Lucerne University of
Applied Sciences and Arts

HOCHSCHULE
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New Frontiers in Quantitative Risk Management

IFZ FinTech Colloquium

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November 27, 2019

Purpose of the Talk

Objective

This presentation is supposed to provide you with

- **selected challenges** that arise in the financial industry,
- an introduction to how these challenges can be tackled by means of **machine learning** techniques.

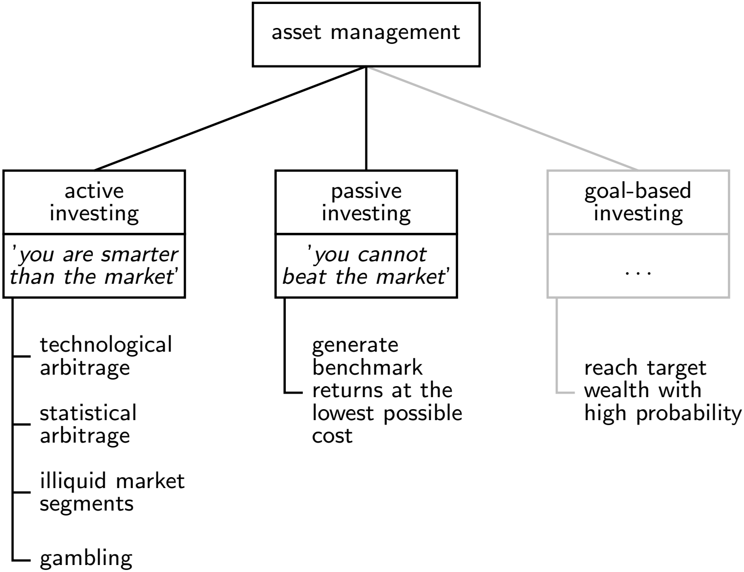
Disclaimer

- This introduction does **not** provide a **comprehensive** overview of how machine learning techniques are applied in the financial industry.
- The presented topics may grant an essential competitive advantage. However, please be aware of **inherent risks**.
- This talk does not disclose any profitable investment strategies.

Outline

1. Challenges
   - Asset Management
   - Pricing and (Over-)Hedging
2. Neural Networks
3. Machine Learning
   - Supervised Learning
   - Re-inforcement Learning
4. Applications

Asset Management



Valuation and (Over-)Hedging

What is a fair price $P(0, T)$ of getting one monetary unit at time $T > 0$ as seen from $t = 0$?

- naive approach:

$$P(0, T) = 1$$

  issues: **inflation risk**, **credit risk**, **liquidity risk**

- static approach:

$$P(0, T) = \frac{1}{(1 + r)^T}$$

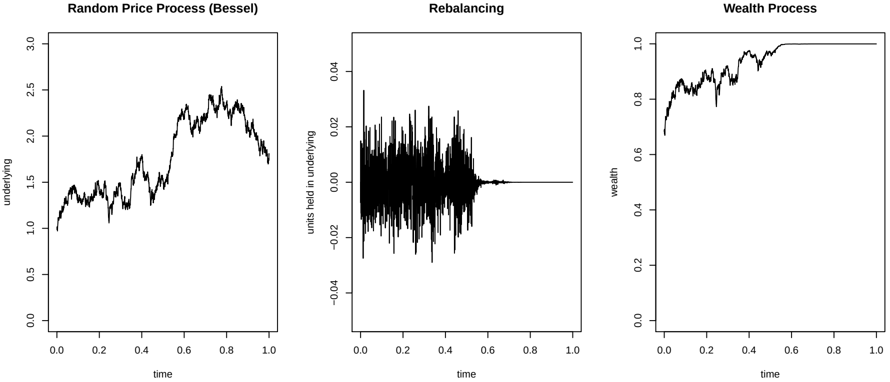
  for some interest rate r

Risk-Adjusted Valuation

$P(0, T)$ is the **minimal cost** to (super-)replicate the desired payoff.

Valuation and (Over-)Hedging

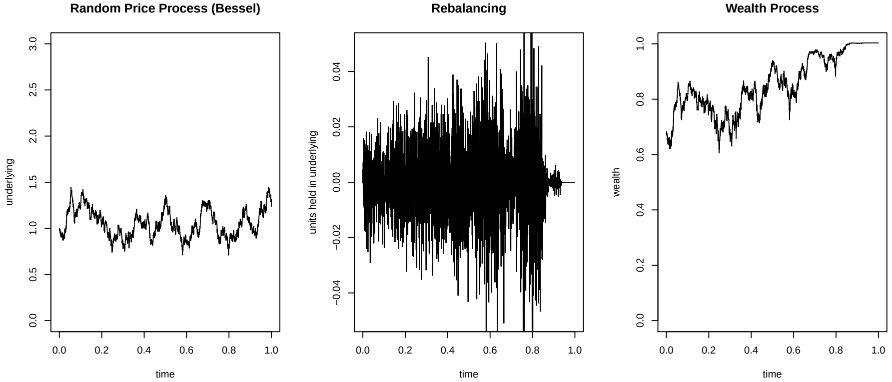
Monte-Carlo



- random price process $S_t = \sqrt{{W_{1,t}}^2 + {W_{2,t}}^2 + {W_{3,t}}^2}$
- (almost) frictionless (delta-)hedging results in minimal super-replication cost of 0.68

Valuation and (Over-)Hedging

Monte-Carlo



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Valuation and (Over-)Hedging

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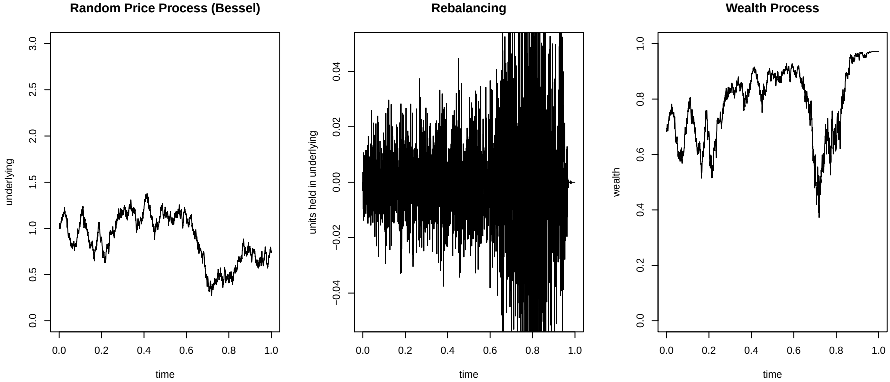

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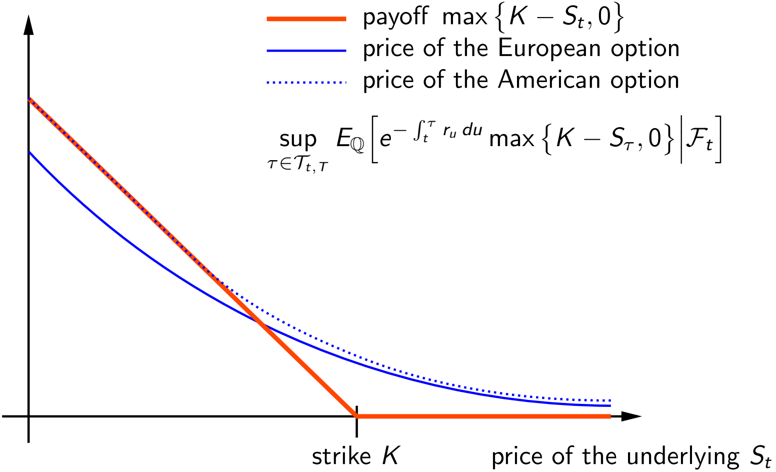
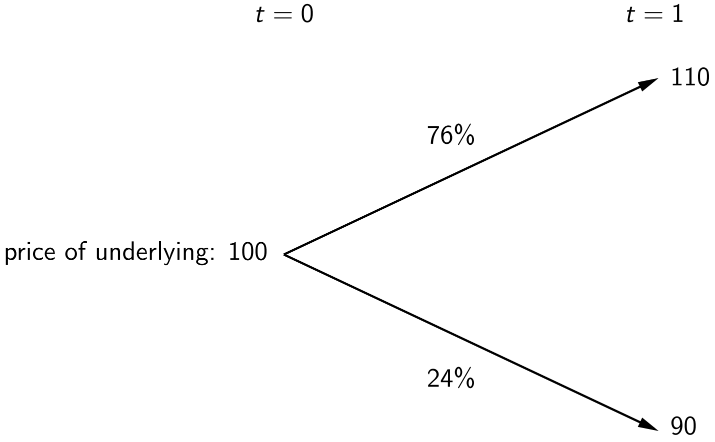
Option Pricing



Valuation and (Over-)Hedging

Dynamic Programming

Discrete World: $K = 110$, $r = 5\%$



Valuation and (Over-)Hedging

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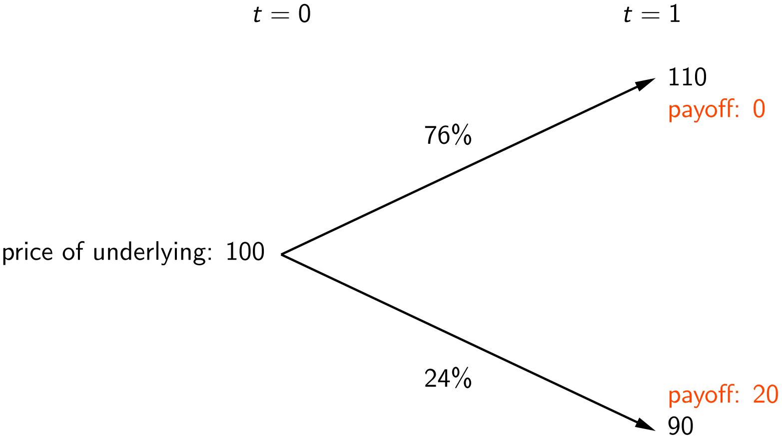

Valuation and (Over-)Hedging

Dynamic Programming

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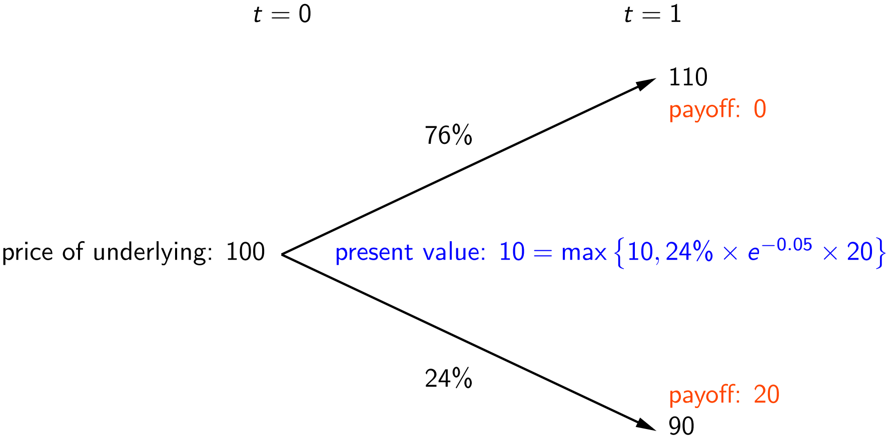
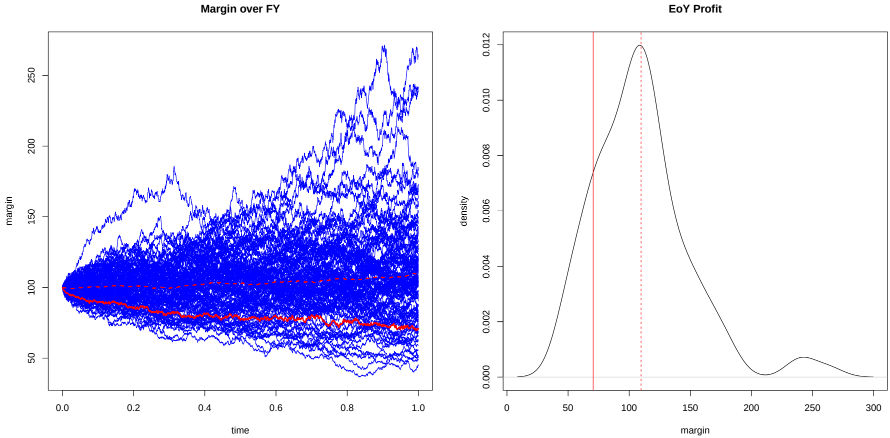


Valuation and (Over-)Hedging

Flaws of Classical Valuation Approaches

- Monte-Carlo-techniques or dynamic programming tend to be **computationally intensive**.
- The **level of sophistication** remains limited.



Valuation and (Over-)Hedging

The Curse of Dimension

No. of Underlyings	Discretisation of Space and Time	Runtime	Scale Unit
1	1 000	1	millisecond
2	1 000 000	1	second
3	1 000 000 000	17	minutes
4	10^{12}	12	days
5	10^{15}	32	years
6	10^{18}	317	centuries
⋮	⋮	⋮	⋮

- Longstaff-Schwartz (2001): 20 underlyings
- Becker-Cheridito-Jentzen (2018): 500 underlyings below 10 minutes with techniques inspired from **machine learning**

Valuation and (Over-)Hedging

Hierarchy of financial assets from the accounting and pricing viewpoint (according to FASB 157):

- **Level 1**: Quotes are readily observable in the market.
- **Level 2**: Prices can be inferred through models and observable quantities.
- **Level 3**: Valuations involve complex models and subjective assumptions.

A professional and well-calibrated valuation platform must meet the following requirements:

- The model reprices level 1 products.
- The model features generally observed market phenomena.
- The model accounts for the significant risk drivers in a realistic manner.

Valuation and (Over-)Hedging

Risk-Adjusted Valuation

What is a fair price π_0 of getting $h(S)$ at time $T > 0$ as seen from $t = 0$, where $S = (S_t)_{0 \leq t \leq T}$ is a d-dimensional underlying risk factor and h some payoff function?

- Finding **realistic dynamics** is almost impossible due to the statical uncertainty.
- The **(super-)replication strategy** is often not known explicitly.
- Trading off **complexity**, mathematical **tractability** and inherent **model risks** is very challenging.
- Analytically, it is very hard to deal with **transaction cost**.
- Maintaining and **automating** a suitable, efficient and well-calibrated valuation platform (e.g., stochastic local volatility models) for several thousand derivatives is tough.

The Game Has Changed

In 2017 a research group of DeepMind published the following results:

White	Black	Wins[3]	Draws	Losses
AlphaZero[1]	*Stockfish*	25	25	0
Stockfish[2]	*AlphaZero*	3	47	0

[1] AlphaZero is an algorithm that learns to play chess from scratch solely by **smart self-play**.

[2] Stockfish is a powerful open-source chess engine and TCEC world champion 2016.

[3] Outcome as seen from AlphaZero's perspective.

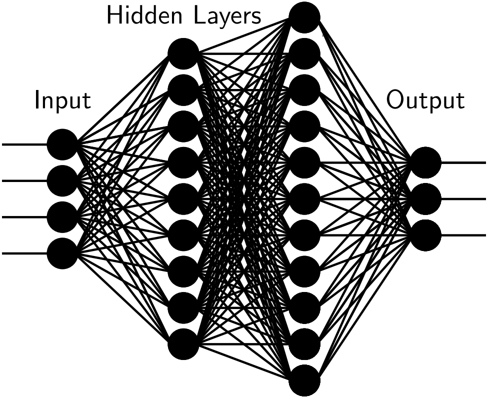
This results stimulates the imagination that quantitative methods for finance enter a new era.

Paradigm

Regarding the presented challenges, what would a **clever** financial agent with a lot of **experience** and a decent **risk appetite** do?

Outline

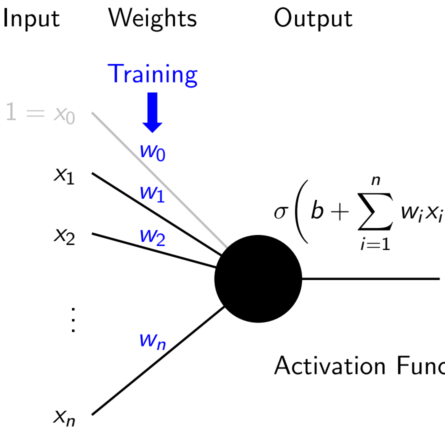
Neural Networks



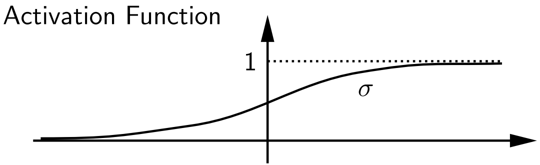
Machine Learning from the Mathematical Viewpoint

Simply put, it is the approximation of a high-dimensional non-linear function in terms of a (deep) neural network (DNN).

Perceptron



$$\sigma\left(b + \sum_{i=1}^{n} w_i x_i\right) = \sigma\left(\sum_{i=0}^{n} w_i x_i\right)$$



Neural Networks

Mathematical Properties

- **Universal Approximation Theorems**: Provided that they are sufficiently large, neural networks can approximate complex functions arbitrarily close.
- Computing the derivative of the network output with respect to the weights is straightforward. Therefore, an incremental **learning process** becomes feasible.

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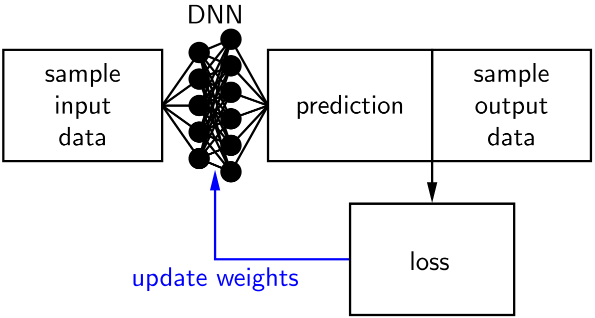
Machine Learning

Supervised Learning

Training: Minimise a Loss Function



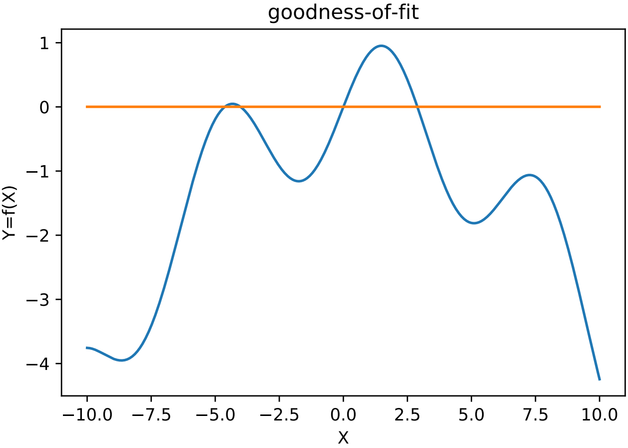
Validation: Check Accuracy of Prediction on Concealed Data

Machine Learning

Supervised Learning

Number of Nodes: 1–30–30–10–10–1

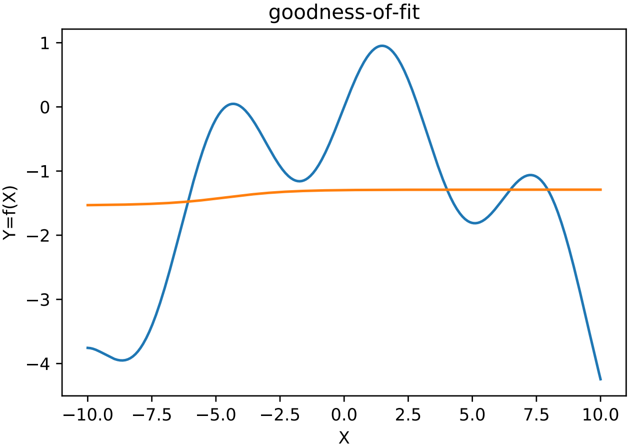
Number of Epochs: 0



Supervised Learning

Number of Nodes: 1–30–30–10–10–1

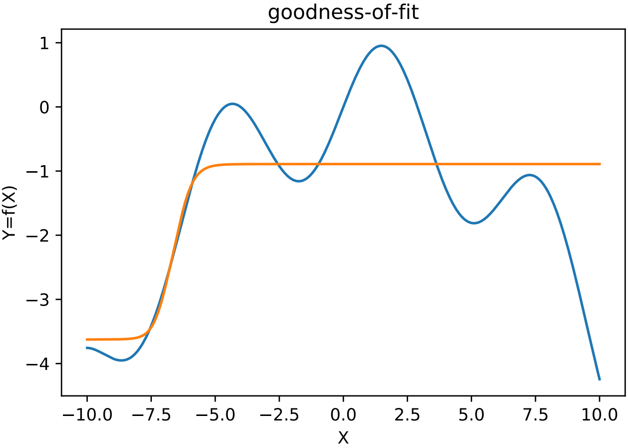
Number of Epochs: 1 000



Supervised Learning

Number of Nodes: 1–30–30–10–10–1

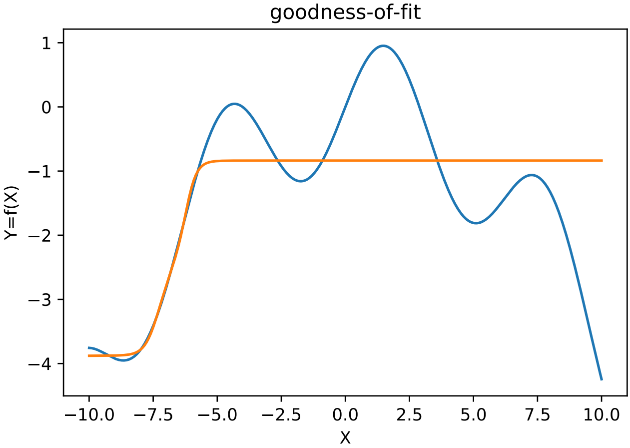
Number of Epochs: 2 000



Supervised Learning

Number of Nodes: 1–30–30–10–10–1

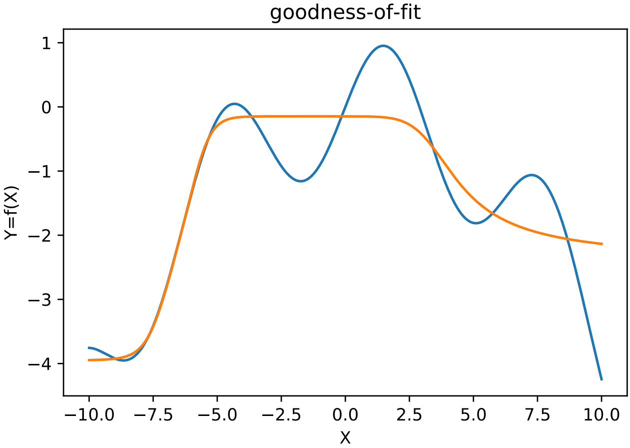
Number of Epochs: 3 000



Supervised Learning

Number of Nodes: 1–30–30–10–10–1

Number of Epochs: 4 000

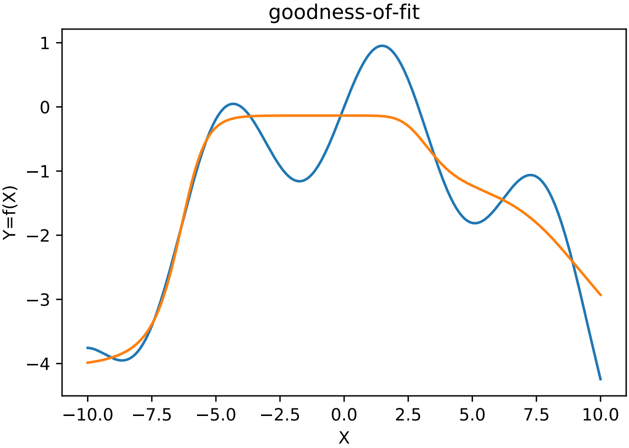


Machine Learning

Supervised Learning

Number of Nodes: 1–30–30–10–10–1

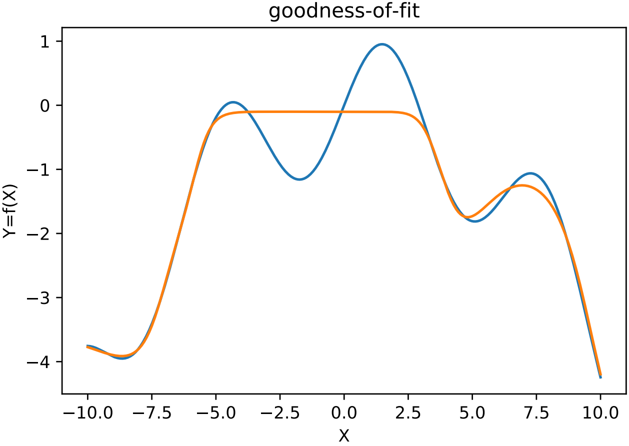
Number of Epochs: 5 000



Supervised Learning

Number of Nodes: 1–30–30–10–10–1

Number of Epochs: 6 000

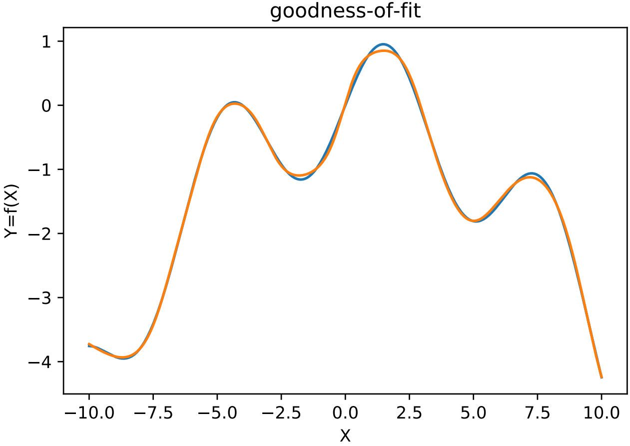


Machine Learning

Supervised Learning

Number of Nodes: 1–30–30–10–10–1

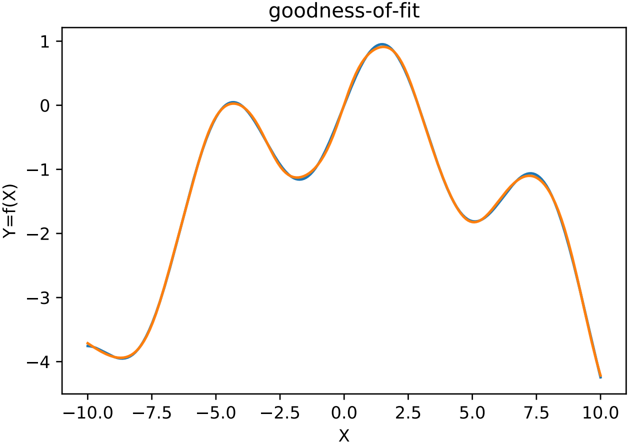
Number of Epochs: 7 000



Supervised Learning

Number of Nodes: 1–30–30–10–10–1

Number of Epochs: 8 000

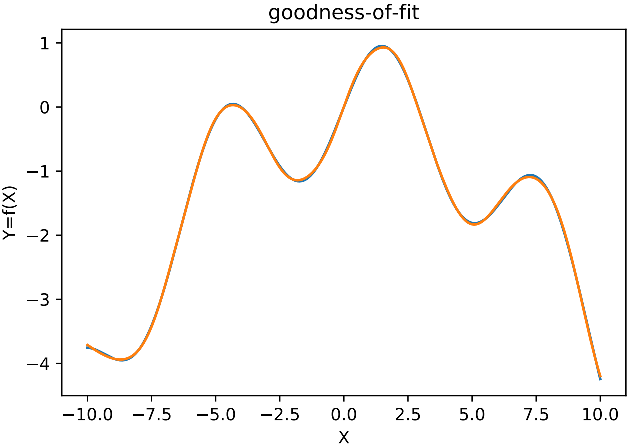


Machine Learning

Supervised Learning

Number of Nodes: 1–30–30–10–10–1

Number of Epochs: 9 000

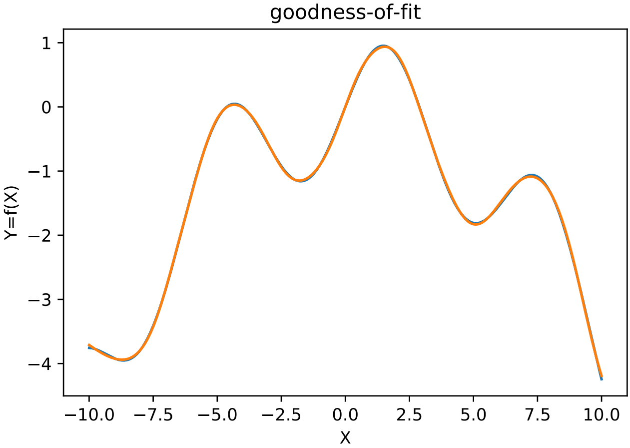


Machine Learning

Supervised Learning

Number of Nodes: 1–30–30–10–10–1

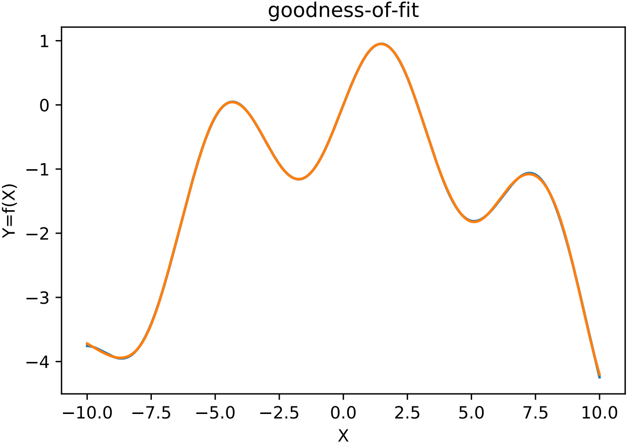
Number of Epochs: 10 000



Supervised Learning

Number of Nodes: 1–30–30–10–10–1

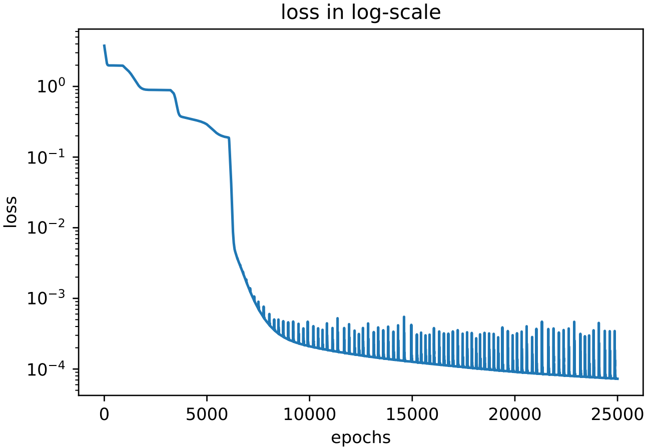
Number of Epochs: 25 000



Supervised Learning

Number of Nodes: 1–30–30–10–10–1

Number of Epochs: 25 000



Machine Learning

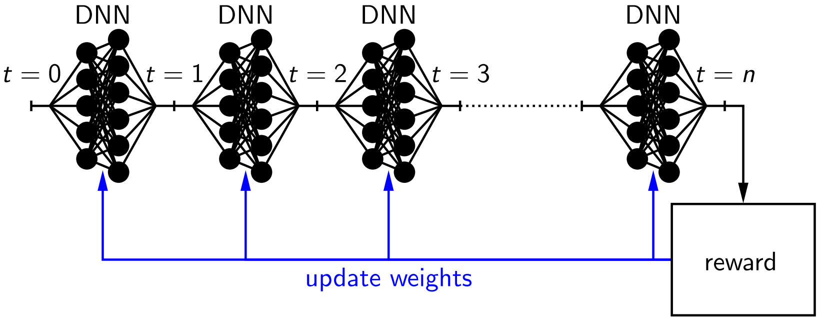
Observations

- The learning process evolves in small and random steps.
- The update of the weights results from the **backpropagation algorithm**. It can be seen as a very smart way of combining Monte-Carlo techniques and dynamic programming.
- Choosing suitable **hyperparameters** for the learning process might be tricky.
- Computing power is crucial.
- Neural networks can be evaluated efficiently by using pertinent software libraries, e.g., **TensorFlow**.
- Storing neural networks requires comparatively little storage space.

Machine Learning

Re-inforcement Learning

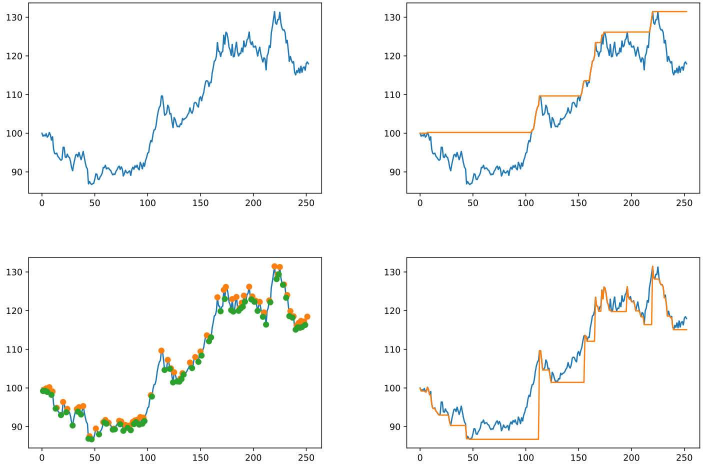
Training: Maximise a Reward Function



Validation: Check Performance of Decisions on New Scenarios

Re-inforcement Learning

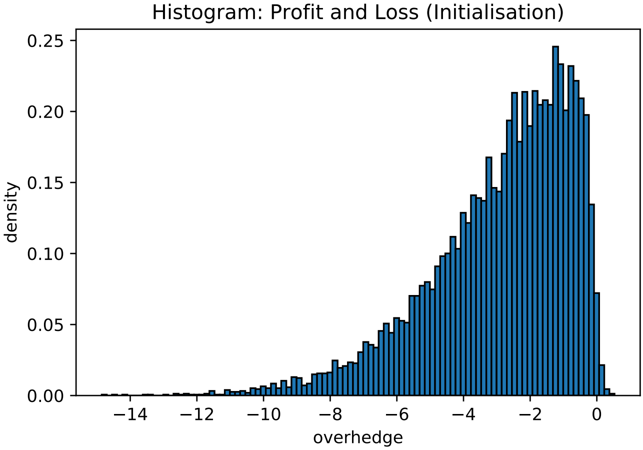
Scenarios, Features and States



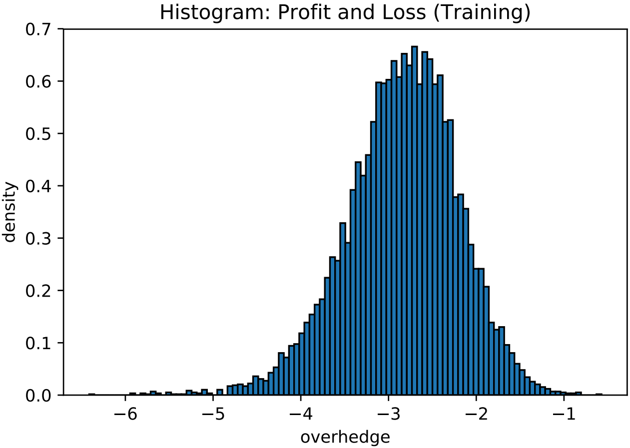
Experiment on Deep Hedging

- Exposure: We issue a call option with payoff $\max\{S_T - K, 0\}$, strike $K = 100$ and maturity $T = 30d$.
- Market Environment:
  - bank account
  - underlying
- Rules:
  - Investment strategies must be **self-financing**.
  - Re-allocations are possible once a day and may involve proportional **transaction cost**.
- Objective: We aim to minimise the quadratic discrepancy between the due payoff and the value of the hedge.
- Training: 10 000 scenarios

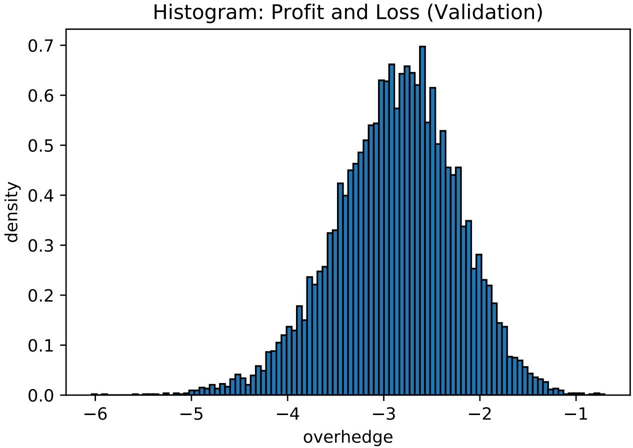
Deep Hedging (without Transaction Cost)



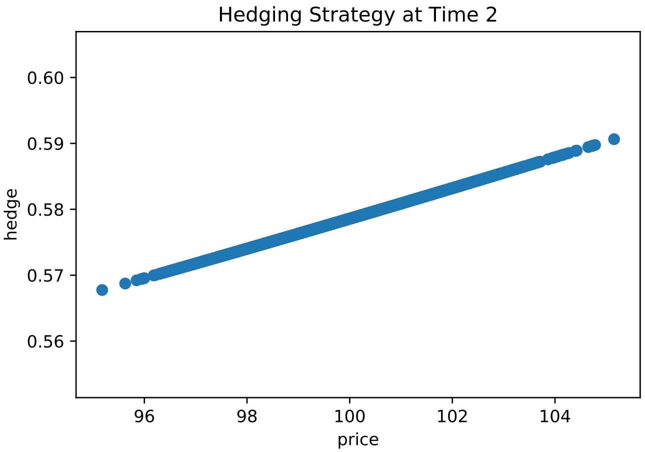
Deep Hedging (without Transaction Cost)



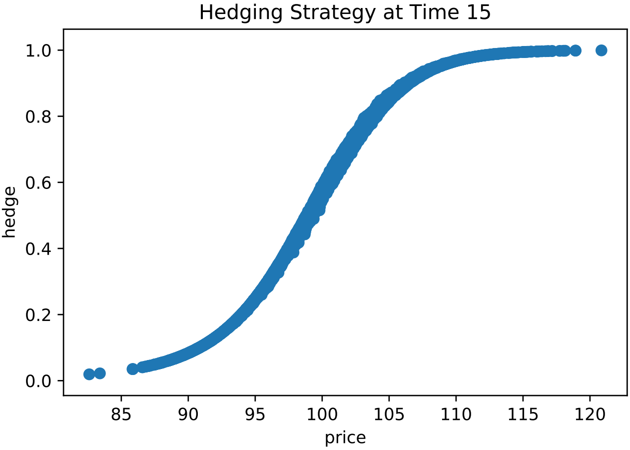
Deep Hedging (without Transaction Cost)



Deep Hedging (without Transaction Cost)



Deep Hedging (without Transaction Cost)



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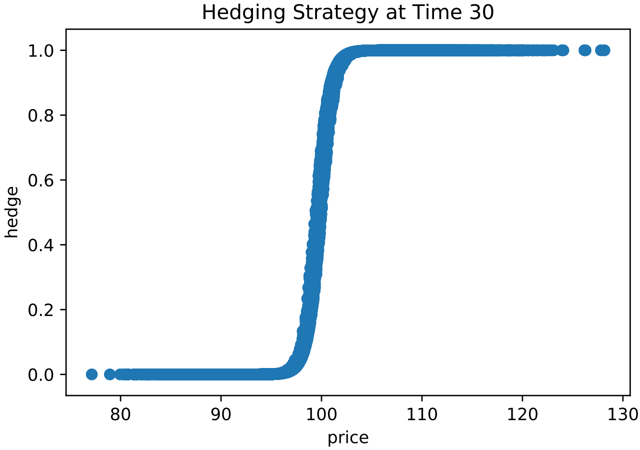
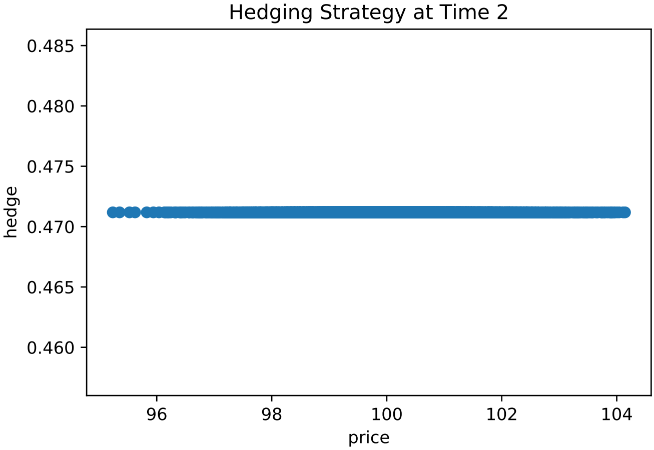
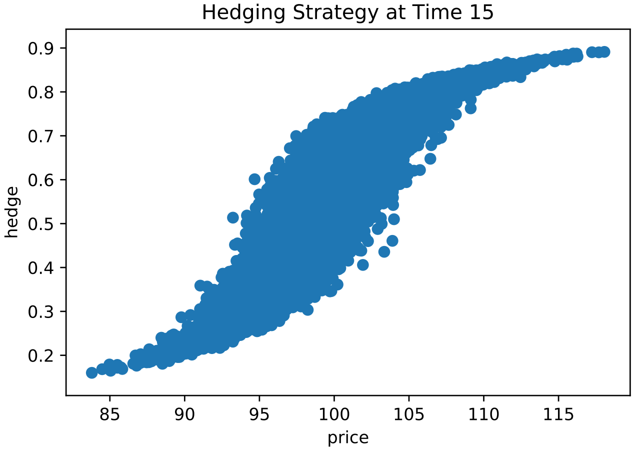


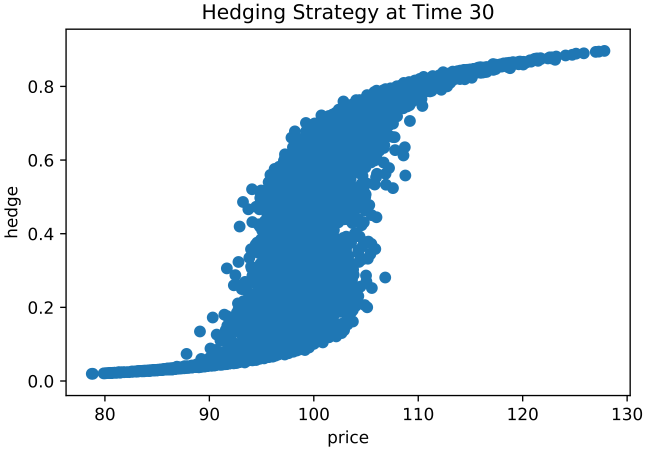
Deep Hedging (with Transaction Cost)



Deep Hedging (with Transaction Cost)



Deep Hedging (with Transaction Cost)



Machine Learning

Summary

Traditional Programming:

$$\text{data} \; + \; \text{program} \; \longrightarrow \; \text{output}$$

Supervised Learning:

$$\text{data} \; + \; \text{output} \; \longrightarrow \; \text{program}$$

Re-inforcement Learning:

$$\text{rules} \; + \; \text{scenarios} \; \longrightarrow \; \text{convincing strategy}$$

Hypothesis

Techniques inspired from re-inforcement learning pave the way for a new era in quantitative risk management from various viewpoints.

1. It is a **disruptive** technology; **high-dimensional** optimisation problems of this kind were not accessible until only recently.
2. It is a very **efficient** and **powerful** technology with
   - super fast requests-on-demand,
   - instantaneous validation (**model risk management**).
3. It is a very **flexible** technology. In a few lines of code, one easily accounts for
   - arbitrary path-dependent payoffs,
   - complex stochastic environments,
   - liquidity squeezes/transaction cost/price impacts,
   - regulatory constraints,
   - risk appetite,

     ⋮

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Applications

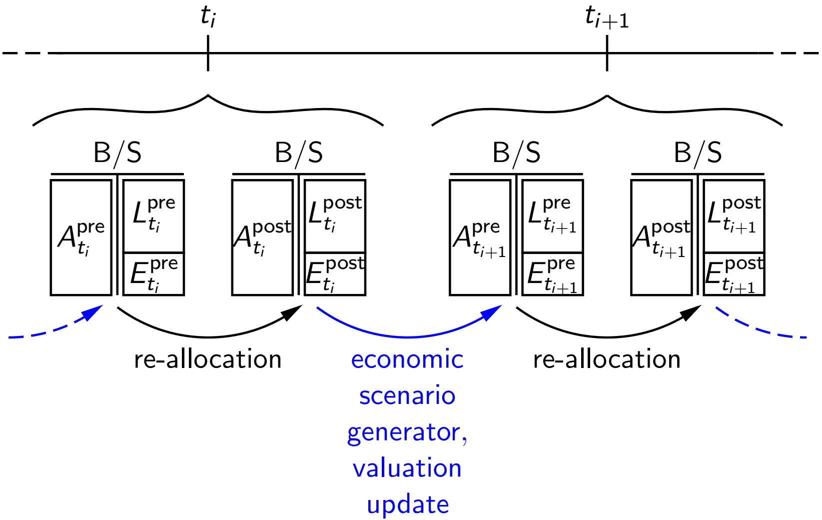
Balance Sheet of an Enterprise in the Financial Industry

Assets	**Liabilities**
Investmentportfolio	Debts
	Equity

Objective

Maximise the expected utility of the **return-on-equity** over different time instances while not exceeding a certain **draw-down** and while guaranteeing the **regulatory constraints** with a high probability.

Balance Sheet Roll-Forward



Applications

Model Ingredients for Re-inforcement Learning

- **economic scenario generator**
  - yield curves
  - credit migrations
  - stock prices
  - client behaviour

    ⋮
- parameterisation of the **states**
- **rule book**
  - constraints
  - eligible re-allocations
  - frictions
- **objective**

Applications

Deep Asset-Liability-Management

Simply put, one solves **high-dimensional** hedging problems with **constraints** in the presence of **frictions** by means of re-inforcement learning techniques.

- **retail bank**: replicating portfolio
- **insurance company**: strategic asset allocation that accounts for the necessary returns and institutional liquidity, optimised re-insurance programme
- **commodity trading**: optimal procurement in the presence of uncertainty, pricing impacts and storage cost
- **pump-storage hydropower plant**: optimised production plan, pricing and hedging in an illiquid environment

Applications

Further Research

- Reach a suitable level of **complexity**.
- Deal with **uncertainty** of model assumptions.
- Model choices and regulisations that promote **robust solutions**.
- Corroborate that sophisticated approach and additional complexity is **profitable**.

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Credits

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