathrm{o}}^{\mathrm{n}-2}\right)
$$

common multiplier and

$$
\left(a_{o}, b_{o}, c_{o}\right)
$$

primitive Pythagorean triplets. Used equation (F), we mat write down the identification of its components:

$$
\begin{aligned}
& \mathbf{a}^{\mathrm{n}}=\mathbf{a}_{\mathrm{o}}^{2} \times \mathbf{D}_{\mathrm{n}} \\
& \mathbf{b}^{\mathrm{n}}=\mathbf{b}_{\mathrm{o}}^{2} \times \mathbf{D}_{\mathrm{n}}
\end{aligned}
$$

$$
c^{\mathrm{n}}=\mathrm{c}_{\mathrm{o}}^{2} \times \mathrm{D}_{\mathrm{n}}
$$

From these identification equations, we derive follow formulas for determining roots:

$$
\begin{aligned}
a & =\sqrt[n]{a_{o}^{2} \times D_{n}}=\sqrt[n]{n_{\alpha}} \\
b & =\sqrt[n]{b_{o}^{2} \times D_{n}}=\sqrt[n]{n_{\beta}} \\
c & =\sqrt[n]{c_{o}^{2} \times D_{n}}=\sqrt[n]{n_{\gamma}}
\end{aligned}
$$

for Fermat's equation

$$
\mathbf{a}^{\mathrm{n}}+\mathbf{b}^{\mathrm{n}}=\mathbf{c}^{\mathrm{n}}
$$

In this article we establish the following unique fact: Exist Invariants numbers

$$
\mathbf{n}_{\alpha}, \mathbf{n}_{\beta}, \mathbf{n}_{\Upsilon}
$$

essence members for endless series equations

$$
\mathbf{n}_{\alpha}+\mathbf{n}_{\boldsymbol{\beta}}=\mathbf{n}_{\boldsymbol{c}}
$$

This - an equivalent of the equation P. Fermat

$$
\mathbf{x}^{\mathbf{n}}+\mathbf{y}^{\mathbf{n}}=\mathbf{z}^{\mathbf{n}}
$$

for all $\mathbf{n} \geq \mathbf{2}$. Here

$$
\begin{gathered}
x^{n} \equiv \mathbf{a}^{n} \\
\mathbf{y}^{n} \equiv b^{n} \\
\mathbf{z}^{n} \equiv \mathbf{c}^{\mathbf{n}} \\
\text { if } \\
\mathbf{a}^{\mathbf{n}}+\mathbf{b}^{\mathbf{n}}=\mathbf{c}^{\mathbf{n}}
\end{gathered}
$$

In Russia was published a books [6] and [7], in Russian and English languages, and in articles [8], [9]. In this books and articles the algorithm of geometrical proof of the Last theorem is described. Algorithm is based on 9 invariant triplets given in the book under numbers (1.6) - (1.14), see [6] and [7]. Those triplets are elements of secondary forms by H . Poincare. Completeness of my proof is characterized by the fact, that it (proof) is finished with formulas, see [10], page7, for calculation of all roots for Fermat's equation

$$
\mathbf{a}^{\mathrm{n}}+\mathbf{b}^{\mathrm{n}}=\mathbf{c}^{\mathrm{n}}
$$

at all even and odd indicators of degree $n$.

Hypothesis by Shimura-Taniyama is wrong and proof of A. Wiles is questionable because there is a great variety of non-modular elliptic curves information about which is in equations by G. Frey [1]:

$$
\mathbf{Y}^{2}=(\mathbf{X}-\mathbf{A}) \times \mathbf{X} \times(\mathbf{X}+\mathbf{B})
$$

This fact is easily illustrated with the help of equation of my elliptical curve

$$
\mathbf{Y}^{2}=\mathbf{a}_{0}^{\mathrm{n}} \times \mathbf{b}_{o}^{\mathrm{n}} \times \mathbf{c}_{\mathrm{o}}^{\mathrm{n}}
$$

which comes from equation of $G$. Frey at following substitutions:

$$
\begin{gathered}
(X-A)=a_{o}^{n} \\
X=b_{o}^{n} \\
(X+B)=c_{o}^{n}
\end{gathered}
$$

here

$$
\begin{gathered}
a_{o}=v^{2}-u^{2} \\
b_{o}=2 v u \\
c_{o}=v^{2}+u^{2}
\end{gathered}
$$

primitive triads by Pythagorean and $v>u$ are natural numbers of different eventy. And $n=2$ or $n>2$ let's envisage properties of my curves. Let's figure out MINIMAL DISCRIMINANT of the my curve - see [4], for first primitive triad, if $\mathrm{v}=2$ and $\mathrm{u}=1$ :

$$
\begin{aligned}
& a_{0}=\left(v^{2}-u^{2}\right)=\left(2^{2}-1^{2}\right)=3 \\
& b_{o}=2 \cdot v \cdot u=(2 \times 2 \times 1)=4 \\
& c_{o}=\left(v^{2}+u^{2}\right)=\left(2^{2}+1^{2}\right)=5
\end{aligned}
$$

at minimal $n=2$ we have discriminant

$$
\Delta=\frac{\left(\mathrm{a}_{\mathrm{o}} \times \mathrm{b}_{\mathrm{o}} \times \mathrm{c}_{\mathrm{o}}\right)^{2 \mathrm{n}}}{2^{8}}=50625
$$

In accord [1], for simple $\mathbf{n}=\mathbf{5}$ and for $\mathbf{a}=\mathbf{a}_{\mathbf{o}}, \mathbf{b}=\mathbf{b}_{\mathbf{o}}$, $\mathbf{c}=\mathbf{c}_{\mathbf{o}}$, we have discriminant

$$
\Delta=\frac{a^{2 q} \cdot b^{2 q} \cdot\left(a^{q}+b^{q}\right)^{2}}{2^{8}}=\frac{(a b c)^{2 q}}{2^{8}}=\frac{(3 \cdot 4 \cdot 5)^{2-5}}{2^{8}}=2.36196 \cdot 10^{15}
$$

As far as discriminants are not equal to zero, curves are NONSINGULAR. So those are ELLIPTICAL CURVES. To this fact also refers the fact that simple $\mathrm{n}=2$ DOES NOT DEVIDE its discriminant

Experts know, why number 16 has a meaning of "litmus paper" in theory of elliptical curves. Without details let's demonstrate this feature of number 16 on definite example for primitive triad (6). At n=5 my curve gets determined expression

$$
Y^{2}=243 \times 1024 \times 3125=777600000
$$

At that:
16 divides 243 with oddment 3,
16 divides 1024 with oddment 0 ,
16 divides 3125 with oddment 5 .
16 divides number

$$
A=\left(b_{o}^{5}-a_{o}^{5}\right)=1024^{5}-243^{5}=1125.0526 \cdot 10^{12}
$$

with oddment, approximate, 5 . It means that numbers forming the given elliptical curve can't be compared by module $\mathrm{d}=16$.

## CONCLUSION:

MY ELLIPTICAL CURVES IS NON-MODULAR HYPOTHESIS BY SHIMURA -TANIYAMA IS WRONG PROOF OF A.WILIS IS UNCERTAIN

## 3. Unique characteristics of congruous Numbers of Pythagoras

Unlike well-known, see Neal Koblitz [11], the Congruent numbers of Pythagorean are calculated as the functions from primitive triplets of Pythagorean:

$$
\begin{equation*}
K_{\mathrm{P}=\mathrm{u}}=\frac{\mathbf{a}_{0} \cdot \mathbf{b}_{0}}{2} \tag{1}
\end{equation*}
$$

here

$$
\begin{equation*}
\left(a_{0}, b_{0}\right) \tag{2}
\end{equation*}
$$

primitive numbers of Pythagorean, which three - tuples calculate on known formulas, see [3]:

$$
\begin{aligned}
& \mathbf{a}_{0}=\mathbf{v}^{2}-\mathbf{u}^{2} \\
& \mathbf{b}_{0}=\mathbf{2} \cdot \mathbf{v} \cdot \mathbf{u} \\
& \mathbf{c}_{0}=\mathbf{v}^{2}+\mathbf{u}^{2}
\end{aligned}
$$

Here numbers

$$
\begin{equation*}
\mathbf{v}>\mathbf{u} \tag{4}
\end{equation*}
$$

are the numbers of various evenness taken from endless series of natural numbers. If go along natural row of numbers

$$
\begin{equation*}
\mathbf{N}=1,2,3,4,5,6,7,8,9,10,11,12,13, \tag{5}
\end{equation*}
$$

possible calculate endless row congruous numbers of Pythagorean

Graduate Texts in Mathematics 97

Neal Koblitz

Introduction to Elliptic Curves and Modular Forms

Springer-Verlag.
New York, Berlin, Heidelberg, Tokyo. 1984
Н. Коблиц

Введение в эллиптические кривые и модулярные формы. Перевод с английского О. В. Огиевецкого под редакцией Ю. И. Манина

Москва «Мир» 1988

Example of the consequent calculation such numbers see below Sequence of the calculations

$$
\begin{aligned}
& \left(\begin{array}{l}
(v=2, u=1) \rightarrow\left(a_{0}=3, b_{0}=4\right) \rightarrow K_{1}=6 \\
(v=4, u=3) \rightarrow\left(a_{0}=7, b_{0}=24\right) \rightarrow K_{2}=84=3 \\
(v=6, u=5) \rightarrow\left(a_{0}=11, b_{0}=60\right) \rightarrow K_{3}=330=6
\end{array}\right)=\sum_{1}=6 \\
& \left(\begin{array}{l}
(v=8, u=7) \rightarrow\left(a_{0}=15, b_{0}=112\right) \rightarrow K_{4}=840=3 \\
(v=10, u=9) \rightarrow\left(a_{0}=19, b_{0}=180\right) \rightarrow K_{5}=1710=9 \\
(v=12, u=11) \rightarrow\left(a_{0}=23, b_{0}=264\right) \rightarrow K_{6}=3036=3
\end{array}\right)=\sum_{2}=6 \\
& \left(\begin{array}{l}
(v=14, u=13) \rightarrow\left(a_{0}=27, b_{0}=364\right) \rightarrow K_{7}=4914=9 \\
(v=16, u=15) \rightarrow\left(a_{0}=31, b_{0}=480\right) \rightarrow K_{8}=7440=6 \\
(v=18, u=17) \rightarrow\left(a_{0}=35, b_{0}=612\right) \rightarrow K_{9}=10710=9
\end{array}\right)=\sum_{3}=6
\end{aligned}
$$

| № | 1. | $v=2 \quad n \quad u=1$ | $\left(\mathrm{a}_{0}=3, \mathrm{~b}_{0}=4\right) \Rightarrow\left(\mathrm{K}_{\mathrm{P}=1}=6\right)$ |
| :---: | :---: | :---: | :---: |
|  | 2. | $v=3$ и $u=2$ | $\left(\mathrm{a}_{0}=5, \mathrm{~b}_{0}=12\right) \Rightarrow\left(\mathrm{K}_{\mathrm{P}=2}=30\right)=3$ |
|  | 3. | $v=4$ и $u=3$ | $\left(a_{0}=7, b_{0}=24\right) \Rightarrow\left(K_{P=3}=84\right)=3$ |
|  | 4. | $v=5$ и $u=4$ | $\left(\mathbf{a}_{\mathbf{0}}=9, \mathbf{b}_{\mathbf{0}}=40\right) \Rightarrow\left(\mathrm{K}_{\mathrm{P}=4}=180\right)=9$ |
|  | 5. | $v=6$ и $u=5$ | $\left(a_{0}=11, b_{0}=60\right) \Rightarrow\left(K_{P=5}=330\right)=6$ |
|  | 6. | $v=7$ и $u=6$ | $\left(\mathbf{a}_{\mathbf{0}}=13, \mathbf{b}_{\mathbf{0}}=84\right) \Rightarrow\left(\mathrm{K}_{\mathrm{P}=6}=546\right)=6$ |
|  | 7. | $v=8$ и $u=7$ | $\left(a_{0}=15, b_{0}=112\right) \Rightarrow\left(K_{P=7}=840\right)=3$ |
|  | 8. | $v=9$ и u=8. | $\left(a_{0}=17, b_{0}=144\right) \Rightarrow\left(K_{\text {r-8 }}=1224\right)=9$ |
|  | 9. | $v=10$ и $u=9$ | $\left(\mathbf{a}_{\mathbf{0}}=19, \mathbf{b}_{\mathbf{0}}=180\right) \Rightarrow\left(\mathrm{K}_{\mathrm{P}=9}=1710\right)=9$ |
|  | 10. | $v=11$ и $u=10$. | $\left(\mathrm{a}_{0}=21, \mathrm{~b}_{0}=220\right) \Rightarrow\left(\mathrm{K}_{\mathrm{P}=10}=2310\right)=6$ |
| № | 11. | $v=12$ и u=11 | $\left(\mathrm{a}_{0}=23, \mathrm{~b}_{0}=264\right) \Rightarrow\left(\mathrm{K}_{\mathrm{P}=11}=3036\right)=3$ |

This example demonstrates important characteristic an congruous numbers of Pythagorean. All numbers of the endless row

$$
\begin{equation*}
6,30,84,180,330,546,840,1224,1710,2310,3036 \text {, } \tag{6}
\end{equation*}
$$

numeralogical grow shorter before numerals, which short numeral 3 :

$$
\begin{gathered}
30=3+0=3 \\
84=8+4=12=1+2=3 \\
180=1+8+0=9
\end{gathered}
$$

$$
\begin{equation*}
330=3+3+0=6 \tag{7}
\end{equation*}
$$

$$
\begin{gathered}
546=5+4+6=15=1+5=6 \\
840=8+4+0=12=1+2=3 \\
1224=1+2+2+4=9
\end{gathered}
$$

In accord [6], [7] exist multivariate congruous numbers of Pythagorean. Exists nine (three three-tuples) the invariants right-angled triangles. Following Photo 1, composition of the formulas for calculation of the areas of a the invariants right-angled triangles of Pythagorean

$$
\begin{array}{r}
\mathbf{n}_{\alpha}=\left[\left(\mathbf{n}_{1}^{\prime}=\frac{\mathbf{a}_{1}^{\prime} \cdot \alpha_{1}^{\prime}}{2}\right)=\left(\mathbf{n}_{2}^{\prime}=\frac{a_{2}^{\prime} \cdot \alpha_{2}^{\prime}}{2}\right)+\left(\mathbf{n}_{3}^{\prime}=\frac{\mathbf{a}_{3}^{\prime} \cdot \alpha_{3}^{\prime}}{2}\right)\right] \\
\mathbf{n}_{\beta}=\left[\left(\mathbf{n}_{1}^{\prime \prime}=\frac{\mathbf{b}_{1}^{\prime} \cdot \beta_{1}^{\prime}}{2}\right)=\left(\mathbf{n}_{2}^{\prime \prime}=\frac{\mathbf{b}_{2}^{\prime} \cdot \boldsymbol{\beta}_{2}^{\prime}}{2}\right)=\left(\mathbf{n}_{3}^{\prime \prime}=\frac{\mathbf{b}_{3}^{\prime} \cdot \beta_{3}^{\prime}}{2}\right)\right] \\
\mathbf{n}_{Y}=\left[\left(\mathbf{n}_{1}^{\prime \prime \prime}=\frac{\mathbf{c}_{1}^{\prime} \cdot Y_{1}^{\prime}}{2}\right)=\left(\mathbf{n}_{2}^{\prime \prime \prime}=\frac{\mathbf{c}_{2}^{\prime} \cdot \Upsilon_{2}^{\prime}}{2}\right)=\left(\mathbf{n}_{3}^{\prime \prime \prime}=\frac{\mathbf{c}_{3}^{\prime} \cdot \Upsilon_{3}^{\prime}}{2}\right)\right] \tag{10}
\end{array}
$$



Photo 1. Note: This photography from books [6] and [7].
Invariants numbers

$$
\mathbf{n}_{\alpha}, \mathbf{n}_{\beta}, \mathbf{n}_{Y}
$$

are multivariate congruous numbers of Pythagorean. The formulas for calculation components

$$
\mathbf{n}_{\alpha}, \mathbf{n}_{\boldsymbol{\beta}}, \mathbf{n}_{Y}
$$

- see (8), (9), (10),

$$
\begin{align*}
& a_{1}^{\prime}=a_{0} \cdot \sqrt{a_{0}^{n-2}}  \tag{11}\\
& \alpha_{1}^{\prime}=\frac{a_{0}}{3} \cdot\left[\frac{a_{0}^{n-2}+b_{0}^{n-2}+c_{0}^{n-2}}{\sqrt{a_{0}^{n-2}}}\right]  \tag{12}\\
& \mathbf{a}_{2}^{\prime}=\mathbf{a}_{\mathbf{o}} \cdot \sqrt{b_{0}^{\mathrm{n}-2}}  \tag{13}\\
& \alpha_{2}^{\prime}=\frac{a_{o}}{3} \cdot\left[\frac{a_{o}^{n-2}+b_{o}^{n-2}+c_{o}^{n-2}}{\sqrt{b_{o}^{n-2}}}\right]  \tag{14}\\
& \mathbf{a}_{3}^{\prime}=\mathbf{a}_{\mathbf{o}} \cdot \sqrt{\mathbf{c}_{\mathbf{o}}^{\mathbf{n}-\mathbf{2}}}  \tag{15}\\
& \alpha_{3}^{\prime}=\frac{a_{o}}{3} \cdot\left[\frac{a_{o}^{n-2}+b_{o}^{n-2}+c_{o}^{n-2}}{\sqrt{c_{o}^{n-2}}}\right]  \tag{16}\\
& \mathbf{b}_{1}^{\prime}=b_{o} \cdot \sqrt{\mathbf{a}_{0}^{\mathrm{n}-2}}  \tag{17}\\
& \beta_{1}^{\prime}=\frac{b_{o}}{3} \cdot\left[\frac{a_{o}^{n-2}+b_{o}^{n-2}+c_{o}^{n-2}}{\sqrt{a_{o}^{n-2}}}\right]  \tag{18}\\
& \mathbf{b}_{2}{ }^{\prime}=\mathbf{b}_{\mathrm{o}} \cdot \sqrt{\mathbf{b}_{\mathrm{o}}^{\mathrm{n}-\mathbf{2}}}  \tag{19}\\
& \boldsymbol{\beta}_{2}^{\prime}=\frac{b_{o}}{3} \cdot\left[\frac{a_{o}^{n-2}+b_{o}^{n-2}+c_{o}^{n-2}}{\sqrt{b_{o}^{n-2}}}\right]  \tag{20}\\
& \mathbf{b}_{3}^{\prime}=\mathbf{b}_{0} \cdot \sqrt{\mathbf{c}_{\mathbf{o}}^{\mathrm{n}-\mathbf{2}}} \tag{21}
\end{align*}
$$

$$
\begin{gather*}
\boldsymbol{\beta}_{3}^{\prime}=\frac{\mathbf{b}_{o}}{3} \cdot\left[\frac{a_{o}^{\mathrm{n}-2}+\mathrm{b}_{o}^{\mathrm{n}-2}+\mathrm{c}_{o}^{\mathrm{n}-2}}{\sqrt{\mathbf{c}_{o}^{\mathrm{n}-2}}}\right]  \tag{22}\\
\mathbf{c}_{1}^{\prime}=\mathbf{c}_{\mathrm{o}} \cdot \sqrt{\mathbf{a}_{o}^{\mathrm{n}-2}}  \tag{23}\\
\Upsilon_{1}^{\prime}=\frac{c_{o}}{3} \cdot\left[\frac{\mathrm{a}_{o}^{\mathrm{n}-2}+\mathrm{b}_{o}^{\mathrm{n}-2}+\mathbf{c}_{o}^{\mathrm{n}-2}}{\sqrt{a_{o}^{\mathrm{n}-2}}}\right]  \tag{24}\\
\left.\mathbf{c}_{2}^{\prime}=\mathbf{c}_{o} \cdot \sqrt{\mathbf{b}_{o}^{\mathrm{n}-2}}\right]  \tag{25}\\
\Upsilon_{2}^{\prime}=\frac{c_{o}}{3} \cdot\left[\frac{\mathrm{a}_{o}^{\mathrm{n}-2}+\mathrm{b}_{o}^{\mathrm{n}-2}+c_{o}^{\mathrm{n}-2}}{\sqrt{b_{o}^{\mathrm{n}-2}}}\right]  \tag{26}\\
\left.\mathbf{c}_{3}^{\prime}=\mathbf{c}_{\mathrm{o}} \cdot \sqrt{\mathbf{c}_{o}^{\mathrm{n}-2}}\right]  \tag{27}\\
\Upsilon_{3}^{\prime}=\frac{c_{o}}{3} \cdot \frac{\left[\mathrm{a}_{o}^{\mathrm{n}-2}+\mathrm{b}_{o}^{\mathrm{n}-2}+\mathrm{c}_{o}^{\mathrm{n}-2}\right]}{\sqrt{c_{o}^{n-2}}}  \tag{28}\\
\text { Resume }
\end{gather*}
$$

Invariants numbers

$$
\mathbf{n}_{\alpha}, \mathbf{n}_{\beta}, \mathbf{n}_{\Upsilon}
$$

essence members for endless series equations

$$
\begin{equation*}
\mathbf{n}_{\alpha}+\mathbf{n}_{\beta}=\mathbf{n}_{c} \tag{29}
\end{equation*}
$$

This - an equivalent of the equation P. Fermat

$$
\begin{equation*}
\mathbf{x}^{\mathbf{n}}+\mathbf{y}^{\mathbf{n}}=\mathbf{z}^{\mathbf{n}} \tag{30}
\end{equation*}
$$

for all $\mathbf{n} \geq 2$.

## Result

Exist to be an identical equality

$$
\begin{equation*}
\mathbf{n}_{\alpha} \equiv \mathbf{F}_{\mathbf{a}}^{*} \tag{31}
\end{equation*}
$$

$$
\begin{align*}
& \mathbf{n}_{\beta} \equiv \mathbf{F}_{\mathrm{b}}^{\prime}  \tag{32}\\
& \mathbf{n}_{Y} \equiv \mathbf{F}_{c}^{\prime} \tag{33}
\end{align*}
$$

and identical formulas for calculation endless series roots of the Fermats equations

$$
\begin{align*}
& \mathbf{x}=\sqrt[n]{\mathbf{n}_{\alpha}}=\sqrt[n]{\mathbf{F}_{a}^{\prime}}  \tag{34}\\
& \mathbf{y}=\sqrt[n]{\mathbf{n}_{\beta}}=\sqrt[n]{\mathbf{F}_{b}^{\prime}}  \tag{35}\\
& \mathbf{z}=\sqrt[n]{\mathbf{n}_{Y}}=\sqrt[n]{\mathbf{F}_{\mathrm{c}}^{\prime}} \tag{36}
\end{align*}
$$

Here, in accord [6] and [7],

$$
\begin{align*}
& \mathbf{F}_{\mathrm{a}}^{\prime}=\frac{1}{3} \cdot\left(\mathbf{a}_{1}^{\prime 2}+\mathbf{a}_{2}^{\prime 2}+\mathbf{a}_{3}^{\prime 2}\right)= \\
& =\frac{\mathrm{a}_{o}^{2}}{3} \cdot\left(\mathbf{a}_{o}^{\mathrm{n}-2}+\mathbf{b}_{o}^{\mathrm{n}-2}+\mathbf{c}_{o}^{\mathrm{n}-2}\right)  \tag{37}\\
& \mathbf{F}_{\mathrm{b}}^{\prime}=\frac{1}{3} \cdot\left(\mathbf{b}_{1}^{\prime 2}+\mathbf{b}_{2}^{\prime 2}+\mathbf{b}_{3}^{\prime 2}\right)= \\
& =\frac{\mathrm{b}_{0}^{2}}{3} \cdot\left(\mathbf{a}_{o}^{\mathrm{n}-2}+\mathbf{b}_{o}^{\mathrm{n}-2}+\mathbf{c}_{o}^{\mathrm{n}-2}\right)  \tag{38}\\
& \mathbf{F}_{\mathrm{c}}^{\prime}=\frac{1}{3} \cdot\left(\mathbf{c}_{1}^{\prime 2}+{\left.c_{2}^{\prime 2}+\mathbf{c}_{3}^{\prime 2}\right)=}^{=\frac{\mathbf{c}_{o}^{2}}{3} \cdot\left(\mathbf{a}_{o}^{\mathrm{n}-2}+\mathbf{b}_{o}^{\mathrm{n}-2}+\mathbf{c}_{o}^{\mathrm{n}-2}\right)}\right.
\end{align*}
$$

Herewith exist to be a correlations, described in work [6] and [7] H. Poincare. In work [10] Poincare confirms:
If three integers of the number but:

$$
\begin{equation*}
\mathbf{a}_{1}, \mathbf{a}_{2}, \mathbf{a}_{3} \tag{40}
\end{equation*}
$$

which mutually simple; that always exists nine integers numbers, which satisfy following condition:

$$
\begin{align*}
& \mathbf{a}_{1} \cdot \alpha_{1}+\mathbf{a}_{2} \cdot \alpha_{2}+\mathbf{a}_{3} \cdot \alpha_{3}=1  \tag{41}\\
& \mathbf{a}_{1}=\beta_{2} \cdot \Upsilon_{3}-\beta_{3} \cdot \Upsilon_{2}  \tag{42}\\
& a_{2}=\beta_{3} \cdot \Upsilon_{1}-\beta_{1} \cdot \Upsilon_{3}  \tag{43}\\
& \mathbf{a}_{3}=\beta_{1} \cdot \Upsilon_{2}-\boldsymbol{\beta}_{2} \cdot \Upsilon_{1} \tag{44}
\end{align*}
$$

Secondary brought forms H. Poincare lead to restrictions parameter A and $B$ in the manner of

$$
\begin{equation*}
\boldsymbol{\lambda}>A_{\text {and }} \boldsymbol{\lambda}<B \tag{45}
\end{equation*}
$$

With reference to such binary form, as equation elliptical crooked G. Fray [1] :

$$
\begin{equation*}
\mathbf{Y}^{2}=(\mathbf{X}-\mathbf{A}) \cdot \mathbf{X} \cdot(\mathbf{X}+\mathbf{B}) \tag{46}
\end{equation*}
$$

this equivalent statement

$$
\begin{gather*}
\mathbf{a}_{1} \neq \beta_{2} \cdot \Upsilon_{3}-\beta_{3} \cdot \Upsilon_{2}=0  \tag{47}\\
\mathbf{a}_{1} \neq \boldsymbol{\beta}_{2} \cdot \Upsilon_{3}-\beta_{3} \cdot \Upsilon_{2}=0  \tag{48}\\
\mathbf{a}_{3} \neq \boldsymbol{\beta}_{1} \cdot \Upsilon_{2}-\beta_{2} \cdot \Upsilon_{1}=0 \tag{49}
\end{gather*}
$$

In this case exist to be three equalities

$$
\begin{equation*}
\mathbf{a}_{\mathbf{1}}^{\prime} \cdot \alpha_{1}^{\prime}+\mathbf{a}_{2}^{\prime} \cdot \alpha_{2}^{\prime}+\mathbf{a}_{3}^{\prime} \cdot \alpha_{3}^{\prime}=\mathbf{1} \tag{50}
\end{equation*}
$$

This form has three equivalent transformations

$$
\begin{align*}
& \frac{\left(\mathbf{a}_{1}^{\prime} \cdot \alpha_{1}^{\prime}+\mathbf{a}_{2}^{\prime} \cdot \alpha_{2}^{\prime}+\mathrm{a}_{3}^{\prime} \cdot \alpha_{3}^{\prime}\right)}{3 \cdot\left(\mathbf{n}_{\alpha} \equiv \mathbf{F}_{\mathrm{a}}^{\prime}\right)}=1  \tag{51}\\
& \frac{\left(\mathbf{b}_{1}^{\prime} \cdot \beta_{1}^{\prime}+\mathbf{b}_{2}^{\prime} \cdot \beta_{2}^{\prime}+\mathbf{b}_{3}^{\prime} \cdot \beta_{3}^{\prime}\right)}{3 \cdot\left(\mathbf{n}_{\boldsymbol{\beta}} \equiv \mathbf{F}_{\mathbf{b}}^{\prime}\right)}=1  \tag{52}\\
& \frac{\left(\mathbf{c}_{1}^{\prime} \cdot \Upsilon_{1}^{\prime}+\mathbf{c}_{2}^{\prime} \cdot \Upsilon_{2}^{\prime}+\mathbf{c}_{3}^{\prime} \cdot \Upsilon_{3}^{\prime}\right)}{3 \cdot\left(\mathbf{n}_{Y} \equiv \mathbf{F}_{\mathrm{c}}^{\prime}\right)}=1 \tag{53}
\end{align*}
$$

As a result - a restriction (45) eliminate.
I form new conditions for binary form (46):

$$
\begin{align*}
& \mathbf{A}=\mathbf{X}-\mathbf{a}_{o}^{\mathrm{n}}=\left(\mathbf{b}_{o}^{\mathrm{n}}-\mathbf{a}_{o}^{\mathrm{n}}\right)  \tag{54}\\
& \mathbf{B}=\mathbf{c}_{o}^{\mathrm{n}}-\mathbf{X}=\mathbf{c}_{o}^{\mathrm{n}}-\mathbf{b}_{o}^{n}=\mathbf{a}_{o}^{n}  \tag{55}\\
& \mathbf{X}=\mathbf{b}_{o}^{n}  \tag{56}\\
& (\mathbf{X}-\mathbf{A})=\mathbf{a}_{o}^{\mathrm{n}}  \tag{57}\\
& (\mathbf{X}+\mathbf{B})=\mathbf{c}_{o}^{n} \tag{58}
\end{align*}
$$

It is conditions for non-modular elliptic curves, see [8]:

$$
\mathbf{Y}^{2}=\mathbf{a}_{\mathrm{o}}^{\mathrm{n}} \times \mathbf{b}_{\mathrm{o}}^{\mathrm{n}} \times \mathbf{c}_{\mathrm{o}}^{\mathrm{n}}
$$

## 4. Physical interpretation

In work [10] Poincare wrote:
Arithmetical study of the uniform forms most of all occupies specialist on geometries In all forms geometry is present in the manner of numbers 3. Number 3 - a sign three-dimensional space of Euclid.

Here is that write about this space authors [12]:
Geometric space depends on time. But changes not space-time. Changes space, three-dimensional space

Authors [12] write: Three-dimensional space adequately physical vacuum Central point consists in following. Started space not at all is empty. It presents itself receptacle of the most tempestuous physical processes.

All elementary particles are born In physical vacuum, in accord study this ensemble elementary particles, see reference № 3 an http://yvsevolod26.narod.ru/index.html, all of these are subordinated stood law three numbers

$$
\begin{equation*}
3=1+2 \tag{59}
\end{equation*}
$$

Known that all systems of the physical units take its begin in three base units. System CGS has a base in the manner 3 of dimensioned units

$$
\begin{align*}
& 1=\operatorname{dim} c m  \tag{60}\\
& \mathbf{1}=\operatorname{dim} g  \tag{61}\\
& \mathbf{1}=\operatorname{dim} s \tag{62}
\end{align*}
$$

Fundamental systems of the physical units Plank and author given messages also have a base as three fundamental units, see [13] :

$$
\begin{align*}
& \mu_{i \mathrm{y}}=\frac{\mu_{i}}{3} \approx 2.6136368 \cdot 10^{-48} \tilde{\mathbf{a}} \\
& \mathrm{r}_{\mathrm{iyy}} \approx 1.6409300 \cdot 10^{-21} \text { ñì } \\
& \tau_{\tilde{\text { yy }}} \approx 5.4735533 \cdot 10^{-32} \tilde{\mathrm{n}} \\
& \rho_{i \grave{y}}=\frac{\mu_{i \ddot{y}}}{\frac{4}{3} \pi \cdot r_{i \ddot{y}}{ }^{3}}=\frac{2.6136368 \cdot 10^{-48}}{1.8507969 \cdot 10^{-62}}=  \tag{63}\\
& =1.41221683 \cdot 10^{14} \tilde{\mathbf{a}} / \tilde{\mathbf{n}} \mathbf{1}^{3} \\
& \tilde{\mathbf{n}} \approx \frac{r_{\text {iì }} \approx 1.6409300 \cdot 10^{-21} \tilde{\mathbf{n}} \mathbf{i}}{\tau_{\text {ì }} \approx 5.4735533 \cdot 10^{22} \tilde{\mathrm{n}}}= \\
& =2.9979246 \cdot 10^{10} \tilde{\mathbf{n}} / \tilde{\mathbf{n}}
\end{align*}
$$

which conjugate at the speed of light in vacuum and with system of the units Max Planck:

$$
\begin{align*}
& \mathbf{M}^{*}=\sqrt{\frac{\hbar \mathbf{c}}{\mathbf{G}}}=2.177 \cdot 10^{-5} \tilde{\mathbf{a}} \\
& \mathbf{L}^{*}=\sqrt{\frac{\hbar \mathbf{G}}{\mathbf{c}^{3}}}=\mathbf{1 . 6 1 6 \cdot 1 0 ^ { - 3 3 } \tilde { \mathbf { n } } \mathbf { i }} \\
& \mathbf{T}^{*}=\sqrt{\frac{\hbar \mathbf{G}}{\mathbf{c}^{5}}}=5.391 \cdot 10^{-44} \mathbf{c} \\
& \mathbf{\rho}^{*}=\frac{\mathbf{M}^{*}}{\mathbf{L}^{* 3}}=\frac{\mathbf{c}^{5}}{\hbar \mathbf{G}^{2}}=5.157 \cdot 10^{93} \tilde{\mathbf{a} / \tilde{\mathbf{n}} \mathbf{i}^{3}}  \tag{64}\\
& \mathbf{c}=\frac{\mathbf{L}^{*}}{\mathbf{T}^{*}}= \\
&=\frac{\sqrt{\frac{\hbar \mathbf{G}}{\mathbf{c}^{3}}}}{\sqrt{\frac{\hbar \mathbf{G}}{\mathbf{c}^{5}}}} \approx 2.997 \cdot 10^{10} \tilde{\mathbf{n}} \mathbf{i} / \tilde{\mathbf{n}}
\end{align*}
$$

## 5. General conclusion

In all forms (37) - (53) dominates number three and vague pair non dimensional numbers. Here all correspond to paradigm of Pyhagorean "Begin whole-unit. The Unit, as prime cause, belongs to the vague двоица; from unit and vague -ness come the numbers; from чиселpoints; from point-lines; from them-flat figures; from flat-threedementional figures; of them - voluptuous perceived bodies, which generate the world animate and reasonable"
Plato, adopted the paradigm of Pythgorean and of Demokrit, created its the conception, resulting from the paradigm of Pythagorean and of Demokrit. The vague-ness compose the Golden ratio (Divina Proportione 0 of Plato). The Vhole so pertains to most, as Big pertains to Minority

$$
\begin{equation*}
\frac{\text { Vhole }}{\text { Big }}=\frac{\text { Big }}{\text { Minority }} \tag{65}
\end{equation*}
$$

Numeric equivale

$$
\begin{equation*}
\frac{3.55555555 \ldots}{1.77777777 \ldots}=\frac{1.7777777 \ldots}{0.88888888 \ldots}=2 \tag{65}
\end{equation*}
$$

If

$$
\begin{equation*}
3.55555555 \ldots=2.66666666 \ldots+0.88888888 \ldots \tag{67}
\end{equation*}
$$

and

$$
\begin{equation*}
2=\lim \frac{1.77777777 \ldots}{0.88888888 \ldots} \tag{68}
\end{equation*}
$$

Integer 2 is basis for Avogadro constant [14]:

$$
\begin{gathered}
\mathrm{N}_{\mathrm{A}}=2 \cdot 2^{78}=\frac{(2 \cdot X \cdot Y \cdot Z)^{16}}{2}=6.0446291 \cdot 10^{23} \\
2^{2}=4, \quad 2^{3}=8, \quad 2^{4}=16,2^{5}=32, \ldots 2^{79}=6.0446291 \cdot 10^{23}, \ldots
\end{gathered}
$$

Here

$$
\begin{align*}
& X=\frac{32}{18}=1.777777777 \ldots  \tag{70}\\
& Y=\frac{18}{8}=2.25  \tag{71}\\
& Z=\frac{8}{2}=4 \tag{72}
\end{align*}
$$

rational numbers, as function electrons envelope for all atoms to actinoids :

$$
\begin{equation*}
32 \cdot \mathbf{m}_{\mathrm{e}} ; 18 \cdot \mathbf{m}_{\mathrm{e}} ; 8 \cdot \mathbf{m}_{\mathrm{e}} ; 2 \cdot \mathbf{m}_{\mathrm{e}} \tag{73}
\end{equation*}
$$

Curious fact

$$
\begin{gather*}
2=\frac{46500}{23250}=\frac{39300}{29650}=\frac{45650}{22825}=\frac{746000}{37300}=\frac{54400}{27200}=\frac{57400}{28700}= \\
=\frac{40500}{20250}=\frac{32200}{16100}=\frac{35400}{17700}=\frac{62700}{31350}=\frac{41500}{20750}=\frac{53400}{26700}=\frac{603550}{301775}=2 \tag{74}
\end{gather*}
$$

This - a data from section of the Number of the Old testament to Bibles Masse of protons:

$$
\begin{align*}
& m_{p}=\left(2 \times \mu_{0 月}^{*}\right) \times\left[2 \times\left(2^{26}\right)^{3}\right]+\left[6.7019968 \times 10^{23} \times\left(2 \times m_{\gamma}\right)\right]= \\
&=1.6735473 \times 10^{-24} \mathrm{r} \tag{75}
\end{align*}
$$

If

$$
\begin{gather*}
\mathrm{m}_{\mathrm{r}}=7 \times 10^{-22} \frac{\mathrm{MeV}}{\mathrm{c}^{2}}= \\
=1.121442 \times 10^{-27} \mathrm{erg}= \\
=1.2485438 \times 10^{-48} \mathrm{~g} \tag{76}
\end{gather*}
$$

the upper case of the mass gamma-quantum, refer to [13]. In formulas (69) and (75) is used number Avogadro

$$
\mathrm{N}_{\mathrm{A}}=2 \times\left(2^{26}\right)^{3}=2^{79}=6.446291 \cdot 10^{23}
$$

and, finally, for the systems hypercomplex numbers possible form two associate Kvaterions of Pythagorean:

$$
\begin{align*}
& H_{P 1}=\left(a_{0}+i b_{0}\right)+\left(b_{0}+i a_{0}\right) \\
& H_{P 2}=\left(a_{0}-i b_{0}\right)+\left(b_{0}-i a_{0}\right) \tag{77}
\end{align*}
$$

from component parts which possible build matrix 2

$$
A=\left(\begin{array}{ll}
\left(a_{0}+i b_{0}\right) & \left(b_{0}+i a_{0}\right)  \tag{78}\\
\left(a_{0}-i b_{0}\right) & \left(b_{0}-i a_{0}\right)
\end{array}\right)
$$

The Determinant of this matrix is

$$
\begin{equation*}
D=2 \cdot \mathbf{i} \cdot c_{0}^{2} \tag{79}
\end{equation*}
$$

In him is marked theorem of Pythagorean

$$
\begin{equation*}
\mathbf{c}_{0}^{2}=\mathbf{a}_{0}^{2}+b_{0}{ }^{2} \tag{80}
\end{equation*}
$$

It is equivalent for unity vector-scalar

$$
\begin{align*}
S=s \cdot s^{*}= & \left(a_{0}+i \cdot b_{0}\right) \cdot\left(a_{0}-i \cdot b_{0}\right)= \\
= & a_{0}^{2}+b_{0}^{2}=c^{2} \tag{81}
\end{align*}
$$

$$
|\mathbf{S}|=\left|\mathbf{s} \times \mathbf{s}^{*}\right|=|\mathbf{s}| \cdot\left|\mathbf{s}^{*}\right| \cdot \sin \mathbf{Q}
$$

Herewith, with reference to to first primitive three-tuple

$$
\begin{equation*}
\left(a_{0}=3, b_{0}=4, c_{0}=5\right) \tag{82}
\end{equation*}
$$

The Duplicated square of the determinant of the mentioned matrix, way numeralical reductions, happens to negative count

$$
\begin{equation*}
2 \cdot D^{2}=2 \cdot\left(2 \cdot i \cdot c_{0}^{2}\right)=-200=-2 \tag{83}
\end{equation*}
$$

This there is manifestation negative sides numbers $310952=2$ of Hipparchos . (190-125 before our era). Known [1], what role has played the ensemble of the matrixes

$$
\left(\begin{array}{ll}
a & b  \tag{84}\\
c & d
\end{array}\right)
$$

in theories an the modular forms, in proof of the hypothesis ShimuraTaniyam and in proof, founded on this hypothesis, the last theorem Fermat. The Author of this article has assumed as a basis their own studies the most simplest matrix

$$
P=\left(\begin{array}{ll}
1 & 2  \tag{85}\\
3 & 4
\end{array}\right)
$$

the determinant which is count

$$
\begin{equation*}
\text { Det } P=(3 \times 2)-(1 \times 4)=2 \tag{86}
\end{equation*}
$$

Decission Problems of G. Gordans and A. Beal - see [15].

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## Yarosh V.S.

# The Riemann hypothesis is finally proved 


#### Abstract

Annotation Author offer the Proof of the known hypothesis of Riemann and two ways of the separation simple numbers are described from endless row natural numbers. Author have offered the algorithm a searching for twin simple numbers by means of the known formula of Fermat's.


## Contents

1. Introduction
2. Sphere of Riemann and her coordinate system
3. Conclusion
4. First way of the separation simple numbers from endless row natural numbers
5. Second way of the separation simple numbers from endless row natural numbers
6. Physical interpreting the formula of Fermat
7. Algorithm of searching for "twin" numbers of Fermat

8 The General conclusion

## 1. Introduction

My proof of the hypothesis of Riemann is founded on Principle general covariations. According to [1], "Each physical value must be described by geometric object (regardless of presence of the coordinates), but all laws physicists must be expressed in the manner of geometric correlations between these geometric object. This standpoint in physicist known as Principle general covariations." Intercoupling to numbers theories and geometries was for the first time installed Pythagoras (570-490, before n. e.) and is used by H. Poincare in his work [2]. The numbers Theory and the Geometry unites in united whole the theorem of Pythagoras .
In accord interpretation of the Clay Mathematics Institute :

## Riemann Hypothesis

Some numbers have the special property that they cannot be expressed as the product of two smaller numbers, e.g., 2, 3, 5, 7, etc. Such numbers are
called prime numbers, and they play an important role, both in pure mathematics and its applications. The distribution of such prime numbers among all natural numbers does not follow any regular pattern, however the German mathematician G.F.B. Riemann (1826-1866) observed that the frequency of prime numbers is very closely related to the behavior of an elaborate function

$$
\begin{equation*}
\zeta(S)=1+\frac{1}{2^{5}}+\frac{1}{3^{5}}+\frac{1}{4^{5}}+\cdots \tag{1}
\end{equation*}
$$

called the Riemann Zeta function. The Riemann hypothesis asserts that all interesting solutions of the equation

$$
\begin{equation*}
\zeta(s)=0 \tag{2}
\end{equation*}
$$

lie on a certain vertical straight line. This has been checked for the first $1,500,000,000$ solutions. A proof that it is true for every interesting solution would shed light on many of the mysteries surrounding the distribution of prime numbers.
Analytically continued $\boldsymbol{\zeta}(\mathbf{s})$-function , if $s \neq 0, s \neq 1$ on determination (5)-(8)), satisfies to equation

$$
\begin{equation*}
\zeta(S)=2^{\mathrm{S}} \pi^{\mathrm{S}-1} \sin \frac{\pi S}{2} \cdot \Gamma(1-S) \cdot \zeta(1-S) \tag{3}
\end{equation*}
$$

Here $\Gamma(z)$ _ gamma-function of Euler. This equation is identified the functional equation of Riemann. I take this equation in its proof of the hypothesis of Riemann, as basis.
As function real variable, $\boldsymbol{\zeta}(\mathbf{S})$ - -function was carried Euler in 1737, which and has indicated her decomposition in product. Then this function was considered Dirichlet and, particularly successfully, Chebyshyev at study of the law of the distribution simple numbers. However the most deep characteristic $\zeta(\mathbf{s})$-functions were discovered later, after working of Riemann (1859), where $\boldsymbol{\zeta}(\mathbf{S})$--function was considered as function complex variable.
In my proof of the hypothesis of Riemann equation

$$
\begin{equation*}
\zeta(S)=2^{S} \pi^{S-1} \sin \frac{\pi S}{2} \cdot \Gamma(1-S) \cdot \zeta(1-S) \tag{4}
\end{equation*}
$$

gains the sense to functions complex variable due to entering the complex invariant:

$$
\begin{align*}
& S=s \cdot s^{*}=\left(a_{o}+i \cdot b_{0}\right) \cdot\left(a_{0}-\mathbf{i} \cdot b_{0}\right)= \\
& ={a_{o}^{2}}^{2}+b_{o}^{2}=c_{o}^{2} \tag{5}
\end{align*}
$$

In this invariant I have connected in united the whole:

1. All are an endless ensemble $C$ complex numbers
2. Theorem of Pythagoras
3. All are an endless ensemble vapour natural numbers $u>v$ different parity
4. All are an endless ensemble primitive three-tuple of Pythagoras, [3]:

$$
\begin{gather*}
a_{o}=u^{2}-v^{2} \\
b_{0}=2 \cdot u \cdot v  \tag{7}\\
c_{0}=u^{2}+v^{2} \tag{8}
\end{gather*}
$$

As a result I have got geometric coordinate system for invariant $S$ and for function

$$
\begin{equation*}
\zeta(S)=2^{\mathrm{S}} \pi^{\mathrm{S}-1} \sin \frac{\pi \mathrm{~S}}{2} \cdot \Gamma(1-\mathrm{S}) \cdot \zeta(1-S) \tag{9}
\end{equation*}
$$



Pic. 1


Pic. 2


Pic. 3, blue colour is marked area of the change associate complex the numbers - vectors


Pic. 4, blue colour is marked area of the change associate complex the numbers -vectors

Base proposed proof of the hypothesis of Riemann form ambiguous characteristic of the vector

$$
\begin{equation*}
\left[\mathbf{s} \cdot \mathbf{s}^{*}\right]=\mathbf{S} \tag{10}
\end{equation*}
$$

The Special role executes the length of this vector, refer to Pic. 3:

$$
\begin{equation*}
|\mathbf{S}|=\left|\mathbf{s}, \mathbf{s}^{*}\right|=|\mathbf{s}| \cdot\left|\mathbf{s}^{*}\right| \cdot \sin \mathbf{Q} \tag{11}
\end{equation*}
$$

This - perpendicular (vertical line), as erection from begin coordinates, refer to Pic. 3.
If the angle $Q$ goes to zero, then according to determination (5), imaginary quantity of complex invariant (5) also go to endless series zero, refer Pic. 1 and Pic.2., if $a_{0} \rightarrow \infty$ the length of the vector (11) will get the uncountable amount of ZERO importances under uncountable amount corresponding to importances associate vectors $s$ and $s^{*}$.
The Riemann hypothesis asserts that all interesting solutions of the equal

$$
\begin{equation*}
\zeta(S)=0 \tag{12}
\end{equation*}
$$

lie on a certain vertical straight line.
Such decision we have easy got, following coordinate system, see Pic. 1 Pic. 5 , complex invariant (5) and functional equation (9).
If in equation dzeta-functions of Riemann as argument will is used vectorinvariant $S$, refer to (5), that dzeta-function of Riemann, refer (9), will generate to zeroes:

$$
\begin{align*}
& \zeta(S)=2^{S} \pi^{S-1} \sin \frac{\pi S}{2} \cdot \Gamma(1-S) \cdot \zeta(1-S)=  \tag{13}\\
& =2^{S} \pi^{S-1} \cdot \sin Q \cdot \Gamma(1-S) \cdot \zeta(1-S)=0
\end{align*}
$$

under any importances function :

$$
\begin{align*}
& \Gamma(1-S)  \tag{14}\\
& \zeta(1-S) \tag{15}
\end{align*}
$$

## 2. Sphere of Riemann and her coordinate system

The Sphere of Riemann, expressed on Pic. 5, has a diameter, equal unit. She (sphere) combine with coordinate system, expressed on Pic. 3 - Pic. 7. Due to such joining, the axis Y , combined with diameter of the sphere, is combined with vertical , which forms the vector::

$$
\begin{equation*}
\mathbf{S}=\left[\mathbf{S}, \mathbf{s}^{*}\right] \tag{16}
\end{equation*}
$$

length of this vector

$$
\begin{equation*}
|\mathbf{S}|=\left|\mathbf{s}, \mathbf{s}^{*}\right|=|\mathbf{s}| \cdot\left|\mathbf{s}^{*}\right| \cdot \sin \mathbf{Q} \tag{17}
\end{equation*}
$$

The Axis Y, penetrating sphere, gets through her (its) pole- south and north. From point of the north pole possible to conduct the ray, which crosses simultaneously surface of the sphere in point:

$$
\begin{equation*}
\mathbf{P}^{\prime}=\mathbf{P}^{\prime}(\xi, \zeta, \mathbf{Y}) \tag{18}
\end{equation*}
$$

and plane $C$ complex numbers:

$$
\begin{equation*}
\mathbf{P}=\mathbf{f}(\mathbf{S}) \tag{19}
\end{equation*}
$$

RIEMANN SPHERE - DIAMETER $D=2 R=1$


On planes C complex numbers-vectors, painted by blue colour, according to Pic. 1, Pic. 2 and Pic. 3 , always stands out the area of the change the complex invariant $S$ and his (its) component. Consequently, each point P , expressed on Pic. 5 , will correspond to mirror reflected point. Herewith, a general real coordinate will always be beside these point on axis of the abscissas, equal

$$
\begin{equation*}
\mathbf{0} \leq \mathbf{a} \leq \infty \tag{20}
\end{equation*}
$$

In the event of, expressed on Pic 6 , point P will become localized only in zone, painted by blue colour.
Half of the sphere of Riemann radius is designed In this zone:

$$
\begin{equation*}
R=\frac{D=1}{2}=\frac{1}{2} \tag{21}
\end{equation*}
$$

RIEMANN SPHERE - DIAMETER $\mathrm{D}=2 \mathrm{R}=1$


Pic. 6
According to determination (21) and Pic. 6, exists to be yellow band width , equal unit. Herewith, in point of the osculation to axis to symmetries of this band with projection of the sphere of the radius $\mathrm{R}=$ $1 / 2$, will exist maximum importance of the real part of coordinates of the point:

$$
\begin{equation*}
0 \leq \operatorname{Re}\left(\mathrm{P}^{\prime}\right) \leq \frac{\mathbf{1}}{\mathbf{2}} \tag{22}
\end{equation*}
$$

We shall Combine the coordinate system, expressed on Pic5, with coordinate system, expressed on Pic.7. In coordinate system, expressed on Pic. 7:

$$
\begin{align*}
\mathbf{Z} & =+\mathbf{i} \cdot \zeta \\
-\mathbf{Z} & =-\mathbf{i} \cdot \zeta \\
\mathbf{X} & =\xi  \tag{23}\\
\mathbf{Y} & =\mathbf{Y}
\end{align*}
$$

Herewith, each point-vector

$$
\begin{equation*}
\mathbf{P}=(\xi, \zeta) \tag{24}
\end{equation*}
$$

on planes $C$ complex numbers this coordinate system is defined by formula

$$
\begin{equation*}
\mathbf{P}=\xi+\mathbf{i} \cdot \zeta \tag{25}
\end{equation*}
$$

The Associate point-vector is defined by formula

$$
\begin{equation*}
\mathbf{P}^{*}=\xi-\mathbf{i} \cdot \zeta \tag{26}
\end{equation*}
$$



Pic. 7
Transition from coordinate system, expressed on Pic.1, Pic. 2 and Pic.3, in coordinate system, expressed on Pic. 7 ( conversely), is realized by means of simple molded:

$$
\begin{align*}
& \xi=\frac{\mathbf{a}}{1+\mathbf{c}} \\
& \zeta=\frac{\mathbf{b}}{1+\mathbf{c}}  \tag{27}\\
& Y=\frac{\mathbf{c}}{1+\mathbf{c}}
\end{align*}
$$

which follow from unit-complex vector-invariant, refer to (26):

$$
\begin{equation*}
\mathbf{S}=\mathbf{s} \cdot \mathbf{s}^{*}=(\mathbf{a}+\mathbf{i} \cdot \mathbf{b}) \cdot(\mathbf{a}-\mathbf{i} \cdot \mathbf{b})=\mathbf{a}^{2}+\mathbf{b}^{2}=\mathbf{c}^{2} \tag{28}
\end{equation*}
$$

We shall Consider these rules on simple example, which illustrates brought below Pic. 8. On this drawing is expressed diametrical section of the sphere of Riemann, which diameter is an unit.

DIAMETER SECTION RIEMANN'S SPHERE


Pic. 8
We shall Calculate the coordinates of the point by means of molded (27). In coordinate system, expressed on Pic. 2, mate, expressed on Pic. 8, correspond to the following data:

$$
\begin{equation*}
\mathbf{a}=\mathbf{0}, \mathbf{b}=\mathbf{1} \tag{29}
\end{equation*}
$$

Herewith, invariant:

$$
\begin{equation*}
\mathbf{S}=\mathbf{s} \cdot \mathbf{s}^{*}=(\mathbf{a}+\mathbf{i} \cdot \mathbf{b}) \cdot(\mathbf{a}-\mathbf{i} \cdot \mathbf{b})=\mathbf{a}^{2}+\mathbf{b}^{2}=\mathbf{c}^{2} \tag{30}
\end{equation*}
$$

gains the following unit-type:

$$
\begin{equation*}
\mathbf{S}=(\mathbf{i} \cdot \mathbf{b}) \cdot(-\mathbf{i} \cdot \mathbf{b})=\mathbf{b}^{2}=\mathbf{c}^{2}=\mathbf{1}^{2} \tag{31}
\end{equation*}
$$

With account these data, the formulas (27) deliver us following coordinates of the point $\mathrm{P}^{\prime \prime}$ :

$$
\begin{align*}
& \xi=\frac{\mathbf{a}}{1+\mathbf{c}}=\frac{\mathbf{0}}{1+1}=\mathbf{0} \\
& \zeta=\frac{\mathbf{b}}{1+\mathbf{c}}=\frac{1}{1+1}=\frac{1}{2}  \tag{32}\\
& Y=\frac{\mathbf{c}}{1+\mathbf{c}}=\frac{1}{1+1}=\frac{1}{2}
\end{align*}
$$

These given exactly comply with given Pic. 8. Now we address to Pic.6. The Pic. 6 graphically illustrate following fact:
All $\zeta(\mathbf{s})=\mathbf{0}$ rest upon vertical direct.
Projection half-sphere of Riemann is marked on Pic. 6 blue colour on plane C complex numbers. Wanderring red ray, going from point of the North pole, crosses surface of the sphere of Riemann in points

$$
\begin{equation*}
\mathbf{P}^{\prime}=\mathbf{P}^{\prime}(\xi, \zeta, \mathbf{Y}) \tag{33}
\end{equation*}
$$

The Projections these points on blue projection of Riemann lies only on this blue projection half-sphere of Riemann. If, according to Pic. 7, angle

$$
\begin{equation*}
\mathbf{Q}=\mathbf{0} \tag{34}
\end{equation*}
$$

then in this case, refer to Pic. 3, and length of the vector of the vector product is a zero:

$$
\begin{equation*}
|\mathbf{S}|=\left|\mathbf{s}, \mathbf{s}^{*}\right|=|\mathbf{s}| \cdot\left|\mathbf{s}^{*}\right| \cdot \sin \mathbf{Q}=\mathbf{0} \tag{35}
\end{equation*}
$$

and vector product itself is a zero

$$
\begin{equation*}
\mathbf{S}=\left[\mathbf{S}, \mathbf{s}^{*}\right]=\mathbf{0} \tag{36}
\end{equation*}
$$

According to Pic.7, all vapour point-vector:

$$
\begin{equation*}
\mathbf{P} \wedge \mathbf{P}^{*} \tag{37}
\end{equation*}
$$

meet, in accordance with determination (26),

$$
\mathbf{S}=\mathbf{S} \cdot \mathbf{s}^{*}=(\mathbf{a}+\mathbf{i} \cdot \mathbf{b}) \cdot(\mathbf{a}-\mathbf{i} \cdot \mathbf{b})=\mathbf{a}^{2}+\mathbf{b}^{2}=\mathbf{c}^{2}
$$

in one point

## a

resting upon axis of the abscissas.
The Amount such poinst strives to $\infty$. To $\infty$ strives the amount of the vector product, definied by form (36). To $\infty$ strives and amount of the zeroes, liing at the beginning initially VERTIKAL, definied by form (35) if importance of the number "a", refer to (38), strives to zero.
If in equation dzeta-functions of Riemann as argument will is used vector-invariant $S$, refer to (26), that dzeta-function of Riemann will generate the zeroes:

$$
\begin{gather*}
\zeta(S)=2^{S} \pi^{S-1} \sin \frac{\pi S}{2} \cdot \Gamma(1-S) \cdot \zeta(1-S)=  \tag{39}\\
=2^{S} \pi^{S-1} \cdot \sin Q \cdot \Gamma(1-S) \cdot \zeta(1-S)=0
\end{gather*}
$$

under any importances function

$$
\begin{align*}
& \Gamma(1-S)  \tag{40}\\
& \zeta(1-S) \tag{41}
\end{align*}
$$

In final count I come in sphere of Riemann, which is built over plane C complex numbers refer Pic. 5, Pic. 6, Pic. 7 and Pic. 8.

RIEMANN SPHERE - DIAMETER $D=2 R=1$


Pic. 5


Pic. 6


Pic. 7

DIAMETER SECTION RIEMANN'S SPHERE


Pic. 8
The Mathematical operations on sphere of Riemann, which built over planes C complex numbers, I shall describe below. Here I offer attention of the readers short analysis the got Result. We passed only half very important way - have built vertical, on which have placed all zeroes dzeta-functions of Riemann. Characteristic primitive three-tuple of Pythagoras has freed us from need analysis whole endless spectrum intervals between prime numbers

| 2-1 $=1$ | 19-17=2 | 51-47 $=2^{2}$ | $79-73=2 \cdot 3$ | 103-101 = 2 |
| :---: | :---: | :---: | :---: | :---: |
| $3-2=1$ | $23-19=2^{2}$ | 53-51-2 | $83-79=2^{2}$ | 107-103 = $\mathbf{2}^{2}$ |
| $5-3=2$ | $31-23=2^{3}$ | 57-53 $=2^{2}$ | $87-83=2^{2}$ | 109-107 = 2 |
| $7-5=2$ | 37-31 $=\mathbf{2} \cdot \mathbf{3}$ | 61-57 $=2^{2}$ | $89-87=2$ | 111-109 = 2 |
| $11-7=2^{2}$ | $41-37=2^{2}$ | $63-61=2$ | $93-89=2^{2}$ | 113-111 = 2 |
| $13-11=2$ | $43-41=2$ | $71-63=2^{2}$ | $97-93=2^{2}$ | $\mathbf{1 1 7 - 1 1 3 =} \mathbf{2}^{\mathbf{2}}$ |
| $\mathbf{1 7 - 1 3}=\mathbf{2}^{\mathbf{2}}$ | $47-43=2^{2}$ | $73-71=2$ | $101-97=2^{2}$ |  |

Shaping primitive three-tuple of Pythagoras, refer (6), (7) and (8), sifts whole endless natural row, through each following unit. That phenomenon primitive three-tuple Pythagoras graphically demonstrates process of the shaping congruent numbers of Pythagoras [4], [5]:

$$
\begin{aligned}
& \left(\begin{array}{l}
(v=2, u=1) \rightarrow\left(a_{0}=3, b_{0}=4\right) \rightarrow K_{1}=6 \\
(v=4, u=3) \rightarrow\left(a_{0}=7, b_{0}=24\right) \rightarrow K_{2}=84=3 \\
(v=6, u=5) \rightarrow\left(a_{0}=11, b_{0}=60\right) \rightarrow K_{3}=330=6
\end{array}\right)=\sum_{1}=6 \\
& \left(\begin{array}{l}
(v=8, u=7) \rightarrow\left(a_{0}=15, b_{0}=112\right) \rightarrow K_{4}=840=3 \\
(v=10, u=9) \rightarrow\left(a_{0}=19, b_{0}=180\right) \rightarrow K_{5}=1710=9 \\
(v=12, u=11) \rightarrow\left(a_{0}=23, b_{0}=264\right) \rightarrow K_{6}=3036=3
\end{array}\right)=\sum_{2}=6 \\
& \left(\begin{array}{l}
(v=14, u=13) \rightarrow\left(a_{0}=27, b_{0}=364\right) \rightarrow K_{7}=4914=9 \\
(v=16, u=15) \rightarrow\left(a_{0}=31, b_{0}=480\right) \rightarrow K_{8}=7440=6 \\
(v=18, u=17) \rightarrow\left(a_{0}=35, b_{0}=612\right) \rightarrow K_{9}=10710=9
\end{array}\right)=\sum_{3}=6
\end{aligned}
$$

Unlike known congruent numbers, I calculate; opened by me congruent of the numbers of Pythagoras on formula :

$$
\begin{equation*}
K_{N}=\frac{a_{o} \cdot b_{o}}{2} \tag{44}
\end{equation*}
$$

Second part of our way consists whole endless ensemble simple numbers in description of the algorithm of the calculation

## 3. Conclusion

Analysing broughted above data, we come to very important conclusion: The Prime numbers are distributed in natural row not regularly refer (16). Primitive three-tuples of Pythagoras are distributed strictly regularly, refer (17). Law of the distribution simple numbers - to search for on ensemble primitive three-tuple of Pythagoras. Such law delivers us matrix A.

Matrix A

$$
\begin{align*}
& A_{1}^{*}=\left(b_{0}{ }^{2}-a_{0}{ }^{2}\right)=\left[b_{0}{ }^{2}+\left(\mathbf{i} \cdot a_{0}\right)^{2}\right] \\
& A^{*}=\left(a_{0}{ }^{2}-b_{0}{ }^{2}\right)=\left[{a_{0}}^{2}+\left(\mathbf{i} \cdot b_{0}\right)^{2}\right] \\
& A_{3}^{*}=\left(b_{0}-a_{0}\right)=b_{0}-\sqrt{S-b_{0}{ }^{2}} \\
& A_{4}^{*}=\left(a_{0}-b_{0}\right)=a_{0}-\sqrt{S-a_{0}{ }^{2}} \\
& A_{5}^{*}=c_{0}=\sqrt{a_{0}{ }^{2}+b_{0}{ }^{2}}=\sqrt{S} \tag{45}
\end{align*}
$$

Here

$$
\begin{align*}
& {A^{*}}_{1}=\left[(2 \mathbf{v u})^{2}-\left(\mathbf{v}^{2}-\mathbf{u}^{2}\right)^{2}\right] \\
& \mathbf{A}_{2}^{*}=\left[\left(\mathbf{v}^{2}-\mathbf{u}^{2}\right)^{2}-(2 \mathbf{v u})^{2}\right] \\
& {A^{*}}_{3}=\left[(2 \mathbf{v u})-\left(\mathbf{v}^{2}-\mathbf{u}^{2}\right)\right] \\
& \mathbf{A}_{4}^{*}=\left[\left(\mathbf{v}^{2}-\mathbf{u}^{2}\right)-(2 \mathbf{v u})\right] \\
& \mathbf{A}_{5}^{*}=\left(\mathbf{v}^{2}+\mathbf{u}^{2}\right) \tag{46}
\end{align*}
$$

From Matrix A but follow three rules for calculation all simple numbers.

## Reasons

1. According to (16), exist three intervals between two nearby prime numbers. Practically all intervals short count, calculate, list 2 , as has served central to known formula P. Fermat.
Prime numbers of the type

$$
\begin{equation*}
p_{k}=2^{2^{k}}+1 \tag{47}
\end{equation*}
$$

have got name an numbers of Fermat
2. In my proof special role executes complex - invariant, refer to (5):
3.

$$
\begin{gather*}
S=s \cdot s^{*}=\left(a_{0}+\mathbf{i} \cdot \mathbf{b}_{0}\right) \cdot\left(a_{0}-\mathbf{i} \cdot b_{0}\right)= \\
={a_{0}}^{2}+b_{0}^{2}=c_{0}^{2} \tag{48}
\end{gather*}
$$

This invariant has a direct attitude to method of the trigonometric amounts.
The Russian academician I.M. Vinogradof created one of the most strong and the general methods to analytical numbers theory - a method of the
trigonometric amounts. Nearly all problems to analytical theory of numbers enough are simply formulated on language of the final amounts composed type:

$$
\begin{equation*}
\cos F\left(x_{1}, \ldots, x_{n}\right)+i \cdot \sin F\left(x_{1}, \ldots, x_{n}\right) \tag{49}
\end{equation*}
$$

where F - real whole-numbers function and

$$
\begin{equation*}
\mathbf{i}=\sqrt{-1} \tag{50}
\end{equation*}
$$

Thereby, the center of gravity many problems is carried on problem of the study of such amounts and, in particular, on problem of the reception possible more proper pricing of the module of such amounts.
The Known formula of Euler:

$$
\begin{equation*}
\exp (a+i \cdot b)=e^{a} \cdot[\cos (b)+i \cdot \sin (b)] \tag{51}
\end{equation*}
$$

is it easy transformed in formula Vinogradof:

$$
\begin{equation*}
\frac{\exp (\mathbf{a}+\mathbf{i} \cdot \mathbf{b})}{\mathbf{e}^{a}}=\cos (\mathbf{b})+\mathbf{i} \cdot(\mathbf{b}) \tag{52}
\end{equation*}
$$

at condition:

$$
\begin{equation*}
\mathbf{b} \Leftrightarrow \mathbf{F} \tag{53}
\end{equation*}
$$

This formula has an associate type

$$
\begin{equation*}
\frac{\exp (\mathbf{a}-\mathbf{i} \cdot \mathbf{b})}{\mathbf{e}^{\mathbf{a}}}=\cos (\mathbf{b})-\mathbf{i} \cdot(\mathbf{b}) \tag{54}
\end{equation*}
$$

The Role whole-numbers function can execute the primitive three-tuples of Pyhagoras:

$$
\begin{align*}
& a=\left(a_{0}=v^{2}-u^{2}\right) \\
& b=\left(b_{0}=2 \mathbf{v} u\right)  \tag{55}\\
& c=\left(c_{0}=v^{2}+u^{2}\right)
\end{align*}
$$

which are built from endless sequence natural numbers:

$$
\begin{equation*}
\mathbf{v}>\mathbf{u} \tag{56}
\end{equation*}
$$

In algorithm calculation simple numbers are used special, created by me, spectral logarithmic functions ::

$$
\begin{align*}
& \xi^{*}(5)=\frac{1}{5 \cdot \ln 1}+\frac{1}{5 \cdot \ln 2}+\frac{1}{5 \cdot \ln 3}+\frac{1}{5 \cdot \ln 4}+\ldots= \\
& =\frac{1}{5} \cdot\left(\frac{1}{\ln 1}+\frac{1}{\ln 2}+\frac{1}{\ln 3}+\frac{1}{\ln 4}+\ldots\right)=\frac{1}{5} \cdot R^{*} \tag{57}
\end{align*}
$$

$$
\begin{align*}
& \xi^{*}(7)=\frac{1}{7 \cdot \ln 1}+\frac{1}{7 \cdot \ln 2}+\frac{1}{7 \cdot \ln 3}+\frac{1}{7 \cdot \ln 4}+\ldots= \\
& =\frac{1}{7} \cdot\left(\frac{1}{\ln 1}+\frac{1}{\ln 2}+\frac{1}{\ln 3}+\frac{1}{\ln 4}+\ldots\right)=\frac{1}{7} \cdot R^{*} \tag{58}
\end{align*}
$$

By means of these spectral function are formed spectral models two typical numbers mathematicians Pifagora and mathematical physicists

$$
\begin{align*}
5 & =\frac{\mathbf{R}^{*}}{\xi^{*}(5)}  \tag{59}\\
7 & =\frac{\mathbf{R}^{*}}{\xi^{*}(7)} \tag{60}
\end{align*}
$$

Through number 5 passes the axis to symmetries of the Triangle of Korneev. But number 7 displays the most most important phenomena in Universe - a magnetism, refer to reference № 31 on http://yvsevolod26.narod.ru/index.httml

## 4. First way of the separation (exclusions) simple numbers from endless row natural numbers

The Spectral numbers:

$$
\begin{align*}
& 5=\frac{\mathbf{R}^{*}}{\xi^{*}(5)}  \tag{61}\\
& 7=\frac{\mathbf{R}^{*}}{\xi^{*}(7)} \tag{62}
\end{align*}
$$

we shall use as numbers-controllers.
For separation simple numbers from endless row primitiv three-tuple of Pythagoras exists three Rules: Rule A , Rule B and Rule C. Triads primitiv three-tuple of Pythagoras are used In these Rule.
4.1. Triads primitive three-tuple of Pythagoras, delivering prime numbers on Rule A
(This - an "sieve" №1, excluding prime numbers without use the most mathematical formalism, described in article Y.V. Matiyasevich "Formulas for simple numbers" )
If $\mathbf{c}_{\mathbf{0}}=\mathbf{A}^{*}{ }_{5}$ simple numbers, then, in accord «Matrix $A »$, refer (45):
for $\mathbf{b}_{\mathbf{0}}>\mathbf{a}_{\mathbf{0}}$, all numbers

$$
\mathbf{A}_{3}^{*}=\left(\mathbf{b}_{0}-\mathbf{a}_{0}\right)
$$

simple. Amongst ensembles numbers

$$
\mathbf{A}_{1}^{*}=\left(\mathbf{b}_{0}{ }^{2}-\mathbf{a}_{0}{ }^{2}\right)
$$

can be simple without fission and after fission by controller

$$
7=\frac{\mathbf{R}^{*}}{\xi^{*}(7)}
$$

For $\mathbf{a}_{\mathbf{0}}>\mathbf{b}_{\mathbf{0}}$, all numbers $\mathbf{A}_{\mathbf{4}}^{*}=\left(\mathbf{a}_{\mathbf{0}}-\mathbf{b}_{\mathbf{0}}\right)_{\text {simple } .}$
All numbers

$$
\mathbf{A}_{2}^{*}=\left(\mathbf{a}_{0}^{2}-\mathbf{b}_{0}^{2}\right)
$$

after divide by controller

$$
7=\frac{\mathbf{R}^{*}}{\xi^{*}(7)}
$$

are a prime numbers
4.2. Triads primitive three-tuple of Pythagoras, delivering prime numbers on Rule B
( This - an "sieve" №2, excluding prime numbers without use the most mathematical formalism, described in article Y.V. Matiyasevich "Formulas for simple numbers" )
If controller

$$
5=\frac{\mathbf{R}^{*}}{\xi^{*}(5)}
$$

the done number $\mathbf{c}_{\mathbf{0}}$, or prime number $\mathbf{c}_{\mathbf{0}}$, that, according to Matrix A: for $\mathbf{b}_{\mathbf{0}}>\mathbf{a}_{\mathbf{0}}$, can be a numbers

$$
\begin{aligned}
& \mathrm{A}_{3}^{*}=\left(\mathrm{b}_{0}-\mathbf{a}_{0}\right) \\
& \mathrm{A}^{*}{ }_{1}=\left(\mathrm{b}_{0}{ }^{2}-\mathbf{a}_{0}{ }^{2}\right)
\end{aligned}
$$

simple, without divisionning by controller $\mathbf{7}=\frac{\mathbf{R}^{*}}{\xi^{*}(\mathbf{7 )}}$ and after divisionning by this controller.

For $\mathbf{a}_{\mathbf{0}}>\mathbf{b}_{\mathbf{0}}$, numbers

$$
\begin{aligned}
& A_{4}^{*}=\left(a_{0}-b_{0}\right) \\
& A^{*}{ }_{2}=\left(a_{0}{ }^{2}-b_{0}{ }^{2}\right)
\end{aligned}
$$

simple
4.3. Triads primitive three-tuple of Pythagoras, delivering prime numbers on Rule C
(This - an "sieve" №3, excluding prime numbers without use the most mathematical formalism, described in article Y.V. Matiyasevich "Formulas for simple numbers" )

If square of the controller

$$
7^{2}=\left(\frac{\mathbf{R}^{*}}{\xi^{*}(7)}\right)^{2}
$$

done difference square

$$
\left(\mathbf{b}_{0}{ }^{2}-\mathbf{a}_{0}{ }^{2}\right)
$$

and controller

$$
7=\frac{\mathbf{R}^{*}}{\xi^{*}(7)}
$$

done difference

$$
\left(b_{0}-a_{0}\right)
$$

Then all numbers

$$
\mathbf{A}_{5}^{*}=\mathbf{c}_{0}
$$

simple
For operational demonstration described above Rules, use the ready primitiv three-tuple of Pythagoras.
Ready, beforehand computable three-tuples of Pythagoras, we find on page 17 remarkable books of Paulo Ribenboym "Last theorem Farm" [4], translated on russian language and published in 2003 in Moscow, the publishers "World". In book [6] of Paulo Ribenboym is accepted following placement primitiv three-tuple:

$$
\left(b_{0}, a_{0}, c_{0}\right):
$$

As a matter of convenience calculations all primitiv triads of Pithagoras me are numbered
№ 1: (4,3,5) №4: (12,5,13) №7: (8,15,17) № $10:(24,7,25)$
№ 2: (20,21,29) №5: (12,35,37) № 8: $(40,9,41)$ № 11: $(28,45,53)$
№3: (60,11,61) №6: (56,33,65) №9: (16,63,65) №12: $(48,55,73)$

Majority of the Triads, computable of Paulo Ribenboym, is calculated on Rule B

Triad № 2 :

$$
\begin{aligned}
& \left(\mathbf{a}_{0}{ }^{2}-\mathbf{b}_{0}{ }^{2}\right)=441-\mathbf{4 0 0}=\mathbf{4 1} \quad \text { simple } \\
& \left(\mathbf{a}_{0}-\mathbf{b}_{0}\right)=\mathbf{2 1}-\mathbf{2 0}=\mathbf{1} \quad \text { simple } \\
& \mathbf{A}^{*}=\mathbf{c}_{0}=\mathbf{2 9} \text { simple }
\end{aligned}
$$

Triad № 4 :
$\left(b_{0}{ }^{2}-a_{0}{ }^{2}\right)=144-25=119 / 7=17$
$\left(b_{0}-\mathbf{a}_{\mathbf{0}}\right)=\mathbf{4 - 3}=\mathbf{1} \quad$ simple
$\mathrm{A}^{*}{ }_{5}=\mathrm{c}_{0}=13$
simple
Триада № 5 :

$$
\begin{aligned}
& \left(\mathbf{a}_{0}^{2}-b_{0}{ }^{2}\right)=\mathbf{1 2 2 5}-\mathbf{1 4 4}=\mathbf{1 0 8 1} \text { simple } \\
& \left(\mathbf{a}_{0}-\mathbf{b}_{0}\right)=\mathbf{3 5}-\mathbf{1 2}=\mathbf{2 3} \text { simple } \\
& \mathbf{A}_{5}^{*}=\mathbf{c}_{0}=\mathbf{3 7} \text { simple }
\end{aligned}
$$

Триада № 7 :

$$
\begin{aligned}
& \left(\mathrm{a}_{0}{ }^{2}-\mathrm{b}_{0}{ }^{2}\right)=\mathbf{2 2 5}-\mathbf{6 4}=161 / 7=23 \text { simple } \\
& \left(\mathrm{a}_{0}-\mathrm{b}_{0}\right)=15-8=7 \text { simple } \\
& \mathrm{A}_{5}^{*}=\mathrm{c}_{0}=17 \text { simple }
\end{aligned}
$$

Triad № 8 :
$\left(b_{0}{ }^{2}-a_{0}{ }^{2}\right)=1600-81=1519 / 7=217$
$\left(b_{0}-a_{0}\right)=40-9=31$
simple
$\mathrm{A}_{5}=\mathbf{c}_{\mathbf{0}}=\mathbf{4 1}$ simple
Triad № 12 :
$\left(\mathrm{a}_{0}{ }^{2}-\mathrm{b}_{0}{ }^{2}\right)=\mathbf{3 0 2 5}-\mathbf{2 3 0 4}=\mathbf{7 2 1} / 7=103$
simple
$\left(\mathbf{a}_{\mathbf{0}}-\mathrm{b}_{\mathbf{0}}\right)=55-48=\mathbf{7}$ simple
$\mathrm{A}^{*}{ }_{5}=\mathbf{c}_{\mathbf{0}}=\mathbf{7 3}$ simple
Triad № 6 :
$\left(\mathrm{c}_{0} / 5\right)=65 / 5=13$
simple
$\left(\mathrm{b}_{0}{ }^{2}-\mathrm{a}_{0}{ }^{2}\right)=\mathbf{3 1 3 6}-\mathbf{1 0 8 9}=\mathbf{2 0 4 7}$ simple
$\left(\mathbf{b}_{\mathbf{0}}-\mathbf{a}_{\mathbf{0}}\right)=\mathbf{5 6}-\mathbf{3 3}=\mathbf{2 3}$ simple
Triad № 9 :
$\mathrm{c}_{0} / 5=65 / 5=13$
simple
$\left(\mathrm{a}_{0}{ }^{2}-\mathrm{b}_{0}{ }^{2}\right)=\mathbf{3 9 6 9}-\mathbf{2 5 6}=\mathbf{3 7 1 3}$ simple
$\left(a_{0}-b_{0}\right)=63-16=47$ simple
Triad № 10 :
$c_{0} / 5=25 / 5=5$
simple
$\left(\mathrm{b}_{0}{ }^{2}-\mathrm{a}_{0}{ }^{2}\right)=\mathbf{5 7 6}-\mathbf{4 9}=\mathbf{5 2 7}$ simple
$\left(b_{0}-a_{0}\right)=\mathbf{2 4}-\mathbf{7}=\mathbf{1 7}$ simple
One triad is calculated on Rule C
Triad № 3 :
If square of the controller

$$
7^{2}=\left(\frac{\mathbf{R}^{*}}{\xi^{*}(7)}\right)^{2}
$$

done difference square

$$
\left(\mathbf{b}_{0}{ }^{2}-\mathbf{a}_{0}{ }^{2}\right) / 49=(3600-121) / 49=3479 / 49=71
$$

that result of the fission prime number. Herewith if controller:

$$
7=\frac{\mathbf{R}^{*}}{\xi^{*}(7)}
$$

done difference

$$
\left(b_{0}-a_{0}\right) / 7=(60-11) / 7=49 / 7=7
$$

which essence fundamental prime number then in this case we calculate the prime number, marked by Matrix A , refer to (45):

$$
\mathbf{A}_{5}^{*}=\mathbf{c}_{0}=61=\sqrt{\mathbf{a}_{0}{ }^{2}+\mathbf{b}_{0}{ }^{2}}=\sqrt{\mathbf{S}}
$$

Other examples
Triad № 13 :
If $v=17$ and $u=14$, then:
$a_{0}=93$
$b_{0}=476$
$c_{0}=485$
Herewith
$\left.\mathbf{( b}_{\mathbf{0}}{ }^{2}-\mathbf{a}_{\mathbf{0}}{ }^{2}\right)=226576-8 \mathbf{6 8 9}=217927$ simple
$\left(b_{0}-a_{0}\right)=(476-93)=383$
simple
$c_{0} / 5=97$
simple
Triad № 14 :
If $v=17$ and $u=12$,then:
$a_{0}=145$
$b_{0}=408$
$c_{0}=433$
In this case:

$$
\begin{aligned}
& \left(b_{0}{ }^{2}-a_{0}{ }^{2}\right)=166464-21025=145439 / 7=20777 \text { simple } \\
& \left(b_{0}-a_{0}\right)=408-145=263 \text { simple }
\end{aligned}
$$

$A^{*} 5=c_{0}=433$ simple

Triad № 15 :
If $\mathrm{v}=18$ and $\mathrm{u}=11$,then:
$\mathrm{a}_{0}=203$
$b_{0}=396$
$c_{0}=445$
In this case:
$\left(b_{0}{ }^{2}-\mathbf{a}_{0}{ }^{2}\right)=156816-41209=115607$ simple
$\left(b_{0}-a_{0}\right)=396-203=193$ simple
$\mathrm{A}^{*}{ }_{5}=\mathbf{c}_{\mathbf{0}} / \mathbf{5}=\mathbf{8 9}$ simple
Triad № 16 :
If $\mathrm{v}=18$ and $\mathrm{u}=13$, then:
$a_{0}=155$
$b_{0}=468$
$c_{0}=493$

In this case:
$\left(b_{0}{ }^{2}-a_{0}{ }^{2}\right)=219024-24025=194999 / 7=27857$ simple
$\left(b_{0}-a_{0}\right)=468-155=313$
simple
$A^{*}{ }_{5}=c_{0}=493$
simple
Et cetera, et cetera

## 5. Second $t$ way of the separation simple numbers from endless row natural numbers

Y.V. Matiyasevich in its article "Formulas for simple numbers" considers long ago known formula of P. Fermat:

$$
\mathbf{p}_{k}=2^{2^{k}}+1
$$

in following type. "Shall Consider presently two formulas, having quite idle time type:

$$
\begin{align*}
& \mathbf{p}_{\leftarrow}=2^{\mathrm{n}}-\mathbf{1}  \tag{63}\\
& \mathbf{p}_{\rightarrow}=2^{\mathrm{n}}+\mathbf{1} \tag{64}
\end{align*}
$$

If $n=0,2^{0}, 2^{1}, 2^{2}, 2^{3}$ and $2^{4}$ formula $\mathbf{p}=\mathbf{2}^{\mathbf{n}}+\mathbf{1}$ really gives prime numbers, Fermat, has voiced the suggestion, as under any $n$ type $2^{k}$ formula $\mathbf{p}=\mathbf{2}^{\mathbf{n}}+\mathbf{1}$ gives the prime number; In his (its) honour of the prime numbers of the type:

$$
\begin{equation*}
p_{k}=2^{2^{k}}+1 \tag{65}
\end{equation*}
$$

have got the name an Fermat's numbers. The Hypothesis of Fermat wrongly to refused Euler, specified on single number:.

$$
\begin{equation*}
p_{5}=2^{2^{5}}+1=4294967297 \tag{66}
\end{equation*}
$$

which is divided on 641. Consequently, number (66) component. General wrong conclusion was made on this single calculation.of Euler. Hereinafter Matiyasevich writes:
"At present known several importances $n$ type $2^{k}$,, under which on formula $\mathbf{p}=\mathbf{2}^{\mathbf{n}}+\mathbf{1}$ are got component numbers, but is not found nor one new prime number Fermat different from specified above." To understand the deep sense simple molded (63) and (64) , bring them to united form, containing in its base number 2 :

$$
\begin{equation*}
2^{\mathbf{n}}=\left(\mathbf{p}_{\leftarrow}+1\right)=\left(p_{\rightarrow}-1\right) \tag{67}
\end{equation*}
$$

Exactly, this ambiguous form bore in mind Fermat.

## 6. Physical interpreting the formula of Fermat

The Form (67) by prescribed Nature in base of the algorithm of the calculation simple numbers not only. She prescribed and in base of the shaping main "brick" of Universe - an nucleons (protons and neutruns) and electrons, which are built from elementary quantum-dynamic systems, refer to [7] , and from which consist all $100 \%$ observed material, refer to references № 3 - № 8 on put http://yvsevolod26.narod.ru/index.html.

This phenomenon of the Universe defines male and feminine beginning and forms the notions parity and non- parity in endless series natural numbersl.
Though in timeses of Fermat about nucleons and electrons did not yet know, but notions parity and non-parity were well known. The Fermat
intuitive (proofless) saw in formula (67) noted above numeric phenomenon of Universe
At factor degree $\mathrm{n}=79$ we get on formula (67) non-dimensional number Avogadro, which serves the key to study of the quantum phenomenas in theories ideal gas not only, but also in nucleus physicist, in subatomic physicist and in physicist of the elementary particles, [7].

The Way to semantic contents of the number Avogadro begins with comprehensions of the paradigm of Pythagoras:
Begin whole - an unit.
Unit, as reason, belongs to vague 2

$$
\begin{equation*}
\frac{2,(6)+0,(8)}{1,(7)}=\frac{1,(7)}{0,(8)}=2 \tag{68}
\end{equation*}
$$

From unit and vague 2 come the numbers;
From numbers-points.
From point-lines;
From them-flat figures;
From flat-threes-dementional figures;
From them-voluptuous perceived bodies."
Number Avogadro scrambled Creator in external spherical layer of the compacted matter, which in the manner of electronic shell (the SPHERICAL SURFACES rather then curvilinear orbits ) surrounds the pulsing kernel an atom material.


Beside heavy atom (36 actinoids) these SPHERICAL layer to matters have a standard set macro-quantum, measured mass electrons [8]:

$$
\begin{equation*}
32 \cdot \mathrm{~m}_{\mathrm{e}} ; 18 \cdot \mathrm{~m}_{\mathrm{e}} ; 8 \cdot \mathrm{~m}_{\mathrm{e}} ; 2 \cdot \mathrm{~m}_{\mathrm{e}} ; \tag{69}
\end{equation*}
$$

Herewith, row electronic numbers :

$$
\begin{equation*}
32 ; 18 ; 8 ; 2 \text {; } \tag{70}
\end{equation*}
$$

forms the group of the relative values::

$$
\begin{align*}
& X=\mathbf{3 2} / \mathbf{1 8}=\mathbf{1 . 7 7 7 7 7 7 7 7 8} \\
& Y=18 / 8=\mathbf{2 . 2 5}  \tag{71}\\
& Z=8 / 2=4
\end{align*}
$$

which exactly define the number Avogadro

$$
\begin{align*}
\mathbf{N}_{\mathrm{A}}= & \frac{\cdot(\mathbf{2} \cdot \mathbf{X} \cdot \mathbf{Y} \cdot \mathbf{Z})^{16}}{2}=\mathbf{2}^{79}= \\
& =\mathbf{2} \cdot\left(\mathbf{2}^{26}\right)^{3}=  \tag{72}\\
= & 6.0446291 \cdot 10^{23}
\end{align*}
$$

This number "handles" not LENGTH direct line, but fresh number - a SQUARE:

$$
\begin{align*}
& 2 \cdot 32=8^{2} \\
& 2 \cdot 18=6^{2} \\
& 2 \cdot 8=4^{2}  \tag{73}\\
& 2 \cdot 2=2^{2}
\end{align*}
$$

This number-square Creator puts in correspondence to geometric area SQUARE, from which can be reflected electromagnetic waves and waves of gravity.
More Detailed information reader will find in article [9] published on put ScyTecLibrary (article and publications). In this article reader will find solving "secrets" of formula ( 67). In brief solving is reduced to the following facts. Main by object of our study will be an endless row natural numbers

$$
\begin{equation*}
1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16, \ldots \tag{74}
\end{equation*}
$$

This row integer чисел is formed by Creator on group sign. All numbers of the natural row form the endless row «NINE»:

$$
\begin{array}{lllllllll}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9
\end{array}
$$

$\begin{array}{lllllll}1011 & 12131415161718\end{array}$

$$
\begin{equation*}
192021 \quad 22 \quad 2324252627 \tag{75}
\end{equation*}
$$

Herewith, all «NINE» numericalu are focused in endless row personal «TWO»;

$$
\begin{array}{rllllllll}
2_{1} & 2_{2} & 2_{3} & 2_{4} & 2_{5} & 2_{6} & 2_{7} & 2_{8} & 2_{9} \\
2_{10} & 2_{11} & 2_{12} & 2_{13} & 2_{14} & 2_{15} & 2_{16} & 2_{17} & 2_{18} \\
2_{19} & 2_{20} & 2_{21} & 2_{22} & 2_{23} & 2_{24} & 2_{25} & 2_{26} & 2_{27} \tag{76}
\end{array}
$$

and in endless row corresponding to personal «NINE»:

$$
\begin{array}{ccccccccc}
9_{1} & 9_{2} & 9_{3} & 9_{4} & 9_{5} & 9_{6} & 9_{7} & 9_{8} & 9_{9} \\
9_{10} & 9_{11} & 9_{12} & 9_{13} & 9_{14} & 9_{15} & 9_{16} & 9_{17} & 9_{18} \\
9_{19} & 9_{20} & 9_{21} & 9_{22} & 9_{23} & 9_{24} & 9_{25} & 9_{26} & 3_{27} \tag{77}
\end{array}
$$

The Focusing of the natural row in numbers $\mathbf{2}_{\mathbf{n}}$ delivers us opening of the doctor A. Korneef. This opening has got the name of the Triangle Korneef - $\mathbf{T K}_{\mathbf{n}}$, refer to Pic. K.

Numerological reduction to first section nine numbers from the row endless series natural numbers

| 1 2 3 4 5 6 7 8 9 |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 5 | 7 | 9 | 2 | 24 | 46 | $6{ }^{6} 8$ |  |
|  | 83 | 37 | 2 | 2 | 6 | 1 | 5 |  |
|  | 2 | 1 | 9 | 8 | 87 | 76 |  |  |
|  |  | 31 | 8 | 8 | 6 | 4 |  |  |
|  |  | 4 | 9 | 5 |  |  |  |  |
|  |  | 4 |  | 5 | 6 |  |  |  |
|  |  |  | 9 | 2 |  |  |  |  |
|  |  |  |  | 2 |  |  |  |  |

## Focus 2

Pic. K.
This amazing object focuses on number 2 not only first "NINE" endless ensemble (74), but also all "NINE" numbers ensemble (75), (76) and
(77). Make sure in this possible way NUMERICAL REDUCTIONS numbers ensemble (75). The Algorithm numerical reductions invariant for the whole natural row numbers. The Essence his - in following. We shall Pay attention to First triangle

$$
\begin{equation*}
\mathbf{T K} \mathrm{K}_{\mathrm{n}}=\mathbf{T} K_{1} \tag{78}
\end{equation*}
$$

expressed on Pic.K
The First, source line of this triangle always remains UNCHANGEABLE for ALL endless ensemble (77).
As example shall form the Second triangle $\mathbf{T K}_{\mathbf{2}}$ with reference to to the second nine natural numbers ensemble (75) and (77).
Summing ttwo first numbers of the second row ensemble (75) ,obeying the rule reductions of the spare numerals:

$$
\begin{equation*}
10+11=21=2+1=3 \tag{79}
\end{equation*}
$$

As a result we have got the first numeral of the second line. $\mathbf{T K}_{\mathbf{2}}$. Hereinafter, summing the second vapour an чисел on rule of the reduction:.

$$
\begin{equation*}
11+12=23=2+3=5 \tag{80}
\end{equation*}
$$

and get the second numeral second line.. Hereinafter on this rule third vapou numbers:.

$$
\begin{equation*}
12+13=25=2+5=7 \tag{81}
\end{equation*}
$$

and get the third numeral of the second line. And so on before the last numeral second line. Such way form the third line, quarter and all ostalinye.

As a result we shall get the copy. $\mathbf{T K}_{\mathbf{1}}$. But "for back" got triangle will stand not numbers of the first line ensemble (75), but numbers of the second line this ensemble "NINE"
Following along whole row natural numbers, we shall get endless ensemble QUALITATIVE differring triangle of Korneef:

$$
\begin{equation*}
\mathrm{TK}_{1} \mathrm{TK}_{2} \mathrm{TK}_{3} \mathrm{TK}_{4} \mathrm{TK}_{5} \mathrm{TK}_{6} \ldots . . \mathrm{TK}_{\mathrm{n} \rightarrow \infty} \tag{82}
\end{equation*}
$$

which delivers us endless row QUALITATIVE differring "TWO" and "NINE", refer to (76) and (77). Such a fundamental characteristic of the endless row natural numbers.

## Algorithm of searching for "twin" numbers of Fermat We Substitute in left part of my formula

$$
\begin{equation*}
\mathbf{N}_{\mathrm{k}, \mathbf{1}}=\mathbf{9} \cdot \mathbf{n}-\mathbf{l} \tag{83}
\end{equation*}
$$

the known fourth prime number of P. Fermat:

$$
\begin{equation*}
p_{4}=\mathbf{2}^{2^{4}}+\mathbf{l}=65537=\mathbf{9} \cdot \mathbf{n}-\mathbf{l} \tag{84}
\end{equation*}
$$

From this formulas we find the determination of the serial number $n$ that "NINE" numbers, in which is localized this prime number of P. Fermat:

$$
\begin{equation*}
\mathrm{n}=\frac{65537+1}{9}=7282 \tag{85}
\end{equation*}
$$

Consequently, this prime number of Fermat dwells in triangle of Korneef

$$
\begin{equation*}
\mathbf{T K}_{\mathrm{n}}=\mathbf{T K}_{7282} \tag{86}
\end{equation*}
$$

The Formula (83) delivers us importance of the additional coordinate of this number:

$$
\begin{gather*}
\mathbf{l}=9 \cdot \mathbf{n}-\mathbf{N}_{\mathrm{k}, \mathrm{l}}= \\
=9 \cdot 7282-65537= \\
=65538-65537=  \tag{87}\\
=1
\end{gather*}
$$

Consequently , in the first row numbers
65531; 65532; 65533; 65534; 65535; 65536;65537;65538;65539
our triangle

$$
\mathbf{T K}_{\mathrm{n}}=\mathbf{T K}_{7282}
$$

the prime number Farm will defend on one step from place-numbers, refer (87), to the left:

$$
\begin{equation*}
65538 \tag{88}
\end{equation*}
$$

Really, left of this numbers we find our familiar number:

$$
\begin{equation*}
p_{4}=65537 \tag{89}
\end{equation*}
$$

The Role of the centre to symmetries in this row чисел natural row executes the number:

$$
\begin{equation*}
65535 \tag{90}
\end{equation*}
$$

The Characteristic to symmetries considered nine numbers predestines the location of the prime number:

65333
which occupies mirror reflected place to place of the number of Fermat. Herewith, the first number and the last number considered nine numbers also are a prime numbers.

As a result, instead of one known prime number of Fermat, we have got the row other earlier unknown mirror reflected simple numbers of Fermat:

## 65531 ; 65533 ; 65537; 65539 ;

For the most complex mathematical formalism Y. Matiyasevich these numbers inaccessible.
For greater persuasiveness of the proposed algorithm of searching for simple numbers, shall consider else two , preceding triangle of Korneef:

$$
\begin{align*}
\mathbf{T K}_{\mathbf{n}} & =\mathbf{T K}_{7281}  \tag{93}\\
\mathbf{T K}_{\mathbf{n}} & =\mathbf{T K}_{7280} \tag{94}
\end{align*}
$$

First nines these triangle are indicative of irregularity of the appearance simple numbers in natural row:
First line for $\mathbf{T K}_{\mathbf{n}}=\mathbf{T K}_{7281}$
65522; 65523; 65524; 65525; 65526; 65527; 65528; $65529 ; 65530$
First line for $\mathbf{T K}_{\mathbf{n}}=\mathbf{T K}_{7280}$
65513 ; 65514; 65515; 65516; 65517; 65518; 65519;65520; 65521
Amongst numbers of the triangle :
$\mathbf{T K}_{7281}$
no simple numbers, refer to (93). The Numbers of the triangle

$$
\begin{equation*}
\mathbf{T K}_{\mathrm{n}}=\mathbf{T K}_{7280} \tag{96}
\end{equation*}
$$

contains the prime numbers

## 65519; 65521;

Using described here algorithm, possible find the big counting ensemble simple numbers - "twin" of numbers of Fermat on all endless row natural numbers.
In my article is formula, refer (83) :

$$
\mathbf{N}_{k, l}=\mathbf{9} \cdot \mathbf{n}-\mathbf{l}
$$

This formula ALWAYS indicates the researcher EIGHTH place abreast NINE numbers of the triangle of Korneef, number which n. Below scheme of this phenomenon for natural row.


If the triangle, refer (96):

$$
\mathbf{T K}_{\mathrm{n}}=\mathbf{T} \boldsymbol{K}_{7280}
$$

contains the prime numbers, refer (97):

## 65519; 65521;

then exists to be only one twin

Nine numbers if $\mathrm{n}=7280$
655136551465515655166551765518655196552065521


## 8. The General Conclusion

In article is offered generalised decision of the hypothesis of Riemann, in which are united in united integer analyst-geometric decision of the
problem and two ways of the separation simple numbers from endless row natural numbers.
The Offered algorithm of the finding чисел - "twin" numbers of Fermat, which amount forms very big counting ensemble.
Confirmed suggestion about that that under greater importances numbers intervals between prime numbers grow and vastly differ from interval, which exist at the beginning initially row natural чисел, refer to (75).

The Offered formulas for searching for any (even and uneven) of the natural row, refer to (75) - (77). Is Once again demonstrated fundamental characteristic of the General law to nonary periodicity, which as one should Nature in base of the Periodic system element D.I. Mendeleev, refer to [6].
Installed unity algorithm shaping simple numbers of Fermat by means of formulas (67) and algorithm of the shaping nucleons, electrons and planceons, which is described in mentionned in given article reference № 3 - № 8 on put http://yvsevolod-26.narod.ru/index.html.
Will semonstrated fundamental characteristic of the Triangle of the doctor A. A. Korneef, author of the new direction in science, got name "Chislonavtika".
Proposed in given article decision hypothesises Rimana opens before reader earlier unknown characteristic of the endless row natural numbers, consisting in that that this row integer numbers is built on group principle, refer to.(75):

$$
\begin{array}{lllllllll}
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\
10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 \\
19 & 20 & 21 & 22 & 23 & 24 & 25 & 26 & 27
\end{array}
$$

$\qquad$

Each group, consisting of 9 numbers, is focused in number 2 by means of Triangle A.A. Korneef:

Numerological reduction to first section nine numbers from the row endless series natural numbers

| 1 2 3 4 5 6 7 8 9 |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 3 | 5 | 7 | 9 | 2 | 4 | 46 | 68 |  |
|  | 83 | 37 | 72 | 6 | 6 | 1 | 5 |  |
|  | 2 | 1 | 9 | 8 | 7 | 76 | 6 |  |
|  |  | 31 | 18 |  | 6 | 4 |  |  |
|  |  | 4 | 9 | 5 |  |  |  |  |
|  |  |  | $4{ }^{4} 5$ |  | 6 |  |  |  |
|  |  |  | 9 | 2 |  |  |  |  |
|  |  |  | 2 | 2 |  |  |  |  |

## Focus 2

As a result whole endless row integer numbers is focused in endless matrix personal dyad - a symbol parity , refer to (76)::

$$
\begin{array}{llllllllll}
2_{1} & 2_{2} & 2_{3} & 2_{4} & 2_{5} & 2_{6} & 2_{7} & 2_{8} & 2_{9} \\
2_{10} & 2_{11} & 2_{12} & 2_{13} & 2_{14} & 2_{15} & 2_{16} & 2_{17} & 2_{18} \\
\mathbf{2}_{19} & 2_{20} & 2_{21} & 2_{22} & 2_{23} & 2_{24} & 2_{25} & 2_{26} & 2_{27}
\end{array}
$$

and in corresponding to endless row Triangleof Korneef,


Dart of time, directed in infinity, carry along endless row natural numbers

$$
\begin{equation*}
N=1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16, \ldots \tag{100}
\end{equation*}
$$

to its never limit:


$$
\begin{equation*}
\lambda^{*}=\left(1+1 / 2^{1}+1 / 2^{2}+1 / 2^{3}+1 / 2^{4}+\ldots+1 / 2^{\mathrm{N}_{\mathrm{A}}}+\ldots\right) \rightarrow 2 \tag{101}
\end{equation*}
$$

about existence which beautifully knew the Pythagorean. This knowledge has found its image in apory of Zenon about Ahille and terrapin:
Spectral invariant $\boldsymbol{\lambda}^{*}=\mathbf{S p} \mathbf{L}^{*} / \mathbf{L}$ by prescribed author in base of the models Inexhaustible Multivariate (spectral) Quantum Material Space, refer to References 5 and 6 on
http://yvsevolood-26.narod.ru/index.html.
As L can "emerge" any object of the Universe, including Photon-hadron White light - an optical radiation of the photoshperes Sun and star, refer to [9] and [10].
The Dart of time, directed to its beyond reach limit, to spectral count, is befitted baby toy -a kaleidoscope, which of two splinters colour flow capable to create the endless ensemble never reiterative scenes.
The Dart of TIME materialised all characteristic an numbers natural row, creating on its way objects micro- and macro-cosmos, objects alive and lifeless nature.
Dart of TIME - dynamic power of the Universe and his energyinformation field, refer to references № 44, № 46 and № 57 on put http://yvsevolood-26.narod.ru/index.html
Here, there is on than think and mathematician, and physicist, and biologist, and geneticist and philosopher.
And, finally, the last. Single CALCULATION of the fifth number of Fermat, executedof Euler, has wound mathematician in dead end. . They started to interpret the formula a Fermat , not as formula for SEPARATION whole ensemble simple numbers from natural row, but as formula , intended for CALCULATION single simple numbers only. Provided in this article data allow to confirm following.Fermat used in their own study general (universal) by formula

$$
\begin{equation*}
\mathbf{2}^{\mathrm{n}}=\left(\mathbf{p}_{\leftarrow}+\mathbf{1}\right)=\left(\mathbf{p}_{\rightarrow}-\mathbf{1}\right) \tag{102}
\end{equation*}
$$

The known formula follows From this dualistic ( ontological) formulas:

$$
\begin{equation*}
p_{n}=2^{n}+1 \tag{103}
\end{equation*}
$$

which under single importance:

$$
\begin{equation*}
\mathrm{n}=2^{2 k} \tag{104}
\end{equation*}
$$

has served the central to discredit of the fundamental opening of Fermat in numbers theories.: In mathematician, either as in Nature, all are engulfed direct and inverse svyazyami. And no nothing amazing in that that my analyst-geometric proof of the hypothesis of Riemann was connected with fundamental opening Fermat in theories numbers and with modern physical given nucleons about construction and electronmain brick of the Universe see references № 2, № 3, № 4, № 8, № 62, № 63, № 64 and № 65 on
http://yvsevolod-26.narood.ru/index.html
The Solved hypothesis of Riemann, we have confirmed real existence organic relationship between endless ensemble simple (indivisible) numbers and endless ensemble of the DIFFERENT zeroes. We also made sure in that that numbers 2,5 and 7 have a special role in Nature, refer to (61), (64), (68), (71), (720, (73), (84), (99) and (101). We made sure in that that all triangles of Korneef, refer to (82), is vested by characteristic to SYMMETRIES, which displays the phenomena to harmonies in the Universe. But, herewith, exists to be an evident disharmony (the ASYMMETRY) in construction material, refer to (71). Why this so?
On this ontological question of the answer does not exist.

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## Series:: METEOROLOGY

Sinitsyn K.N.

# The idea of manage tornado through the technology of «atmospheric magnetic bubbles» 

## Annotation

The present situation with phenomenon tornado research is allow make predictions through meteorological forecast, which have warning time between 1 and 3 hours only. In the modern theory not take into account of influence Earth's magnetic field structure to form tornado at all and theirs open field lines in partially. Author proposes to create new technology of «atmospheric magnetic bubbles» to make tornado tame, the atmosphere cleaning and managing of global climate changing. By author's opinion this technology using material results could be achieved in middle-time outlook even (from 12 to 24 months). Prior beliefs for that are created simulation tornado models as well deployed networks of weather forecast and weather observation. To create and development the technology author looks for investors. Approximate cost could be about $\$ 1.200 .000$ (one million two hundreds thousands US dollars). Cover of these investments could be calculating as percentage from tornado damages prevention service. Because annual losses from tornado are calculated as several billions US dollars, thus annual profit for investors after develop the technology could be about hundreds millions US dollars as minimum. But if take into account this longtime outlook (or more then 24 months after the technology using), additional profit increasing could be between $30 \%$ and $50 \%$.

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1. Introduction.
2. Brief history of the problem and modern point for the tornado phenomenon.
3. Last observations data for magnetic field structure
4. Description of the idea «atmospheric magnetic bubbles»
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References.

## 1. Introduction.

A problem for human in present days is phenomenon of powerful vortex (or tornado).A multiple attempts to prevent tornado damages are not successful until the time.

The modern theory allow make simulation of tornado forming process, but it is not possible to make all necessary measurements to do this simulation enough accurate.

An observations for last 10 years connected with this phenomenon allow to make forecasts its appearance in time from 1 to 3 hours as well to make forecasts for initiation of the phenomenon on distance until some hundreds kilometers [1, 2]. But it could be not enough for power tornado cases (F3-F5 types) and caused damages, consequently, remains still as fate and unpleasant fact.

## 2. Brief history of the problem and modern point for the tornado phenomenon.

The first description for tornado is dated of 1643 , but dependence between atmosphere pressure and cyclones behavior was noted by German scientist G. Leibniz on 1690. Though, scientists gave possibility to simulate a tornado phenomenon, in connection with mesometeorology area, close to end of 20th century only.

Most important researches until 1998 are:
S.A. Arseniev, A. Yu. Gubarev, V.N. Nikolaevskiy theory, which suppose that nature of tornado formation is connected to mesocyclone with start wind velocity from $1 \mathrm{~m} / \mathrm{s}$; and basic mass of mesocyclone contains from vortex as a result of convection or instability of atmospheric fronts for frontal clouds;
B.N. Belousov's and A.M.Zhabotinskiy's mathematic model created for typical dissipative structures with high degree of symmetry, but in phase of far condition from thermodynamic equilibrium.

The phenomenon modeling as well measurements for this phenomenon on last 10 years allow well describe a model for formation. And several papers [3...14] have well agreements for observations data as well for research interpretations of these data volumes [15...24].

These data volumes consist from features measurements sound waves for several ranges as between $0,1 \mathrm{~Hz}$ and $10-17 \mathrm{~Hz}$, as well between 100 kHz and 300 MHz . And additional measurements are data from Doppler radars as well some confirmation information received from satellites.

In this way prognosis model use features of atmospheric pressure gradient changing related with dense (or mass) and square (or volume) indicators for funnel of vortex. But calculations make for various heights data.

Because computer equipment resources are limited, turbulent flow interaction with debris physics is not taking into account at all or could be take into account with limitations.

Possibilities to calculate of turbulent flow in near-surface tornado structure are limited too. As a result these calculations are different from real processes actually.

Due to other limitations for computer equipment resources calculations make basically for so called «quasi-static condition» only. Generally it is beginning tornado structure formation when «tornado funnel» is not connected to earth's surface. But in these limitations even computer modeling working for condition one funnel [6] or the same velocity parameters for several funnels (as rule, not more two) [10]. But real observation practice of tornado presents a lot of cases for several funnels quite frequently.

Any way, the current situation is allow make forecasts for tornado in terms of quantity and quality only. Such additional factors use for this forecast as impulse generator of electromagnetic waves in tornado structure (a warning time between 30 minutes and 1,5 hours); electromagnetic perturbations in ionosphere, connected to formation and moving vortex, as well sound wave registrations in ranges of between 0,1 Hz and $10-17 \mathrm{~Hz}, 16 \mathrm{~Hz}$ and $16 \mathrm{kHz}, 100 \mathrm{kHz}$ and 300 MHz (a warning time between 25 minutes and 3 hours).

Attempts to impact on vortex using some chemical agents were not successful until the time. In the meanwhile, in accordance to research and observations, tornado threats are actual and frequent phenomenon for USA, South Canada, north-west and south-east area of Europe, Italy, west and south-east area of Australia, New Zealand. Annual losses for budgets these countries and areas are about tens billions US dollars as
minimum. And injured persons quality are about tens thousands humans there.

Additionally, in accordance to prognosis until 2050 in connection with global climate changing problems, as frequency of tornado occasions and as theirs damages volume will be increase. But in last time these problems actuality would be not excluded for central Europe area too in connection with global climate changing.

## 3. Last observations data for magnetic field structure

In this part of the article some of last observations considered, but these data are not connected to tornado observations directly. Nevertheless, by the author opinion, it could be key not only to uncover of puzzle the phenomenon tornado, but and well key to change value of forecasts basically, as well make reliable managing of atmospheric fronts in near our future.

In last astrophysical observations noted that active stars have complicated structure of theirs magnetic fields; actually it is noted about open field lines. Such structure is cause to burst some plasma substance into outer space.

As consequence, magnetic fields interactions of interstellar gas and plasma create abnormal ionization areas in distant space from active stars (usually it is several radius of the star distance). These zones have time instability and theirs «life time» is limited about few seconds.

As a result, interstellar gas particles have accelerated projection. From the Earth it looks like «bubble burst», what's why such effect was called as «magnetic bubble».

Rather like effect was observed inside distant Earth's magnetic field area (the distance is about 10-15 radii of our planet) in so called «zero zones». Such zones formed in Earth «shadow area» where summary magnitude of the magnetic field is very weak. In this area magnetic field have strong stretched form, and in «tail» magnetic field lines are open (i.e. not connected to Earth's surface). These field lines have reconnection process under influence of «sunny wind».

Vector of the reconnection process have rotation (i.e. precession), which determined by current value of summary gravitational field for our planetary system in area of «shadow» of the Earth. This vector has determination in many parameters. Thus, as Sun move in the Galaxy, as this vector change direction on law which close to chaotic motion. Consequently, the function which describes open field lines reconnection process has law close to chaotic motion too.

It is mean that ionized and charged particles moving inside a «zero zones» have space-time path, which could be described by chaotic motion law. It has confirmation through observations in «zero zones» some structures which close to vortex.

## 4. Description of the idea «atmospheric magnetic bubbles»

In accordance to modern theory of our planet magnetic field formation, a liquid magma core located in centre of the Earth.

Above this core a continental platforms move, and theirs interaction is cause for observed tectonic activity. Continental platforms have secondary magnetic field formed by core magnetic field as well due to partial ionization caused temperature influence from magma to bottom of platforms.

When magma go up to our planet surface, their magnetic field is interact with magnetic field of continental platform. As a result, on nearsurface area in the atmosphere this interaction creates some magnetic field formations which close to «magnetic bubbles» in the structure. Such structures have possibly extension for heights until 2000-3000 meters above sea level. Thus, such structures could be called for convenience «atmospheric magnetic bubbles».

Because «atmospheric magnetic bubbles» zones have instability, like «magnetic bubbles» areas, as additional effects from «atmospheric magnetic bubbles» formations should be observed abnormal ionization air particles and abnormal water condensation of air masses. In first step of such influence the atmospheric pressure reduction should be observed. But, in second step, should be observed abnormal air masses concentration inside the area of «atmospheric magnetic bubbles» as well clouds structure formations. And this concentration should have rotation as well turbulent motion due to mechanism considered in section 3 this article.

On the third step, as a result abnormal air masses concentration inside «atmospheric magnetic bubbles» area should be observed creation increased atmospheric pressure zone. As a result this, it should be observed redistribution of formed atmospheric fronts.

In other terms, in accordance to author assumption, the zone of creation and evolution for «atmospheric magnetic bubbles» is possible to connect to zone of creation and evolution of cyclones/anti-cyclones.

But in our assumption the temperature of ejected on Earth's surface magma have not high value relatively interstellar plasma. Thus, on nearsurface area must be created not too much quantity of open field lines
and theirs reconnection process must going more slowly than for space observed examples (see section 3 this article).

Consequently, «life time» for «atmospheric magnetic bubbles» might be limited by range substantially greater than several seconds. Thus, these processes might have influence to form increased atmospheric pressure zones in Earth's atmosphere until heights supposedly 2000-3000 meters.

## 5. Conclusion.

Based on assumptions made above, propose the technology included possibilities to create zones of increased/decreased atmospheric pressure. A model for simulations of creation and evolution tornado structure is developed and partially verified. Thus, author supposes that the technology could be ready to use in short-time outlook perspective (not more 12 months for full cycle of creation, development and field examination).

Additionally author hopes that same technology mechanism will allow to start manage of atmospheric fronts distribution process, as well to start process to clean atmosphere from harmful gas concentrations, as well, partially, some process to manage global climate (in middle-time and long-time outlooks for mentioned above processes, i.e. in time between 12 and 24 months for full cycle of creation, development and field examination).

As author expect, finance expenses could be minimized if take into account a possibilities of usable simulation models as well weather observation stations network in frame of exist projects.

Coverage of associated investments could be calculating as percentage from tornado damages prevention service.

Because annual losses from tornado are calculated as several billions US dollars, thus annual profit for investors after develop the technology could be about hundreds millions US dollars as minimum. But if take into account this long-time outlook (or more then 24 months after the technology using), additional profit increasing could be between $30 \%$ and $50 \%$.

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Solomon I. Khmelnik

## Principle extremum of full action

## Annotation

A new variational principle extremum of full action is proposed, which extends the Lagrange formalism on dissipative systems. It is shown that this principle is applicable in electrical engineering, mechanics, taking into account the friction forces. Its applicability to electrodynamics and hydrodynamics is also indicated. The proposed variational principle may be considered as a new formalism used as an universal method of physical equations derivation, and also as a method for solving these equations. The formalism consists in building a functional with a sole saddle line; the equation that describes it presents the equation with dynamic variables for a certain domain of physics. The solution method consists in a search for global saddle line for given conditions of a physical problem.

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## Introduction

Here we formulate principle extremum of full action, allowing to construct a functional for various physical systems and, which is most important, for dissipative systems.

The first step in building such functional is to write for a certain physical system an equation of energy conservation or an equation of powers balance. There we take into account as the energy losses (for example, for friction or heating), as also the energy flow into the system and from it.

Here we shall describe this principle applied to electric engineering and mechanics.

## 1. The Principle Formulation

The Lagrange formalism is widely known - it is an universal method of deriving physical equations from the principle of least action. The action here is determined as a definite integral - functional

$$
\begin{equation*}
S(q)=\int_{t_{1}}^{t_{2}}(K(q)-P(q)) d t \tag{1}
\end{equation*}
$$

from the difference of kinetic energy $K(q)$ and potential energy $P(q)$, which is called Lagrangian

$$
\begin{equation*}
\Lambda(q)=K(q)-P(q) \tag{2}
\end{equation*}
$$

Here the integral is taken on a definite time interval $t_{1} \leq t \leq t_{2}$, and $q$ is a vector of generalized coordinates, dynamic variables, which, in their turn, are depending on time. The principle of least action states that the extremals of this functional (i.e. the equations for which it assumes the minimal value), on which it reaches its minimum, are equations of real dynamic variables (i.e. existing in reality).

For example, if the energy of system depends only on functions $q$ and their derivatives with respect to time $q^{\prime}$, then the extremal is determined by the Euler formula [1]

$$
\begin{equation*}
\frac{\partial(K-P)}{\partial q}-\frac{d}{d t}\left(\frac{\partial(K-P)}{\partial q^{\prime}}\right)=0 \tag{3}
\end{equation*}
$$

As a result we get the Lagrange equations.
The Lagrange formalism is applicable to those systems where the full energy (the sum of kinetic and potential energies) is kept constant. The principle does not reflect the fact that in real systems the full energy
(the sum of kinetic and potential energies) decreases during motion, turning into other types of energy, for example, into thermal energy $Q$, i. e. there occurs energy dissipation. The fact, that for dissipative systems (i.e., for system with energy dissipation) there is no formalism similar to Lagrange formalism, seems to be strange: so the physical world is found to be divided to a harmonious (with the principle of least action) part, and a chaotic ("unprincipled") part.

The author puts forward the principle extremum of full action, applicable to dissipative systems. We propose calling full action a definite integral - the functional

$$
\begin{equation*}
\Phi(q)=\int_{t_{1}}^{t_{2}} \mathfrak{R}(q) d t \tag{4}
\end{equation*}
$$

from the value

$$
\begin{equation*}
\mathfrak{R}(q)=(K(q)-P(q)-Q(q)) \tag{5}
\end{equation*}
$$

which we shall call energian (by analogy with Lagrangian). In it $Q(q)$ is the thermal energy. Further we shall consider a full action quasiextremal, having the form:

$$
\begin{equation*}
\frac{\partial(K-P)}{\partial q}-\frac{d}{d t}\left(\frac{\partial(K-P)}{\partial q^{\prime}}\right)-\frac{\partial Q}{\partial q}=0 \tag{6}
\end{equation*}
$$

Functional (4) reaches its extremal value (defined further) on quasiextremals. The principle extremum of full action states that the quasiextremals of this functional are equations of real dynamic processes.

Right away we must note that the extremals of functional (4) coincide with extremals of functional (1) - the component corresponding to $Q(q)$, disappears

Let us determine the extremal value of functional (5). For this purpose we shall "split" (i.e. replace) the function $q(t)$ into two independent functions $x(t)$ and $y(t)$, and the functional (4) will be associated with functional

$$
\begin{equation*}
\Phi_{2}(x, y)=\int_{t_{1}}^{t_{2}} \mathfrak{R}_{2}(x, y) d t \tag{7}
\end{equation*}
$$

which we shall call "split" full action. The function $\mathfrak{R}_{2}(x, y)$ will be called "split" energian. This functional is minimized along function $x(t)$ with a fixed function $y(t)$ and is maximized along function $y(t)$ with a fixed function $x(t)$. The minimum and the maximum are sole ones. Thus, the extremum of functional (7) is a saddle line, where one group of
functions $x_{O}$ minimizes the functional, and another - $y_{O}$, maximizes it. The sum of the pair of optimal values of the split functions gives us the sought function $q=x_{O}+y_{O}$, satisfying the quasiextremal equation (6). In other words, the quasiextremal of the functional (4) is a sum of extremals $x_{O}, y_{O}$ of functional (7), determining the saddle point of this functional. It is important to note that this point is the sole extremal point - there is no other saddle points and no other minimum or maximum points. Therein lies the essence of the expression "extremal value on quasiextremals". Our statement 1 is as follows:

In every area of physics we may find correspondence between full action and split full action, and by this we may prove that full action takes global extremal value on quasiextremals.
Let us consider the relevance of statement 1 for several fields of physics.

## 2. Energian in Electrical Engineering

Full action in electrical engineering takes the form (1.4, 1.5), where

$$
\begin{equation*}
K(q)=\frac{L q^{\prime 2}}{2}, \quad P(q)=\left(\frac{S q^{2}}{2}-E q\right), Q(q)=R q^{\prime} q \tag{1}
\end{equation*}
$$

Here stroke means derivative , $q$ - vector of functions-charges with respect to time, $E$ - vector of functions-voltages with respect to time, $L$ - matrix of inductivities and mutual inductivities, $R$ - matrix of resistances, $S$ - matrix of inverse capacities, and functions $K(q), P(q), Q(q)$ present magnetic, electric and thermal energies correspondingly. Here and further vectors and matrices are considered in the sense of vector algebra, and the operation with them are written in short form. Thus, a product of vectors is a product of column-vector by row-vector, and a quadratic form, as, for example, $R q^{\prime} q$ is a product of row-vector $q^{\prime}$ by quadratic matrix $R$ and by column-vector $q$.

The equation of quasiextremal (1.6) in this case takes the form:

$$
\begin{equation*}
S q+L q^{\prime \prime}+R q^{\prime}-E=0 \tag{2}
\end{equation*}
$$

Substituting (1) to (1.5), we shall write the Energian (1.5) in expanded form:

$$
\begin{equation*}
\mathfrak{R}(q)=\left(\frac{L q^{\prime 2}}{2}-\frac{S q^{2}}{2}+E q-R q^{\prime} q\right) \tag{3}
\end{equation*}
$$

Let us present the split energian in the form

$$
\mathfrak{R}_{2}(x, y)=\left[\begin{array}{l}
\left(L y^{\prime 2}-S y^{2}+E y-R x^{\prime} y\right)-  \tag{4}\\
\left(L x^{\prime 2}-S x^{2}+E x-R x y^{\prime}\right)
\end{array}\right]
$$

Here the extremals of integral (1.7) by functions $x(t)$ and $y(t)$, found by Euler equation, will assume accordingly the form:

$$
\begin{align*}
& 2 S x+2 L x^{\prime \prime}+2 R y^{\prime}-E=0  \tag{5}\\
& 2 S y+2 L y^{\prime \prime}+2 R x^{\prime}-E=0 \tag{6}
\end{align*}
$$

By symmetry of equations $(5,6)$ it follows that optimal functions $x_{0}$ and $y_{0}$, satisfying these equations, satisfy also the condition

$$
\begin{equation*}
x_{0}=y_{0} \tag{7}
\end{equation*}
$$

Adding the equations (5) and (6), we get equation (2), where

$$
\begin{equation*}
q=x_{o}+y_{o} \tag{8}
\end{equation*}
$$

Consequently, conditions $(5,6)$ are necessary for the existence of a sole saddle line. In [2, 3] showed that sufficient condition for this is that the matrix $L$ has a fixed sign, which is true for any electric circuit.

Thus, the statement 1 for electrical engineering is proved. From it follows also statement 2:

Any physical process described by an equation of the form (2), satisfies the principle extremum of full action.

Note that equation (2) is an equation of the circuit without knots. However, in [2,3] has shown that to a similar form can be transformed into an equation of any electrical circuit (with any accuracy).

## 3. Energian in mechanics

Here we shall discuss only one example - line motion of a body with mass $m$ under the influence of a force $f$ and drag force $k q^{\prime}$, where $\boldsymbol{k}$ - known coefficient, $q$ - body's coordinate. It is well known that

$$
\begin{equation*}
f=m q^{\prime \prime}+k q^{\prime} \tag{1}
\end{equation*}
$$

In this case the kinetic, potential and thermal energies are accordingly:

$$
\begin{equation*}
K(q)=m q^{\prime 2} / 2, \quad P(q)=-f q, Q(q)=k q q^{\prime} \tag{2}
\end{equation*}
$$

Let us write the energian (1.5) for this case:

$$
\begin{equation*}
\mathfrak{R}(q)=m q^{\prime 2} / 2+f q-k q q^{\prime} \tag{3}
\end{equation*}
$$

The equation for energian in this case is (1)/
Let us present the split energian as:

$$
\mathfrak{R}_{2}(x, y)=\left[\left(\begin{array}{l}
\left.m y^{\prime 2}+f y-k x^{\prime} y\right)-  \tag{4}\\
\left.m x^{\prime 2}+f x-k x y^{\prime}\right)
\end{array}\right]\right.
$$

It is easy to notice an analogy between energians for electrical engineering and for this case, whence it follows that Statement 1 for this case is proved. However, it also follows directly from Statement 2.

## 4. Mathematical Excursus

Let us introduce the following notations:

$$
\begin{equation*}
y^{\prime}=d y / d t, \quad \hat{y}=\int_{0}^{t} y d t \tag{1}
\end{equation*}
$$

There is a known Euler's formula for the variation of a functional of function $f\left(y, y^{\prime}, y^{\prime \prime}, \ldots\right)$ [1]. By analogy we shall now write a similar formula for function $f\left(\ldots, \hat{y}, y, y^{\prime}, y^{\prime \prime}, \ldots\right)$ :

$$
\begin{align*}
& f\left(\ldots, \hat{y}, y, y^{\prime}, y^{\prime \prime}, \ldots\right):  \tag{2}\\
& \operatorname{var}=\ldots-\int_{0}^{t} f_{\hat{y}}^{\prime} d t+f_{y}^{\prime}-\frac{d}{d t} f_{y^{\prime}}^{\prime}+\frac{d^{2}}{d t^{2}} f_{y^{\prime \prime}}^{\prime \prime}-\ldots \tag{3}
\end{align*}
$$

In particular, if $f()=x y^{\prime}$, then $\operatorname{var}=-x^{\prime}$; if $f()=x \hat{y}$, then $\operatorname{var}=-\hat{x}$. The equality to zero of the variation (1) is a necessary condition of the extremum of functional from function (2).

## 5. Action for Powers

Further we shall refer to the power of energy (kinetic, potential, thermal) as to the variation of this energy in a time unit. We shall consider these powers as the functions of integral generalized coordinates $\hat{i}=q$ - integrals $i$ from generalized coordinates $q$. We shall denote these powers as $\hat{K}(i), \hat{P}(i), Q(i)$. It is important to note the
following. The energy functions contain as an argument the generalized coordinates $q$ and their derivatives $q^{\prime}, q^{\prime \prime}$. The energy functions contain as their arguments the integral generalized coordinates $i$, their derivatives $i^{\prime}$ and their integrals $\hat{i}$.

Let us consider action-2 for powers and define it as a definite integral - functional

$$
\begin{equation*}
\hat{S}(i)=\int_{t_{1}}^{t_{2}}(\hat{K}(i)+\hat{P}(i)) d t \tag{1}
\end{equation*}
$$

from the sum of kinetic and potential powers

$$
\begin{equation*}
\hat{\Lambda}(i)=\hat{K}(i)+\hat{P}(i) \tag{2}
\end{equation*}
$$

and we shall call this sum Lagrangian-2.
The principle of minimal action may be extended also on action-2, i.e. assert that the extremals of functional (1) are equations of real physical processes over the same integral generalized coordinates as quasiexstremals. But the extremals should be calculated by the formulas (4.3).

Example 1. Let us consider the example from Section 3, for which the equation (3.1) is applicable, or, if the thermal losses are absent,

$$
\begin{equation*}
f=m \cdot i^{\prime} \tag{3}
\end{equation*}
$$

In this case the kinetic and potential powers are accordingly:

$$
\begin{equation*}
\hat{K}(i)=m \cdot i \cdot i^{\prime}, \quad \hat{P}(i)=-f \cdot i \tag{4}
\end{equation*}
$$

Let us write the Lagrangian-2 (2) for this case:

$$
\begin{equation*}
\mathfrak{R}(i)=m \cdot i \cdot i^{\prime}-f \cdot i \tag{5}
\end{equation*}
$$

The equation of extremal for functional (1) in this case coincides with equation (3).

Example 2. Let us consider the example from Section 2, for which the equation (2.2) is applicable, or, if the thermal losses are absent,

$$
\begin{equation*}
S \hat{i}+L i^{\prime}-E=0 \tag{6}
\end{equation*}
$$

In this case the kinetic and potential powers are accordingly:

$$
\begin{equation*}
\hat{K}(i)=L \cdot i \cdot i^{\prime}, \quad \hat{P}(i)=S \cdot \hat{i} \cdot i-E \cdot i \tag{7}
\end{equation*}
$$

Let us write the Lagrangian-2 (2) for this case:

$$
\begin{equation*}
\hat{\mathfrak{R}}(i)=L \cdot i \cdot i^{\prime}+S \cdot \hat{i} \cdot i-E \cdot i \tag{8}
\end{equation*}
$$

The equation of extremal for functional (1) in this case may be obtained by formula (4.3) and coincides with equation (6).

## 6. Full Action for Powers

In this case full action-2 is a definite integral - functional

$$
\begin{equation*}
\hat{\Phi}(i)=\int_{t_{1}}^{t_{2}} \hat{\mathfrak{R}}(i) d t \tag{1}
\end{equation*}
$$

from the value

$$
\begin{equation*}
\mathfrak{R}(i)=(\hat{K}(i)+\hat{P}(i)+\hat{Q}(i)) \tag{2}
\end{equation*}
$$

which we shall call Energian-2. In this case we shall define full action quasiextremal-2 as

$$
\begin{equation*}
\frac{\partial\left(\frac{\hat{Q}}{2}+\hat{P}+\hat{K}\right)}{\partial i}=0 \tag{3}
\end{equation*}
$$

Functional (1) assumes extremal value on these quasiextremals. The principle extremal of full action-2 asserts that quasiextremals of this functional are equations of real dynamic processes over integral generalized coordinates $i$.

Let us now determine the extremal value of functional (1, 2). For this purpose we, as before, will "split" the function $i(t)$ to two independent functions $x(t)$ and $y(t)$, and put in accordance to functional (1) the functional

$$
\begin{equation*}
\hat{\Phi}_{2}(x, y)=\int_{t_{1}}^{t_{2}} \hat{R}_{2}(x, y) d t \tag{4}
\end{equation*}
$$

which we shall call "split full action-2. We shall call the function $\mathfrak{R}_{2}(x, y)$ "split " Energian--2. This functional is being minimized by the function $x(t)$ with fixed function $y(t)$ and maximized by function $y(t)$ with fixed function $x(t)$. As before, the quasiextremal (3) of functional (1) is a sum $i=x_{O}+y_{O}$ of extremals $x_{O}, y_{O}$ of the functional (4), determining the saddle point of this functional.

## 7. Energian-2 in mechanics

As in Section 3 we shall consider an example, for which the equation (3.1) is applicable, or

$$
\begin{equation*}
f=m \cdot i^{\prime}+k \cdot i \tag{1}
\end{equation*}
$$

In this case the kinetic, potential and thermal powers are accordingly:

$$
\begin{equation*}
\hat{K}(i)=m \cdot i \cdot i^{\prime}, \quad \hat{P}(i)=-f \cdot i, \quad \hat{Q}(q)=k \cdot i^{2} \tag{2}
\end{equation*}
$$

Let us write the energian-2 (6.2) for this case:

$$
\begin{equation*}
\mathfrak{\Re}(i)=m \cdot i \cdot i^{\prime}-f \cdot i+k \cdot i^{2} \tag{3}
\end{equation*}
$$

Уравнение квазиэкстремали в этом случае принимает вид (1).

## 8. Energian-2 in Electrical Engineering

Let us consider an electrical circuit which equation has the form, (2.2) or

$$
\begin{equation*}
S \cdot \hat{i}+L \cdot i^{\prime}+R \cdot i-E=0 \tag{1}
\end{equation*}
$$

In this case the kinetic, potential and thermal powers are accordingly:

$$
\begin{equation*}
\hat{K}(i)=L \cdot i \cdot i^{\prime}, \quad \hat{P}(i)=S \cdot \hat{i} \cdot i-E \cdot i, \quad Q(i)=R \cdot i^{2} \tag{2}
\end{equation*}
$$

Let us write the energian-2 (6.2) for this case:

$$
\begin{equation*}
\mathfrak{\mathfrak { R }}(i)=L \cdot i \cdot i^{\prime}+S \cdot \hat{i} \cdot i-E \cdot i+R \cdot i^{2} \tag{3}
\end{equation*}
$$

The equation of quasiextremal in this case assumes the form (1).
Let us now present the "split" Energian-2 as

$$
\hat{\mathfrak{R}}_{2}(x, y)=\left[\begin{array}{l}
S(x \hat{y}-\hat{x} y)+L\left(x y^{\prime}-x^{\prime} y\right)+  \tag{4}\\
+R\left(x^{2}-y^{2}\right)-E(x-y)
\end{array}\right]
$$

The extremals of integral (6.4) by the functions $x(t)$ and $y(t)$, found according to equation (4.3), will assume accordingly the form:

$$
\begin{align*}
& 2 S \hat{y}+2 L y^{\prime}+2 R x-E=0  \tag{5}\\
& 2 S \hat{x}+2 L x^{\prime}+2 R y-E=0 \tag{6}
\end{align*}
$$

From the symmetry of equations $(5,6)$ it follows that optimal functions $x_{0}$ and $y_{0}$, satisfying these equations, satisfy also the condition

$$
\begin{equation*}
x_{0}=y_{0} \tag{7}
\end{equation*}
$$

Adding together the equations (5) and (6), we get the equation (1), where

$$
\begin{equation*}
q=x_{o}+y_{o} \tag{8}
\end{equation*}
$$

Therefore, the equation (1) is the necessary condition of the existence of saddle line. In [2,3] it is shown that the sufficient condition for the existence of a sole saddle line is matrix $L$ having fixed sign, which is true for every electrical circuit.

## Conclusion

The functionals (1.7) and (6.4) have global saddle line and therefore the gradient descent to saddle point method may be used for calculating physical systems with such functional. As the global extremum exists, then the solution also always exists. Such method applied to electrical engineering and electro mechanics is described in $[2,3]$.

The author has applied the full time extremum principle for powers, and also the calculation method applicable for electrodynamics $[2,3]$ and hydrodynamics $[4,5]$.

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## Solomon I. Khmelnik

# The existence and the search method for global solutions of Navier-Stokes equation 

## Annotation

We formulate and prove the variational extremum principle for viscous incompressible and compressible fluid, from which principle follows that the Naviet-Stokes equations represent the extremum conditions of a certain functional. We describe the method of seeking solution for these equations, which consists in moving along the gradient to this functional extremum. We formulate the conditions of reaching this extremum, which are at the same time necessary and sufficient conditions of this functional global extremum existence.

Then we consider the so-called closed systems. We prove that for them the necessary and sufficient conditions of global extremum for the named functional always exist. Accordingly, the search for global extremum is always successful, and so the unique solution of NavietStokes is found.

We contend that the systems described by Naviet-Stokes equations with determined boundary solutions (pressure or speed) on all the boundaries, are closed systems. We show that such type of systems include systems bounded by impenetworkrable walls, by free space under a known pressure, by movable walls under known pressure, by the socalled generating surfaces, through which the fluid flow passes with a known speed.

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## Introduction

In his previous works [6-8] the author presented the full action extremum principle, allowing to construct the functional for various physical systems, and, which is most important, for dissipative systems. In $[4,5,9]$ this principle is described applied to the hydrodynamics of incompressible fluid. There one may find multiple examples of this principle use for the solution of specific problems. In this paper the author is using a more strict extension of this principle for the powers [6] and considers also the hydrodynamics of incompressible fluid.

Here we are discussing the Navier-Stokes equations for viscous incompressible fluid. We show that these equations are the conditions of a certain functional's extremum. A solution method for these equation is described - it consists of moving by the gradient in the direction of this functional's extremum. The conditions of reaching this extremum are formulated - they are simultaneously necessary and sufficient conditions of the existence of this functional's global extremum.

Then we separate the so-called closed systems. For them it is proved that the necessary and sufficient conditions of the existence of this functional's global extremum are always valid Thereafter, the method of searching for global extremum always gives a positive result, and hence the sole solution of the Navier-Stokes equations is found.

It is stated that the systems described by Navier-Stokes equations, having definite boundary conditions (of pressure or speed) on all the
boundaries, are closed systems. It is shown that such systems include the systems bounded by

- impenetrable walls,
- free surfaces that are under known pressure,
- movable walls that are under known pressure,
- so-called generating surfaces through which the flows passes with known speed.
In this way we have shown that the Navier-Stokes equations have only one solution.


## 1. Viscous incompressible fluid

### 1.1. Hydrodynamic equations for viscous incompressible fluid

The hydrodynamic equations for viscous incompressible liquid are as follows [2]:

$$
\begin{align*}
& \operatorname{div}(v)=0  \tag{1}\\
& \rho \frac{\partial v}{\partial t}+\nabla p-\mu \Delta v+\rho(v \cdot \nabla) v-\rho F=0 \tag{2}
\end{align*}
$$

where
$\rho=$ const is constant density,
$\mu$ - coefficient of internal friction,
$p$ - unknown pressure,
$v=\left\lfloor v_{x}, v_{y}, v_{z}\right\rfloor$ - unknown speed, vector,
$F=\left\lfloor F_{x}, F_{y}, F_{z}\right\rfloor$ - known mass force, vector,
$x, y, z, t$ - space coordinates and time.

### 1.2. The power balance

Umov [1] discussed for the liquids the condition of balance for specific (by volume) powers in a liquid flow. For a non-viscous and incompressible liquid this condition is of the form (see (56) in [1])

$$
\begin{equation*}
P_{1}(v)+P_{5}(v)+P_{4}(p, v)=0 \tag{3}
\end{equation*}
$$

and for viscous and incompressible liquid - another form (see (80) in [1])

$$
\begin{equation*}
P_{1}(v)+P_{5}(v)+P_{2}(p, v)=0 \tag{4}
\end{equation*}
$$

where

$$
\begin{equation*}
P_{1}=\frac{\rho}{2} \frac{\partial W^{2}}{\partial t} \tag{5}
\end{equation*}
$$

$$
\begin{align*}
& P_{2}=\left\{\begin{array}{l}
v_{x}\left(\frac{d p_{x x}}{d x}+\frac{d p_{x y}}{d y}+\frac{d p_{x z}}{d z}\right)+ \\
v_{y}\left(\frac{d p_{x y}}{d x}+\frac{d p_{y y}}{d y}+\frac{d p_{y z}}{d z}\right)+ \\
v_{z}\left(\frac{d p_{x z}}{d x}+\frac{d p_{y z}}{d y}+\frac{d p_{z z}}{d z}\right)
\end{array}\right\}  \tag{6}\\
& P_{4}=v \cdot \nabla p,  \tag{7}\\
& P_{5}=\frac{1}{2} \rho\left(v_{x} \frac{d W^{2}}{d x}+v_{y} \frac{d W^{2}}{d y}+v_{z} \frac{d W^{2}}{d z}\right)  \tag{8}\\
& W^{2}=\left(v_{x}^{2}+v_{y}^{2}+v_{z}^{2}\right) \tag{9}
\end{align*}
$$

$p_{x y}$ and so on - tensions (see [2]).
Here $P_{1}$ is the power of energy variation, $P_{4}$ is the power of work of pressure variation, $P_{5}$ - the power of variation of energy variation for direction change, and the value

$$
\begin{equation*}
P_{7}(p, v)=P_{5}(v)+P_{4}(p, v) \tag{10}
\end{equation*}
$$

is, as it was shown by Umov, the variation of energy flow power through a given liquid volume - see (56) и (58) в [1]. In [2] it was shown, that for incompressible liquid the following equality is valid

$$
\left(\begin{array}{l}
\left(\frac{d p_{x x}}{d x}+\frac{d p_{x y}}{d y}+\frac{d p_{x z}}{d z}\right)  \tag{11}\\
\left(\frac{d p_{x y}}{d x}+\frac{d p_{y y}}{d y}+\frac{d p_{y z}}{d z}\right) \\
\left(\frac{d p_{x z}}{d x}+\frac{d p_{y z}}{d y}+\frac{d p_{z z}}{d z}\right)
\end{array}\right)=\nabla p-\mu \cdot \Delta v
$$

From this it follows that

$$
\begin{equation*}
P_{2}=v(\nabla p-\mu \cdot \Delta v) \tag{12}
\end{equation*}
$$

or, subject to (6),

$$
\begin{equation*}
P_{2}=P_{4}-P_{3} \tag{13}
\end{equation*}
$$

where

$$
\begin{equation*}
P_{3}=\mu \cdot v \cdot \Delta v \tag{14}
\end{equation*}
$$

- power of change of energy loss for internal friction during the motion. Therefore, we rewrite (4) in the form

$$
\begin{equation*}
P_{1}(v)+P_{5}(v)+P_{4}(p, v)-P_{3}(v)=0 \tag{15}
\end{equation*}
$$

We shall supplement the condition (15) by mass forces power

$$
\begin{equation*}
P_{6}=\rho F v \tag{16}
\end{equation*}
$$

Then for every viscous incompressible liquid this balance condition is of the form

$$
\begin{equation*}
P_{1}(v)+P_{5}(v)+P_{4}(p, v)-P_{3}(v)-P_{6}(v)=0 \tag{17}
\end{equation*}
$$

Taking into condition(1) and formula (p1a) let us rewrite (7) in the form

$$
\begin{equation*}
P_{4}=\operatorname{div}(v \cdot p) \tag{18}
\end{equation*}
$$

Taking into account ( p 9 a ), condition(1) and formula (p1a) let us rewrite (8) in the form

$$
\begin{equation*}
P_{5}=\operatorname{div}\left(v \cdot W^{2}\right) \tag{19}
\end{equation*}
$$

From $(18,19)$ and Ostrogradsky formula (p28) we find:

$$
\begin{align*}
& \iiint_{V} P_{4} d V=\iiint_{V}^{\circ} \operatorname{div}(v \cdot p) d V=\iint_{S} p \cdot v_{n} \cdot d S  \tag{20}\\
& \iiint_{V} P_{5} d V=\iiint_{V} \operatorname{div}\left(v \cdot W^{2}\right) d V=\iint_{S} W^{2} \cdot v_{n} \cdot d S \tag{20a}
\end{align*}
$$

or, subject to (p15),

$$
\begin{equation*}
\iiint_{V} P_{5} d V=\iiint_{V}(v \cdot G(v)) d V=\iint_{S} W^{2} \cdot v_{n} \cdot d S \tag{21}
\end{equation*}
$$

Returning again to the definitions of powers $(7,8)$, we will get

$$
\begin{align*}
& \iiint_{V}(v \cdot \nabla p) d V=\iint_{S} p_{S} \cdot v_{n} \cdot d S  \tag{21a}\\
& \iiint_{V}\left(v \cdot \nabla\left(W^{2}\right)\right) d V=\iint_{S} W^{2} \cdot v_{n} \cdot d S \tag{21в}
\end{align*}
$$

or

$$
\begin{equation*}
\iiint_{V}(v \cdot G(v)) d V=\iint_{S} W^{2} \cdot v_{n} \cdot d S \tag{21c}
\end{equation*}
$$

### 1.3. Energian-2 and quasiextremal

For further discussion we shall assemble the unknown functions into a vector

$$
\begin{equation*}
q=[p, v]=\left\lfloor p, v_{x}, v_{y}, v_{z}\right\rfloor \tag{22}
\end{equation*}
$$

This vector and all its components are functions of $(x, y, z, t)$. We are considering a liquid flow in volume $V$. The full action-2 [6] in hydrodynamics takes a form

$$
\begin{equation*}
\Phi=\int_{0}^{T}\left\{\int_{V} \mathfrak{R}(q(x, y, z, t) d V\} d t\right. \tag{23}
\end{equation*}
$$

Having in mind (17) the definition of energian-2 in [6], let us write the energian-2 in the following form:

$$
\begin{equation*}
\mathfrak{R}(q)=P_{1}(v)-\frac{1}{2} P_{3}(v)+P_{4}(q)+P_{5}(v)-P_{6}(v) \tag{24}
\end{equation*}
$$

Below in Supplement 1 will be shown - see (p4, p13, p15):

$$
\begin{align*}
& P_{1}=\rho \cdot v \frac{d v}{d t}  \tag{25}\\
& P_{5}=\rho \cdot v \cdot G(v) \tag{26}
\end{align*}
$$

where

$$
\begin{equation*}
G(v)=(v \cdot \nabla) v \tag{27}
\end{equation*}
$$

Taking this into account let us rewrite the energian-2 (24) in a detailed form

$$
\begin{equation*}
\mathfrak{R}(q)=\rho \cdot v \frac{d v}{d t}-\frac{1}{2} \mu \cdot v \cdot \Delta v+\operatorname{div}(v \cdot p)+\rho \cdot v \cdot G(v)-\rho F v \tag{28}
\end{equation*}
$$

Further we shall denote the derivative computed according to Ostrogradsky formula (p23), by the symbol $\frac{\partial_{O}}{\partial v}$, as distinct from ordinary derivative $\frac{\partial}{\partial v}$. Taking this into account (p16), we get

$$
\begin{align*}
& \frac{\partial}{\partial v}\left(P_{1}\left(v, \frac{d v}{d t}\right)\right)=\rho \frac{d v}{d t} ; \frac{\partial_{o}}{\partial v}\left(P_{3}(v)\right)=\mu \cdot \Delta v \\
& \frac{\partial}{\partial q}\left(P_{4}(q)\right)=\left\lvert\, \begin{array}{l}
\operatorname{div}(v) \\
\nabla(p)
\end{array}\right. ; \quad \frac{\partial}{\partial v}\left(P_{5}(v, G(v))\right)=\rho(v \cdot \nabla) v ;  \tag{29}\\
& \frac{\partial_{o}}{\partial v}\left(P_{6}(v)\right)=\rho F
\end{align*}
$$

In accordance with [6] we write the quasiextremal in the following form:

$$
\left[\begin{array}{l}
\frac{\partial}{\partial v}\left(P_{1}\left(v, \frac{d v}{d t}\right)\right)+\frac{1}{2} \frac{\partial_{o}}{\partial v}\left(P_{3}(v)\right)+\frac{\partial}{\partial q}\left(P_{4}(q)\right)  \tag{30}\\
+\frac{\partial}{\partial v}\left(P_{5}(v, G(v))\right)-\frac{\partial_{o}}{\partial v}\left(P_{6}(v)\right)
\end{array}\right]=0
$$

From (29) it follows that the quasiextremal (30) after differentiation coincides with equations (1, 2).

### 1.4. The split energian-2

Let us consider the split functions (22) in the form

$$
\begin{align*}
& q^{\prime}=\left[p^{\prime}, v^{\prime}\right]=\left[p^{\prime}, v_{x}^{\prime}, v_{y}^{\prime}, v_{z}^{\prime}\right]  \tag{31}\\
& q^{\prime \prime}=\left[p^{\prime \prime}, v^{\prime \prime}\right]=\left[p^{\prime \prime}, v_{x}^{\prime \prime}, v_{y}^{\prime \prime}, v_{z}^{\prime \prime}\right] \tag{32}
\end{align*}
$$

Let us present the split energian-2 taking into account the formula (p15) in the form

$$
\Re_{2}\left(q^{\prime}, q^{\prime \prime}\right)=\left\{\begin{array}{l}
\rho \cdot\left(v^{\prime} \frac{d v^{\prime \prime}}{d t}-v^{\prime \prime} \frac{d v^{\prime}}{d t}\right)-\mu \cdot\left(v^{\prime} \Delta v^{\prime}-v^{\prime \prime} \Delta v^{\prime \prime}\right)  \tag{33}\\
+2\left(\operatorname{div}\left(v^{\prime} \cdot p^{\prime \prime}\right)-\operatorname{div}\left(v^{\prime \prime} \cdot p^{\prime}\right)\right)+ \\
\rho \cdot\left(v^{\prime} G\left(v^{\prime \prime}\right)-v^{\prime \prime} G\left(v^{\prime}\right)\right)-\rho \cdot F\left(v^{\prime}-v^{\prime \prime}\right)
\end{array}\right\} .
$$

Let us associate with the functional (23) functional of split full action-2

$$
\begin{equation*}
\Phi_{2}=\int_{0}^{T}\left\{\int_{V} \mathfrak{R}_{2}\left(q^{\prime}, q^{\prime \prime}\right) d V\right\} d t \tag{34}
\end{equation*}
$$

With the aid of Ostrogradsky formula (p23) we may find the variations of functional (34) with respect to functions $q^{\prime}$. In this we shall take into account the formulas (p21), obtained in the Supplement 1. Then we have:

$$
\begin{align*}
& \frac{\partial_{o} \mathfrak{R}_{2}}{\partial p^{\prime}}=b_{p^{\prime}}  \tag{35}\\
& \frac{\partial_{o} \mathfrak{R}_{2}}{\partial v^{\prime}}=b_{v^{\prime}}  \tag{36}\\
& b_{p^{\prime}}=2 \operatorname{div}\left(v^{\prime \prime}\right) \tag{37}
\end{align*}
$$

$$
b_{v^{\prime}}=\left\{\begin{array}{l}
2 \rho \cdot \frac{d v^{\prime \prime}}{d t}-2 \mu \cdot \Delta v^{\prime}+2 \nabla\left(p^{\prime \prime}\right)  \tag{38}\\
+2 \rho \cdot\left[G\left(v^{\prime \prime}, \frac{\partial v^{\prime \prime}}{\partial X}\right)+G\left(v^{\prime}, \frac{\partial v^{\prime \prime}}{\partial X}\right)\right]-\rho \cdot F
\end{array}\right\} .
$$

So, the vector

$$
\begin{equation*}
b^{\prime}=\left\lfloor b_{p^{\prime}}, b_{v^{\prime}}\right\rfloor \tag{39}
\end{equation*}
$$

is a variation of functional (34), and the condition

$$
\begin{equation*}
b^{\prime}=\left\lfloor b_{p^{\prime}}, b_{v^{\prime}}\right\rfloor=0 \tag{40}
\end{equation*}
$$

is the necessary condition for the existence of the extremal line. Similarly,

$$
\begin{equation*}
b^{\prime \prime}=\left\lfloor b_{p^{\prime \prime}}, \quad b_{v^{\prime \prime}}\right\rfloor=0 \tag{41}
\end{equation*}
$$

The equations $(40,41)$ are necessary condition for the existence of a saddle line. By symmetry of these equations we conclude that the optimal functions $q_{0}^{\prime}$ and $q_{0}^{\prime \prime}$, satisfying these equations, satisfy also the condition

$$
\begin{equation*}
q_{0}^{\prime}=q_{0}^{\prime \prime} \tag{42}
\end{equation*}
$$

Subtracting in couples the equations $(40,41)$ taking into consideration $(37,38)$, we get

$$
\begin{align*}
& 2 \operatorname{div}\left(v^{\prime}+v^{\prime \prime}\right)=0  \tag{43}\\
& \left\{\begin{array}{l}
2 \rho \cdot \frac{d\left(v^{\prime}+v^{\prime \prime}\right)}{d t}-2 \mu \cdot \Delta\left(v^{\prime}+v^{\prime \prime}\right)+2 \nabla\left(p^{\prime}+p^{\prime \prime}\right)-2 \rho \cdot F \\
\left.+2 \rho \cdot\left[G\left(v^{\prime \prime}, \frac{\partial v^{\prime \prime}}{\partial X}\right)+G\left(v^{\prime}, \frac{\partial v^{\prime \prime}}{\partial X}\right)+G\left(v^{\prime}, \frac{\partial v^{\prime}}{\partial X}\right)+G\left(v^{\prime \prime}, \frac{\partial v^{\prime}}{\partial X}\right)\right]\right\}=0
\end{array}\right. \tag{44}
\end{align*}
$$

For $v^{\prime}=v^{\prime \prime}$ according to (p12), we have

$$
\begin{equation*}
\left[G\left(v^{\prime \prime}\right)+G\left(v^{\prime}, \frac{\partial v^{\prime \prime}}{\partial X}\right)+G\left(v^{\prime}\right)+G\left(v^{\prime \prime}, \frac{\partial v^{\prime}}{\partial X}\right)\right]=G\left(v^{\prime}+v^{\prime \prime}\right) \tag{45}
\end{equation*}
$$

Taking into account $(27,45)$ and reducing $(43,44)$ by 2 , получаем we get the equations $(1,2)$, where

$$
\begin{equation*}
q=q_{O}^{\prime}+q_{O}^{\prime \prime} \tag{46}
\end{equation*}
$$

- see (22, 31, 32), i.e. the equations of extremal line are Naviet-Stokes equations.


### 1.5. About sufficient conditions of extremum

Let us rewrite the functional (34) in the form

$$
\begin{equation*}
\Phi_{2}=\int_{0}^{T}\left\{\int_{z}\left\{\int_{y}\left\{\int_{x} \mathfrak{R}_{2}\left(q^{\prime}, q^{\prime \prime}\right) d x\right\} d y\right\} d z\right\} d t \tag{47}
\end{equation*}
$$

где векторы $q^{\prime}, q^{\prime \prime}$ определенны по (31, 32), $X=(x, y, z, t)$ - вектор независимых переменных. Далее будем варьировать только функции $q^{\prime}(X)=\left[p^{\prime}(X), v^{\prime}(X)\right]$.

Vector $b$, defined by (39), is a variation of functional $\Phi_{2}$ by the function $q^{\prime}$ and depends on function $q^{\prime}$, i.e. $b=b\left(q^{\prime}\right)$. Here the function $q^{\prime \prime}$ here is fixed.

Let $S$ be an extremal, and subsequently, the gradient in it is $b_{S}=0$. To find out which type of extremum we have, let us look at the sign of functional's increment

$$
\begin{equation*}
\delta \Phi_{2}=\Phi_{2}(S)-\Phi_{2}(C) \tag{48}
\end{equation*}
$$

where $C$ is the line of comparison, where $b=b_{c} \neq 0$. Let the values vector $q^{\prime}$ on lines $S$ и $C$ differ by

$$
\begin{equation*}
q_{C}^{\prime}-q_{S}^{\prime}=q^{\prime}-q_{S}^{\prime}=\delta q^{\prime}=a \cdot b \tag{49}
\end{equation*}
$$

where $b$ is the variation on the line $C, a-$ a known number. Thus,

$$
q^{\prime}=q_{S}^{\prime}+a \cdot b=\left|\begin{array}{c}
p_{S}^{\prime}  \tag{50}\\
v_{S}^{\prime}
\end{array}\right|+a\left|\begin{array}{c}
b_{p} \\
b_{v}
\end{array}\right|
$$

where $b_{p}, b_{v}$ are determined by $(35,36)$ accordingly, and do not depend on $q^{\prime}$.

If

$$
\begin{equation*}
\delta \Phi_{2}=a \cdot A \tag{51}
\end{equation*}
$$

where $A$ has a constant sign in the vicinity of extremal $b_{S}=0$, then this extremal is sufficient condition of extremum. If, furthermore, $A$ is of constant sign in all definitional domain of the function $q^{\prime}$, then this extremal determines a global extremum.

From (48) we find

$$
\begin{equation*}
\delta \mathfrak{R}_{2}=\mathfrak{R}_{2}(S)-\mathfrak{R}_{2}(C)=\mathfrak{R}_{2}\left(q_{S}^{\prime}\right)-\Re_{2}\left(q^{\prime}\right) \tag{52}
\end{equation*}
$$

or, taking into account $(33,50)$,

$$
\delta \mathfrak{R}_{2}=\left\{\begin{array}{l}
-\rho \cdot\left(\left(v_{S}^{\prime}+a b_{v}\right) \frac{d v^{\prime \prime}}{d t}-v^{\prime \prime} \frac{d\left(v_{S}^{\prime}+a b_{v}\right)}{d t}\right)  \tag{53}\\
-\mu \cdot\left(\left(v_{s}^{\prime}+a b_{v}\right) \Delta\left(v_{S}^{\prime}+a b_{v}\right)-v^{\prime \prime} \Delta\left(v^{\prime \prime}\right)\right) \\
+2\left(\left(v_{S}^{\prime}+a b_{v}\right) \cdot \nabla\left(p^{\prime \prime}\right)-v^{\prime \prime} \cdot \nabla\left(p_{S}^{\prime}+a b_{p}\right)\right) \\
+2 \rho \cdot\left(\left(v_{S}^{\prime}+a b_{v}\right) G\left(v^{\prime \prime}\right)-v^{\prime \prime} G\left(v_{S}^{\prime}+a b_{v}\right)\right) \\
-\rho \cdot F\left(\left(v_{S}^{\prime}+a b_{v}\right)-v^{\prime \prime}\right)
\end{array}\right\}
$$

Taking into account (p20), we get:

$$
\begin{equation*}
G\left(v_{S}^{\prime}+a b_{v}\right)=G\left(v_{S}^{\prime}\right)+a\left[G_{1}\left(v_{S}^{\prime}, b_{v}\right)+G_{2}\left(v_{S}^{\prime}, b_{v}\right)\right]+a^{2} G\left(b_{v}\right) \tag{54}
\end{equation*}
$$

Here (53) is transformed into

$$
\begin{equation*}
\delta \mathfrak{R}_{2}=\mathfrak{R}_{20}+\mathfrak{R}_{21} a+\mathfrak{R}_{22} a^{2} \tag{55}
\end{equation*}
$$

where $\mathfrak{R}_{20}, \mathfrak{R}_{21}, \mathfrak{R}_{22}$ are functions not dependent on $a$, of the form

$$
\begin{gather*}
\mathfrak{R}_{20}=\left\{\begin{array}{l}
\rho \cdot\left(v_{S}^{\prime} \frac{d v^{\prime \prime}}{d t}-v^{\prime \prime} \frac{d\left(v_{S}^{\prime}\right)}{d t}\right) \\
-\mu \cdot\left(v_{S}^{\prime} \Delta\left(v_{s}^{\prime}\right)-v^{\prime \prime} \Delta\left(v^{\prime \prime}\right)\right)+2\left(v_{S}^{\prime} \nabla\left(p^{\prime \prime}\right)-v^{\prime \prime} \cdot \nabla\left(p_{S}^{\prime}\right)\right) \\
+2 \rho \cdot\left(v_{S}^{\prime} G\left(v^{\prime \prime}\right)-v^{\prime \prime} G\left(v_{S}^{\prime}\right)\right)-\rho \cdot F\left(v_{S}^{\prime}-v^{\prime \prime}\right)
\end{array}\right\},  \tag{56}\\
\mathfrak{R}_{21}=\left\{\begin{array}{l}
\rho \cdot\left(b_{v} \frac{d v^{\prime \prime}}{d t}-v^{\prime \prime} \frac{d b_{v}}{d t}\right)-\mu \cdot\left(b_{v} \Delta v_{S}^{\prime}+v_{S}^{\prime} \Delta\left(b_{v}\right)\right) \\
+2\left(b_{v} \cdot \nabla\left(p^{\prime \prime}\right)-v^{\prime \prime} \cdot \nabla\left(b_{p}\right)\right)+ \\
2 \rho\left(b_{v} G\left(v^{\prime \prime}\right)-v^{\prime \prime}\left(G_{1}\left(v_{S}^{\prime}, b_{v}\right)+G_{2}\left(v_{S}^{\prime}, b_{v}\right)\right)\right)-\rho \cdot F \cdot b_{v} \\
\mathfrak{R}_{22}=-\mu b_{v} \Delta\left(b_{v}\right)-2 \rho v^{\prime \prime} G\left(b_{v}\right) .
\end{array}\right\}, \tag{57}
\end{gather*}
$$

Now we must find

$$
\begin{equation*}
\frac{\partial^{2}\left(\delta \mathfrak{R}_{2}\right)}{\partial a^{2}}=\mathfrak{R}_{22} \tag{59}
\end{equation*}
$$

This function depends on $q^{\prime}$. To prove that the necessary condition (40) is also a sufficient condition of global extremum of the functional (47) with respect to function $q^{\prime}$, we must prove that the integral

$$
\begin{equation*}
\frac{\partial^{2} \Phi_{2}}{\partial a^{2}}=\int_{0}^{T}\left\{\int_{V} \partial \Re_{2}\left(q^{\prime}, q^{\prime \prime}\right) d V\right\} d t \tag{60}
\end{equation*}
$$

or, which is the same, the integral

$$
\begin{equation*}
\frac{\partial^{2} \Phi_{2}}{\partial a^{2}}=\int_{0}^{T}\left\{\int_{V} \Re_{22} d V\right\} d t \tag{61}
\end{equation*}
$$

is of constant sign. Similarly, to prove that the necessary condition (41) is also a sufficient condition of a global extremum of the functional (47) with respect to function $q^{\prime \prime}$, we have to prove that the integral similar to (60) is also of the same sign.

Specifying the concepts, we will say that the Navier-Stokes equations have a global solution, if for them there exists a unique nonzero solution in a given domain of the fluid existence.

In the above-cited integrals the energy flow through the domain's boundaries was not taken into account. Hence the above-stated may be formulated as the following lemma

Lemma 1. The Navier-Stokes equations for incompressible fluid have a global solution in an unlimited domain, if the integral $(61,58)$ has constant sign for any speed of the flow.

## 2. Boundary conditions

The boundary conditions determine the power flow through the boundaries, and, generally speaking, they may alter the power balance equation. Let us view some specific cases of boundaries.

### 2.1. Absolutely hard and impenetrable walls

If the speed has a component normal to the wall, then the wall gets energy from the fluid, and fully returns it to the fluid. (changing the speed direction). The tangential component of speed is equal to zero (adhesion effect). Therefore such walls do not change the system's energy. However, the energy reflected from walls creates an internal energy flow, circulating between the walls. So in this case all the abovestated formulas remain unchanged, but the conditions on the walls (impenetrability, adhesion) should not be formulated explicitly - they
appear as a result of solving the problem with integrating in a domain bounded by walls. Then the second lemma is valid:

Lemma 2. The Navier-Stokes equations for incompressible fluid have a global solution in a domain bonded by absolutely hard and impenetrable walls, if the integral $(61,58)$ is of the same sign for any flow speed.

### 2.2. Systems with a determined external pressure

In the presence of external pressure the power balance condition (17) is supplemented by one more component - the power of pressure forces work

$$
\begin{equation*}
P_{8}=p_{s} \cdot v_{n} \tag{62}
\end{equation*}
$$

where
$p_{S}$ - external pressure,
$S$ - surfaces where the pressure determined,
$v_{n}$ - normal component of flow incoming into above surface,
In this case the full action-2 is presented as follows:

$$
\begin{equation*}
\Phi=\int_{0}^{T}\left\{\int _ { V } \Re \left(q(x, y, z, t) d V+\int_{S} P_{8}(q(x, y, z, t) d V\} d t\right.\right. \tag{63}
\end{equation*}
$$

For convenience sake let us consider the functions $Q$, determined on the domain of the flow existence and taking zero value in all the points of this domain, except the points belonging to the surface $S$. Then the restraint (63) may be written in the form

$$
\begin{equation*}
\Phi=\int_{0}^{T}\left\{\int_{V} \hat{R}(q(x, y, z, t) d V\} d t\right. \tag{64}
\end{equation*}
$$

where energian-2

$$
\begin{equation*}
\hat{\mathfrak{R}}(q)=\mathfrak{R}(q)+Q \cdot P_{8}\left(v_{n}\right) . \tag{65}
\end{equation*}
$$

One may note that here the last component is identical to the power of body forces - in the sense that both of them depend only on the speed. So all the previous formulas may be extended on this case also, by performing substitution in them.

$$
\begin{equation*}
F \Rightarrow F+Q \cdot p_{s} / \rho \tag{66}
\end{equation*}
$$

Therefore the following lemma is true:

Lemma 3. The Navier-Stokes equations for incompressible fluid have a global solution in a domain bounded by surfaces with a certain pressures, if the integral $(61,58)$ has constant sign for any flow speed.

Such surface may be a free surface or a surface where the pressure is determined by the problem's conditions (for example, by a given pressure in the pipe section).

Note also that the pressure $p_{S}$ may be included in the full action functional formally, without bringing in physical considerations. Indeed, in the presence of external pressure there appears a new constraint (21a). In [3] it is shown that such problem of a search for a certain functional with integral constraints (certain integrals of fixed values) is equivalent to the search for the extremum of the of the sum of our functional and integral constraint. More precisely, in our case we must seek for the extremum of the following functional:

$$
\begin{align*}
& \Phi=\int_{0}^{T}\left\{\int_{V} \hat{\mathfrak{R}}(q(x, y, z, t)) d V\right\} d t  \tag{67}\\
& \hat{R}\left(q(x, y, z, t)=\left\{\begin{array}{l}
\mathfrak{R}(q(x, y, z, t))+ \\
\lambda \cdot\left(-v \cdot \nabla p+Q \cdot p_{s} \cdot v_{n}\right)
\end{array}\right\},\right. \tag{68}
\end{align*}
$$

where $\lambda$ - an unknown scalar multiplier. It is determined or known initial conditions [3]. For $\lambda=1$ after collecting similar terms the energian-2 (68) again assumes the form (65), which was to be proved.

### 2.3. Systems with generating surfaces

There is a conception often used in hydrodynamics of a certain surface through which a flow enters into a given fluid volume with a certain constant speed, i.e., NOT dependent on the processes going on in this volume. The energy entering into this volume with this flow, evidently will be proportional to squared speed module and is constant. We shall call such surface a generating surface (note that this is to some extent similar to a source of stabilized direct current whose magnitude does not depend on the electric circuit resistance).

If there is a generating surface, the power balance condition (17) is supplemented by another component - the power of flow with constant squared speed module.

$$
\begin{equation*}
P_{9}=W_{s}^{2} \cdot v_{n} \tag{69}
\end{equation*}
$$

где
$W_{S}$ - squared module of input flow speed,
$S$ - surfaces where the pressure determined,
$v_{n}$-normal component of flow incoming into above surface,
One may notice a formal analogy between $W_{S}$ and $p_{S}$. So here we also may consider the functional (64), where the energian-2 is

$$
\begin{equation*}
\mathfrak{R}(q)=\mathfrak{R}(q)+Q \cdot P_{9}\left(v_{n}\right), \tag{70}
\end{equation*}
$$

and then perform the substitution

$$
\begin{equation*}
F \Rightarrow F+Q \cdot W_{s}^{2} / \rho \tag{71}
\end{equation*}
$$

Consequently, the following lemma is true:
Lemma 4. The Navier-Stokes equations for incompressible fluid have a global solution in a domain bounded by generating surface with a certain pressure , if the integral $(61,58)$ has constant sign for any flow speed.

Note also that $W_{S}$ the pressure $p_{S}$ may be included in the full action-2 functional formally, without bringing in physical considerations.(similar with pressure $p_{S}$ ). Indeed, in the presence of external pressure there appears a new constraint - (21c). Including this integral constraint into the problem of the search for functional's extremum, we again get energian-2 (70).

### 2.4. Closed systems

We will call the system closed if it is bounded by

- absolutely hard and impenetrable walls,
- surfaces with certain external pressure,,
- generating surfaces, or
- not bounded by anything.

In the last case the system will be called absolutely closed. Such case is possible. For example, local body forces in a bondless ocean create such a system, and we shall discuss this case later. There is a possible case when the system is bounded by walls, but there is no energy exchange between fluid and walls. An example - a flow in endless pipe under the action of axis body forces Such example will also be considered below.

In consequence of Lemmas 1-4, the following theorem is true:
Theorem 1. The Navier-Stokes equations for incompressible fluid have a global solution in a given domain, if

- the domain of fluid existence is closed,
- the integral $(61,58)$ has constant sign for any flow speed.

The free surface, which is under certain pressure, may also be the boundary of a closed system. But the boundaries of this system are changeable, and the integration must be performed within the fluid volume. It is well known that the fluid flow through a certain surface $S$ is determined as

$$
\begin{equation*}
w_{S}=\iint_{S} \rho \cdot \operatorname{div}(v) \cdot d \Theta \tag{72}
\end{equation*}
$$

Thus, the boundary conditions in the form of free surface are fully considered, by the fact that the integration must be performed within the changeable boundaries of the free surface.

We have indicated above, that the power of energy flow change is determined by (10). In a closed system this power is equal to zero. Therefore for such system the energian-2 (24) or (28) turns into energian-2 (accordingly)

$$
\begin{align*}
& \mathfrak{R}(q)=P_{1}(v)+P_{3}(v)-P_{6}(v),  \tag{73}\\
& \mathfrak{R}(q)=\rho \cdot v \frac{d v}{d t}+\mu \cdot v \cdot \Delta v-\rho F v . \tag{74}
\end{align*}
$$

For such systems the Navier-Stokes equations take the form (1) and

$$
\begin{equation*}
\rho \frac{\partial v}{\partial t}-\mu \Delta v-\rho F=0 \tag{75}
\end{equation*}
$$

Some examples of such system will be cited below.

## 3. Modified Navier-Stokes equations

From (p19a) we find that

$$
\begin{equation*}
(v \cdot \nabla) \cdot v=\Delta\left(W^{2}\right) / 2 \tag{76}
\end{equation*}
$$

Substituting (76) in (2), we get

$$
\begin{equation*}
(v \cdot \nabla) \cdot v=\Delta\left(W^{2}\right) / 2 \tag{77}
\end{equation*}
$$

Let us consider the value

$$
\begin{equation*}
D=\left(p+\frac{\rho}{2} W^{2}\right) \tag{78}
\end{equation*}
$$

which we shall call quasipressure. Then (77) will take the form

$$
\begin{equation*}
\rho \frac{\partial v}{\partial t}-\mu \cdot \Delta v+\nabla D-\rho \cdot F=0 \tag{79}
\end{equation*}
$$

The equations system $(1,79)$ will be called modified Navier-Stokes equations. The solution of this system are functions $v, D$, and the pressure may be determined from $(9,78)$. It is easy to see that the equation (79) is much simpler than (2).

The above said may be formulated as the following lemma.
Lemma 5. If a given domain of incompressible fluid is described by Navier-Stokes equations, then it is also described by modified NavierStokes equations, and their solutions are similar.

Physics aside, we may note that from mathematical point of view the equation (79) is a particular case of equation (2), and so all the previous reasoning may be repeated for modified Navier-Stokes equations. Let us do it.

The functional of split full action-2 (34) contains modified split energian-2

$$
\mathfrak{R}_{2}\left(q^{\prime}, q^{\prime \prime}\right)=\left\{\begin{array}{l}
-\rho \cdot\left(v^{\prime} \frac{d v^{\prime \prime}}{d t}-v^{\prime \prime} \frac{d v^{\prime}}{d t}\right)-\mu \cdot\left(v^{\prime} \Delta v^{\prime}-v^{\prime \prime} \Delta v^{\prime \prime}\right)  \tag{80}\\
+\left(\operatorname{div}\left(v^{\prime} \cdot D^{\prime \prime}\right)-\operatorname{div}\left(v^{\prime \prime} \cdot D^{\prime}\right)\right)-\rho \cdot F\left(v^{\prime}-v^{\prime \prime}\right)
\end{array}\right\}
$$

- see (33). Gradient of this functional with respect to function $q^{\prime}$ is (37) and

$$
\begin{equation*}
b_{v^{\prime}}=\left\{2 \rho \cdot \frac{d v^{\prime \prime}}{d t}-2 \mu \cdot \Delta v^{\prime}+2 \nabla\left(D^{\prime \prime}\right)-\rho \cdot F\right\} \tag{81}
\end{equation*}
$$

- see (38). The components of equation (55) take the form

$$
\begin{align*}
& \mathfrak{R}_{21}=\left\{\begin{array}{l}
-\rho \cdot\left(b_{v} \frac{d v^{\prime \prime}}{d t}-v^{\prime \prime} \frac{d b_{v}}{d t}\right)-\mu \cdot\left(b_{v} \Delta v_{s}^{\prime}+v_{s}^{\prime} \Delta\left(b_{v}\right)\right) \\
+2\left(b_{v} \cdot \nabla\left(D^{\prime \prime}\right)-v^{\prime \prime} \cdot \nabla\left(b_{p}\right)\right)-\rho \cdot F \cdot b_{v}
\end{array}\right\},  \tag{82}\\
& \mathfrak{R}_{22}=-\mu b_{v} \Delta\left(b_{v}\right) \tag{83}
\end{align*}
$$

Thus, for modified Navier-Stokes equations by analogy with Theorem 1 we may formulate the following theorem

Theorem 2. Modified Navier-Stokes equations for incompressible fluid have a global solution in the given domain, if

- the fluid domain of existence is a closed system
o the integral $(61,83)$ has the same sign for any fluid flow speed.
Lemma 6. Integral $(61,83)$ always has positive value.
Proof. Consider the integral

$$
\begin{equation*}
J=\mu \int_{0}^{T}\left\{\int_{V} v \cdot \Delta(v) d V\right\} d t \tag{84}
\end{equation*}
$$

This integral expresses the thermal energy, evolved by the liquid due to internal friction. This energy is positive not depending on what function connects the vector of speeds with the coordinates. A stricter proof of this statement is given in Supplement 3. Hence, integral (84) is positive for any speed. Substituting in (84) $v=b_{v}$, we shall get integral ( 61,83 ), which is always positive, as was to be proved.

From Lemmas 5, 6 and Theorem 2 there follows a following.
Theorem 3. The equations of Navier-Stokes for incompressible fluid always have a solution in a closed domain.

The solution of equation $(1,79)$ permits to find the speeds. Calculation of pressures inside the closed domain with known speeds is performed with the aid of equation (78) or

$$
\begin{equation*}
\nabla p+\rho(v \cdot \nabla) v=0 \tag{85}
\end{equation*}
$$

## 4. Conclusions

1. Among the computed volumes of fluid flow the closed volumes of fluid flow may be marked, which do not exchange flow with adjacent volumes - the so-called closed systems.
2. The closed systems are bounded by:

- Impenetworkrable walls,
- Surfaces, located under the known pressure,
- Movable walls being under a known pressure,
- So-called generating surfaces through which the flow passes with a known speed.

3. It may be contended that the systems described by Naviet-Stokes equations, and having certain boundary conditions (pressures or speeds) on all boundaries, are closed systems.
4. For closed systems the global solution of modified Navier-Stokes equations always exists.
5. The solution of Navier-Stokes equations may always be found from the solution of modified Navier-Stokes equations. Therefore, for closed systems there always exists a global solution of modified NavierStokes equations.

## 5. Computational Algorithm

The method of solution for hydrodynamics equations with a known functional, having a global saddle point, is based on the following outlines $[7,8]$. For the given functional from two functions $q_{1}, q_{2}$ two more secondary functionals are formed from those functions
$q_{1}, q_{2}$. Each of these functionals has its own global saddle line. Seeking for the extremum of the main functional is substituted by seeking for extremums of two secondary functionals, and we are moving simultaneously along the gradients of these functionals. In general operational calculus should be used for this purpose. However, in some particular cases the algorithm may be considerably simplified.

Another complication is caused by the fact that in the computations we have to integrate over all the flow area. But the area may be infinite, and full integration is impossible. Nevertheless, the solution is possible also for an infinite area, if the flow speed is damping.

The solution method consists in moving along the gradient towards saddle point of the functional generated from the power balance equation. The obtained solutions:
a. may be interpreted as experimentally found physical effects (for instance, the walls impermeability, "sticking" of fluid to the walls, absence of energy flow through a closed system),
b. coincide with solutions obtained earlier with the aid of other methods (for instance, the solution of Poiseille problem),
c. may ue seen as generalization of known solutions (for instance, a generalization of Poiseille problem solution for pipes with arbitrary form of section and/or with arbitrary form of axis line),
d. belong to unsolved (as far as the author knows) problems (for instance, problems with body as the functions of speed, coordinates and time).
Here we shall discuss only these particular cases.
Various examples are given in [4, 5].

## 6. Stationary Problems

Note that in stationary mode the equations $(2.1,2.2)$ assumes the form

$$
\left\{\begin{array}{l}
\operatorname{div}(v)=0,  \tag{1}\\
\nabla p-\mu \Delta v+\rho(v \cdot \nabla) v-\rho F=0
\end{array}\right.
$$

The modified equations $(1,79)$ in stationary mode take the form:

$$
\left\{\begin{array}{l}
\operatorname{div}(v)=0  \tag{2}\\
-\mu \cdot \Delta v+\nabla D-\rho \cdot F=0
\end{array}\right.
$$

In Appendix 6 we considered the discrete version of modified NavierStokes equations for stationary systems (2). It was shown that for
stationary closed systems the solution of modified Navier-Stokes equations is reduced to a search for quadratic functional minimum (and not a saddle points, as in general case). After solving these equations the pressure is calculated by the equation (2.78), i.e.

$$
\begin{equation*}
p=D-\frac{\rho}{2} W^{2} \tag{3}
\end{equation*}
$$

or

$$
\begin{equation*}
\nabla p=\nabla D-\rho(v \cdot \nabla) v=0 \tag{4}
\end{equation*}
$$

The equation (75) for absolutely closed systems in stationary mode takes the form

$$
\begin{equation*}
-\mu \Delta v-\rho F=0 \tag{5}
\end{equation*}
$$

The solution of equation (2) has been discussed in detail in Supplement 4. After solving it the pressures are calculated by the equation (4).

## 7. Dynamic Problems

### 7.1. Absolutely closed systems

Let us consider the equation (2.75) for absolutely closed systems and rewrite is as

$$
\begin{equation*}
\frac{\partial v}{\partial t}-\eta \Delta v-F=0 \tag{1}
\end{equation*}
$$

where

$$
\begin{equation*}
\eta=\frac{\mu}{\rho} \tag{2}
\end{equation*}
$$

Assuming that time is a discrete variable with step $d t$, we shall rewrite (1) as

$$
\begin{equation*}
\frac{v_{n}-v_{n-1}}{d t}-\eta \Delta v_{n}-F_{n}=0 \tag{3}
\end{equation*}
$$

where $n=1,2,3, \ldots-$ the number of a time point. Let us write (3) as

$$
\begin{equation*}
\frac{v_{n}}{d t}-\eta \cdot \Delta v_{n}-F_{n 1}=0 \tag{4}
\end{equation*}
$$

where

$$
\begin{equation*}
F_{n 1}=\left(F_{n}+\frac{v_{n-1}}{d t}\right) \tag{5}
\end{equation*}
$$

For a known speed $v_{n-1}$ the value $v_{n}$ is determined by (4). Solving this equation is similar to solving a stationary problem. On the whole the algorithm of solving a dynamic problem for a closed system is as follows

## Algorithm 1

1. $v_{n-1}$ and $F_{n}$ are known
2. Computing $v_{n}$ by $(4,5)$.
3. Checking the deviation norm

$$
\begin{equation*}
\varepsilon=\frac{\partial v_{n}}{\partial t}-\frac{\partial v_{n-1}}{\partial t} \tag{6}
\end{equation*}
$$

and, if it doesn't exceed a given value, the calculation is over. расчет заканчивается. Otherwise we assign

$$
\begin{equation*}
v_{n-1} \Leftarrow v_{n} \tag{7}
\end{equation*}
$$

and go to p. 1 .
Example 1. Let the body forces on a certain time point assume instantly a certain value - there is a jump of body forces. Then in the initial moment the speed $v_{O}=0$, and on the first iteration we assign $v_{n-1}=0$. Further we perform the computation according to Algorithm 1.

### 7.2. Closed systems with variable mass forces and external pressures

Consider the modified equation $(1,79)$ in the case when the mass forces are sinusoidal functions of time with circular frequency $\omega$. In this case equations $(1,79)$ take the form of equations with complex variables:

$$
\left\{\begin{array}{l}
\operatorname{div}(v)=0  \tag{8}\\
j \cdot \omega \cdot \rho \cdot v-\mu \cdot \Delta v+\nabla D-\rho \cdot F=0
\end{array}\right.
$$

where $j$ - the imaginary unit.
In $[4,5]$ the discrete version of these equations is considered. There it is shown that their solution is reduced to the search of saddle point of a certain function of complex variables. After solving these equations the pressure is calculated by equation(4).

## 8. Compressible fluid

In this section we shall use this principle for the Navier-Stokes equations describing compressible fluid.

Navier-stokes equation for viscous compressible fluid are considered. It is shown that these equations are the conditions of a certain functional's extremum. The method of finding the solution of these equations is described. It consists of moving along the gradient towards the extremum of his functional. The conditions of reaching this extremum are formulated - they are simultaneously necessary and sufficient conditions of the existence of this functional's global extremum

### 8.1. The equations of hydrodynamics

In contrast with the equations for viscous incompressible fluid, the equations for viscous compressible fluid have the following form [2]:

$$
\begin{align*}
& \frac{\partial \rho}{\partial \mathrm{t}}+\operatorname{div}(\rho \cdot v)=0,  \tag{4}\\
& \rho \frac{\partial v}{\partial t}+\nabla p-\mu \cdot \Delta v+\rho \cdot G(v)-\rho \cdot F-\frac{\mu}{3} \Omega(v)=0, \tag{5}
\end{align*}
$$

where

$$
\begin{equation*}
\Omega(v)=\nabla(\nabla v) \tag{6}
\end{equation*}
$$

В приложении функции (3) и (6) представлены в развернутом виде см. (p14, p29, p30). Для сжимаемой жидкости плотность является известной функцией давления:

$$
\begin{equation*}
\rho=f(p) \tag{7}
\end{equation*}
$$

Further the reasoning will be by analogy with the previous. In this case we have to consider also the power of energy loss variation in the course of expansion/compression due to the friction.

$$
\begin{equation*}
P_{8}(v)=\frac{\mu}{3} v \cdot \Omega(v) \tag{9}
\end{equation*}
$$

We have also:

$$
\begin{equation*}
\frac{\partial}{\partial v}\left(P_{8}(v)\right)=\frac{\mu}{3} \Omega(v) \tag{10}
\end{equation*}
$$

We may note that the function $\Omega(v)$ in the present context behaved in the same way as the function $\Delta(v)$. This allows to apply the proposed method also for compressible fluids.

### 8.2. Energian-2 and quasiextremal

By analogy with previous reasoning we shall write the formula for quasiextremal for compressible fluid in the following form:

$$
\left\{\begin{array}{l}
\frac{\partial}{\partial v}\left(\rho \cdot v \frac{d v}{d t}\right)-\frac{1}{2} \mu \cdot \frac{\partial_{o}}{\partial v}(v \cdot \Delta v)+\frac{\partial}{\partial q}\left(\frac{1}{\rho} \operatorname{div}(\rho \cdot p \cdot v)\right)+  \tag{11}\\
+\frac{\partial}{\partial v}(\rho \cdot v \cdot G(v))-\frac{\partial_{O}}{\partial v}(\rho \cdot F \cdot v)- \\
-\frac{\partial}{\partial p}\left(\frac{p}{\rho} \frac{\partial \rho}{\partial t}\right)-\frac{1}{2} \frac{\mu}{3} \cdot \frac{\partial_{o}}{\partial v}(v \cdot \Omega(v))
\end{array}\right\}=0
$$

### 8.3. The split energian-2

By analogy with previous reasoning we shall write the formula for split energian-2 for compressible fluid in the following form:

$$
\mathfrak{R}_{2}\left(q^{\prime}, q^{\prime \prime}\right)=\left\{\begin{array}{l}
\rho \cdot\left(v^{\prime} \frac{d v^{\prime \prime}}{d t}-v^{\prime \prime} \frac{d v^{\prime}}{d t}\right)-\mu \cdot\left(v^{\prime} \Delta v^{\prime}-v^{\prime \prime} \Delta v^{\prime \prime}\right)  \tag{12}\\
+\frac{2}{\rho}\left(\left(\operatorname{div}\left(\rho \cdot v^{\prime} \cdot p^{\prime \prime}\right)-\operatorname{div}\left(\rho \cdot v^{\prime \prime} \cdot p^{\prime}\right)\right)\right)+ \\
\rho \cdot\left(v^{\prime} G\left(v^{\prime \prime}\right)-v^{\prime \prime} G\left(v^{\prime}\right)\right)-\rho \cdot F\left(v^{\prime}-v^{\prime \prime}\right)- \\
\frac{2}{\rho}\left(p^{\prime} \frac{d \rho}{d t}-p^{\prime \prime} \frac{d \rho}{d t}\right)-\frac{\mu}{3} \cdot\left(v^{\prime} \Omega\left(v^{\prime}\right)-v^{\prime \prime} \Omega\left(v^{\prime \prime}\right)\right)
\end{array}\right\}
$$

With the aid of Ostrogradsky formula (p23) we may find the variations of functional of spilt full action-2 with respect to functions $q^{\prime}$ :

$$
\begin{align*}
& \frac{\partial_{o} \mathfrak{R}_{2}}{\partial p^{\prime}}=b_{p^{\prime}}  \tag{13}\\
& \frac{\partial_{o} \Re_{2}}{\partial v^{\prime}}=b_{v^{\prime}} \tag{14}
\end{align*}
$$

These variations are determined by varying the functions $p^{\prime}$ and $v^{\prime}$, whereas the functions $\rho, p^{\prime \prime}, v^{\prime \prime}$ do not change. Then we shall get:

1) $\frac{\partial}{\partial v^{\prime}}\left[\rho \cdot\left(v^{\prime} \frac{d v^{\prime \prime}}{d t}-v^{\prime \prime} \frac{d v^{\prime}}{d t}\right)\right]=2 \rho \frac{d v^{\prime \prime}}{d t}$,
2) $\frac{\partial}{\partial v^{\prime}}\left[-\mu \cdot\left(v^{\prime} \Delta v^{\prime}-v^{\prime \prime} \Delta v^{\prime \prime}\right)\right]=-2 \mu \cdot \Delta v^{\prime}$,
3) $\frac{\partial}{\partial v^{\prime}}\left[\rho\left(v^{\prime} G\left(v^{\prime \prime}\right)-v^{\prime \prime} G\left(v^{\prime}\right)\right)\right]=2 \rho \cdot\left[G\left(v^{\prime \prime}, \frac{\partial v^{\prime \prime}}{\partial X}\right)+G\left(v^{\prime}, \frac{\partial v^{\prime \prime}}{\partial X}\right)\right]$,
4) $\frac{\partial}{\partial v^{\prime}}\left[-\rho \cdot F\left(v^{\prime}-v^{\prime \prime}\right)\right]=-\rho \cdot F$,
5) $\frac{\partial}{\partial v^{\prime}}\left[-\frac{\mu}{3} \cdot\left(v^{\prime} \Omega\left(v^{\prime}\right)-v^{\prime \prime} \Omega\left(v^{\prime \prime}\right)\right)\right]=-\frac{2 \mu}{3} \cdot \Omega\left(v^{\prime}\right)$,
6) $\frac{\partial}{\partial v^{\prime}}\left[\frac{2}{\rho}\left(\operatorname{div}\left(\rho \cdot v^{\prime} \cdot p^{\prime \prime}\right)-\operatorname{div}\left(\rho \cdot v^{\prime \prime} \cdot p^{\prime}\right)\right)\right]=2 \operatorname{grad}\left(p^{\prime \prime}\right)$,
7) $\frac{\partial}{\partial p^{\prime}}\left[\frac{2}{\rho}\left(\operatorname{div}\left(\rho \cdot v^{\prime} \cdot p^{\prime \prime}\right)-\operatorname{div}\left(\rho \cdot v^{\prime \prime} \cdot p^{\prime}\right)\right)\right]=-\frac{2}{\rho} \operatorname{div}\left(\rho \cdot v^{\prime \prime}\right)$,
8) $\frac{\partial}{\partial p^{\prime}}\left[-\frac{2}{\rho}\left(p^{\prime} \frac{d \rho}{d t}-p^{\prime \prime} \frac{d \rho}{d t}\right)\right]=-\frac{2}{\rho} \frac{d \rho}{d t}$.

Remarks for these formulas:
$1,2,3,4)$ - the derivation is given below,
5) - is similar to formula 2),

6,7 ) - the derivation is given in the Supplement 1 - see (p34, p35) accordingly
Then we have:

$$
\begin{align*}
& b_{p^{\prime}}=-2 \frac{d \rho}{d t}-2 \operatorname{div}\left(\rho \cdot v^{\prime \prime}\right),  \tag{16}\\
& b_{v^{\prime}}=\left\{\begin{array}{l}
2 \rho \cdot \frac{d v^{\prime \prime}}{d t}-2 \mu \cdot \Delta\left(v^{\prime}\right)-\frac{2 \mu}{3} \cdot \Omega\left(v^{\prime}\right)+2 \nabla\left(p^{\prime \prime}\right) \\
+2 \rho \cdot\left[G\left(v^{\prime \prime}, \frac{\partial v^{\prime \prime}}{\partial X}\right)+G\left(v^{\prime}, \frac{\partial v^{\prime \prime}}{\partial X}\right)\right]-\rho \cdot F
\end{array}\right\} . \tag{17}
\end{align*}
$$

As was shown above, the condition

$$
\begin{equation*}
b^{\prime}=\left\lfloor b_{p^{\prime}}, b_{v^{\prime}}\right\rfloor=0 \tag{18}
\end{equation*}
$$

and the similar condition

$$
\begin{equation*}
b^{\prime \prime}=\left\lfloor b_{p^{\prime \prime}}, \quad b_{v^{\prime \prime}}\right\rfloor=0 \tag{19}
\end{equation*}
$$

Are necessary conditions for the existence of a saddle line. From the symmetry of these equations it follows that the optimal functions $q_{0}^{\prime}$ and $q_{0}^{\prime \prime}$, satisfying the equations $(18,19)$, must satisfy also the condition

$$
\begin{equation*}
q_{0}^{\prime}=q_{0}^{\prime \prime} \tag{20}
\end{equation*}
$$

Subtracting in pairs the equations $(18,19)$ taking into account $(16,17)$, we get

$$
\begin{equation*}
-2 \frac{\mathrm{~d} \rho}{\mathrm{dt}}-2 \operatorname{div}\left(v^{\prime}+v^{\prime \prime}\right)=0 \tag{21}
\end{equation*}
$$

$$
\left\{\begin{array}{l}
+2 \rho \cdot \frac{d\left(v^{\prime}+v^{\prime \prime}\right)}{d t}-2 \mu \cdot \Delta\left(v^{\prime}+v^{\prime \prime}\right)-\frac{2 \mu}{3} \cdot \Omega\left(v^{\prime}+v^{\prime \prime}\right)+ \\
+2 \nabla\left(p^{\prime}+p^{\prime \prime}\right)-2 \rho \cdot F+2 \rho \cdot\left[\begin{array}{l}
\left.G\left(v^{\prime \prime}, \frac{\partial v^{\prime \prime}}{\partial X}\right)+G\left(v^{\prime}, \frac{\partial v^{\prime \prime}}{\partial X}\right)+\right] \\
+G\left(v^{\prime}, \frac{\partial v^{\prime}}{\partial X}\right)+G\left(v^{\prime \prime}, \frac{\partial v^{\prime}}{\partial X}\right)
\end{array}\right]=0 \cdot(22)
\end{array}\right.
$$

Учитывая $(1.45)$ и сокращая $(34,35)$ на 2 , получаем уравнения $(4,5)$, где

$$
\begin{equation*}
q=q_{o}^{\prime}+q_{o}^{\prime \prime} \tag{23}
\end{equation*}
$$

Taking into account $(1.45)$ and cancelling $(34,35)$ by 2 , we get the equations (4, 5), where

### 8.4. About sufficient conditions of extremum

Above we have proved for incompressible fluid, that the necessary conditions $(18,19)$ of the existence of extremum for the full action-2 functional are also sufficient conditions, if the integral

$$
\begin{equation*}
I=\int_{0}^{T}\left\{\int_{V} \mathfrak{R}_{22} d V\right\} d t \tag{24}
\end{equation*}
$$

has constant sign, where

$$
\begin{equation*}
\Re_{22}=-\mu b_{v} \Delta\left(b_{v}\right)-2 \rho v^{\prime \prime} G\left(b_{v}\right) \tag{25}
\end{equation*}
$$

For compressible fluid the necessary conditions $(18,19)$ of the existence of extremum for the full action-2 functional are also sufficient conditions, if the integral (24) has constant sign, where, contrary to (25),

$$
\begin{equation*}
\mathfrak{R}_{22}=-\mu b_{v} \Delta\left(b_{v}\right)-\frac{\mu}{3} b_{v} \Omega\left(b_{v}\right)-2 \rho v^{\prime \prime} G\left(b_{v}\right) . \tag{26}
\end{equation*}
$$

For closed systems with a flow of систем incompressible fluid we have shown above that the value (25) assumes the form

$$
\begin{equation*}
\mathfrak{R}_{22}=-\mu b_{v} \Delta\left(b_{v}\right) \tag{27}
\end{equation*}
$$

Similarly, for closed systems with a flow of compressible fluid the value (26) assumes the form

$$
\begin{equation*}
\Re_{22}=-\mu b_{v} \Delta\left(b_{v}\right)-\frac{\mu}{3} b_{v} \Omega\left(b_{v}\right) \tag{28}
\end{equation*}
$$

Let us consider now, similarly to (24), the integral

$$
\begin{equation*}
J=\int_{0}^{T}\left\{\int_{V} \Re_{22}^{\prime} d V\right\} d t \tag{29}
\end{equation*}
$$

where

$$
\begin{equation*}
\mathfrak{R}_{22}^{\prime}=-\mu \cdot v \cdot \Delta(v)-\frac{\mu}{3} v \cdot \Omega(v) \tag{30}
\end{equation*}
$$

(i.e. in this formula instead of the function $b_{v}$ there is the function of speed). As the proof of the integral's constancy of sign must be valid for any function, it is enough to prove the constancy of sign of integral (29) with speeds. For this we must note that:

- the first term in (30) expresses the heat energy exuded by the fluid as the result of internal friction,
- the second tem in (30) is the heat energy exuded/absorbed by the fluid as the result of expansion \compression.
The first energy is positive regardless to the value of vector-function of speed with respect to the coordinates (A more exact proof of this fact for the first term is given in $[4,5])$. The second term is equal to zero (as in our statement the temperature is not taken into account, i.e. assumed to be constant). Therefore, integral $(24,30)$ is positive on any iteration, which was required to show.

Thus, the Navier-Stokes equations for incompressible fluid have a global solution.

## 9. Discussion

Physical assumptions are often built on mathematical corollary facts. So it may be legitimate to build mathematical assumption on the base of physical facts. In this book there are several such places
2. The equations are derived on the base of the presented principle of general action extremum.
3. The main equation is divided into two independent equations based on a physical fact - the absence of energy flow through a closed system.
4. The exclusion of continuity conditions for closed systems is based in the physical fact - the continuity of fluid flow in a closed system
5. Usually in the problem formulation we indicate the boundaries of solution search and the boundary conditions - for speed, acceleration pressure on the boundaries These conditions usually are formed on the base of physical facts, for example the fluid "adhesion" to the walls, the walls hardness, etc. In the presented method we do not include the boundary conditions into the problem formulation - they are found in the process of solution.

We may point also some possible directions of this approach development, for example
i. for problems of electro- and magneto-hydrodynamics
ii. for free surfaces dynamics (in changing boundaries for constant fluid volume).

The proof of global solution existence belongs to closed systems Practically, we must analyze the bounded and closed systems. Therefore some methods of formal transformation of non-closed systems into closed ones are also proposed, such as:

1. long pipe as the limit of ring pipe,
2. transformation of a limited pipe segment into closed system

## Supplement. Certain formulas

Here we shall consider the proof of some formulas that were used in the main text. First of all we must remind that

$$
\begin{align*}
& \operatorname{div}(v)=\left[\frac{\partial v_{x}}{\partial x}+\frac{\partial v_{y}}{\partial y}+\frac{\partial v_{z}}{\partial z}\right]  \tag{p1}\\
& \nabla p=\operatorname{grad}(p)=\left[\frac{\partial p}{\partial x}, \frac{\partial p}{\partial y}, \frac{\partial p}{\partial z}\right] \tag{p2}
\end{align*}
$$

$$
\begin{align*}
& \Delta v_{x}=\frac{\partial^{2} v_{x}}{\partial x^{2}}+\frac{\partial^{2} v_{x}}{\partial y^{2}}+\frac{\partial^{2} v_{x}}{\partial z^{2}},  \tag{p3}\\
& \Delta v=\left[\begin{array}{l}
\frac{\partial^{2} v_{x}}{\partial x^{2}}+\frac{\partial^{2} v_{x}}{\partial y^{2}}+\frac{\partial^{2} v_{x}}{\partial z^{2}} \\
\frac{\partial^{2} v_{y}}{\partial x^{2}}+\frac{\partial^{2} v_{y}}{\partial y^{2}}+\frac{\partial^{2} v_{y}}{\partial z^{2}} \\
\frac{\partial^{2} v_{z}}{\partial x^{2}}+\frac{\partial^{2} v_{z}}{\partial y^{2}}+\frac{\partial^{2} v_{z}}{\partial z^{2}}
\end{array}\right]  \tag{p4}\\
& (v \cdot \nabla)=\left[\begin{array}{l}
v_{x} \frac{\partial}{\partial x}+v_{y} \frac{\partial}{\partial y}+v_{z} \frac{\partial}{\partial z}
\end{array}\right],  \tag{p5}\\
& (v \cdot \nabla) v=\left[\begin{array}{l}
v_{x} \frac{\partial v_{x}}{\partial x}+v_{y} \frac{\partial v_{x}}{\partial y}+v_{z} \frac{\partial v_{x}}{\partial z} \\
v_{x} \frac{\partial v_{y}}{\partial x}+v_{y} \frac{\partial v_{y}}{\partial y}+v_{z} \frac{\partial v_{y}}{\partial z} \\
v_{x} \frac{\partial v_{z}}{\partial x}+v_{y} \frac{\partial v_{z}}{\partial y}+v_{z} \frac{\partial v_{z}}{\partial z}
\end{array}\right] . \tag{p6}
\end{align*}
$$

From (2.5, 2.7a) it follows that

$$
\begin{equation*}
P_{1}=\frac{\rho}{2} \frac{d}{d t}\left(v_{x}^{2}+v_{y}^{2}+v_{z}^{2}\right) \tag{p7}
\end{equation*}
$$

i.e.

$$
\begin{equation*}
P_{1}=\rho v \frac{d v}{d t} \tag{p8}
\end{equation*}
$$

Let us consider the function (2.7) or

$$
\frac{P_{5}}{\rho}=\frac{1}{2}\left(\begin{array}{c}
v_{x} \frac{d}{d x}\left(v_{x}^{2}+v_{y}^{2}+v_{z}^{2}\right)  \tag{p9}\\
+v_{y} \frac{d}{d y}\left(v_{x}^{2}+v_{y}^{2}+v_{z}^{2}\right) \\
+v_{z} \frac{d}{d z}\left(v_{x}^{2}+v_{y}^{2}+v_{z}^{2}\right)
\end{array}\right)
$$

or

$$
\begin{equation*}
P_{5}=\frac{\rho}{2} v \cdot \Delta\left(W^{2}\right) \tag{p9a}
\end{equation*}
$$

Differentiating, we shall get:

$$
\frac{P_{5}}{\rho}=\left\{\begin{array}{l}
v_{x}\left(v_{x} \frac{d v_{x}}{d x}+v_{y} \frac{d v_{y}}{d x}+v_{z} \frac{d v_{z}}{d x}\right)+  \tag{p10}\\
v_{y}\left(v_{x} \frac{d v_{x}}{d x}+v_{y} \frac{d v_{y}}{d x}+v_{z} \frac{d v_{z}}{d x}\right)+ \\
v_{z}\left(v_{x} \frac{d v_{x}}{d x}+v_{y} \frac{d v_{y}}{d x}+v_{z} \frac{d v_{z}}{d x}\right)
\end{array}\right\}
$$

After rearranging the items, we get

$$
\frac{P_{5}}{\rho}=\left\{\begin{array}{l}
v_{x}\left(v_{x} \frac{d v_{x}}{d x}+v_{y} \frac{d v_{x}}{d y}+v_{z} \frac{d v_{x}}{d z}\right)+  \tag{p11}\\
v_{y}\left(v_{x} \frac{d v_{y}}{d x}+v_{y} \frac{d v_{y}}{d y}+v_{z} \frac{d v_{y}}{d z}\right)+ \\
v_{z}\left(v_{x} \frac{d v_{z}}{d x}+v_{y} \frac{d v_{z}}{d y}+v_{z} \frac{d v_{z}}{d z}\right)
\end{array}\right\} .
$$

Let us denote:

$$
\begin{align*}
& g_{x}=\left(v_{x} \frac{d v_{x}}{d x}+v_{y} \frac{d v_{x}}{d y}+v_{z} \frac{d v_{x}}{d z}\right) \\
& g_{y}=\left(v_{x} \frac{d v_{y}}{d x}+v_{y} \frac{d v_{y}}{d y}+v_{z} \frac{d v_{y}}{d z}\right)  \tag{p12}\\
& g_{z}=\left(v_{x} \frac{d v_{z}}{d x}+v_{y} \frac{d v_{z}}{d y}+v_{z} \frac{d v_{z}}{d z}\right)
\end{align*}
$$

Let us consider the vector

$$
G=\left\{\begin{array}{l}
g_{x}  \tag{p13}\\
g_{y} \\
g_{y}
\end{array}\right\}
$$

or

$$
G=\left[\begin{array}{l}
v_{x} \frac{\partial v_{x}}{\partial x}+v_{y} \frac{\partial v_{x}}{\partial y}+v_{z} \frac{\partial v_{x}}{\partial z}  \tag{p14}\\
v_{x} \frac{\partial v_{y}}{\partial x}+v_{y} \frac{\partial v_{y}}{\partial y}+v_{z} \frac{\partial v_{y}}{\partial z} \\
v_{x} \frac{\partial v_{z}}{\partial x}+v_{y} \frac{\partial v_{z}}{\partial y}+v_{z} \frac{\partial v_{z}}{\partial z}
\end{array}\right] .
$$

Note that

$$
\begin{equation*}
\frac{1}{2} G(v)=2 G(v / 2) \tag{p14a}
\end{equation*}
$$

From (p11-p14) we get

$$
\begin{align*}
& P_{5} / \rho=v \cdot G  \tag{p15}\\
& \frac{\partial P_{5}(v, G(v))}{\partial v}=\rho G(v) \tag{p16}
\end{align*}
$$

Comparing (p6) and (p14), we find that

$$
\begin{equation*}
G(v)=(v \cdot \nabla) v . \tag{p18}
\end{equation*}
$$

Thus,

$$
\begin{equation*}
\frac{\partial P_{5}(v, G)}{\partial v}=\rho(v \cdot \nabla) v \tag{p19}
\end{equation*}
$$

Comparing ( $\mathrm{p} 9 \mathrm{a}, \mathrm{p} 15, \mathrm{p} 18$ ), we find that

$$
\begin{equation*}
\Delta\left(W^{2}\right)=2 \cdot(v \cdot \nabla) \cdot v \tag{p19a}
\end{equation*}
$$

As dynamic pressure is determined [2] by

$$
\begin{equation*}
P_{d}=\rho W^{2} / 2 \tag{p19c}
\end{equation*}
$$

then from ( $\mathrm{p} 18, \mathrm{p} 19 \mathrm{a}$ ) it follows that the gradient of dynamic pressure is

$$
\begin{equation*}
\Delta\left(P_{d}\right)=\rho \cdot G \tag{p19d}
\end{equation*}
$$

Let us consider also

$$
\begin{equation*}
G(v+b)=G(v)+G(b)+G_{1}(v, b)+G_{2}(v, b), \tag{p20}
\end{equation*}
$$

where

$$
\begin{align*}
& G_{1}(v, b)=\left[\begin{array}{l}
v_{x} \frac{\partial b_{x}}{\partial x}+v_{y} \frac{\partial b_{x}}{\partial y}+v_{z} \frac{\partial b_{x}}{\partial z} \\
v_{x} \frac{\partial b_{y}}{\partial x}+v_{y} \frac{\partial b_{y}}{\partial y}+v_{z} \frac{\partial b_{y}}{\partial z} \\
v_{x} \frac{\partial b_{z}}{\partial x}+v_{y} \frac{\partial b_{z}}{\partial y}+v_{z} \frac{\partial b_{z}}{\partial z}
\end{array}\right],  \tag{p20a}\\
& G_{2}(v, b)=\left[\begin{array}{l}
b_{x} \frac{\partial v_{x}}{\partial x}+b_{x} \frac{\partial v_{x}}{\partial y}+b_{x} \frac{\partial v_{x}}{\partial z} \\
b_{y} \frac{\partial v_{y}}{\partial x}+b_{y} \frac{\partial v_{y}}{\partial y}+b_{y} \frac{\partial v_{y}}{\partial z} \\
b_{z} \frac{\partial v_{z}}{\partial x}+b_{z} \frac{\partial v_{z}}{\partial y}+b_{z} \frac{\partial v_{z}}{\partial z}
\end{array}\right] . \tag{р20в}
\end{align*}
$$

If $b=a \cdot b_{v}$, then

$$
G\left(v+a \cdot b_{v}\right)=G(v)+a^{2} G\left(b_{v}\right)+a G_{1}\left(v, b_{v}\right)+a G_{2}\left(v, b_{v}\right) \cdot(\mathrm{p} 21)
$$

We have

$$
\begin{align*}
& \frac{\partial_{O}}{\partial v^{\prime}}\left(v^{\prime \prime} \frac{d v^{\prime}}{d t}\right)=-\frac{d v^{\prime \prime}}{d t}, \frac{\partial_{O}}{\partial v^{\prime \prime}}\left(v^{\prime \prime} \frac{d v^{\prime}}{d t}\right)=\frac{d v^{\prime}}{d t} \\
& \frac{\partial_{O}}{\partial v^{\prime}}\left(v^{\prime} \Delta v^{\prime}\right)=2 \Delta v^{\prime \prime} \\
& \frac{\partial_{O}}{\partial v^{\prime}}\left(v^{\prime \prime} G\left(v^{\prime}\right)\right)=-G\left(v^{\prime}, \frac{\partial v^{\prime \prime}}{\partial X}\right), \frac{\partial_{O}}{\partial v^{\prime}}\left(v^{\prime} G\left(v^{\prime \prime}\right)\right)=G\left(v^{\prime \prime}\right) \\
& \frac{\partial_{O}}{\partial v^{\prime}}\left(v^{\prime} \cdot \nabla\left(p^{\prime \prime}\right)\right)=\nabla\left(p^{\prime \prime}\right), \frac{\partial_{O}}{\partial p^{\prime \prime}}\left(v^{\prime} \cdot \nabla\left(p^{\prime \prime}\right)\right)=-\operatorname{div}\left(v^{\prime}\right) \\
& \frac{\partial_{O}}{\partial v^{\prime}} \operatorname{div}\left(v^{\prime} \cdot p^{\prime \prime}\right)=\nabla\left(p^{\prime \prime}\right), \frac{\partial_{O}}{\partial p^{\prime \prime}} \operatorname{div}\left(v^{\prime} \cdot p^{\prime \prime}\right)=-\operatorname{div}\left(v^{\prime}\right) \text {-see. (p31). } \tag{p22}
\end{align*}
$$

The necessary conditions for extremum of functional from the functions with several independent variables - the Ostrogradsky equations [3] have for each of the functions the form

$$
\begin{equation*}
\frac{\partial_{o} f}{\partial v}=\frac{\partial f}{\partial v}-\sum_{a=x, y, z, t}\left[\frac{\partial}{\partial a}\left(\frac{\partial f}{\partial(d v / d a)}\right)\right]=0, \tag{p23}
\end{equation*}
$$

where $f$ - the integration element, $v(x, y, z, t)$ - the variable function, $a_{-}$ independent variable.

$$
\begin{align*}
& \Omega(v)=\left[\frac{\partial(\operatorname{div}(v))}{\partial x}, \frac{\partial(\operatorname{div}(v))}{\partial y}, \frac{\partial(\operatorname{div}(v))}{\partial z}\right]  \tag{p29}\\
& \Omega(v)=\left[\begin{array}{l}
\frac{\partial^{2} v_{x}}{\partial x^{2}}+\frac{\partial^{2} v_{y}}{\partial x \partial y}+\frac{\partial^{2} v_{z}}{\partial x \partial z} \\
\frac{\partial^{2} v_{x}}{\partial x \partial y}+\frac{\partial^{2} v_{y}}{\partial y^{2}}+\frac{\partial^{2} v_{z}}{\partial y \partial z} \\
\frac{\partial^{2} v_{x}}{\partial x \partial z}+\frac{\partial^{2} v_{y}}{\partial y \partial z}+\frac{\partial^{2} v_{z}}{\partial z^{2}}
\end{array}\right] \tag{p30}
\end{align*}
$$

If $\rho, p$ are scalar fields, and $v$ is a vector field, then

$$
\begin{align*}
& \operatorname{div}(\rho \cdot v)=v \cdot \operatorname{grad}(\rho)+\rho \cdot \operatorname{div}(v)  \tag{p31}\\
& \operatorname{div}(\rho \cdot p \cdot v)=\rho \cdot v \cdot \operatorname{grad}(p)+p \cdot \operatorname{div}(\rho \cdot v) \tag{p32}
\end{align*}
$$

i.e.
$\operatorname{div}(\rho \cdot p \cdot v)=\rho \cdot v \cdot \operatorname{grad}(p)+p \cdot v \cdot \operatorname{grad}(\rho)+p \cdot \rho \cdot \operatorname{div}(v) . \quad(\mathrm{p} 33)$
Consider $\operatorname{div}\left(\rho \cdot p^{\prime} \cdot v^{\prime \prime}\right)$ and suppose that the extremum of a certain functional is determined or by varying the function $p^{\prime}$, or by varying the function $v^{\prime \prime}$. Then, differentiating the last expression by Ostrogradsky formula ( p 23 ), we shall find:
$\frac{\partial_{O}}{\partial p^{\prime}}\left[\operatorname{div}\left(\rho \cdot p^{\prime} \cdot v^{\prime \prime}\right)\right]=0+v^{\prime \prime} \cdot \operatorname{grad}(\rho)+\rho \cdot \operatorname{div}\left(v^{\prime \prime}\right)$,
$\frac{\partial_{o}}{\partial v^{\prime \prime}}\left[\operatorname{div}\left(\rho \cdot p^{\prime} \cdot v^{\prime \prime}\right)\right]=\rho \cdot \operatorname{grad}\left(p^{\prime}\right)+p^{\prime} \cdot \operatorname{grad}(\rho)-p^{\prime} \cdot \operatorname{grad}(\rho)$
or

$$
\begin{align*}
& \frac{\partial_{O}}{\partial p^{\prime}}\left[\operatorname{div}\left(\rho \cdot p^{\prime} \cdot v^{\prime \prime}\right)\right]=\operatorname{div}\left(\rho \cdot v^{\prime \prime}\right)  \tag{p34}\\
& \frac{\partial_{O}}{\partial v^{\prime \prime}}\left[\operatorname{div}\left(\rho \cdot p^{\prime} \cdot v^{\prime \prime}\right)\right]=\rho \cdot \operatorname{grad}\left(p^{\prime}\right) \tag{p35}
\end{align*}
$$

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## Solomon I. Khmelnik

# Principle extremum of full action in electrodynamic 

## Annotation

Here we are going to formulate and prove variational extremum principle for electrodynamics, asserting that there exists a functional that depends on powers. This functional always has a single extremum, and the necessary and sufficient conditions of this extremum existence are represented by Maxwell equations. This principle is realized also in the case when the system contains magnetic charges and magnetic currents. Besides, this principle is valid also if there are heat losses in the system. The method for solving the Maxwell equations system by gradient descent to extremum is indicated.

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## Introduction

1. The power balance of electromagnetic field
2. Building the functional for Maxwell equations
2.1. Maxwell equations
2.2. Principle extremum of full action-2
3. Splitting the functional for Maxwell equations References

## Introduction

In his previous works [1,2] the author has presented the principle extremum of full action, permitting to construct a functional for various physical systems, and, which is most important, for dissipative systems. Herewith the action is determined as an integral from kinetic, potential and heat energies. The principle has been applied in implicit form [3, 4] to electrical engineering, electromechanics, electrodynamics. In this paper the author is using a stricter extension of this principle for powers [2] applied to electrodynamics.

It is known [5], that Maxwell equations are deducted from the least action principle. For this purpose it is necessary to introduce the concept of vector magnetic potential and formulate a certain functional with respect to such potential and to scalar electrical potential, and this
functional will be called action. Then by varying the action with respect to vector magnetic potential and to scalar electrical potential the conditions of this functional's minimum may be found. Further (after certain reductions) it is shown that this condition (with regard to the potentials) is equivalent to equations system with respect to electric and magnetic intensities. The obtained equation system corresponds only to four of Maxwell equations. It is evident, since the vector magnetic potential and electric scalar potential provide only four varying functions. But such partial result permits authors to conclude that all Maxwell equations (with respect to the intensities) are the consequences of least action principle as the above determined functional. But all Maxwell equations do not follow from this functional !

Furthermore, the Maxwell equations are dealing with currents in a medium with a certain electroconductivity. As a consequence, there are heat losses, i.e. energy dissipation. It means that, for the sake of the least action principle in addition to electromagnetic energy, the thermal energy should be also included in the functional; but this energy is not a part of Lagrangian. Therefore the Lagrange formalism is in principle not applicable to Maxwell equations.

Thus, the above conclusion, which has some cognitive value, does not demonstrate a triumph of the least action principle. And, all the more, this functional cannot be used for direct solution of technical problems (using the above described method of descent along the functional) So it turns out that the Lagrange formalism is insufficient for the deduction of Maxwell equations.

The matter becomes complicated also because for symmetrical form of Maxwell equations (figuring magnetic and magnetic current), an electromagnetic field cannot be described by vector potential that is continuous in all the space Therefore the symmetrical Maxwell equations cannot be deducted from variational least action principle, where the action is an integral of difference between kinetic and potential energies.

## 1. The power balance of electromagnetic field

The equation of electromagnetic field power balance in differential form is well known [6]. It has the following form

$$
\begin{equation*}
P_{\Pi}+P_{E H}+P_{Q}+P_{C}=0 \tag{1}
\end{equation*}
$$

where
$P_{\Pi}$ - the density of power flow through a certain surface ,
$P_{E H}$ - the density of electromagnetic power of an electromagnetic field,
$P_{Q}$ - the density of heat loss power,
$P_{C}$ - the density of outside current sources power.
Also

$$
\begin{equation*}
P_{\Pi}=\operatorname{div}[E \times H] \tag{2}
\end{equation*}
$$

or, according to a known formula of vector analysis,

$$
\begin{align*}
& P_{\Pi}=E \cdot \operatorname{rot}(H)-H \cdot \operatorname{rot}(E)  \tag{3}\\
& P_{E H}=\mu H \frac{d H}{d t}+\varepsilon E \frac{d E}{d t}  \tag{4}\\
& P_{Q}=J_{1} E  \tag{5}\\
& P_{C}=J_{2} E \tag{6}
\end{align*}
$$

where
$\mathcal{E}$ - absolute permittivity,
$\mu$ - absolute magnetic permeability,
$J_{1}$ - the density of conduction current,
$J_{2}$ - the current density of outside current source.
Here and further the three-component vectors

$$
H, \frac{d H}{d t}, E, \frac{d E}{d t}, J_{1}, J_{2}, \operatorname{rot}(H), \operatorname{rot}(E)
$$

are considered vectors in the sense of vector algebra. So the operations of multiplication for them may be written in simplified form. For instance, a product of vectors $E \cdot \operatorname{rot}(H)$ is a product of columnvector $E$ by row-vector $\operatorname{rot}(H)$.

Let us denote

$$
\begin{align*}
& J=J_{1}+J_{2}  \tag{7}\\
& P_{J}=P_{Q}+P_{C}  \tag{8}\\
& J=\operatorname{grad}(K) \tag{9}
\end{align*}
$$

где $K$ - scalar potential. From (5-9) that electric current power

$$
\begin{equation*}
P_{J}=E \cdot \operatorname{grad}(K) \tag{10}
\end{equation*}
$$

The charges in scalar potential field have got potential energy. The appropriate power

$$
\begin{equation*}
P_{\rho}=K \rho / \varepsilon \tag{11}
\end{equation*}
$$

where $\rho$-distribution density of summary (free and outside) charges.

Let us assume now, that there exist magnetic charges with density distribution $\sigma$ and magnetic currents

$$
\begin{equation*}
M=\operatorname{grad}(L) \tag{12}
\end{equation*}
$$

where $L$ is a scalar parameter. Then by symmetry we should assume that there exists magnetic current power

$$
\begin{equation*}
P_{M}=H \cdot \operatorname{grad}(L) \tag{13}
\end{equation*}
$$

potential energy of magnetworkic charges and the appropriate energy

$$
\begin{equation*}
P_{\sigma}=L \sigma / \mu \tag{14}
\end{equation*}
$$

where $\sigma$ - distribution density of magnetic charges.
Let us denote also the summary currents power (electric and magnetic)

$$
\begin{equation*}
P_{J M}=P_{J}+P_{M}=E \cdot \operatorname{grad}(K)+H \cdot \operatorname{grad}(L) \tag{15}
\end{equation*}
$$

and the summary charges power (electric and magnetic)

$$
\begin{equation*}
P_{\rho \sigma}=P_{\rho}+P_{\sigma}=K \rho / \varepsilon+L \sigma / \mu \tag{16}
\end{equation*}
$$

Then the equation of power balance of electromagnetic field takes the form

$$
\begin{equation*}
P_{\Pi}+P_{E H}+P_{J M}+P_{\rho \sigma}=0 \tag{17}
\end{equation*}
$$

where the components are determined as $(3,4,15,16)$ accordingly.

## 2. Building the functional for Maxwell equations

### 2.1. Maxwell equations

It is known that Heaviside was the first who introduced magnetic charges and magnetic currents to Maxwell's electrodynamics. At present there have been found some materials whose properties may be interpreted as properties of materials in which there are monopoles and magnetic [8]. Therefore we shall further consider a symmetrical Maxwell equations system with magnetic charges and currents - see, for instance, [3, 4].
Denote:
$E$ - electric field intensity,
$H$ - magnetic field intensity,
$\mu$ - magnetic permeability,
$\varepsilon$ - dielectric permittivity,
$\varphi$ - electric scalar potential,
$\vartheta$ - electrical conductivity,
$\rho$ - electric charge density
$\sigma$ - magnetic charge density,
Let us take into consideration a Maxwell equations system in Cartesian coordinate system of the following form:

| 1. | $\frac{\partial H_{z}}{\partial y}-\frac{\partial H_{y}}{\partial z}-\varepsilon \frac{\partial E_{x}}{\partial t}-\frac{d K}{d x}=0$ |
| :---: | :---: |
| 2. | $\frac{\partial H_{x}}{\partial z}-\frac{\partial H_{z}}{\partial x}-\varepsilon \frac{\partial E_{y}}{\partial t}-\frac{d K}{d y}=0$ |
| 3. | $\frac{\partial H_{y}}{\partial x}-\frac{\partial H_{x}}{\partial y}-\varepsilon \frac{\partial E_{z}}{\partial t}-\frac{d K}{d z}=0$ |
| 4. | $\begin{equation*} \frac{\partial E_{z}}{\partial y}-\frac{\partial E_{y}}{\partial z}+\mu \frac{\partial H_{x}}{\partial t}+\frac{d L}{d x}=0 \tag{1} \end{equation*}$ |
| 5. | $\frac{\partial E_{x}}{\partial z}-\frac{\partial E_{z}}{\partial x}+\mu \frac{\partial H_{y}}{\partial t}+\frac{d L}{d y}=0$ |
| 6. | $\frac{\partial E_{y}}{\partial x}-\frac{\partial E_{x}}{\partial y}+\mu \frac{\partial H_{z}}{\partial t}+\frac{d L}{d z}=0$ |
| 7. | $-\frac{\partial E_{x}}{\partial x}-\frac{\partial E_{y}}{\partial y}-\frac{\partial E_{z}}{\partial z}+\frac{\rho}{\varepsilon}=0$ |
| 8. | $\frac{\partial H_{x}}{\partial x}+\frac{\partial H_{y}}{\partial y}+\frac{\partial H_{z}}{\partial z}-\frac{\sigma}{\mu}=0$ |

where

$$
\begin{align*}
& K=-\vartheta \varphi .  \tag{2}\\
& L=-\varsigma \phi, \tag{3}
\end{align*}
$$

Let us introduce also
$j$ - electric current density,
$m$ - magnetic current density
Then we get

$$
\begin{align*}
& \mathrm{j}=\operatorname{grad}(K)  \tag{4}\\
& \mathrm{m}=\operatorname{grad}(L) \tag{5}
\end{align*}
$$

In abbreviated form the system (1) looks as follows:

$$
\begin{align*}
& \operatorname{rot} H-\varepsilon \frac{d E}{d t}-\operatorname{grad}(K)=0,  \tag{6}\\
& \operatorname{rot} E+\mu \frac{d H}{d t}+\operatorname{grad}(L)=0,  \tag{7}\\
& \operatorname{div} E-\frac{\rho}{\varepsilon}=0  \tag{8}\\
& \operatorname{div} H-\frac{\sigma}{\mu}=0 . \tag{9}
\end{align*}
$$

Let us note some particular features of the equation system (1):

1. the existence of magnetic charges and currents is assumed,
2. instead of electric and magnetic currents we shall introduce scalar potentials and conductivities, not only electrical, but also magnetic ones.
3. it is assumed that the densities of electric and magnetic charges vary with time
4. these equations are extended also to physical systems containing microscopic bearers of electric and magnetic charges (as that we may consider, for instance, the ends of the permanent magnets).
The introduction of electric and magnetic potentials allows considering the system of 8 Maxwell equations as 8 unknown functions 6 intensities и 2 scalar differentials. In the solution of these equations there appear the products $\vartheta \varphi$ and $\varsigma \phi$. A reader who doesn't accept the conception of magnetic resistancy $\zeta$ of the environment and scalar magnetic potential $\phi$, may notice that for $\varsigma=\infty, \phi=0$ the value of the product $\zeta \phi$ is not determined, and may be taken equal to any value required by the problem's conditions. But here another paradox appears: the existence of magnetic current in the absence of magnetic conductivity and scalar magnetic potential. Nevertheless, accepting further the conception of magnetic resistancy and scalar magnetic potential we shall be able to solve some problems with definite physical meaning. We may note also that materials with high permeability $\mu$, as, for example, soft iron, behave approximately as magnetic conductors.

### 2.2. Principle extremum of full action-2

The principle extremum of full action-2 [8] asserts that for this field of physics it is possible to construct a functional with respect to powers, and quasiextremals of this functional are the equations of real dynamic
processes with respect to integral generalized coordinates $q$. In [1] some examples of such functionals in mechanics and electrical engineering are given. In these cases there exists a single independent variable - time on which all the generalized coordinates $q$ depend.

For electrodynamics such functional should include the powers depending on generalized coordinates $q$, which, in their turn, depend not only on time, but on space coordinates.

For further discussion we shall merge the unknown functions of Maxwell equations into a vector of generalized coordinates

$$
\begin{equation*}
q=\left\lfloor E_{x}, E_{y}, E_{z}, H_{x}, H_{y}, H_{z}, K, L\right\rfloor \tag{10}
\end{equation*}
$$

This vector and all its components are functions of $(x, y, z, t)$. We are considering an electromagnetic field in volume $V$, bounded by surface $S$. Full action-2 [1] in electrodynamics we shall submit in the form

$$
\begin{equation*}
\Phi=\int_{0}^{T}\left\{\int_{V} \mathfrak{R}(q(x, y, z, t) d V\} d t\right. \tag{11}
\end{equation*}
$$

Having in mind the definition of Energian-2 in [1], we shall write Energian-2 in the following form:

$$
\begin{equation*}
\mathfrak{R}(q)=\left\{P_{E H}-P_{\Pi}-P_{J M}-P_{\rho \sigma}\right\} \tag{12}
\end{equation*}
$$

Taking into account the formulas (1.3, 1.4, 1.17), we get:

$$
\mathfrak{R}(q)=\left\{\begin{array}{l}
H \cdot \operatorname{rot}(E)-E \cdot \operatorname{rot}(H)+\mu H \frac{d H}{d t}+\varepsilon E \frac{d E}{d t}-  \tag{13}\\
-\left(E \cdot \operatorname{grad}(K)+\frac{K \rho}{\varepsilon}\right)-\left(H \cdot \operatorname{grad}(L)+\frac{L \sigma}{\mu}\right)
\end{array}\right\} .
$$

Let us remind that the necessary conditions of extremum for a functional from functions of several independent variables Ostrogradsky equations [7] have for each function the form

$$
\begin{equation*}
\frac{\partial f}{\partial q}-\sum_{a=x, y, z, t}\left[\frac{\partial}{\partial a}\left(\frac{\partial f}{\partial(d q / d a)}\right)\right]=0 \tag{14}
\end{equation*}
$$

where $f$ - the integrand, $q(x, y, z, t)$ - the variable function, $a-$ independent variable. Further we shall denote the derivative, computed
by this formula, by the symbol $\frac{\partial_{O}}{\partial v}$, contrary to ordinary partial derivative $\frac{\partial}{\partial v}$.

Let us write quasiextremal of the functional (11) for each $i$ component $q_{i}$ of vector $q$

$$
\left\{\begin{array}{l}
\frac{\partial P_{J M}}{\partial q_{i}}-\sum_{a=x, y, z, t}\left[\frac{d}{d a}\left(\frac{\partial P_{J M}}{\partial\left[d q_{i} / d a\right]}\right)\right]  \tag{15}\\
+\frac{\partial P_{\rho \sigma}}{\partial q_{i}}+\frac{\partial P_{\Pi}}{\partial q_{i}}+\frac{\partial P_{E H}}{\partial q_{i}}
\end{array}\right\}=0
$$

The first four terms here correspond to the Ostrogradsky equation (14), and two others are ordinary partial derivatives. Differentiating (28), we get:

- by variable $E=\left\lfloor E_{x}, E_{y}, E_{z}\right\rfloor$ - equation (6),
- by variable $H=\left\lfloor H_{x}, H_{y}, H_{z}\right\rfloor$ - equation (7),
- by variables $K, L$ - equations $(8,9)$ accordingly.


## 3. Splitting the functional for Maxwell equations

Let us associate with the functional (2.11) the functional oa split full action-2

$$
\begin{equation*}
\Phi_{2}=\int_{0}^{T}\left\{\int_{z}\left\{\oint_{y}\left\{\int_{x} \Re_{2}\left(q^{\prime}, q^{\prime \prime}\right) d x\right\} d y\right\} d z\right\} d t \tag{1}
\end{equation*}
$$

Let us present the split energian in the form

$$
\mathfrak{R}_{2}\left(q^{\prime}, q^{\prime \prime}\right)=\left\{\begin{array}{l}
\frac{1}{2}\left(H^{\prime} \cdot \operatorname{rot}\left(E^{\prime}\right)+E^{\prime} \cdot \operatorname{rot}\left(H^{\prime}\right)\right)  \tag{2}\\
-\frac{1}{2}\left(H^{\prime \prime} \cdot \operatorname{rot}\left(E^{\prime \prime}\right)+E^{\prime \prime} \cdot \operatorname{rot}\left(H^{\prime \prime}\right)\right)+ \\
\frac{\mu}{2}\left(H^{\prime} \frac{d H^{\prime \prime}}{d t}-H^{\prime \prime} \frac{d H^{\prime}}{d t}\right)-\frac{\varepsilon}{2}\left(E^{\prime} \frac{d E^{\prime \prime}}{d t}-E^{\prime \prime} \frac{d E^{\prime}}{d t}\right) \\
-\left(E^{\prime} \cdot \operatorname{grad}\left(K^{\prime}\right)+\frac{K^{\prime} \rho}{\varepsilon}\right)+\left(E^{\prime \prime} \cdot \operatorname{grad}\left(K^{\prime \prime}\right)+\frac{K^{\prime \prime} \rho}{\varepsilon}\right) \\
-\left(H^{\prime} \cdot \operatorname{grad}\left(L^{\prime}\right)+\frac{L^{\prime} \sigma}{\mu}\right)+\left(H^{\prime \prime} \cdot \operatorname{grad}\left(L^{\prime \prime}\right)+\frac{L^{\prime \prime} \sigma}{\mu}\right)
\end{array}\right\}
$$

The extremals of integral (1) by functions $q^{\prime}, q^{\prime \prime}$ found from Ostrogradsky equation, are the necessary conditions of the existence of a sole saddle line. It's possible also to prove that these extremals are sufficient condition of this saddle line existence [3, 4]. The optimal functions $q_{O}^{\prime}, q_{O}^{\prime \prime}$ satisfy these extremals.

We may also see $[3,4]$, that optimal functions $q_{O}^{\prime}, q_{O}^{\prime \prime}$, satisfy also the condition

$$
\begin{equation*}
q_{O}^{\prime}=q_{O}^{\prime \prime}, \tag{3}
\end{equation*}
$$

and the sum of these extremals forms the Maxwell equations system (2.1), where

$$
\begin{equation*}
q=q_{o}^{\prime}+q_{o}^{\prime \prime} \tag{4}
\end{equation*}
$$

Thus, the quasiextremal (2.15) coincides with Maxwell equations system and is the necessary and sufficient condition of the existence of a single saddle line of the functional (2.11, 2.12). Consequently, the Maxwell equations system determines the single and always existing extremum of the functional $(2.11,2.12)$.

The existence of a single extremum of the functional allows to solve the Maxwell equations system by the method of gradient motion to extremum [3, 4].

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## Konstantin D. Ryndyuk

# Space-time properties and their influence on Universe modeling 


#### Abstract

In this article, we have to touch upon the very difficult problem in Modern Astronomy - is a distance measurement in the Universe. Lack of knowledge about the space-time properties has led to give birth such concepts like are: "the Expanding Universe", "the Big Bang", "the Dark Matter" and "the Dark Energy". The purpose of this article is a presentation "a fresh opinion" on the Universe evolution, which has appeared during the study of the early-unknown space-time properties. The conclusion, that have been received after considering this "new" space-time properties, bring us to review the correctness of such concepts as "Universe expansion", "the Big Bang", "the Dark Matter" and "the Dark Energy", which have been prevailing in the scientific community now.


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## Introduction

The purpose of this article is to present "a fresh opinion" (as far as possible), or speaking more precisely "a sound" judgment, without any misconceptions or illusions concerning the Universe evolution; this judgment has appeared during the study of the
early-unknown space-time properties (so we shall discreetly call them). The conclusions, that have been received after considering this "new" space-time properties, bring us to review the correctness of such concepts as "Universe expansion", "the Big Bang", "the Dark Matter" and "the Dark Energy", which have been prevailing in the scientific community now.

Let us shortly explain the essence of the concepts, which we are going to criticize:

This theory assumes in its basis that in the beginning (or before the beginning, if you prefer) the matter in the Universe was concentrated inside the negligibly small volume at indefinitely great temperature and pressure. Then, according to the script, this matter blew up with monstrous force. This explosion produced superbeated ionized gas, or plasma. Plasma was expanding homogeneously until it cooled down up to such a degree that it turned into a common gas. Many galaxies appeared inside of this cooling cloud of expanding gas, and inside of these galaxies, there were generations of stars. Then planets developed around some stars, as well as our Earth did. [8]

## 1. Preliminary provisions (vocabulary)

Although concepts such as "expanding Universe" and "the Big Bang" are being disproved by the conclusions of the present research, we shall use terminology of these hypotheses, by virtue of established traditions of the scientific community. We shall particularly apply such terms as "ray velocity", "receding", "speed of receding" and red shift is expressed in kilometers per second.
Now we shall explain the essence of this work:

We found out that the space-time possesses some unique properties, which are very similar to the complex space properties, nevertheless the author does not dare to assert it definitely that the very complex space. As for the properties of a space-time, described in this article, though properly speaking not a complex one, we shall apply some terms referring to complex concept due to their accuracy and expressiveness. Examples of such terms are "real" and "imaginary", and the term "complex" meaning "composite, compound".

## 2. The "complex" space

We found out that the Observed distance of space consists of the vector sum of Real and Imaginary components. Not to frighten unsophisticated readers by using such "odd terms" as complex space, let us explain the reasons of dividing the space into Real and Imaginary parts. This division is made in order to avoid confusion between the Observed and Real distances between the Objects and the Observer of the space. By the way, this very confusion resulted in illusion about the matter deficit in the Universe, which, in its turn, brought forth a hypothesis about the presence of the Dark matter; this issue will be discussed below.

In theory, Real and Imaginary components may be considered, but in practice, we deal only with their complex resultant - Observed distance. Forestalling this question, we shall note that the illusory spacetime properties take theirs effect only on large intergalactic distances, which gave an impulse for revising the views on universal evolution.

Above we mentioned a term "vector sum". This vector sum, according to the Pythagorean Theorem, means the hypotenuse length of a right triangle, which is equal to square root out of sum of square sides. Let us show it on the fig. 1 (hypotenuse length (AC) is always larger than side length of a triangle ( $\mathbf{A B}$ ); this property of hypotenuse will be required during our further explanation).


Fig. 1.

Above we have mentioned "the complex space". If we were speaking about the true complex space, then we would take a root of difference of squares instead of a sum, by virtue of the complex space properties

$$
\begin{aligned}
& Z=a+i b \\
& |Z|=\sqrt{a^{2}-b^{2}}
\end{aligned}
$$

Now we shall imagine, that the segment (AB) is laid on the horizontal line and presents a real component of distance, and perpendicular to it, i. e. vertically, lies the segment (BC) - an imaginary component of distance. Then the Observed distance (AC) is a hypotenuse of the right-angled triangle, which will be equal to the vector sum of triangle sides $(\mathbf{A B})$ and $(\mathbf{B C})$, as shown below

$$
(A C)=\sqrt{(A B)^{2}+(B C)^{2}}
$$

$\mathbf{A}, \mathbf{B}, \mathbf{C}$ points are laid in the same plane, that means that through these three points possible to pass a plane. As a complex plane, it has Real Direction - (we have agreed earlier to consider that it will be the horizontal direction) and Imaginary one (perpendicular to the horizontal line - vertical direction).

## 3. The "new" properties of the space-time

Above there were some cautious hints about "early-unknown space-time properties". What do we know about them? (Some factors influencing these space-time properties are given in Appendix 1.)

These "new" space-time properties have physical interpretation: "As all objects in the Universe radiate energy, space-time properties around these objects are being changed. These changes of properties exert themselves as increasing time delay of signals coming from the Object to the Observer. Increasing time delay of a signal is assimilated with the larger distance to the Object. The red shift effect also contributes to increasing of a signal time delay. Long-term effect of this factor leads to seemingly increasing distance to the object, i. e. "receding" from the observer".

Now we shall place the Observer in point $\mathbf{O}$ of this complex plane, who observes object $\mathbf{M}$ located in point $\mathbf{P}$, (this object can be a star, a galaxy, a quasar, etc.) at real distance $\mathbf{R}=$ (OP) from the Observer. Let us focus on the fact that this object does not move away from the Observer, but is observed as "moving away". So we can consider Object $\mathbf{M}$ as moving on a complex plane from point $\mathbf{P}$ in imaginary direction (perpendicular to real direction, or vertically) to point
$\mathbf{P}^{*}$. Here, as we see, the Object $\mathbf{M}$ has made an imaginary moving $\mathbf{S}^{*}=$ (PP*), and the Observed Distance $\mathbf{S}$ from Object $\mathbf{M}$ towards the Observer located in point $\mathbf{O}$ will be equal to a vector sum of Real distance $\mathbf{R}=(\mathbf{O P})$ and Imaginary moving $\mathbf{S}^{*}=\left(\mathbf{P P}{ }^{*}\right)$ (Fig. 2).

$$
O P=\sqrt{(O P)^{2}+\left(P P^{*}\right)^{2}} \text { Or } S=\sqrt{R^{2}+\left(S^{*}\right)^{2}}
$$

Let's draw an arc with the centre in point $\mathbf{O}$ and radius equal to $\mathbf{S}=$ (OP*) up to the Horizontal line upon which segment (OP) is laid, and thus let's find the $\mathbf{P}_{1}$ point position.
This is the starting position of the system "Object - observer".


Fig. 2.
The observer "sees" how the Object $\mathbf{M}$ has "moved" from point $\mathbf{P}$ along the eye-beam to the point $\mathbf{P}_{1}$, and now this Object $\mathbf{M}$ is "seen" further from the Observer, on the distance $\mathbf{S}$. Once again, let us focus on the fact that this Object has remained on Real distance $\mathbf{R}$ from the Observer, but observed "further" - on the distance $\mathbf{S}$. In reference to Object $\mathbf{M}$, we have two distances: Real one $\mathbf{R}$ and Observed one $\mathbf{S}$, as well as Imaginary moving - the distance of "Receding" $\mathbf{S}^{*}=\left(\mathbf{P P}{ }^{*}\right)$.

Important note. As mentioned above, in the course of abstract theoretical construction we can show that the Object has "receded" from the Observer along the eye-beam; as a rule, in practice we cannot, however, see the process of the Object moving away from the Observer, even if this observation is carried out for many centuries. It is impossible to see from Earth the galaxies moving even with the most powerful telescopes. We can only see motionless pictures. We can record that red shift of the Object spectrum has increased. This increasing red shift, as well as red shift effect itself in the context of the "Universe expansion" hypothesis may be explained as receding of the Object from the Observer.

## 4. Factors causing the red shift

Now we shall pay attention to the "new" properties of the spacetime that we are aiming to research. Let us describe in details the factors, the properties depend on.

We shall start from mentioning that the red shift in radiation spectrum may be caused by the following reasons:

- receding of the source of radiation from the Observer, i.e. Doppler's principle.
- movement of light against gravitational field, with partial loss of energy and occurring of the red shift (Appendix 2).
- some early-unknown space-time properties may cause red shift effect, which depends on emission power, to put it more precisely from a relative emission power.

The following statement was presented above "Because all objects in the Universe have radiated energy around themselves, so space-time properties around these objects are being changed...".

Let's have a detailed look at the system consisting of the Physical body (the Object) radiating energy around itself, and the space surrounding this body. As the body is radiating energy around itself, so the whole energy of this system obviously depends on time. Therefore, partial derivative by time should be added to the Lagrange function, which is determining the system movement, that is $\frac{\partial L}{\partial t} \neq 0$ (formula derivation is in Appendix 3).
As a result, we have the following:

$$
\ddot{q}=\ddot{q}^{m}+\ddot{q}^{n}=\Gamma_{i k}^{m} \dot{q} \dot{q}^{i} \dot{q}^{k}+\Gamma_{j k}^{n} \dot{q^{j}} \dot{q}^{k}
$$

If the system is not depended on time obviously, then its partial derivative by time equals to zero $\frac{\partial L}{\partial t}=0$, so the Lagrange function defining the system condition has simplier view:

$$
\ddot{q}=\ddot{q}=\Gamma_{\mu}^{n} q^{j} \dot{q^{k}}
$$

Let us take this formula and compare it with the foregoing formula.
There are the following designations in these formulas:
$\ddot{q}$ is particle acceleration (the second derivative of the coordinate change by time) in this space area.

- ${ }^{i} \cdot{ }^{j} \cdot{ }^{k}$
$q, q, q$-generalized velocities of particle motion, where indexes $\mathbf{i}, \mathrm{j}, \mathrm{k}$ have accepted values as $\mathbf{i}, \mathfrak{j}, \mathbf{k}=1,2, \mathbf{3}$.
.. ${ }^{m}$
$q$ - particle acceleration under the influence of stationary curved spacetime, which exists around the Heavy Load of Matter. accordingly:
$\Gamma_{i k}^{m}$ - connectedness (Christoffel symbols) determining the space-time curve under the influence of the Matter heavy load.
$\ddot{q}^{n}$ - additional particle acceleration under the influence of energy change factor in this space-time volume.
accordingly:
$\Gamma_{j k}^{n}$ - connectedness determining the space-time curve under the
influence of energy change factor in this space-time volume.
The analysis and comparison of these formulas shows qualitatively us that if the body began to radiate energy, then the environment (the space-time) around it changes its properties simultaneously - as shows additional connectedness. (Here has been given the "qualitatively", estimation, but one is making any practical calculations with these formulas not possible obviously).

Now we shall specify the previously mentioned formulation of "new" space-time properties:
"... If the body radiates energy, i. e. the energy liberation process is taking place, and more intensive the energy liberation (or absorption) process in this point (area) of the space is, the more significant the change of the space-time properties become in this point (area) of the space especially. That is all space-time properties changes directly depend on relative emission power in this point (area) of the energy space..."

## 5. Relative emission power

Let's specify that "the relative emission power" (we shall designate it as $\mathbf{H}$ ) is ratio of the power of energy liberation process $\mathbf{N}$ in this space area, to the total energy $-\mathbf{E}$ existing in same area (volume) of the space $H=\frac{N}{E}$, where is $\quad N=\frac{d E}{d t}$ ( $\frac{\text { joule }}{\text { sec }}$ or watt $)$ - the power of energy liberation process (energy emission), $\mathbf{E}$ - energy (Joule) of this area (volume) of the space, $H=\frac{d E}{d t} \cdot \frac{1}{E}=\frac{N}{E}\left(\frac{1}{\sec }\right)$-dimension of the relative emission power.

## 6. Relative emission power and geometrical properties of the space-time

It was obtained, that the space-time properties change show themselves as downsizing of this body volume, in other words its compression. It is so because the parallelepiped volume change, made up on a basis of metric tensor determinant of the space-time, also depends on the relative emission power in direct ratio to flow of energy in this space area. (Appendix \#4)

In ordinary words: - "If the body radiates energy around itself, so space-time around this body is undergone by some changes. That is this space area is compressed (it decreases in volume), therefore this body is occupying same the space volume compressed too"
This functional dependence is expressed by formula:

$$
\frac{d E}{d t} \cdot \frac{1}{E}=-\frac{\partial \sqrt{h}}{\partial t} \text { or } \frac{N}{E}=-\frac{\partial \sqrt{h}}{\partial t}
$$

or $H=-\frac{\partial \sqrt{h}}{\partial t}$, where $\frac{\partial \sqrt{h}}{\partial t}$ - is expresses space-time properties change, that is unit parallelepiped volume change made up on a basis of a metric tensor. The minus sign "-" shows us that the volume decreases at the expression and together with its reduction of volume the body linear values (extension) decreases too.

And so on, it is necessary to pay special attention: General Relativity (GR) is formed on a principle of "conservation of an interval constancy. " $\partial I=0$ If the interval spatial component is changed, then the interval time component does not remain without changes too
$I^{2}=\left(x^{0}\right)^{2}-\left(x^{1}\right)^{2}-\left(x^{2}\right)^{2}-\left(x^{3}\right)^{2}, \quad$ or $\left(x^{0}\right)^{2}=\left(x^{1}\right)^{2}+\left(x^{2}\right)^{2}+\left(x^{3}\right)^{2}, \quad$ where $x^{0}=c \cdot d \tau-$ is the interval time component.
In ordinary words: - "the volume decreases, while time interval increases." The signal delay time increases from the Object up to the Observer. This increase of the signal delay time is shown as red shift effect. A.L. Zelmanov pointed at this phenomenon in his work: [3] (Appendix \#5)
literally - " ... this non-relativistic effect is similar to Doppler's caused by the reference system deformation ... "

## 7. Relative emission power and red shift

In general, that turns out such long "chain" of the cause-andeffect relation:
" ... That is any object radiates energy

- Thereof, the space-time properties varied around of this object;
- This space-time properties change is shown as increase in signal time delay from the Object up to the observer in particular;
- The Observer will record this time increase as the demonstration of the red shift effect in a Object radiated spectrum ... ".

Naturally, the question arises as to whether - " Could it be such formula? Which would show us how the Object radiation influences directly on its object red shift effect "?
Such formula was found. (Appendix \#6)
Here is $\frac{d E}{d t} \cdot \frac{1}{E}=\frac{c}{\omega} \cdot \frac{\partial \omega}{\partial u}$.
Where is $\partial u=c \cdot \partial \tau$ - the Distance from the Object up to the Observer,
$\mathbf{c}$ - velocity of light in vacuum, $\frac{d E}{d t} \cdot \frac{1}{E}=H$ - relative emission power,
$\omega$ - cyclic frequency,
$\frac{\partial \omega}{\omega}=z$ - red shift; the red shift is defined in wave-length (wave frequency).

In this formula shown, that the red shift effect $\frac{\Delta \omega}{\omega}$ is directly proportional to the relative emission power $H \Rightarrow \frac{\Delta \omega}{\omega} \approx H$, as well as, that this value $\frac{\Delta \omega}{\omega}$ to a first approximation proportional to distance $(\partial u)$ from the Object up to the Observer.

## 8. The formula of the red shift and the Hubble law

Now we shall do with this formula the following "cunning" transformations:

Let express $\quad \partial u$ through $\quad \tau: \quad \partial u=c \cdot \partial \tau=r$,
$\frac{d E}{d t} \cdot \frac{1}{E}$ we shall write as $\mathbf{H}$, that is $\frac{d E}{d t} \cdot \frac{1}{E}=H ; \frac{\Delta \omega}{\omega}$ we shall write through $\mathbf{z}$. For one's turn, we are known that $c \cdot z=V^{*}$ - is the object "Ray velocity" (in the expanding Universe concept).

As a result, we shall receive the following $H=\frac{c \cdot z}{r}$. Then we shall continue the transformation: we shall replace the expression $C \cdot Z$ on $V^{*}$ and this both reformed expression parts multiply by the $\mathbf{r}$, so we shall receive the $H \cdot r=V^{*}$ expression, one is swapping both parts - we shall finally receive the expression $V^{*}=H \cdot r$.

Which shows us, that - "the object ray velocity $V^{*}$ measured by means of red shift, directly proportional to the distance $\mathbf{r}$ up to the Object. As well as directly proportional to the relative emission power $\mathbf{H}$ of this Object radiation.

Now we compare it (this formula) to the Hubble law.
Which shows us, that is - "the Galaxy (Object) ray velocity measured by means of red shift is proportional to the distance $\mathbf{r}$ up to it " - that is $V^{*}=r \cdot H_{\text {Hubble }}$.

Are these both formulas so very similar? As it will be shown below - the Hubble constant (parameter) is the mean value of the relative emission power of all Objects in the Universe. In other words, it
represents itself the mean value of the relative emission power of the Universe as a whole. Therein lays a problem of its exact calculation.

Let us mention some supplementary information:
However, there are many various values of the Hubble constant, which were received by different scientists in different year. There is the Hubble constant value was 500 in 1929 year. There is it was equal 550 in 1931 year. There is it was equal 520 or 526 in 1936 year. In 1950 year, it has been given as 260, that is has considerably fallen. In 1956 year, it has fallen up to 176 or 180 . In 1958 year it has fallen, still more downwards, up to 75, but in 1968 it has jumped up back up to 98. In 1972, by the bighest standards, it reached from 50 down to 130. Today, the Hubble constant value bas been accepted as 55. Nevertheless, the different observers receive the various Hubble constant values yet. Tammann and Sandage give 55 plus or a minus 5. Abell and Eastmond come to 47 plus or a minus 5. Then Van den Bergh has calculated between 93 and 111. As an illustration, Heidmann bas given the Hubble constant value as 100. De V aucoulers bas come to 100 plus or a minus 10. (The Hubble constant is counted in kilometers per second on megaparsec).

## 9. Features of the "new" space-time's property

Let sum it up and we shall answer on the earlier question

## " What is a manifestations of the "new" space-time properties? What are the factors which it depend on "?

- This "new" space-time properties influence begins to manifest itself when is taking place a flow of energy in any spatial region. That is, any body (any Object) starts to radiate (or Absorb) energy around itself.
- The exterior Observer "is seeing" (if he will observe an inordinate length of time, an endless amount of millions years) that a body (an Object) starts to be compressed, that is it decreases in volume, as well as the body visible lateral dimension $-\mathbf{d}$ decreases too

$$
d=d_{0} \cdot \sqrt{1-\left(\frac{H \cdot r}{c}\right)^{2}}, d=d_{0} \cdot \sqrt{1-(H \cdot \Delta t)^{2}} .
$$

The body lateral dimensions are practically not observed in Astronomy, but it calculated by indirect method, so check-up the aforecited formulas are becoming complicated because of it.

- There is also an increase of the signal time delay $-\tau$ from this Object up to the Observer.
- Red shift has appeared in the Object radiated spectrum.

Comparing with two last observations is as - " an increase of the signal time delay " and " an increase of the red shift effect " the observer has drawn a conclusion that this Object "has moved away" from him.
All this above-named properties is directly proportional to the relative emission power of the flow of energy, which is taking place in this spatial region. That is it depends on the Object relative emission power $\mathbf{H}$ and lifetime of this factor $\Delta t$. As is easy to see, that all this change (i.e. an increase signal time delay) "is operating" with a progressive total, being accumulated and summed up.
Let to precise this computation. We shall do the following operation with the aforecited formula

$$
H=\frac{c \cdot z}{r}
$$

we shall replace the variable $\mathbf{r}$ on the expression

$$
r=c \cdot \Delta t
$$

We shall receive

$$
H=\frac{c \cdot z}{c \cdot \Delta t}
$$

Our next operation is to cancellation on $\mathbf{c}$,

$$
H=\frac{c \cdot z}{c \cdot \Delta t} \Rightarrow H=\frac{z}{\Delta t}
$$

We shall transfer the value $\Delta t$ to the left-hand side of this equation, as a result, we shall receive the following:

$$
H \cdot \Delta t=z \text { or } z=H \cdot \Delta t .
$$

Let have a look more attentively at last expression

$$
H=\frac{c \cdot z}{c \cdot \Delta t}
$$

Evidently - the red shift $\mathbf{z}$ is product of two variables - $\mathbf{H}$, which can change itself on a large scale (a star development cycle is it from its "birth" to its "death") and a variable $\Delta t$, which grows permanently faithfully to present this formula as the sum $z=\sum_{H} \sum_{\Delta t} H \cdot \Delta t$ and passing from summation $\Delta t \Rightarrow d t$ on to integration, then possible to write the following $z=\int H d t$.

However, this space-time properties change is so little in itself becoming visible only after the expiration of long time (Millions and Billions years). For example, our Sun data

$$
H_{\otimes}=2.141 \cdot 10^{-21} 1 / \mathrm{cek}
$$

By the way, this (ability for accumulation and summation) is an explanation for such phenomenon as "acceleration of the Universe expansion".

## 10. Two temporal dimensions and the relative emission power

How to compute these changes used for that the aforecited formulas such as $H=\frac{c \cdot z}{r}, V^{*}=H \cdot r$ and $V^{*}=c \cdot z$ ?
Let finds the ratio for two velocities.
There are the formulas connecting the interval time change from the rate of movement $\tau=\frac{\tau}{\sqrt{1-\beta^{2}}}$ in the Special Relativity (SR), where the factor
$\beta$ - is the ratio $\beta=V / c$ of the $\mathbf{V}$ rate of movement to the $\mathbf{c}$ velocity of light. We shall substitute the $V^{*}$ instead of the $V$.
Now we shall receive the following formula

$$
\frac{\tau_{0}}{\tau}=\sqrt{1-\left(\frac{H \cdot r}{\mathrm{c}}\right)^{2}}
$$

having replaced the $\mathbf{z}$ values instead of the $\beta$. In case is to substitute the expression $r=c \cdot \Delta t$ instead of the $\mathbf{r}$, then we shall receive a certain remarkable formula,

$$
\frac{\tau_{0}}{\tau}=\sqrt{1-(H \cdot \Delta t)^{2}}
$$

which is connecting two temporal dimensions are such as $\mathbf{t}$ and $\tau$, where $\mathbf{t}$ - acts as "an Object age", $\mathcal{\tau}_{\text {- is a time lag. By the way, in his works [1] }}$ and [2], the astronomer Halton Arp suggested the following statements that made without any mathematical justification:

- That the Red shift (i.e. the red shift effect) is not the manifestation of the Doppler effect.
- The red shift effect is closely connected to the Object state.
- Halton Arp in his cosmological model has proposed to apply the Galaxy age as in parameter.


## 11. Optical phenomenon of the "new" spacetime property

Let us have a "look" at this "new" space-time properties from "another point of view", in order to check-up the accuracy of the given above formulas. What is it turns out? There is the volume decrease and the time dilation (the increase of time). We are "seeing" here the whole of a characteristic of "the curved space" (the deformed space), which it acts against our "usual" "not deformed" space as a denser optical medium. We shall explain that the $\mathbf{n}$ refraction coefficient is the ratio of electromagnetic constant to velocity of light in medium

$$
n=\frac{c}{V}
$$

In other words, light goes more slowly in denser optical medium than in vacuum. That is, the light will overcome the same distance in denser optical medium for longer time. Therefore, we can conceive a refraction coefficient as a ratio of two time lags.
Here is $\tau_{0}$ - a time lag, which would be required to light to overcome any certain distance if it (light) moved in vacuum. Whereas is $\tau$ - a time lag, which would do in denser optical medium.

Let us go to the background and "shed light on" that triumphal experiment. Arthur Stanley Eddington is English astronomer who has observed the full solar eclipse in 1919 thereby has confirmed the General Relativity conclusions (GR).
" ...One way of doing examination the GR conclusions about the space-time distortion near massive bodies is studying of a beam deviation of the light coming near Sun. One photo of the starry sky is made during a solar eclipse, and another - after balf a year of the same site of the sky. Then both photos are matched to define visible shift of stars.
"... The shift of some hundreds stars positions were measured and it turned out, that a deviation of light is equal $2^{\prime \prime}$ on average. The General Relativity value is predicted as $1,75{ }^{\prime \prime} "$. This experiment has certainly proved the GR prediction, but has remained the undecided $\mathbf{0 , 2 5}$ " of a beam deviation which has not found its explanation when this epochmaking experiment was carried out. (So, the author of this article with his theory, which you a dear reader can read at present time, seriously pretends to this "bonus" $\mathbf{0 , 2 5}$ ").

Let us recollect from the school physics course (Fig. 3) that the refraction coefficient $\mathbf{n}$ is the ratio of a sine of the light angle $\alpha_{1}$ to a sine of the angle of refraction $\alpha_{2}$ too


Fig. 3.
How could these angles measure up? We shall consider that on the surface, nearby at it, of our nearest star that is the Sun (Fig. 4), this "deformed" space is available there. A light beam from that distant star is passed on the tangent to surface that is nearby to the Sun surface. Thereby an incident ray is coming at an angle of ninety degrees that is at right angles $\mathbf{9 0}$. (If the light beam would enter at right angle to a surface that is on a normal, the angle would be equal to $\mathbf{0}^{0}$ (zero degree)). The light beam enters to surface (an angle $\mathbf{9 0}^{\mathbf{0}}$ ), and leaves its surface under little bit smaller angle, than $\mathbf{9 0}^{\mathbf{0}}$. Now we shall count the difference between of these two angles. As we know, the sine value of the angle $90^{\circ}$ is equal to 1 .

$$
\alpha_{1}=90^{\circ} \sin \alpha_{1}=\sin 90^{\circ}=1
$$

$$
n=\frac{\sin \alpha_{1}}{\sin \alpha_{2}}=\frac{\tau}{\tau_{0}}=\frac{1}{\sqrt{1-(H \cdot \Delta t)^{2}}} \quad \alpha_{2}=\arcsin \left(\sqrt{1-\left(H_{\infty} \cdot \Delta t_{\alpha}\right)^{2}}\right)
$$

Here are:
$\mathbf{H}_{\mathrm{a}}$ - is the relative emission power of our Sun,
$H_{a}=2,14110^{-21} 1 / \mathrm{sec}$,
$\Delta \mathbf{t}_{\mathrm{a}}$ - is a "life time" of the Sun,
$\Delta \mathrm{t}_{\mathrm{a}}=3600 \times 24 \times 365 \times 5,4 \cdot 10^{9}=1,70294 \cdot 10^{17}$ seconds, $\alpha=90^{0}-\alpha_{2}=90^{0}-\arcsin \left(\sqrt{1-\left(H_{\infty} \cdot \Delta t_{\alpha}\right)^{2}}\right)$,
$\boldsymbol{\alpha}=\mathbf{0 , 2 1}$ " that required deviation which has not earlier provided its explanation.


Fig. 4.

## 12. "Hot" and "Cold" Power Sources

Now we shall show, as this "reopened" space-time properties " finds its reflection " at an observation over the Universe Objects. In order to making the better presentation of the next narration, we shall invent such terms as "hot" and "Cold" Objects. We shall name the "Cold" Objects, the objects that are having the relative emission power similar to our Sun. However, we shall name the "hot" Objects, the objects that are having relative emission power, similar to quasars, for example the quasar 3C273.

By the example of our nearest star - the Sun, we shall do the following: we shall divide a numerical value of the solar luminosity $\mathbf{L}_{\otimes}$ is expressed in watts into the solar mass $\mathbf{M}_{\otimes}$ is expressed in kilogram accordingly, thereby we shall find the specific power- $\mathbf{N}^{*} \otimes$. Evidently, this specific power value is not big - a household electric heater has much higher values of it. This specific power can be compared with the power that it released by the carrion leaves, which are collected in heaps after the autumn fall of the leaves.

$$
N_{\otimes}^{*}=\frac{L_{\otimes}}{M_{\otimes}}=\frac{3.826 \cdot 10^{26} \mathrm{Watt}}{1.989 \cdot 10^{30} \mathrm{~kg}}=1.924 \cdot 10^{-4} \frac{\mathrm{Watt}}{\mathrm{~kg}}
$$

For comparison, we shall result the characteristics of one well-known quasar 3C 273: mass $\mathrm{M}_{\mathrm{Q}} \sim 1 \mathbf{1 0}^{8}$ solar mass, Luminosity - $\mathrm{L}_{\mathrm{Q}} \sim 10^{39}$ Watt. Just as in aforecited example, we shall find its specific power.

$$
N_{Q}^{*}=\frac{L_{Q}}{M_{Q}}=\frac{1 \cdot 10^{39} \mathrm{Watt}}{10^{8} \cdot 1.989 \cdot 10^{30} \mathrm{~kg}} \approx 5 \frac{\mathrm{Watt}}{\mathrm{~kg}}
$$

Further, if the specific power value to multiply on the multiplier, which is

$$
\frac{1}{r^{2}}
$$

equal to an inverse square of a value of the velocity of light $c$ equal

$$
\frac{1}{\left(2.998 \cdot 10^{8} \mathrm{~m} / \mathrm{sec}\right)^{2}}=1.113 \cdot 10^{-17} \mathrm{Sec}^{2} / \mathrm{m}^{2}
$$

that we shall receive $\mathbf{H}$ is the relative emission power value

$$
H=\frac{N}{E}=N^{*} \cdot \frac{1}{c^{2}}=\frac{L}{M \cdot c^{2}}, L=N_{\text {and }} E=M \cdot c^{2}
$$

that the dimension of the relative emission power is - a minus second

$$
H=\frac{N}{E} \sim 1 / \mathrm{sec}
$$

Both the specific power and the relative emission power differ between themselves in only $\frac{1}{c^{2}}$ the constant factor. For instance, for our Sun $\mathbf{H}_{\otimes}$ is the solar relative emission power

$$
H_{\otimes}=\frac{N_{\otimes}}{E_{\otimes}}=1.924 \cdot 10^{-4} \frac{\mathrm{Watt}}{\mathrm{~kg}} \times 1.113 \cdot 10^{-17} \frac{\mathrm{sec}^{2}}{\mathrm{~m}^{2}}=2.141 \cdot 10^{-21} 1 / \mathrm{sec}
$$

For the Quasar 3C $\mathbf{2 7 3} \mathbf{H}_{\mathrm{Q}}$ is the relative emission power

$$
H_{Q}=\frac{N_{Q}}{E_{Q}}=5 \frac{\mathrm{Watt}}{\mathrm{~kg}} \times 1.113 \cdot 10^{-17} \mathrm{sec}^{2} / \mathrm{m}^{2}=5.565 \cdot 10^{-17} 1 / \mathrm{sec}
$$

## 13. The Hubble constant

Now we shall consider the Hubble constant (or the Hubble parameter). On average, the Hubble constant has its numerical value

$$
55 \frac{\mathrm{~km}}{\mathrm{sec}} \text { on } 1 \text { Megaparsec }\left(50 \div 100 \frac{\mathrm{~km}}{\mathrm{sec}} \text { on } 1 \text { Megaparsec }\right)
$$

Let divide kilometers per second on Megaparsec, thereby we shall receive the $H_{\text {Hubble }} \approx 1.7 \cdot 10^{-18} 1 / \mathrm{sec}$. The dimension of that obtained expression is - a minus second. Comparing an earlier received values of

$$
\begin{gathered}
H_{\text {Hubble }} \approx 1,7 \cdot 10^{-18} 1 / \mathrm{sec} \\
H_{Q}=2,141 \cdot 10^{-211} / \mathrm{sec} \\
H_{Q}=5,565 \cdot 10^{-17} 1 / \mathrm{sec}
\end{gathered}
$$

The relative emission power for our Sun, the quasar 3C 273 and Hubble constant, possible to dare say, that the Hubble constant, in its deep essence, expresses a mean value of the relative emission power of the Universe.

What is "a mean value of the relative emission power of the Universe "? For this purpose, we shall perform a rapid calculation, which in no circumstances, one ought not be counted exactly. Let $\mathbf{x}$-are a number of the "cold" objects in the Universe, similar to our Sun, having the relative emission power $-\mathbf{H}_{\otimes}$. Whereas $\mathbf{y}$-are a number of the "hot" objects in the Universe, similar to the quasar 3C 273, having the relative emission power $\mathbf{H}_{\mathrm{Q}}$. We shall make a ratio of these values:

$$
H_{\text {Hubble }}=\frac{x \cdot H_{\odot}+y \cdot H_{o}}{x+y}
$$

In that case, the Hubble constant will represent itself the mean value of " a blend" of some "cold" and some "hot" objects. We shall find a numerical ratio from the aforecited expression between a number of the objects similar to our Sun and a number of objects which are similar to the quasar 3C 273. Here are

$$
\frac{x}{y}=\frac{H_{Q}-H_{\text {Hubble }}}{H_{\text {Hubble }}-H_{\otimes}}, \frac{x}{y} \approx 32, y \approx 3.1 \%
$$

Evidently according to this estimation that a number of quasars (and the "hot" objects similar to quasars) in the Universe should make not less than $3 \%$ from all objects in it. Nevertheless some astronomical observations show us, that such objects are much less in its number. What is the matter? The fact is that these "hot" as well as "the super heavy" objects have ceased to be visible!" The paradox consist in - an object radiates enormous number of energy; instead of this not visible! How should we understand it! To explain this paradox is used such method as analogy. (Ones shall notice, that the analogy is not exact the mathematical proof, only a train of thought so quite suitable in given article).

## 14. Explanation of the paradox

For that purpose we shall result a conclusion of the formula of an imaginary velocity of the "seeming" receding $\mathbf{V}^{*}$. For this purpose in Hubble's law, that shows that - " the Galaxy (Object) ray velocity measured by means of red shift is proportional to the distance $\mathbf{r}$ up to it"

$$
V=H_{\text {Hubble }} \cdot r
$$

We shall replace the value $\mathbf{r}$ on the expression $c \cdot \Delta t$, that is $r=c \cdot \Delta t$ and let substitute it in this formula, next we shall substitute the Hubble constant for the value of the relative emission power $\mathbf{H}$, in a result we shall receive:

$$
V^{*}=c \cdot H \cdot \Delta t
$$

Let us write out also one more formula from a school course of Physics. A well-known simplest formula, which bound up with the value of a rate of movement with acceleration of this body $V=a \cdot \Delta t$. Here is

V - velocity (rate of movement),
$a$ - acceleration,
$\Delta t$ - time.
How in our case should we find acceleration? For that, let multiply the value of the relative emission power $\mathbf{H}$ on a multiplier $\mathbf{c}$ (c the velocity of light in vacuum) we shall receive that very imaginary acceleration which the object "is receding" from the observer.

$$
V^{*}=c \cdot \frac{N}{E} \cdot \Delta t^{\prime}
$$

Here is $\mathbf{V}^{\mathbf{*}}$ - ray velocity of the object imaginary receding from the observer, $a=c \cdot \frac{N}{E}$. imaginary acceleration. These three values for our previously mentioned triple will be the following:

$$
\begin{gathered}
a_{Q}=H_{Q} \cdot \mathrm{c}=6,418 \cdot 10^{-13} \mathrm{~m} / \mathrm{sec}^{2} \\
a_{Q}=H_{Q} \cdot \mathrm{c}=1,668 \cdot 10^{-8} \mathrm{~m} / \mathrm{sec}^{2} \\
a_{H u b b l e}=H_{H u b b l e} \cdot \mathrm{c}=5,096 \cdot 10^{-10} \mathrm{~m} / \mathrm{sec}^{2}
\end{gathered}
$$

Right now, we shall ask a question - how much time is needed to any object to move with such "acceleration" for an achievement of the value of velocity of light? Any object becomes invisible for observation on reaching the velocity of light, as we know, i.e. it "will disappear" from "field of vision". For a finding the object time value - we shall divide the value of velocity of light on the object acceleration value

$$
\begin{gathered}
t=\frac{V}{a} \Rightarrow T=\frac{c}{a}, \quad V \Rightarrow c \\
T=\frac{c}{a} \Rightarrow \frac{c}{c \cdot H}=\frac{1}{H}
\end{gathered}
$$

As may be seen from these calculations, this time is equal to reciprocal of the value of the relative emission power $\mathbf{H}$. Let makes the following calculations for our "triple" of objects.

$$
\begin{aligned}
& T_{\otimes}=\frac{c}{a_{\otimes}}=\frac{c}{c \cdot H_{\otimes}}=\frac{1}{H_{\otimes}}=\frac{1}{2.141 \cdot 10^{-21} 1 / \mathrm{sec}}=4.671 \cdot 10^{20} \mathrm{sec} \\
& T_{Q}=\frac{c}{a_{Q}}=\frac{c}{c \cdot H_{Q}}=\frac{1}{H_{Q}}=\frac{1}{5.565 \cdot 10^{-17} 1 / \mathrm{sec}}=1.797 \cdot 10^{16} \mathrm{sec} \\
& T_{\text {Huble }}=\frac{c}{c \cdot H_{\text {Hubble }}}=\frac{1}{H_{\text {Hublle }}}=\frac{1}{1.7 \cdot 10^{-18} 1 / \mathrm{sec}}=5.882 \cdot 10^{17} \mathrm{sec}
\end{aligned}
$$

Let have a look more closely at last expression - $\mathbf{T}_{\text {Hubble }}$. In front of us is the very age of the Universe, it is calculated based on "the Big Bang" data! Further, we shall find for our "triple" of objects "the horizon of visibility" $\mathbf{D}$, for this purpose we shall multiply the value of time $\mathbf{T}$ by the velocity of light in vacuum - c. Again, we shall look at last expression $\mathbf{D}_{\text {Hubble }}$. "the extension" of the Universe - according to " the Big Bang" data!

$$
\begin{aligned}
& D_{\otimes}=T_{\otimes} \cdot c=4.671 \cdot 10^{20} \mathrm{sec} \cdot 2.998 \cdot 10^{8} \mathrm{~m} / \mathrm{sec}=1.400 \cdot 10^{29} \mathrm{~m} \\
& D_{Q}=T_{Q} \cdot c=1.797 \cdot 10^{16} \mathrm{sec} \cdot 2.998 \cdot 10^{8} \mathrm{~m} / \mathrm{sec}=5.387 \cdot 10^{24} \mathrm{~m} \\
& D_{\text {Hublle }}=T_{\text {Hublle }} \cdot c=5.882 \cdot 10^{17} \mathrm{sec} \cdot 2.998 \cdot 10^{8} \mathrm{~m} / \mathrm{sec}=1.763 \cdot 10^{26} \mathrm{~m}
\end{aligned}
$$

The analysis of these values shows to us that if the value the horizon of visibility $\mathbf{D}$ is less than value of horizon of visibility $\mathbf{D}_{\text {Hubble }}$ for the Universe, such object not "visible" or it becomes "invisible"! Paradoxical! Just fancy a super-power and super heavy "hot" object, which radiates huge energy, and this object becomes "invisible"! While some rather small and "cold" objects "do not disappear" anywhere! (One of the "comic" proofs is presented in the appendix \#7). However, those not numerous quasars, which else continue to open, just observed on border of a visible part of the Universe. Therefore, we see only the "cold" and limited part of the Universe. The story is not complete. A light that is the Universe objects radiation, which has remained beyond the "horizon of visibility", nevertheless to us accessible. Some photons are overcoming this "horizon of visibility" barrier owing to its quantum features. A relict radiation is also a "light" of that Universe that broken through this "horizon of visibility". For now, this effect waits for its researchers.

## 15. Formula for calculating the Observed distance length

For the further explanation, we will need the formula for the

Calculation of Observed distance, for its conclusion such method as analogy is used also. For this purpose, we shall write out from a school course of Physics some body accelerated motion formulas

$$
V=a \cdot \Delta t, S=R_{0}+\frac{a \cdot(\Delta t)^{2}}{2}
$$

Here are: $\mathbf{V}$ - a velocity, $\boldsymbol{\alpha}$ - an acceleration, $\mathbf{R}_{0}$ - an initial remote distance, $\mathbf{S}$ - a passed distance, $\Delta \mathbf{t}$ - a traveling time.

We shall re-arrange some equations so that they have velocity and acceleration in them (because they are accessible to measurement) $S=R_{0}+\frac{a \cdot(\Delta t)^{2}}{2}$. Above it was shown how to find an imaginary acceleration $\alpha=c \cdot H, V^{*}=c \cdot H \cdot \Delta t$. Then we shall receive on a complex plane the followings (Fig. 5):

$$
S^{*}=\frac{\left(V^{*}\right)^{2}}{2 \cdot c \cdot H}
$$



Fig. 5.
Here is "an Imaginary moving" of the object that moving in an imaginary direction on a vertical, $\mathbf{S}$ - is the Observed distance, $\mathbf{R}_{0}$ - is the object initial remote distance from the observer (the real distance)

$$
S=\sqrt{\left(R_{0}\right)^{2}+\left(S^{*}\right)^{2}=\sqrt{\left(R_{0}\right)^{2}+\left(\frac{\left(V^{*}\right)^{2}}{2 \cdot c \cdot H}\right)^{2}} \text {. }}
$$

Here is the formula for calculation of a length of the Observed distance.

## 16. "Imaginary" experiment No. 1

Right now, we shall do one "mental" experiment, with the purpose of showing an illusion of "the Universe exfoliation". In fact, why such super-power Energy sources as quasars "are observed" at the edge of a "visible" part of the Universe, besides them are "moved away" from us with a huge velocity? Why they (the quasars) are not close by us? In
fact, the cosmological principle says that the Universe is homogeneous and isotropic. Where from such "Exfoliation", heterogeneity is appears?


Fig. 6.
We shall place the Observer in the center (Fig. 6). We shall place "hot" objects on any equal distance $\mathbf{R}_{1}=\mathbf{R}_{0}$ (The Real distance) around of him. As it is well known "hot" objects are super-power energy sources are designated at numbers \#1 in the foreground of the picture. Let locate the "cold" objects which are designated at numbers \#2 on the double distance $\mathbf{R}_{2}=\mathbf{2} \mathbf{R}_{0}$ in the background of the picture. The space-time properties around of all these objects will change after the expiry of a long time i.e. plenty of millions and millions years. A greater change will take place near to the "hot" objects rather than it will do around of the "cold" objects. As it turned out earlier, the "hot" objects have moved away considerable "farther" rather than their "cold" neighbours have done. Our Observer "will see" the following: the "cold" objects will "be" in the foreground and the "hot" objects will "be" on the background. That is to say, opposite to the original position. Because $\mathbf{H}_{1}>\mathbf{H}_{2}$ (The relative emission power of the "Hot" objects is higher then the relative emission power of the "cold" objects). $\mathbf{V}_{1}{ }^{*}>\mathbf{V}_{2}{ }^{*}$ (Speed of receding of the "Hot" objects is greater than speed of receding of the "Cold" objects.) So $\mathbf{S}_{\mathbf{1}}^{*}>\mathbf{S}_{2}^{*}$ (distances of "receding" the "Hot" objects is greater than the "Cold" objects.) It follows from this that $\mathbf{S}_{1}>\mathbf{S}_{2 \text {. }}$ (The Observed distance $\mathbf{S}_{1}$ of the "Hot" objects is greater than the Observed distance $\mathbf{S}_{2}$ of the "Cold" objects). On the expiry still any long time (Figure 7), our Observer will not find out the "hot" objects, and then the turn "to
disappear" will approach to "cold" objects. It is an explanation why the quasars and other "hot" energy sources are observed at the edge of a visible part of the Universe!


Fig. 7.
After a while the Observer "will see" the next picture.
The same "mental" experiment is on a "complex" plane (Fig. 8).

The Real distance -R


Fig. 8.

## 17. "Imaginary" experiment No. 2

Now we shall carry through one more "imaginary" experiment \#2 in order to give an explanation to the anomalous red shift from Halton Arp's observations - "he reports that he has found an Object with the great red shift located in a close proximity to another one with a small red shift". According to the aspects theory of the expanding Universe, the Object with the small red shift should be located comparatively closer to us while the Object with the big red shift - much farther. Therefore, two objects located in a close proximity shall be defined by approximately the same red shift. However, Arp exemplifies the following: the Spiral Galaxy NGC7603 is connected to the next galaxy with the gleamy bridge, and nevertheless the next galaxy has the red shift bigger for $\mathbf{8 0 0 0}$ kilometers per second than the spiral Galaxy. If judge on a difference of their red shift, galaxies should be in significant distances from each other, definitely the next galaxy should be on 478 than
millions light years are farther - already strange, in fact two galaxies are close enough for physical contact. For comparison, Our Galaxy (the Milky Way) is away from nearest "neigbour" a galaxy M31 (NGC224) whom is located in Andromeda constellation at 2,9 million light years only. This is one more the Arp disputable discovery: the quasar Makarian 205 is near to the spiral galaxy NGC4319, visually connected to this galaxy by means of the luminous bridge. The galaxy has red shift of 1700 kilometers per second, corresponding to distance about 107 million light years. The quasar has red shift 21000 kilometers per second that should mean, that on the distance of 1,24 billion light years. However, Arp has assumed that objects are definitely connected. [For example, the disturbed galaxy NGC4319 and the nearby quasar Makarian 205 have very different Redshifts ( $\mathbf{C Z}=1,700$ and 21,000 respectively), get anyone can see from the photographs that they are connected. Thus, the quasar is close to the galaxy in space, not at its red shift distance according to the Hubble law. Despite much criticism, several independent lines of evidence have confirmed this result, which plainly contradicts conventional assumptions.] For an explanation, we shall apply the same pattern of reasoning is consisting from the Observer, the "hot" and "cold" objects. So that these two galaxies are connected by the luminous bridge that both them are in physical contact there, most likely they are away from us (Observer) on the same Real distance. Repeating all aforecited judgments, the observer will see the following picture that these two galaxies are removed from each other on significant distance on the expiry any long time. Just like this conclusion, possible to suggest as an explanation of an Observed picture - the Galaxies NGC4319 and the quasar Makarian 205. With all due evidence these examples show us the distinctions between the Observed and the Real distances from the Object up to the observer. This article was writing for the purpose of an elimination of confusion between two these distances. The further narration will be proceed about that how this "substitution" of the Real distance for the Observed distance has led to an appearance of an illusion of a "shortage" of matter, that is "the Dark Matter's" hypothesis.

## 18. An illusion of shortage of matter

To understand the problem essence let us return to its background. Above we marked that excess of the Observed distance over the Real distance takes its effect only on the big intergalactic scales of distances. By the way, there is Fritz Zwicky (1933) "has found" "the

Dark Matter" and Mordechai Milgrom (1987) has suggested making changes to the Newton law in same intergalactic scales.

Here is some supplemental information:
Many years the scientists were in the big difficulty in an explanation of galaxies' movement dynamics in terms of the law of gravity. Jan Oort (1933) bas noticed that our galaxy stars move too quickly in order that their attractive interaction has not allowed them to scatter. Friť Zwicky and Sinclair Smith measured the galactic cluster velocity in constellations of Berenice's Hair and Virgo. According to an orbit received from prospective velocity, the galaxies should be much more massive. For an explanation of an absent mass of the bodies, not sacrificing the law of gravity, astronomers assume an existence of the enormous invisible dark matter. Some people speak that $90 \%$ of the Universe' mass is invisible.

Fritz Zwicky (1933) studied a rotation the remote galaxy around of a galactic cluster. If possible, we can reduce simplistically this system to a Keplerian problem(Fig. 9) in which the remote galaxy rotates around of the center mass of a galactic cluster; in which one body (Object) $\mathbf{M}_{2}$ rotates around more massive body $\mathbf{M}_{1}$. $\mathbf{F}_{\text {in }}$ - a centrifugal force of inertia of the small body in mass $\mathbf{M}_{2}$ is moving in a circle of around of massive body $\mathbf{M}_{1} . \mathbf{F}_{\text {in }}$ - is counterbalanced by force of their attractive interaction $\mathbf{F}_{\mathbf{G r}}$ - under the action of the law of Newton (the law of gravity).

$$
F_{i n}=\frac{M_{2} \cdot V^{2}}{R} ; F_{G r}=G \frac{M_{1} \cdot M_{2}}{R^{2}} ; F_{i n}=F_{G r} \Rightarrow \frac{M_{2} \cdot V^{2}}{R}=G \cdot \frac{M_{1} \cdot M_{2}}{R^{2}}
$$

Fig. 9.
Here is $\mathbf{G}$ - a gravitational constant, $\mathbf{R}$ - a distance (The Real) between these two bodies, $\mathbf{V}$ - a rate of movement of body $\mathbf{M}_{2}$.
In his observation, Fritz Zwicky has found out that these two forces are not equal (as if accountant would be say - the balance has not coincided with oneself - the debit with the credit). What is the matter? How to solve it? Earlier we have shown that value of the Observed distance (only it we can observe) represents itself a complex value. Thereby it has
generated this problem, but at the same time, it contains a key to its decision. There is a value distance $\mathbf{R}$ at a denominator

$$
F_{u u t}=\frac{M_{2} \cdot V^{2}}{R}, F_{2 p}=G \cdot \frac{M_{1} \cdot M_{2}}{R^{2}}
$$

in these two formulas. As earlier, we hinted that all the matter is that there was a "substitution" of - $\mathbf{R}$ the Real distances for $\mathbf{S}$ - the Observed distance, which obviously is greater always. If to substitute the value of the Real distance - $\mathbf{R}$ for the value of the Observed distance - $\mathbf{S}$ in these formulas, than for balance of forces will be necessary to increase the value of mass $\mathbf{M}_{1}$ in the term of fraction also.

$$
V=\sqrt{G \cdot \frac{M_{1}}{R}}
$$

Where is an additional mass to take? These values of mass $\mathbf{M}_{1}$ and $\mathbf{M}_{2}$ are found from the "mass-luminosity" diagram. From which moreover greater than is observed (that is shines), impossible to take anything extra. Then as expected the so-called " The dark matter ", or " a shortage of the luminous Matter " problem appears, which, as it turned out, possesses surprising properties - it cannot be observed, however interacts with all Objects in the Universe by means of the gravity force.
Let add our figure we shall place the Observer and from him to objects $\mathbf{M}_{1}$ and $\mathbf{M}_{2}$ we shall draw the segments of the Real distances $-\mathbf{R}_{1}$ and $\mathbf{R}_{2}$. (Fig. 10) Now we shall imagine, that a plane of this figure - $\mathbf{R}$ represents itself our real three-dimensional space. The $\mathbf{Q}_{1}$ and $\mathbf{Q}_{2}$ planes of an imaginary direction (the gamma motion) i.e. the distances of "receding" will settle down at right angle to the plane $\mathbf{R}$ of this figure. Let draw a complex plane $\mathbf{Q}_{1}$ at right angle to plane of this figure $\mathbf{R}$ through a point $\mathbf{O}$ in which there is an Observer and point $\mathbf{P}_{1}$ in which Object $\mathbf{M}_{1}$ settles down. Right now, we shall repeat all those arguments by which we did for similar constructions - that is - " ... because the Object radiates energy - around of this Object the space-time properties has varied - and so on $\ldots \mathrm{l}$. We shall draw a perpendicular from point $\mathbf{P}_{1}$ in this complex plane $\mathbf{Q}_{1}$, at right angle to the Real plane $\mathbf{R}$. This perpendicular is a segment $-S_{1}^{*}=P_{1} P_{1}^{*}$ an imaginary moving of object $\mathbf{M}_{1}$. Let join the point $\mathbf{O}$ with the point $P_{1}^{*}$ - thus we shall receive the Observed Distance $\mathbf{S}_{1}$. Further, we shall draw an arch from point $\mathbf{P}_{1}{ }_{1}$ with the center in the point $\mathbf{O}$ and radius equal to distance $\mathbf{S}_{1}$ even to crossing this arch with the real plane $\mathbf{R}$ thus we shall find position of the point $P_{1}^{\prime}$. The same procedure we shall carry out to the Object $\mathbf{M}_{2}$, thus we shall find a position of point $\mathbf{P}^{\prime}{ }_{2}$. Then we shall draw the $S_{12}^{\prime}$ segment between the
points $\mathbf{P}^{\prime}{ }_{1}$ and $\mathbf{P}^{\prime}{ }_{2}$ which it will represent the Observed distance between the Objects $\mathbf{M}_{1}$ and $\mathbf{M}_{2}$. As we see in figure, with all due evidence $S_{12}^{*} \triangleright R_{12}$ that the Observed distance is greater then the Real distance. Again, we shall repeat that " The dark matter " problem is " a shortage of the luminous matter " has taken place owing to a "substitution" of the Real distance for Observed distance. All the values of the distances in the Universe (in intergalactic scales) have turned out to be oversized because of this "new" space-time properties manifestation.


Fig. 10.

## 19. Hubble law correction

There is a natural question - "Is it possible to regard seriously to the data received during an observation at present? Could we "trust our own eyes"? Could it be to trust the values of the distances received based on the red shift? We shall answer to it unambiguously - " possible ones taking into account the correction data ". For problem solving, by way of illustration, we shall result again our "triangle" (Fig. 11).

$$
S=\sqrt{\left(R_{0}\right)^{2}+\left(S^{*}\right)^{2}}
$$



Fig. 11.
Here (we shall repeat again) $\mathbf{S}$ - is the Observed distance, $\mathbf{R}_{0}$ - is the Real distance, $\mathbf{S}^{*}$ - is the Imaginary moving, the distance of "receding" $S^{*}=\frac{\left(V^{*}\right)^{2}}{2 \cdot c \cdot H}$. Let find the $R_{0}=\sqrt{S^{2}-\left(S^{*}\right)^{2}}$ There is the aforecited formula for calculations $\mathbf{S}^{*}$ - imaginary moving, the distance of "receding". We shall find the Observed distance $\mathbf{S}$ - by means of the Hubble law

$$
S=\frac{V^{*}}{H}, V^{*}=H \cdot r \Rightarrow V^{*}=H \cdot S
$$

Where $\mathbf{H}$ - is the relative emission power, $\mathbf{c}$ - is velocity of light, $\mathbf{V}^{*}$ - is ray velocity. In that case the Real distance $\mathbf{R}_{0}$ will be

$$
R_{0}=\sqrt{\left(\frac{V^{*}}{H}\right)^{2}-\left(\frac{\left(V^{*}\right)^{2}}{2 \cdot c \cdot H}\right)^{2}}
$$

It has been shown above that the ray velocity $V^{*}=c \cdot H \cdot \Delta t$ in direct proportion depends on the Object "age" is $-\Delta t$. Then this formula can be presented as

$$
R_{0}=\frac{V^{*}}{H} \sqrt{1-\frac{1}{4} \cdot(H \cdot \Delta t)^{2}}
$$

Doing the analysis of these formulas, possible to come to a conclusion (in the context of the " Expanding Universe" hypothesis) that the excess of the Object Observed distance above its Real distance, increases with increase this Object Ray velocity, so with increase the Observed distance or according to this Object "ageing". The presented amendment allows us to measure the distance up to the Objects in the Universe more precisely, taking into account the Object "distinctiveness" that is its
relative emission power, rather than the Hubble's Law is used in the "Expanding Universe" hypothesis do it. The Hubble constant $\mathbf{H}_{\text {Hubble }}$ is an averaged value of the relative emission power of the Universe, therefore, that is resulted in emergence of matter shortage illusion (i.e. "The dark matter's" hypothesis) and another "absurdity" such as Halton Arp abnormal observation.

## 20. Some critical remarks about the red shift origin hypothesis

Now we shall consider the existing Hypotheses in the Cosmology. To prove or deny ones or others hypotheses, we shall result some notorious facts, based on them we shall argue the reasons for and against for each of these hypothesis.

## The facts are as follows:

- There is a red shift at the Universe object radiated spectrum;
- This red shift is increasing in the course of time;
- Cosmic microwave background were discovered, which interpreted as residual "relic" radiation in an expanding model of the Universe.

Now there are three explanations to the Red shift phenomenon:

- There is a hypothesis of "ageing" of photons. It agrees to it - photons, being repeatedly absorbed and being radiated, overcoming a huge distances in the Universe, "being squeezed one's way" through clouds of an interstellar dust and atomic hydrogen - "become frayed", that is lose a part of energy and owing to all it "redden" and "grow old". According to this conception, the longer the photon, "flying" to us through an open space of the Universe, exists, the "older" also "becomes redder" it.
- The following explanation of a red shift origin is a well-known hypothesis of "the Big Bang". The red shift effect is interpreted as a manifestation of the Doppler effect.
- The author puts forward the third explanation to the red shift origin. The red shift effect is internal inseparable objects' radiation parameter and the reason of the early-unknown space-time properties.

Let us state critical remarks concerning a hypothesis "an Ageing of Photons". French astrophysicist Jean Pierre Vigier from the Institute Henri Poincare has suggested, that there are a certain variety of hypothetical particles in the intergalactic space, which cooperate with light in such a way, that these particles take off a part of energy of light. There are the following remarks in this cause:

- If it would that variety of hypothetical particles be, by which Jean Pierre Vigier refers, which took off energy at light photons, then according to

Thermodynamics - entropy would grow and it would lead to heat death of the Universe.

- According to quantum mechanics canons - E-field radiation is let out and absorbed by portions - quantum. The photon as quantum of energy is let out and absorbed completely, that is entirely and without any "remains". (There is radiation intensity decreases but not
Wavelength (wave frequency) does). So it (a photon, a quantum of
E-field radiation) is indivisible; consequently, it has no internal structure. Therefore in it (photon) there are no those "parts", which could be separated from it.
- The Special Relativity (SR) is grounded upon four-dimensional pseudoEuclidean Minkowski space-time in which photons are moved on its four-dimensional isotropic line. A movement on its isotropic line according to these space-time properties occurs instantly, i.e. outside of time. So the photon does not have that "a huge number of time" over which it can "grow old and redden".
- Alteration of radiation frequency - is such well-studied effect in the nonlinear optics, showing us a manifestation of the photon quantum properties. This effect is shown, for example, how laser coherent laser emission is interacted with the substance, thus an additional spectrum lines are aroused but not red shift effect. There is a diversion of a laser beam to be observed also.


## 21. Phases of formation of the expanding Universe concept

Now let in retrospect return to those "turning points" of a natural science advancement where the scientific idea "has made" an incorrect direction which now with all acuteness was showed as deadlock. We shall make a brief survey about how the concept of the expanding Universe was formed.

In 1913, the American astronomer Vesto Melvin Slipher started to study the spectra of light, which are arriving from ten known nebulas. He also has noticed that a line the certain elements in spectra of galaxies have been displaced in a direction of the red end of a spectrum. Slipher bas explained the red shift by means of Doppler effect and has decided that Galaxies should leave from us. The next step bringing us to belief in Universe expansion bas been made in 1917 when Einstein bas published the theory of Relativity. According to Einstein's theory a set of forms can be shaped in space. One of them is - the closed space-time without the borders, similar to a spherical surface; another is - a negatively curved space which infinite extended in all directions. Einstein supposed that the Universe is static and he has adapted bis equation for this
purpose. Almost at the same time, Danish Astronomer Willem de Sitter bas found the solution of the Einstein equation that predicted a fast expansion of the Universe. Such space geometry has to change in the course of time. De Sitter work has caused an interest among the astronomers in whole world. Edvin Hubble was among them. He attended the American Astronomical Society conference in 1914 when Slipher reported on his original discoveries in the galaxies' movement. In 1928, Hubble being in Mt. Wilson observatory has started for his work in an attempt to join the Sitter theory about the expanding Universe with the Slipher observation of the moved away galaxies. Hubble argued about like this: - " in the expanding Universe, you should expect receding of galaxies from each other. The distant galaxies will move faster away from each other. It should mean that the Observer should see from any point including the Earth, that all other galaxies move away from him, however the distant galaxies should move faster on the average from bim ... ". He observed that in the most galaxies' spectra are having a place the red shift effect and the greatest distant galaxies from us have the greater red shift. Hubble has proved this proportional dependence between the distance to galaxy and a degree of red shift in their spectrum, now known as Hubble law (the red shift law or the velocity-distance relation).

## 22. Some critical remarks about the concept of expanding Universe

(There is certainly very difficult and unpleasant question: - " How Hubble could learn as far as everyone is given Galaxy moved away from us "? A very difficult question for Hubble and it heretofore remains difficult for modern astronomers too. Eventually, there is no measuring scale, which could reach the stars. Generally, the distance measurement is very difficult problem and one might say - a "thorny subject" in Astronomy.)

However, we shall return to that initial point, when in 1913 Vesto Melvin Slipher has taken as interpretation the Doppler effect for the galaxies' red shift discovered by him. In this work, we have shown that the overwhelming majority of the Objects' red shift are the internal properties of radiation and not connected to the Objects' rate of movement Correspondingly, the Doppler effect means the presence of the very Real not Imaginary movement of any Objects among themselves. If these movements are real and not imaginary, then we have to confront with a fundamental problem - is a violation of energy conservation law. (There is a certain calculation about " the nonpresent power " is given in the appendix \#8, which "is lacking" for the Universe stability maintenance under the concept of the expanding Universe.) The "expanding Universe" concept has confronted with it and so to speak for
a "repair" has been invented up a "patch" - is the "Dark Energy " hypothesis. Right now, let contrary evidence (there is such a proof in the mathematics) that all movements of "receding" are real and not imaginary. We shall look that it turns out to be. The Objects in the Universe (i.e. galaxies, stars, quasars, planets, etc.) possess mass (real) and are connected among them by universal gravitation forces (under the Newton's law), that is are in the universal gravitational field. These Objects play "a part" as any gravitational charges in this field. As stated above let assume that the movement of "receding" are real and not imaginary, therefore these "Charges" are moving apart from each other (as we notice the Universe extends, the galaxies move away from each other) with some various velocities (according to the latest information received moving with acceleration). As is well known in physics that for moving the charges away from each other in Potential field in which they are, performing of work is necessary. And it does not make any difference whether these charges are electric in electric field or gravitational charges in gravitational field. All the same, it is necessary to do work. The spatial scales, quantity of charges and field intensity do not sweep aside necessity of fulfillment of work. Energy is necessary for fulfillment of work! Where do we get energy necessary for performance of this work on moving the charges from? Where its source is? Otherwise, the energy conservation law will not be realized! (The conservation law observance is a distinction between an advancement of science from the flight of fancy). So here "the Dark Energy " hypothesis became as "source" of a missing energy in the extending Universe concept. As it proved by hypothesis, " the Dark Energy " possesses such surprising properties - they are not registered by devices, but "move apart" the Universe space, forces "to run up" galaxies in this concept. Here is a pretty kettle of fish! As it turns out that ones should assume for a moment that a movement of "receding" is real, we so to say figurative "we get it in the neck" - is a violation of the conservation law.

Summed up to the aforesaid, possible to add that all this flight of fancy has appeared thereof for red shift interpretations in the Objects' radiation spectrum has been chosen the incorrect basing - is Doppler effect. One more vulnerability of "the Universe expansion" hypothesis also its fundamental problem is that velocity of the "flying rocks" should not increase with the distance from " the place of explosion " otherwise it contradicts to the law of conservation of energy-momentum.

## 23. The "fresh opinion" at evolution of the Universe

In the beginning of this article, we were promised to give "the fresh opinion", "the sensible view " at evolution of the Universe. We shall try to reproduce this "view" in the form of theses now as we have divided all distances on Real or Observed one, so we have two various consistent "view" at the Universe:

It represents in the Real part of the Universe (where a real distance) the followings:

- The Newton Universe - is infinite, homogeneous, and eternal.
- The Universe is static it does not extend and is not compressed.
- All objects in the Universe (objects can be - galaxies, Quasars, stars, planets and so on) make own movements according to Newton's law and Kepler celestial mechanics.
- There is a red shift at the objects' radiation spectrum, which is the Object internal parameter.
- This red shift in direct proportion to the relative emission power of object radiation and its "age".

The so-called an imaginary component has been appeared owing to wrong interpretation of red shift as manifestations of Doppler effect, which it actually is not present because we have thought up it in fact. This "seeming", an illusory idea was imposed on " a solid basis " of the Real part; in a result we have received the Observed which is combination of Real and Imaginary. In an Observed part, we have everything that the concept of the expanding Universe represents, namely:

- The Universe extends.
- All Objects in the Universe move from each other with the velocity proportionate to their distances among themselves.
- The Universe is limited on the extension and its age, Hubble law reflects this restriction.
- In the past, the Universe had the compact size, indefinitely big temperature and pressure, huge density.
- All Objects began to move away from each other by virtue of the Universal Cataclysm is " the Big Bang" which is took place 20 billion years ago.
- There is " the Dark Energy " which causes the Universe "accelerated" expansion.
- As well as there is " the Dark Matter " which explains the discrepancy between theoretical calculations on the gravitational influence and the Observed data in the Universe.
In this article, we have to touch upon the very difficult problem and one might say - a "thorny subject" in Modern Astronomy - is a distance measurement in the Universe. Lack of knowledge about the space-time properties has led to give birth such concepts like: " the Expanding Universe ", " the Big Bang", " the Dark Matter " and " the Dark Energy " which are proved to be "the speculative" ideas and better to say illusions.


## 24. Background materials:

Solar Mass
Solar emittance (Luminosity)
Electromagnetic constant (Velocity of light in vacuum)

G gravitational constant

$$
\begin{aligned}
& M_{\otimes}=1.989 \cdot 10^{30} \mathrm{~kg} \\
& L_{\otimes}=3.826 \cdot 10^{26} \text { watt }
\end{aligned}
$$

$$
c=2.99792458 \cdot 10^{8} \mathrm{~m} / \mathrm{sec}
$$

The gravitational field equation is - the Einstein equation:

$$
R_{i k}-\frac{g_{i k} \cdot R}{2}=\frac{8 \pi}{c^{4}} \cdot T_{i k} \quad R_{i k}=\frac{8 \pi}{c^{4}} \cdot\left(T_{i k}-\frac{g_{i k} \cdot T}{2}\right), \text { or } \quad R=\frac{8 \pi}{c^{4}} \cdot T
$$

where
$\mathbf{T}_{\mathrm{ik}}, \mathbf{T}$ - energy-momentum tensor,
$\mathbf{R}_{\mathrm{ik}}, \mathbf{R}$ - Ricci tensor (transformed curvature tensor),
$\mathbf{g}_{\text {ik }}$ - space-time metric tensor.

## Appendixes

Appendix 1. In the beginning of this article, we promised to come out with a suggestion about the surrounding space-time properties. What do we know about these properties? What are the "perturbing factors" that have an influence upon them, which we know from a school Geometry course?

Let list these "perturbing factors":

- A huge Mass of Substance.
- A subluminal velocity of Movement.
- In addition to those above-listed factors, the author states an idea that this factor is the Energy Source described by such parameter as Power.

Now we shall consider an influence of each factor on properties of the Space-time individually:

- If Mass of Substance small and Velocity of Movement not high as such example can serve our circumterrestrial Space. The distortions in it are so insignificant that in practice of them cannot take into account. Such space is possible to consider as Euclidean space.
- If Velocity of Movement come nearer to velocity of light that possible "to speak" already about relativistic effects of the Special Relativity. The task tool here will be 4-dimensional pseudo-Euclidean Minkowski space-time.
- On the contrary in case Mass of Substance are significant in any spatial region that possible "to speak" already about a space-time curvature near to this huge body. . " The more Mass is, than the more curvatures is ". This influence on space-time properties is terrifically shown in Einstein's Equations, which connecting the space-time curvature tensor with mass distribution of Substance in the form of energy-momentum tensor in the formula. The task tool in this curved space will be no Euclidean geometry yet but Riemann geometry.
- "In consideration of" such space-time properties - as ability to distort itself around of the big Mass of Substance, for some reason earlier one was not taken into account such "an essential" fact that the big Mass of Substance i.e. - stars, nucleus of galaxies, quasars, Galaxies are radiating energy. At all points, they are the sources of energy, which are characterized by such parameter as power - $\mathbf{N}$. The more powerfully source of energy is, the more significant its influence on surrounding space will be. Significant that this influence in the cosmological space is shown as followings: " the more powerfully source of energy is, the faster it "moves away" from us and consequently also "is observed" further ".

Appendix 2. We shall compute the value of gravitational red shift on a basis of the neoclassical ideas. Photon mass $\boldsymbol{m}$ is radiated from a surface of some star.

$$
m=\frac{h v_{0}}{c^{2}}
$$

The consumed energy $\mathbf{E}$ is spent by Photon to overcome the gravitational attraction of a star:

$$
E=\frac{G M m}{r_{0}}
$$

where
$\mathbf{M}$ - is the mass of a star, $\mathbf{r}_{0}$ - is its radius,
$\mathbf{G}$ - is a gravitational constant.

The same energy will change the photon frequency from $\mathbf{v}$ up to $\mathbf{v}_{\mathbf{0}}$ $E=h\left(v_{0}-v\right)$ Substituting $m=\frac{h v_{0}}{c^{2}}$ in $E=\frac{G M m}{r_{0}}$ then it equating both parts $E=h\left(v_{0}-v\right)$, we can find an expression for relative change of frequency variation of a spectral line after some transformations $z=\frac{\boldsymbol{v}_{0}-\boldsymbol{v}}{\boldsymbol{v}}$. This expression is $z=\frac{1}{\frac{c^{2} \cdot r_{0}}{G M}-1}$. Substituting $z=\frac{\boldsymbol{v}_{0}-\boldsymbol{v}}{\boldsymbol{v}}$ in the
numerical data for the Sun, we shall find that the red shift for it will make $2 \cdot 10^{-6}$.

Appendix 3. There is a system consisting of a physical body (object) radiating energy around itself and the space surrounding this body. Therefore, energy of all this System is emplicity dependent on time. Therefore, the partial time derivative $\frac{\partial L}{\partial t}$ is added to Lagrange function, which is determined the system motion. We shall find the value $\mathbf{L}$ from the

Expressions such are

$$
L=\sum_{i} \dot{q}_{i} \cdot \frac{\partial L}{\partial \dot{q}_{i}}-\text { const }
$$

$$
\text { and } \frac{\partial L}{\partial t}=\frac{\partial\left(\sum_{j}^{\bullet} \cdot \frac{\partial L}{\dot{q}_{j}}-\text { const }\right)}{\partial t} \text {. }
$$

The derivative of a constant is equal to zero. We substitute the value $\frac{\partial L}{\partial t}$ in expression $\frac{d L}{d t}=\sum_{i} \frac{\partial L}{\partial q_{i}} \cdot \dot{q}_{i}+\sum_{i} \frac{\partial L}{\partial q_{i}} \cdot \ddot{q}_{i}+\frac{\partial L}{\partial t}$, as a result we will get the
followings $\frac{d L}{d t}=\sum_{i} \frac{\partial L}{\partial q_{i}} \cdot \dot{q}_{i}+\sum_{i} \frac{\partial L}{\dot{\bullet} q_{i}} \cdot \ddot{q}_{i}+\frac{\partial\left(\sum_{j} \dot{q}_{j} \cdot \frac{\partial L}{\dot{\partial} q_{j}}\right)}{\partial t}$. We shall substitute last item for $\frac{\partial}{\partial t}\left(\sum_{j} \dot{q}_{j} \cdot \frac{\partial L}{\partial q_{j}}\right)=\sum_{j} \frac{\partial L}{\partial q_{j}} \cdot \ddot{q}_{j}+\sum_{j} \dot{q}_{j} \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial q_{j}}\right)$ expression then we shall finally receive the following. There is getting summation by $\mathbf{i}$ under the first bracket, also there is getting summation by $\mathfrak{j}$ under the
second bracket. Let express the value $\frac{d}{d t} \cdot\left(\frac{\partial L}{\dot{\partial q}}\right)$ by way of $\frac{\partial L}{\partial q}$ from the equation of motion $\frac{\partial L}{\partial q}-\frac{d}{d t} \cdot\left(\frac{\partial L}{\dot{\partial q}}\right)=0$.
Then the general equation of motion will become:

$$
\frac{d L}{d t}=\underbrace{\sum_{i} \frac{\partial L}{\partial q_{i}} \cdot \dot{q}_{i}+\sum_{i} \frac{\partial L}{\partial q_{i}} \cdot \ddot{q}_{i}+\underbrace{\sum_{j} \frac{\partial L}{\partial \dot{q}_{j}} \cdot \ddot{q}_{j}+\sum_{j} \dot{q}_{j} \cdot \frac{\partial L}{\partial q_{j}}}_{j} .}_{i}
$$

The Lagrange function and its partial derivatives shall look like:
$L=\frac{1}{2} \cdot \sum g_{i k}(q) \cdot \dot{q}^{i} \cdot \dot{q}^{k}-U(q), \frac{\partial L}{\partial q_{i}}=\frac{1}{2} \cdot \frac{\partial g_{i k}}{\partial q_{i}} \cdot \dot{q}^{i} \cdot \dot{q}^{k}$,
$\frac{\partial L}{\partial q_{i}}=-\frac{1}{2} \cdot \frac{\partial g^{i k}}{\partial q_{i}} \cdot \dot{q}_{i} \cdot \dot{q}_{k}, \frac{\partial L}{\dot{\partial q_{i}}}=g_{i k} \cdot \dot{q}^{k}=\dot{q}^{i}$
We shall substitute it in our equation
$\frac{d L}{d t}=\sum_{i}\left(-\frac{1}{2} \cdot \frac{\partial g^{i k}}{\partial q^{\prime}} \cdot \dot{q}_{i} \cdot \dot{q}_{\dot{k}} \dot{q}^{{ }^{\prime}}+\cdot \dot{q}_{i} \dot{q}^{i}\right)+\sum_{j}\left(-\frac{1}{2} \cdot \frac{\partial g^{j k}}{\partial q^{\prime}} \cdot \dot{q}_{j} \cdot \dot{q}_{\dot{k}} \dot{q}^{{ }^{\prime}}+\ddot{q}_{j} \cdot \dot{q}^{j^{\prime}}\right)$
Taking out the multiplier $\dot{q}^{l}$ of brackets and in order to rising of index we multiply by $\quad g^{i j}$ all expression $\quad \ddot{q_{i}} \cdot g^{i j}=\ddot{q}^{j}$, $\frac{d L}{d t}=\dot{q}^{i} \cdot g^{i j} \cdot \sum_{i}\left(-\frac{1}{2} \cdot \frac{\partial g^{i k}}{\partial q_{i}} \cdot \dot{q}_{i} \cdot \dot{q}_{k}+\ddot{q}^{i}\right)+\dot{q}^{i} \cdot g^{i j} \cdot \sum_{j}\left(-\frac{1}{2} \cdot \frac{\partial g^{i k}}{\partial q_{l}} \cdot \dot{q}_{j} \cdot \dot{q}_{k}+\ddot{q}^{j}\right)$
and knowing a condition $\frac{d L}{d t}=0$ of the momentum conservation law.
Then one is separating the derivative of order $\mathbf{n}=\mathbf{1}$ from the derivative of order $\mathbf{n}=\mathbf{2}$ and having made replacement of partial derivative of the metric tensor for sound Christoffel symbols. (Known that in Christoffel
symbols are $\quad \Gamma_{k i}^{\prime}=\frac{1}{2} \cdot g^{i m} \cdot\left(\frac{\partial g_{m k}}{\partial x^{i}}+\frac{\partial g_{m i}}{\partial x^{k}}-\frac{\partial g_{k i}}{\partial x^{m}}\right)$ and $\Gamma_{i j, l}=g_{i k} \cdot \Gamma_{i j}^{k}$ one swapping the indexes m and i , in the third and first member of equation, we see that both members in brackets are mutually canceled, so $\Gamma_{k i}^{\prime}=\frac{1}{2} \cdot g^{i m} \cdot \frac{\partial g_{i m}}{\partial x^{k}}$. In our case, we shall do "the inverse operation", we shall substitute the partial derivative of the metric tensor $\frac{1}{2} \cdot \frac{\partial g_{k}}{\partial q^{m}}$ for the sound Christoffel symbols $\Gamma_{i k}^{m}$. Then one taking out from under the brackets the second derivatives of the generalized coordinates $\ddot{q}^{n}$ and decreasing it on $\dot{q}^{\prime}$, in a result we shall receive

$$
\ddot{q}=\ddot{q}^{i}+\ddot{q}^{j}=\Gamma_{i k}^{m} \cdot \dot{q}^{i} \cdot \dot{q}^{k}+\Gamma_{j k}^{n} \cdot \dot{q}^{j} \cdot \dot{q}^{k},
$$

where $\ddot{q}^{i}$ is the particle acceleration is taking place under the influence of a stationary curved space-time, which one is existed around the Heavy

## Load of Matter.

$\ddot{q}^{j}$ here is the additional particle acceleration under the influence of energy change factor in this space-time volume.
$\Gamma_{i k}^{m}$ here is the connectedness (the Christoffel symbols) - which is determining the space-time curvature under the influence of the Heavy Load of Matter.
$\Gamma_{j k}^{n}$ here is the connectedness - which is determining the space-time curvature is taking place under the influence of an energy change factor in this space-time volume.

Appendix 4. As is well known, in the absence thereof a gravitational field - the law of conservation of energy and conservation of momentum (along with electromagnetic field) is expressed by the equation $\frac{\partial T^{k}}{\partial x^{k}}=0$ [5]
(94.7), p. 362 The equation $T_{i, k}^{k}=\frac{1}{\sqrt{-g}} \cdot \frac{\partial\left(T_{i}^{k} \sqrt{-g}\right)}{\partial x^{k}}-\frac{1}{2} \cdot \frac{\partial g_{k}}{\partial x^{i}} \cdot T^{h}=0 \quad$ is the generalization of this equation in the presence of a gravitational field [5] (96.1)

Making some simple transformations i.e. separating variables, possible to receive the following equation: $T^{k l}=g^{i l} \cdot T_{i}^{k}$, then $\frac{1}{T_{i}^{k} \sqrt{-g}} \cdot \frac{\partial\left(T_{i}^{k} \sqrt{-g}\right)}{\partial x^{k}}=\frac{1}{2} \cdot g^{i} \cdot \frac{\partial g_{k}}{\partial x^{i}}$, or $\frac{\partial \ln }{\partial x^{k}}\left(T_{i}^{k} \sqrt{-g}\right)=\frac{1}{2} \cdot g^{i l} \cdot \frac{\partial g_{k l}}{\partial x^{i}}$ [5].
As can be seen from this equation that change of state of a matter (changes of the Energy-momentum tensor) and changes of the Gravitational Field (the expressions made from the metric tensor derivatives) occurs simultaneously. That is the state of a matter varies and varies its ambient field (the surrounding Gravitational field) at the same time.

It is possible to show by the example of ambient field influence on a state of a matter how the particle motion in the alternating gravitational field. In these equations is very evidently shown how the particles' energy and its impulse vary according to ambient field. Here given an example, when the free particle is moving in a gravitational field in which it (particle) receives acceleration, which its projections onto Coordinate are expressed as:

$$
\begin{aligned}
& \frac{d^{2} x^{\alpha}}{d t^{2}}=-c \cdot \frac{\partial \gamma_{\alpha 0}}{\partial t}+\frac{c^{2}}{2} \cdot \frac{\partial \gamma_{00}}{\partial x^{\alpha}}-\frac{\partial \gamma_{\alpha \beta}}{\partial t} \cdot \frac{d x^{\beta}}{d t}+ \\
& +c \cdot\left(\frac{\partial \gamma_{\beta 0}}{\partial x^{\alpha}}-\frac{\partial \gamma_{\alpha 0}}{\partial x^{\beta}}\right) \cdot \frac{d x^{\beta}}{d t}-\frac{1}{2} \cdot \frac{\partial \gamma_{00}}{\partial t} \cdot \frac{d x^{\alpha}}{d t}
\end{aligned}
$$

As evident, this acceleration depends on the particle location, on time and on its rate of movement also.
$\frac{d^{2} x^{\alpha}}{d t^{2}}$ here is particle acceleration (a projection of acceleration onto Coordinate),
$\frac{d x^{\alpha}}{d t}$ here is rate of particle motion (a projection of velocity onto Coordinate).
$\frac{\partial \gamma_{\alpha 0}}{\partial t}, \frac{\partial \gamma_{00}}{\partial t}, \frac{\partial \gamma_{\alpha \beta}}{\partial t}$ here is the metric tensor component
variation on time.
$\frac{\partial \gamma_{\beta 0}}{\partial x^{\alpha}}, \frac{\partial \gamma_{\alpha 0}}{\partial x^{\beta}}, \frac{\partial \gamma_{00}}{\partial x^{\alpha}}$ here is the metric tensor component variation on distance.

In a gravitational field, which does not vary in the course of time (a stationary case), all metric tensor partial derivatives on time are equal to zero, so in that case an expression of the particle acceleration will become:

$$
\frac{d^{2} x^{\alpha}}{d t^{2}}=\frac{c^{2}}{2} \cdot \frac{\partial \gamma_{00}}{\partial x^{\alpha}}-c \cdot\left(\frac{\partial \gamma_{\beta 0}}{\partial x^{\alpha}}-\frac{\partial \gamma_{\alpha 0}}{\partial x^{\beta}}\right) \cdot \frac{d x^{\beta}}{d t}
$$

In addition to it if the gravitational field has the central symmetry, that is its $\frac{\partial \gamma_{\beta 0}}{\partial x^{\alpha}}, \frac{\partial \gamma_{\alpha 0}}{\partial x^{\beta}}$ components are equal to zero, then acceleration of particle motion accepts a classical kind: $\frac{d^{2} x^{\alpha}}{d t^{2}}=\frac{c^{2}}{2} \cdot \frac{\partial \gamma_{00}}{\partial x^{\alpha}}$, where $\frac{\partial \gamma_{00}}{\partial x^{\alpha}}$ - is a gravitational field gradient [7]
(7) $\frac{d E}{d \tau}+m D_{i j} v^{i} v^{j}-m F_{i} v^{i}=\xi_{i} v^{i}$
(8) $\frac{d p^{k}}{d \tau}+\Delta_{i j} p^{i} v^{j}+2 m\left(D_{i}^{k}+A_{i}^{k}\right) \cdot v^{i}-m F^{k}=\xi^{k}$

Here $\frac{d E}{d \tau}, \frac{d p^{k}}{d \tau}$ are the changes of energy and impulse accordingly,
Here $D_{i j}, A_{i}^{k}$ are the changes of the Gravitational Field (the expressions made from the metric tensor derivatives) [4] .
Although, these aforecited equations also show the physical meaning of occurring processes, but they "are not convenient" for the further operations. We need such equation that would show us that not only the field influences on a state of a matter, but also a state of a matter influences also to a field accordingly. Such equation was found the continuity equation and it looks like: $\frac{\partial}{\partial t}(\rho \sqrt{h})=0^{\text {[3] }}$.
Here $\boldsymbol{\varrho}$ - is a density of mass, $\sqrt{h}$ - is a unit parallelepiped volume, which made from the metric tensor determinant. $\frac{\partial \sqrt{h}}{\partial t} \neq 0 h=\left|h_{i k}\right|, \sqrt{h} \cong 1$. (There is $\sqrt{h} \equiv 1$ in Euclidean space). Making the simple transformations i.e. multiplying both member of an equation by the factor $\mathbf{k}$ is equal to product of an elementary volume $\boldsymbol{\nu}$ and the squared velocity of light $\boldsymbol{c}^{2},\left(\boldsymbol{k}=\boldsymbol{c}^{2}\right)$. Further we shall receive the followings $\frac{\partial E}{\partial t} \cdot \sqrt{h}+E \frac{\partial \sqrt{h}}{\partial t}=0$, separating the
variables, in a result we shall receive the joint variation equation of energy of any elementary volume of space and change of the metric tensor of its space (volume), in which this process of change of energy is taking place. There is the remark 1. The volume of a unit parallelepiped is equal to a root square of the determinant module by definition.
There is the remark 2. One has been mentioned an expression is "Metric tensor" (not to frighten the reader off from continuation of reading this article), we shall explain that by definition " a terrible word " is - " metric tensor " of the 4-dimensional spaces-time represents itself the square array of $\mathbf{1 6}$ numbers are straddling in a special way, or 9 numbers for usual 3-dimensional space-time.
Metric tensor $X_{i j}$ will consist of a constant component ${ }^{0} g_{i j}$ and its variable component $y_{i j}$, which is a deviation of metric tensor from Galilean metrics $X_{i j}=\stackrel{0}{g}_{i j}+y_{i j}$. For one's turn, possible to present $\gamma_{i j}$ as product of metric tensor time derivative and time lag. $y_{i j}=\frac{\partial \gamma_{i j}}{\partial t} \cdot \partial t$ Or in general:

$$
X_{i j}=\stackrel{0}{g}_{i j}+\frac{\partial \gamma_{i j}}{\partial t} \cdot \partial t
$$

Appendix 5. Now we shall draw an attention to motion of light in free space. Let $K^{\alpha}$ is world phase vector, $\omega^{- \text {is chronometric invariant of }}$ cyclic frequency. Then
c $K_{0} \cdot\left(g_{00}\right)^{-1 / 2}=\omega, c K^{i}=\omega \alpha^{i}, \alpha^{i}=d x^{i} / d u, c d \tau=d u$. We have
(9) $\frac{1}{\omega} \cdot \frac{d \omega}{d u}+\frac{1}{c} \cdot D_{i j} \alpha^{i} \alpha^{j}-\frac{1}{c^{2}} \cdot F_{i} \alpha^{i}=0$
(10) $\frac{1}{\omega} \cdot \frac{d\left(\omega \alpha^{k}\right)}{d u}+\Delta_{i j}^{k} \alpha^{i} \alpha^{j}+\frac{2}{c}\left(D_{i}^{k}+A_{i}^{k}\right) \alpha^{i}-\frac{1}{c^{2}} \cdot F^{k}=0$

Here $D_{i j}$ - is the system strain rate of reference frame.
This nonrelativistic effect is similar to Doppler effect it caused by reference system deformation. Being limited by the macroscopic metrics, we shall consider $\Delta \omega / \omega$ in directions, for which in vantage point (a Point of Observation) $D_{i j} \alpha^{i} \alpha^{j} \neq 0, F_{i} \alpha^{i} \neq 0$. Then from (9) we shall find, that in each given direction, the value $\Delta \omega / \omega$ as a first
approximation is proportional to distance $(d u)$ the Source from a Point of Observation, and for the given distance in any two opposite directions the half-sum of values $\Delta \omega / \omega$ gives the value of Doppler effect [4].

Appendix 6. We shall write out the equations (7) and (9) from work [4]
(7) $\frac{d E}{d r}+m D_{i j} \cdot v^{i} \cdot v^{j}-m F_{i} \cdot v^{i}=\xi_{i} \cdot v^{i}$
(9) $\frac{1}{\omega} \cdot \frac{d \omega}{d u}+\frac{1}{c} \cdot D_{i j} \cdot \alpha^{i} \cdot \alpha^{j}-\frac{1}{c^{2}} \cdot F_{i} \cdot \alpha^{i}=0$, Here are $D_{i j}=\frac{\partial x^{i}}{\partial t}$,
$\alpha^{i}=\frac{d x^{i}}{c \cdot d r}=\frac{1}{c} \cdot v^{i}, \frac{\partial}{\partial t}=\frac{c}{\sqrt{g_{00}}} \cdot \frac{\partial}{\partial x^{0}}, v^{i}=\frac{d x^{i}}{d \tau}, d u=c \cdot d \tau$.
Taking into account the aforecited transformations, we shall write down the equation (9) as (9) $\frac{1}{\omega} \cdot \frac{d \omega}{d u}+\frac{1}{c} \cdot D_{i j} \cdot \frac{v^{i}}{c} \cdot \frac{v^{j}}{c}-\frac{1}{c^{2}} \cdot F_{i} \cdot \frac{v^{i}}{c}=0$.
Right now, from each equation (7) and (9) we shall deduct expression $D_{i j} \cdot v^{i} \cdot v^{j}$, that is
(7) $D_{i j} \cdot v^{i} \cdot v^{j}=\frac{1}{m} \cdot \xi_{i} \cdot v^{i}-\frac{1}{m} \cdot \frac{d E}{d \tau}+F_{i} \cdot v^{i}$
(9) $D_{i j} \cdot v^{i} \cdot v^{j}=F_{i} \cdot v^{i}-\frac{c^{3}}{\omega} \cdot \frac{d \omega}{d u}$

Further, we shall deduct the equations (7) (that is we shall add with negative sign) from the equation (9)

$$
\frac{1}{m} \cdot \xi_{i} \cdot v^{i}-\frac{1}{m} \cdot \frac{d E}{d \tau}+F_{i} \cdot v^{i}-F_{i} \cdot v^{i}+\frac{c^{3}}{\omega} \cdot \frac{d \omega}{d u}=0
$$

Let reduce it on

$$
F_{i} \cdot v^{i}, \frac{1}{m} \cdot \xi_{i} \cdot v^{i}-\frac{1}{m} \cdot \frac{d E}{d \tau}+\frac{c^{3}}{\omega} \cdot \frac{d \omega}{d u}=0
$$

Let multiply it term by term by $\mathbf{m}$

$$
\xi_{i} \cdot v^{i}-\frac{d E}{d \tau}+\frac{c^{3} \cdot m}{\omega} \cdot \frac{d \omega}{d u}=0
$$

Since a selection of reference frame is bearing a random character, possible to choose such system of reference frame in which $\xi_{i}$ not gravitational force is equal to zero, that is expression $\xi_{i}=0$. We shall
remember about mass-energy equivalence $E=m \cdot c^{2}$, further we shall continue our transformations. $\frac{c \cdot E}{\omega} \cdot \frac{d \omega}{d u}-\frac{d E}{d \tau}=0 \Rightarrow \frac{1}{E} \cdot \frac{d E}{d \tau}=\frac{c}{\omega} \cdot \frac{d \omega}{d u}$

Appendix 7. As a case in point, possible to result the comic "Proof" of this statement. See as Energy source a usual electric bulb also we shall leave it " to blaze " such long time for example in an entrance of a house. According to our calculations, this bulb after a while - a certain number of millions years - "will disappear" from "field of vision", will drop out of sight. However, our routine day-to-day experience speaks us that bulbs "disappear" much earlier in particular from entrances of houses.

## Appendix No. 8.

$\mathbf{M}_{\text {Mw }}$ - is Mass of the Milky Way (our Galaxy is - Milky Way galaxy) (Evans and Wilkinson, 2000)

$$
M_{M W}=1.9 \cdot 10^{12} M_{\otimes}=1.9 \cdot 10^{12} \cdot 1.989 \cdot 10^{30} \mathrm{~kg}=3.78 \cdot 10^{42} \mathrm{~kg}
$$

$\mathbf{M}_{\mathbf{M} 31}$ - is Mass of the Galaxy in Andromeda constellation M31 (NGC 224)
$M_{M 31}=1.23 \cdot 10^{12} M_{\otimes}=1.23 \cdot 10^{12} \cdot 1.989 \cdot 10^{30} \mathrm{~kg}=2.45 \cdot 10^{42} \mathrm{~kg}$
$\mathbf{V}^{*}{ }_{\text {M31 }}$ - is the range rate of recession of Galaxy $\mathbf{M 3 1}-\mathbf{V}_{\text {M31 }}^{*}=\mathbf{3 0 0} \pm 4$ $\mathrm{km} / \mathrm{sec}$ on (NED) data.
$\mathbf{R}_{\mathrm{M} 31}$ - is the distance up to Galaxy M31
$\mathbf{R}_{\mathrm{M} 31}=2900000$ light year $=2.74 \cdot 10^{22} \mathrm{~m}$
F - is the attractive force between Milky Way (our Galaxy) and Galaxy M31

$$
F=G \cdot \frac{M_{w w} \cdot M_{w 31}}{R^{2}}, F=6.67 \cdot 10^{-11} \frac{3.78 \cdot 10^{42} \mathrm{~kg} \cdot 2.45 \cdot 10^{42} \mathrm{~kg}}{\left(2.74 \cdot 10^{22} \mathrm{~m}\right)^{2}}=9.12 \cdot 10^{29} \mathrm{~N}
$$

$\mathbf{N}^{*}$ - is Power which would be required for "taking apart" these two galaxies in the opposing sides

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## Authors



Ryndyuk Konstantin D., Russia.
Rynduke@mail.ru
I was born in 1963 in Dushanbe Tajikistan, have graduated Omsk Tank Higher Engineering School in 1985 with honest degree. Now I am living in Naryan-Mar.

Sinitsyn Konstantin N., Russia.
koscmp@yahoo.com
Date of birth: 21 st of November, 1960.
Education: High technical education: 1978-1984 Moscow High Technical School named N.E. Bauman (today is Moscow State Technical University named N.E. Bauman). IT manager, LLC Lear. Professional activity: automation, IT, staff management. Sattelite navigation and car motion tracking. Hobby astrophysic, gravity. In 1999 and 2000 had participation in international conferences (Moscow, Russia, 1999; Kiev, Ukraine, 2000). I have several publications on site www.n-t.org. From November, 2004 to May 2007 had participation in UN mission in Kosovo as a technician expert (Regional Kosovo Police Telephone Network project).

Yarosh Vsevolod S., Russia.

yvsevolod-26@yandex.ru
Candidate of the technical sciences. Has Finished in 1951 Harikovskiy Aviainstitut. The Veteran of the Great Domestic war. 84 years.
Author of the scientific opening № OT-11681 Phenomena of the forming the masses-energy rest photon of the electromagnetic radiations Author to alternative against Big bang theory, as highenergy vacuum, which structure is built from cool photons-hadrons, rather then from non-observe quarks, gluons, colours, aroma and higgs. Author patent to Russian federation № 2145742 - Way of the industrial mining to internal energy-masses of the vacuum matter, freeing from multivariate quantrm world power system "Vacuum materialnuclids" and device for his realization. Author patent to Russian federation № 2097517- Way of protection from multiple seismic influence sunk in soil of the object and device for his realization Author report for international congress ISISSYMMETRY about united symmetry micro and macro-cosmos (USA -1995, IISRAIL-1998, POLAND -2009). Author two books and row articles on
http://www.patentstorm.us/patents/5377129.htm I

