Order in the particle zoo

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Abstract

The standard model of particle physics classifies particles into elementary leptons and hadrons composed of quarks. There exists an alternate ordering principle based on a second order differential equation giving a convergent series of particle energies, to be quantized as a function of the fine-structure constant, α, with limits given by the energy values of the electron and the Higgs boson. The series expansion of the energy equation provides quantitative terms for Coulomb, strong and gravitational interaction. The value of α is given by the gamma functions of the integrals for calculating particle energy in a photon and a point charge expression, representing their corresponding symmetries.

The basic terms of the model may be derived directly from the framework of the Einstein field equations and can be expressed without use of free parameters.

1 Introduction

Particle zoo is the informal though fairly common nickname to describe what was formerly known as "elementary particles". The standard model of particle physics [1] divides these particles into leptons, considered to be fundamental "elementary particles" and the hadrons, composed of quarks.

Well hidden in the data of particle energies lies another ordering principle based on a second order differential equation. Its solution, the exponential function Ψ(r), may be used as equivalent of a probability amplitude applied to an electromagnetic field. This yields a convergent series of particle energies to be quantized as a function of α, the fine-structure constant. α can be calculated form the Γ-functions of the integrals for calculating particle energy in a photon and a point charge representation, suggesting to interpret α as a geometric constant connecting 1D and 3D features that might be expressed by O(1) and O(3) symmetry [1 2].

The expansion of the incomplete gamma function appearing in the integral for calculating particle energy gives quantitative terms for Coulomb, strong and gravitational interaction. The latter provides a quantitative link between the electron and the Planck energy, allowing to identify the electron as ground state. The upper limit of the convergent particle energy series coincides with the vacuum expectation value / Higgs boson energy. The relation with Planck terms allows to express the equations of the model “ab initio” as function of elementary charge, e, electric constant, ε, and gravitational constant G.

The physical constants used should preferably be expressed in natural units and a particularly compact description may be obtained by using the following unit system, attributing the value of the speed of light, c_0, to the inverse value of electric and magnetic constant, ε_c and μ_c. In addition the units will be chosen to yield the elementary charge, e_c, in units of energy. Using SI units for length, time and energy this will result in:

$$c_0^2 = (\varepsilon_c \mu_c)^{-1} \tag{1}$$

with $\varepsilon_c = (2.998E+8\ [m^2/Jm]\ )^{-1} = (2.998E+8)^{-1}\ [J/m]$

$\mu_c = (2.998E+8\ [Jm/s^2]\ )^{-1} = (2.998E+8)^{-1}\ [s^2/Jm]$

From the Coulomb term $b_0 = e^2/(4\pi\varepsilon_0) = e_c^2/(4\pi\varepsilon_c) = 2.307E-28$ [Jm] follows for the square of the elementary charge: $e_c^2 = 9.671E-36\ [J^2]$. In the following $e_c = 3.110E-18$ [J] and $e_c/\varepsilon_c = 9.323E-10$ [m] may be used as natural unit of energy and length.

Within this unit system the necessary parameters of the model will be reduced to e_c and ε_c.

The model originates from a heuristic “ad hoc” approach guided by principles of quantum mechanics and focusing on particle energies. Since this approach has been elaborated in more detail it will be described first in chapter 2. The quantitative relationship with Planck and gravitational terms inherent in its equations

1 Considering the E-vector only; the attribution to symmetry is not unambiguos, O(1) would represent +/- orientation of E; considering the spatial extension of the E-field, Schoenflies $C_{\infty v}$ in place of O(1) and K in place of O(3) might be more appropriate; emphasizing rotation would suggest to use SO(2) (E-vector of the photon as projected in propagation direction) and SO(3); a more general classification might be U(1) and SO(3) / SU(2).

2 The relation of the masses e, μ, π with α was noted in 1952 by Y.Nambu [2]. M.MacGregor calculated particle mass and constituent quark mass as *multiples* of α and related parameters [3].

strongly suggests to examine a possible connection with the theory of general relativity and in chapter 3 it will be demonstrated that the basic equations can be derived from the Einstein field equations. Chapter 4.3 discusses a possible extension to a 5D model.

For both approaches it might be useful to visualize a particle as a rotating electromagnetic field with the E-vector constantly pointing towards the origin [3]. Neutral particles are supposed to exhibit nodes separating corresponding equal volume elements of opposite polarity.

To focus on the more fundamental relationships some minor aspects of the model are exiled to an appendix, related topics to be marked as [A]. The model may be used to calculate additional particle properties, see [4]. Typical accuracy of the calculations presented is in the order of 0.001 [4]. QED corrections are not considered in this model.

2 Ad hoc approach

2.1 Energy terms

The model may be essentially based on a single assumption:

Particles can be described by using an appropriate exponential wave function, Ψ(r), that acts as a probability amplitude on an electromagnetic field.

An appropriate form of Ψ can be deduced from three boundary conditions:

1.) To be able to apply Ψ to a point charge Ψ(r = 0) = 0 is required.

2.) To ensure integrability an integration limit is needed.

3.) Ψ should be applicable regardless of the expression chosen to describe the electromagnetic object. In particular requiring a point charge and a photon representation of a localized electromagnetic field (particle) to have the same energy results in an exponent of 3 for r in the equation below (see 2.2).

Condition 1.) to 3.) are met by an expression (corresponding differential equation see [A1]):

$$\Psi_n(r) = \exp\left(-\left(\frac{\beta_n/2}{r^3}+\left[\left(\frac{\beta_n/2}{r^3}\right)^2-4\frac{\beta_n/2}{\sigma r^3}\right]^{0.5}\right)/2\right) \tag{2}$$

Up to the limit of the real solution of (2), r = r_n, with

$$r_n = (\sigma\, \beta_n/8)^{1/3} \tag{3}$$

in all integrals over Ψ(r) given below equ. (4) may be used as approximation for (2):

$$\Psi_n(r<r_n) \approx \exp\left(\frac{-\beta_n/2}{r^3}\right) \tag{4}$$

Phase will be neglected on this approximation level, properties of particles will be calculated by the integrals over $\Psi(r)^2$ (hence factor 2 in (2)ff) times some function of r which can be given by:

$$\int_0^{r_n} \Psi(r)^2 r^{-(m+1)} dr \approx \int_0^{r_n} \exp(-\beta/r_n^3) r^{-(m+1)} dr = \Gamma(m/3, \beta/r_n^3)\ \frac{\beta^{-m/3}}{3} = \int_{\beta_n/r_n^3}^{\infty} t^{\frac{m}{3}-1} e^{-t} dt\ \frac{\beta^{-m/3}}{3} \tag{5}$$

with m = {..-1;0;1;..}. The term $\Gamma(m/3, \beta/r_l^3)$) denotes the upper incomplete gamma function, given by the Euler integral of the second kind with $\beta_n/r_n^3 = 8/\sigma$ as lower integration limit [5]. For m ≥ 1 the complete gamma function $\Gamma_{m/3}$ is a sufficient approximation, for m ≤ 0 the integrals have to be integrated numerically.

Coefficient β_n may be given as partial product of a value for a ground state particle, β_{GS}, carrying a dimensional term, β_{dim} [m^3], that will be demonstrated to have a particular useful expression using the unit system defined in chpt. 1 as [see A4]:

$$\beta_{dim} = \frac{1}{(4\pi)^2}\left(\frac{e_c}{\varepsilon_c}\right)^3 = 5.131\text{E-30 [m}^3] \tag{6}$$

times particle specific dimensionless coefficients, α_n, of succeeding particles representing the ratio of β_n and β_{n+1}:

3 with vectors of B-field and propagation velocity perpendicular to it;

4 Including e.g. errors due to the numerical approximation of Γ-functions;

5 Euler integrals yield positive values, the absolute sign used for $|\Gamma_{-1/3}|$ is due to the sign convention of Γ-functions.

$$\beta_n = \beta_{GS}\Pi_{k=1}^{n}\alpha_k = 2\sigma\alpha_{GS}\beta_{dim}\Pi_{k=1}^{n}\alpha_k = 2\sigma\alpha_{GS}\beta_{dim}\Pi_{\beta,n} \qquad n = \{1;2;..\} \quad [6] \tag{7}$$

Index n will indicate solutions of (2) and serve in the following as equivalent of a radial quantum number. For the angular terms of Ψ(r, ϑ, φ), to be indicated by index l, only rudimentary results exist, their contribution has to be incorporated in parameter σ = 1.772E+8 [-] (see 2.4).
Particle energy is expected to be equally divided into electric and magnetic part, $W_n = 2W_{n,el} = 2W_{n,mag}$. To calculate energy, the integral over the electrical field E(r) of a point charge is used as a first approximation. Using (5) for m = 1 gives:

$$W_{pc,n} = 2\varepsilon_0\int_0^\infty E(r)^2\Psi_n(r)^2 d^3r = 2b_0\int_0^{r_n}\Psi_n(r)^2 r^{-2}dr = 2\, b_0\, \Gamma(1/3, \beta_n/r_n^3)\, \beta_n^{-1/3}/3 \approx 2\, b_0\, \Gamma_{1/3}\, \beta_n^{-1/3}/3 \tag{8}$$

Using equation (5) for m = -1 to calculate the Compton wavelength, λ_C, gives:

$$\lambda_{C,n} \approx \int_0^{\lambda_{C,n}}\Psi_n(r)^2 dr = \int_{\beta/\lambda_{C,n}^3}^{\infty} t^{-4/3}e^{-t}\,dt\ \ \beta_n^{1/3}/3 = \Gamma(-1/3, \beta_n/\lambda_{C,n}^3)\, \beta_n^{1/3}/3 \approx 36\,\pi^2\, |\Gamma_{-1/3}|\, \beta_n^{1/3}/3 \quad [7] \tag{9}$$

to be used in in the expression for the energy of a photon, hc_0/λ_C :

$$W_{Phot,n} = hc_0/\lambda_{C,n} = \frac{hc_0}{\int^{\lambda_{C,n}}\Psi_n(r)^2 dr} \approx \frac{3hc_0}{36\pi^2|\Gamma_{-1/3}|\beta_n^{1/3}} \tag{10}$$

2.2 Fine-structure constant, α

The energy of a particle is assumed to be the same in both photon and point charge description. Equating (8) with (10) and rearranging to emphasize the relationship of α with the gamma functions ($\Gamma_{1/3}$ = 2.679; $|\Gamma_{-1/3}|$ = 4.062) gives as first approximation (note: h => ħ):

$$\frac{4\pi\Gamma_{1/3}|\Gamma_{-1/3}|}{0.998} = \frac{9hc_0}{18\pi b_0} = \frac{\hbar c_0}{b_0} = \alpha^{-1} \quad [8] \tag{11}$$

In (11) $\Gamma_{1/3}$ represents the limit of $\Gamma(_{1/3,}\ \beta_n/r_n^3)$ for the lower bound of integration approaching zero, β/r^3-> 0. Using the analog limit for $\Gamma(_{-1/3,}\ \beta_n/\lambda_{C,n}^3)$ gives a more precise expression depending on $\lambda_{C,n}$ and β_n (see [A3]):

$$\frac{\Gamma_{1/3}\lambda_{C,n}}{3\pi\beta_n^{1/3}} = \alpha^{-1} \tag{12}$$

In this case the precision for the calculation of α is identical with the precision for calculating particle energy with the respective β_n, e.g. with β_e of (65) α_{calc} = 1.0001 α.

The relationship of α with the Γ-functions according to (11) can be found in the expressions for coefficient σ and angular momentum as well, see 2.4. In 4.3 a possible extension of the 3D formalism to 4D will be discussed, yielding an expression for α_{weak}.

2.3 Quantization with powers of 1/3^n over α

Inserting (7) in the product of the point charge and the photon expression of energy, (8) and (10), gives for $W_n^2 = W_{pc,n} * W_{phot,n}$

$$W_n^2 = 2b_0hc_0\frac{\int^{r_n}\Psi_n(r)^2r^{-2}dr}{\int^{\lambda_{C,n}}\Psi_n(r)^2dr} \sim \frac{1}{\beta_n^{2/3}} \sim \frac{\alpha_1^{1/3}\alpha_2^{1/3}.....\alpha_n^{1/3}}{\alpha_1\alpha_2....\alpha_n} \tag{13}$$

The last expression of (13) is obtained by expanding the product $\Pi_n^{-2/3}$ included in $\beta_n^{-2/3}$ of (7) with $\Pi_n^{1/3}$.
From this term it is obvious that a relation $\alpha_{n+1} = \alpha_n^{1/3}$ yields the only non-trivial solution for W_n^2 where all intermediate particle coefficients cancel out and W_n becomes a function of coefficient α_1 only. Identifying α_1 as $\alpha_1 = \alpha_\mu = \alpha^3$ would give an expression using the muon as reference state:

6 $\Pi_{\beta n}$ denoting the sum of all particle coefficients except the one of the ground state particle (electron, see below).
7 Factor ≈ 355 ≈ 36 π^2 is calculated numerically from the Euler integral (5) for m = -1, using β_n of (18), (65) or from a fit of particle energy and angular momentum.
8 As with all calculations in this work the calculation for α^{-1} refers to a rest frame and thus corresponds to an IR limit.

$$\Pi_{k=1}^{n}\left(\frac{\alpha\wedge(3/3^n)}{\alpha\wedge(9/3^n)}\right) = \frac{\alpha\wedge(3/3^n)}{\alpha^3} \qquad n = \{1;2;..\} \quad [9] \qquad (14)$$

and with its root

$$\left(\frac{\alpha\wedge(3/3^n)}{\alpha^3}\right)^{0.5} = \frac{\alpha\wedge(1.5/3^n)}{\alpha^{1.5}} = \Pi_{k=1}^{n}\alpha\wedge(-3/3^k) \qquad n = \{1;2;..\} \qquad (15)$$

the corresponding term for particle energies would be:

$$W_n = W_\mu \ \Pi_{k=2}^{n}\alpha\wedge(-3/3^k) \qquad n = \{2;3..\} \quad [10] \qquad (16)$$

for spherical symmetry.

In equation (16) no state is singled out in particular as a ground state in the equations. The partial product of (16) may be extended to include the electron by inserting *ad hoc* an additional factor ≈ 3/2 to represent the irregularity due to the energy ratio of e, μ, W_μ / W_e = 1.5088 α^{-1} (see 2.4, [A2]). In chpt. 2.8 it will be demonstrated that a fundamental relationship exists between the electron and the Planck energy [11], implying the electron to correspond to a ground state term. With the ratio of electron and Planck energy given as:

$$W_e/W_{Pl} = 4.903\text{ E-22} = \alpha_o \quad [12] \qquad (17)$$

β_{GS} of the ground state, the electron, can be approximated in a particular simple expression:

$$\beta_{GS} = \beta_e = \sigma^*\alpha_0\ \beta_{\,dim} = \frac{\sigma^*\alpha_0}{(4\pi)^2}\left(\frac{e_c}{\varepsilon_c}\right)^3 = 1.286\text{E-43 [m}^3] \quad [13] \qquad (18)$$

With W_e as ground state W_n would be given by (13) as [14] (n = {1;2;...}):

$$W_n = \frac{3}{2}\left(\frac{4\pi b_0^2}{\alpha}\frac{\int^{r_n}\Psi_n(r)^2 r^{-2}dr}{\int_{\lambda_{C,n}}\Psi_n(r)^2 dr}\right)^{0.5} = \frac{3}{2}\left(\frac{4b_0^2\Gamma_{1/3}^2}{9[\alpha 4\pi\,\Gamma_{1/3}|\Gamma_{-1/3}|]\beta_n^{2/3}}\right)^{0.5} = \frac{3}{2}\left(\frac{\mathbf{2b_0\Gamma_{1/3}(4\pi)^{2/3}}}{\mathbf{3\ (\sigma^*\alpha_0)^{1/3}}}\left(\frac{\boldsymbol{\varepsilon_c}}{\boldsymbol{e_c}}\right)\right)\frac{\alpha\wedge(1.5/3^n)}{\alpha^{1.5}} \qquad (19)$$

and with adding a factor y_l^m for the contribution of non-spherical symmetric states:

$$W_n/W_e \approx 3/2\,y_l^m\frac{\alpha\wedge(1.5/3^n)}{\alpha^{1.5}} = 3/2\,y_l^m\,\Pi_{k=0}^{n}\alpha\wedge(-3/3)^k = 3/2\,y_l^m\,\Pi_{W,n} \qquad n = \{1;2;..\} \qquad (20)$$

2.4 Angular momentum, coefficients σ and α

A simple relation with angular momentum J for spherical symmetric states will be given by applying a semi-classical approach using

$$J = r_2\,x\,p(r_1) = r_2 W_n(r_1)/c_0 \qquad (21)$$

with $W_{kin,n}$=1/2 W_n , using term $2b_0$ of (8) as constant factor, integrating over a circular path of radius $|r_2| = |r_1|$ and setting r_n of (3), 8/σ as integration limits. This will give:

$$|J| = \int_0^{r_n}\int_0^{2\pi} J_n(r)\,d\varphi\,dr = 4\pi\frac{b_0}{c_0}\int_0^{r_n}\Psi_n(r)^2 r^{-1}dr \qquad (22)$$

From (5) follows for m = 0:

$$\int_0^{r_n}\Psi_n(r)^2 r^{-1}dr = 1/3\int_{8/\sigma}^{\infty} t^{-1}e^{-t}dt = \frac{\alpha^{-1}}{8\pi} \approx 5.45 \approx \Gamma_{1/3}|\Gamma_{-1/3}|/2 \qquad (23)$$

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9 For illustration purposes with n = 4: $\frac{\alpha^1\alpha^{1/3}\alpha^{1/9}\boldsymbol{\alpha}^{\mathbf{1/27}}}{\boldsymbol{\alpha}^{\mathbf{3}}\alpha^1\alpha^{1/3}\alpha^{1/9}} = \frac{\boldsymbol{\alpha}^{\mathbf{1/27}}}{\boldsymbol{\alpha}^{\mathbf{3}}}$

10 Series starts with n=2 since n=1, i.e. α-coefficient of μ already included in W_μ.

11 as defined by (32);

12 Note: The extension of (16) gives as coefficient for the electron: $\alpha_e \approx (3/2)^3\alpha^9 \approx 3\alpha_0\,\alpha^{-1}$.

13 $\sigma^* = \sigma/1.5133^3$, see 2.4. With equ. (32) for W_{Pl} this will give $W_e = 1.0085\ W_{e,\,exp}$

14 factor 3/2 added ad hoc; expanding by $\Gamma_{1/3}$ and using (11) to eliminate term in square brackets; W_e given in bold;

Inserting (23) in (22) gives:

$$|J| = 4\pi \frac{b_0}{c_0} \frac{\alpha^{-1}}{8\pi} = 1/2\ [\hbar] \tag{24}$$

For r >> $\beta^{1/3}$ the integral for calculating energy in (8) depends only weakly on r, thus it is integral (23) that determines a specific particle radius and value for σ. As a consequence in addition to the mandatory term of $|\Gamma_{-1/3}|\ \beta_n^{1/3}/3$ of the integrals (5) for m=-1, r_n, σ contain a semi-empirical factor ≈1.51 α^{-1}, very close to the ratio W_μ/W_e = 206.8 = 1.5088 α^{-1}. The exact value of 1.5133 for ≈1.51 has been chosen due to the 3rd power relationship between 1.5088 and 1.5133 (see [A2]) and a possible geometric interpretation of the terms in σ:

$$1.51\ \alpha^{-1}\ |\Gamma_{-1/3}|\ /3 \approx\ |\Gamma_{-1/3}|\ /\Gamma_{1/3}\ \ 4\pi\ |\Gamma_{-1/3}|\ \Gamma_{1/3}/0.998\ \ |\Gamma_{-1/3}|/3\ \approx\ \frac{4\pi\,|\Gamma_{-1/3}|^3}{3}\ =\ (\sigma/8)^{1/3} \quad ^{15} \tag{25}$$

Both (23) and (25) corroborate the significance of the term for α of (11) independent of the derivation in 2.2.

The various useful terms for σ may be summed up as:

$$\sigma = 8\ r_n^3\ /\beta_n = (1.5133\ \alpha^{-1}\ 2/3\ |\Gamma_{-1/3}|)^3\ = 1.5133^3\ \sigma^* =\ 8\left(\frac{4\pi\,|\Gamma_{-1/3}|^3}{3}\right)^3\ = 1.772\text{E+8}\ [-] \quad ^{16} \tag{26}$$

2.5 Upper limit of energy

According to the geometrical interpretation given in 2.4 non-spherical particles should exhibit lower values of σ (and r_n). The variable part in σ is given by the term $(1.5133\ \alpha^{-1})^3$, leaving the minimum for σ, defined by $(2/3\ |\Gamma_{-1/3}|)^3$, i.e. the term in the integral expression for r, and the integers in the square bracket of equ.(2) [17]:

$$\sigma_{min} = (2/3\ |\Gamma_{-1/3}|)^3 \tag{27}$$

The maximum angular contribution to W_{max} would be :

$$\Delta W_{max,\ angular} = 1.5133\ \alpha^{-1} \tag{28}$$

The limit of the partial product in (20) is given by $\alpha^{-1.5}$, the limit term of ≈ 3/2 by 1.5066 [A2], thus according to (20) and (28), the maximum energy will be $W_{max} = W_e$ 1.5066*1.5133 $\alpha^{-2.5}$ = 4.103E-8 [J] (=1.041 vacuum expectation value, 246GeV=3.941E-8 [J]).

In the simple visualization sketched in the introduction the "rotating E-vector" might be interpreted to cover the whole angular range in the case of spherical symmetric states while an object with one angular node, as represented by the spherical harmonic Y_1^0 or an atomic p-orbital, might be interpreted as forming a double cone. Increasing the number of angular nodes would close the angle of the cone leaving in the angular limit case, l -> ∞, a state of minimal angular extension representing the original vector, however, extending in both directions from the origin and featuring parity p= -1, giving the energy of the VEV. Considering only „half" such a state, extending in one direction only and having p = +1, would notably feature an energy of 1.024 W_{Higgs}, suggesting the energy value of the Higgs boson as possible high energy end for particle energy of the series (20).

2.6 Non-spherical symmetric states

Except for the limit case of 2.5 angular solutions for particle states are not known yet and to extend the model to such states assumptions have to be made.

Assuming the angular part of Ψ to be related to spherical harmonics and exhibiting the corresponding nodes would give the analog of an atomic p-state for the 1st angular state, Y_1^0. According to the very simple geometric interpretation of chpt. 2.5 this would give a factor of 3 for energiy of such a state relative to spherical states, applying an additional factor 1/2, as in the case of the Higgs boson, would give 3/2.

Alternatively it might be assumed that $W_{n,l} \sim 1/r_{n,l} \sim 1/V_{n,l}^{1/3}$ (V = volume) would be applicable for non-spherically symmetric states as well, giving $W_1^0/W_0^0 = 3^{1/3} = 1.44$.

A second partial product series of energies corresponding to these values approximately fits the data, for W_{calc} in table 1 factor 1.44 has been used.

A change in angular momentum has to be expected for a transition from Y_0^0 to Y_1^0 which is actually observed with ΔJ = ± 1 except for the pair μ/π with Δ J = 1/2.

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15 See 4.2 as well. 4π $|\Gamma\text{-}1/3|^3/3$ is used for σ in all calculations.

16 Factor 1.5133 is also part of a minor term depending on the radial quantum number, n, (see [A2, 4]). Thus in the following β_n may be split into σ* = σ/1.5133³ = 5.112E+7 [-] and α(n)-terms containing factor 1.5133³.

17 For the approximation (4) to hold a minimum value of σ >> 1 is required in (2) as well.

	n, l	$W_{n,Lit}$ [MeV]	α-coefficient (energy) equ (20)	α-coefficient in ß equ (7)	W_{calc}/W_{Lit}	J	r_n [fm]
Planck	(-1,∞)	1.0 E+21*	$(2/3\ \alpha^{-3})^3\ 3/2\ \alpha^{-1}\ 2$ source term, relative to e !		0.9994 rel. to e !	-	-
e^{+-}	**0, 0**	**0.51**	**$2/3\ \alpha^{-3}$**	$(3/2)^3$ **α^9**	**1.0001**	**1/2**	**1412**
μ^{+-}	**1, 0**	**105.66**	**$\alpha^{-3}\alpha^{-1}$**	**$\alpha^9\alpha^3$**	**1.0001**	**1/2**	**6.83**
π^{+-}	1, 1	139.57	$\alpha^{-3}\alpha^{-1}$ 1.44	$\alpha^9\alpha^3/3$	1.0919	0	4.74
K		495	see [A5]	see [A5]		0	
η^0	**2, 0**	**547.86**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}$**	**$\alpha^9\alpha^3\alpha^1$**	**0.9934**	**0**	**1.32**
ρ^0	2, 1	775.26	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3})$ 1.44	$\alpha^9\alpha^3\alpha^1/3$	1.0124	1	0.92
ω^0	2, 1	782.65	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3})$ 1.44	$\alpha^9\alpha^3\alpha^1/3$	1.0029	1	0.92
K*		894				1	
p^{+-}	**3, 0**	**938.27**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}$**	**$\alpha^9\alpha^3\alpha^1\alpha^{1/3}$**	**1.0017**	**1/2**	**0.76**
n	**3, 0**	**939.57**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}$**	**$\alpha^9\alpha^3\alpha^1\alpha^{1/3}$**	**1.0004**	**1/2**	**0.76**
η'		958	see [A5]	see [A5]		0	
Φ^0		1019	see [A5]	see [A5]		1	
Λ^0	**4, 0**	**1115.68**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}$**	**$\alpha^9\alpha^3\alpha^1\alpha^{1/3}\alpha^{1/9}$**	**1.0107**	**1/2**	**0.63**
Σ^0	**5, 0**	**1192.62**	**$\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}\alpha^{-1/81}$**	**$\alpha^9\alpha^3\alpha^1\alpha^{1/3}\alpha^{1/9}\alpha^{1/27}$**	**1.0047**	**1/2**	**0.61**
Δ	**∞, 0**	**1232.00**	**$\alpha^{-9/2}$**	**$\alpha^{27/2}$**	**1.0026**	**3/2**	**0.59**
Ξ		1318				1/2	
Σ^{*0}	3, 1	1383.70	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9})$ 1.44	$\alpha^9\alpha^3\alpha^1\alpha^{1/3}/3$	0.9797	3/2	0.53
Ω^-	4, 1	1672.45	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27})$ 1.44	$\alpha^9\alpha^3\alpha^1\alpha^{1/3}\alpha^{1/9}/3$	0.9725	3/2	0.45
N(1720)	5, 1	1720.00	$(\alpha^{-3}\alpha^{-1}\alpha^{-1/3}\alpha^{-1/9}\alpha^{-1/27}\alpha^{-1/81})$1.44	$\alpha^9\alpha^3\alpha^1\alpha^{1/3}\alpha^{1/9}\alpha^{1/27}/3$	1.0047	3/2	0.43
tau^{+-}	∞, 1	1776.82	$(\alpha^{-9/2})$1.44	$\alpha^{27/2}/3$	1.0025	1/2	0.40
Higgs	∞,∞ **	1.25 E+5	$(\alpha^{-9/2})\ 3/2\ \alpha^{-1}/2$	$(\alpha^{27/2})/(3/4\ \alpha^{-1})^3$	1.0230	0	0.006
VEV	∞,∞ **	2.46 E+5	$(\alpha^{-9/2})\ 3/2\ \alpha^{-1}$	$(\alpha^{27/2})/(3/2\ \alpha^{-1})^3$	1.04	0	0.003

Table 1: Particle energies for $\mathbf{y_0^0}$ **(bold)**, y_1^0 [18]; col. 2: radial, angular quantum number; col. 3: energy values of [6] except* (see (32)); col. 4: α-coefficient according to the energy terms of (20), including (2/3) α^{-3} of electron; col. 5: coefficients in β of (7); col. 6: W_{calc} calculated using the slightly more precise [A4 (64)f] in place of (20); ** see 2.5;
Blanks in the table are discussed in [A5].

2.7 Expansion of the incomplete gamma function $\Gamma(1/3,\ \beta_n/r^3)$

The series expansion of $\Gamma(1/3,\beta_n/r^3)$ in the equation for calculating particle energy (8) gives [7]:

$$\Gamma(1/3,\ \beta_n/(r^3)) \approx \Gamma_{1/3} - 3\left(\frac{\beta_n}{r^3}\right)^{1/3} + \frac{3}{4}\left(\frac{\beta_n}{r^3}\right)^{4/3} = \Gamma_{1/3} - 3\frac{\beta_n^{1/3}}{r} + \frac{3}{4}\frac{\beta_n^{4/3}}{r^4} \tag{29}$$

and for $W_n(r)$:

$$W_n(r) \approx W_n - 2b_0\frac{3\beta_n^{1/3}}{3\beta_n^{1/3}r} + 2b_0\frac{3}{4}\frac{\beta_n^{4/3}}{3\beta_n^{1/3}r^4} = W_n - \frac{2b_0}{r} + b_0\frac{\beta_n}{2r^4} \quad {}^{19} \tag{30}$$

The 2nd term in (30) drops the particle specific factor β_n and gives twice [20] the electrostatic energy of two elementary charges at distance r. The 3rd term is an appropriate choice for the 0th order term of the differential equation [A1]. It is thus supposed to be responsible for the localized character of a particle state and may be identified with the "strong force" of the standard model.

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18 up to Σ'^0 all resonance states given in [6] as **** included; Exponents of -9/2, 27/2 for Δ and tau are equal to the limit of the partial products in (7) and (20); r_n calculated with (3); 1.5133 approximated by 3/2;
19 Signs not adapted to conventional definition.
20 Due to the assumption used in (8): W_n is supposed to be $W_n = 2W_{n,el} = 2W_{n,mag} = W_{n,el} + W_{n,mag}$

2.8 Gravitation
2.8.1 Planck scale
Expressing energy/mass in essentially electromagnetic terms suggests to test if mass interaction i.e. gravitational attraction can be derived from the corresponding terms. Assuming the expansion of the incomplete Γ-function for the integral over r^{-2}, $\Gamma(1/3,\beta_n/r^3)$ (29)f, might be an adequate starting point for gravitational attraction as well, implies that the Coulomb term b_0 will be part of the expression for F_G, i.e. the ratio between gravitational and Coulomb force, e.g. for the electron, $F_{G,e}/F_{C,e}$ = 2.41E-43, should be a completely separate, self-contained term.

This is equivalent to assume that gravitational interaction is a higher order effect with respect to electromagnetic interaction and as such should be of less or equal strength compared to the latter. This suggests to use the expression

$$b_0 = G\, m_{Pl}^2 = G\, W_{Pl}^2 / c_0^4 \tag{31}$$

as definition for Planck terms, giving for the Planck energy, W_{Pl}:

$$W_{Pl} = c_0^2 (b_0/G)^{0.5} = c_0^2 (\alpha h c_0/G)^{0.5} = 1.671\text{ E+8 [J]} \tag{32}$$

Using (32) and expanding relationship (19)f to higher powers of α i.e. $\alpha_e^3 = (1.5133^3\,\alpha^9)^3$ multiplied by the angular limit factor according to (28) [21] gives a quantitative relationship for the ratio of W_e and W_{Pl}:

$$1.0006\frac{W_e}{W_{Pl}} = 1.5133^2\alpha^{10}/2 = 4.903\text{ E-22} = \alpha_o \tag{33}$$

In the next chapter a derivation for this relation will be suggested originating in the third term of the energy expansion (30).

Using (62) to express factor 1.5133 gives (F_G, F_C = gravitational, Coulomb forces):

$$\left(\frac{W_e}{W_{Pl}}\right)^2 = \left(\frac{F_{G,e}}{F_{C,e}}\right)_{calc} \approx \left(\frac{1.5133^3\alpha^9}{1.5133\,\alpha^{-1}2}\right)^2 = \left(\frac{(4\pi)^2|\Gamma_{-1/3}|^4\alpha^{12}}{2}\right)^2 = 1.00075^2\left(\frac{F_{G,e}}{F_{C,e}}\right)_{\exp} = \frac{G W_e^2}{c_0^4 b_0} = \alpha_0^2 \tag{34}$$

Using (11) and [A4 (66)] for calculating W_e would turn G into a coefficient based on electromagnetic constants:

$$G_{calc} \approx \frac{c_0^4}{4\pi\varepsilon_c}\left(\frac{1}{3\pi^{2/3}}\left(\frac{|\Gamma_{-1/3}|}{\Gamma_{1/3}}\right)^4\alpha^{12}\right)^2 \approx \frac{c_0^4}{4\pi\varepsilon_c}\,\frac{2}{3}\,\alpha^{24} = 1.0008\,G_{\exp} \tag{35}$$

2.8.2 Virtual superposition states
Within this model particles might interact via direct contact in place of boson-mediated interaction. The particles are not expected to exhibit a rigid radius. Within the limits of charge and energy conservation a superposition of many states might be conceivable, extending the particle in space with radius ~ r_n, $\lambda_{C,n}$ etc. appropriate for energy of each virtual particle state (VS) [22], providing a source of energy at a distance r_{VS} from the primary particle and in turn contributing to the stress-energy tensor responsible for curvature of spacetime that manifests itself in gravitational attraction.

In general VS are not supposed to consist of analogs of e.g. spherical symmetric states covering the complete angular range of 4π but to be an instantaneous, short term extension of the E-vector thus requiring the angular limit factor of (28).

A long range effect of the 3rd, the strong interaction term, of (30) may be exerted via virtual particle states. To estimate such an effect in first approximation the following will be used:

- the 3rd term of the energy expansion equ. (30) with β according to (7), (18),
- the angular limit state of σ^*_{min} according to (27), $\sigma^*_{min} \approx 1$,
- $\beta_{dim} = (4\pi)^{-2}(e_c/\varepsilon_c)^3 \approx (\alpha^{-1} r_e)^3$, which might be considered to represent the cube of a natural unit of length, R.

For any VS at $r = \alpha^{-1}\, r_{VS} = \Pi_{VS}^{1/3}(\alpha^{-1} r_e)$, i.e. the radius of the VS in natural units, R_{VS}, equ. (36) will hold:

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21 The latter factor may be interpreted as a sum of minor factors of a more detailed analysis of β as well [A4 (64)f].

22 The superposition states considered here would be not virtual in a Heisenberg sense, the energy is provided by the primary particle.

$$W_{VS}(r)\approx \frac{b_0\beta_{VS}/2}{(\alpha^{-1}\ \ r_{VS})^4} \approx \frac{b_0\alpha_0\Pi_{W,VS}(\alpha^{-1}r_e)^3}{(\alpha^{-1}r_{VS})^3(\alpha^{-1}r_{VS})} \approx \frac{b_0\alpha_0\Pi_{W,VS}(\alpha^{-1}r_e)^3}{(\Pi_{W,VS}^{1/3}\alpha^{-1}r_e)^3(\alpha^{-1}r_{VS})} = \frac{b_0\alpha_0}{(\alpha^{-1}r_{VS})} = \frac{b_0}{R_{VS}}\left(\frac{F_{G,e}}{F_{C,e}}\right)^{0.5} \quad (36)$$ [23]

Considering that the composition of the stress-energy tensor from virtual states is expected to be based on a much more complex mechanism requiring consideration of all possible virtual states at a particular point and appropriate averaging, (36) has to be a first approximation. The crucial factor that turns the r^{-4} dependence of the strong interaction term into r^{-1} of gravitational interaction is the proportionality of β_n to the cube of any characteristic particle length, r_n, $\lambda_{C,n}$ etc. which is valid for each particle state subject to the relations of this model.

Equ. (36) is a representation of the gravitational energy of the electron, terms for other particles may be obtained by inserting their energy values relative to the electron according to (20) in (36) which might be interpreted as the intensity/frequency of emergence of virtual states being proportional to the energy of the primary particle.

Consequently the highest possible particle energy value attributable to (36) will be α_0^{-1}, i.e. the value of the Planck energy relative to the electron. This is the fundamental cause for relation (33) and in turn corroborates the assumption used in the definition of equ. (31)f.

3 Derivation from the Einstein field equation

The quantitative relationship of the model for calculating particle energies with gravitational interaction via a mechanism that provides energy at a distance from a primary particle and thus a contribution to the stress-energy-tensor and curvature of spacetime suggests to test if the equations of this model may be derived directly from the Einstein field equations.

The minute factor G/c_0^4 in the Einstein field equation (EFE) is responsible for this equation not being particularly suited to attempt a calculation of particle energies based on this formalism. The interpretation of gravitation as a higher order effect with respect to electromagnetism suggests to replace G/c_0^2 [m/kg] or G/c_0^4 [m/J] by an equivalent electromagnetic term. A term of order $1/\varepsilon_c$ [m/J] may provide the appropriate units and the necessary order of magnitude, suggesting to use a substitution such as:

$$(8\pi)G/c_0^4 \quad => \quad \approx\frac{4\pi}{\varepsilon_c} \quad (37)$$

In the following the central concept will be the “rotating E-vector” of the introduction, i.e. a photon with its intrinsic angular momentum visualized as having its E-vector rotating around a central axis of propagation (symmetry SO(2) as projected in propagation direction) will be transformed to an object that has the -still rotating- E-vector constantly oriented to a fixed point, the origin of the local coordinate system used, resulting in a SO(3) object with point charge properties.

The basic question will be: What kind of metric will yield an undisturbed photon propagation according to Maxwell's equations that manifests itself as a localized object in flat spacetime ?

In a spherical coordinate system the rotation of an object with extension in angular direction will result in some kind of self interaction increasing with r ->0 *unless* space(time) is curved in such a way as to prevent that. This will be the case if the r^2-term in the angular coordinates is canceled, implying positive curvature and an expansion of curved spacetime with r^2 at any given r, i.e. the Ricci scalar should be $R(r) \sim -1/r^2$.

The general approach will be to set all terms in an appropriately constructed Ricci scalar to be zero, except a $1/r^2$ component, thus obtaining a homogenous 2nd order differential equation.

In a simple 4-D metric of type $g_{\mu\nu} = (+1, -1, -r^2, -r^2\sin^2\vartheta)$ [24] a factor of -1 arises in the Ricci components R_{22} and R_{33} due to the derivative of the term $\Gamma_{23}^3 = \Gamma_{32}^3 = \cot\vartheta$ with respect to ϑ, resulting in in a term $+1/r^2$ in the Ricci scalar. Changing the sign of $+1/r^2$ in R in such a 4D-metric can be formally achieved by changing the sign in x_2,x_3 to $-r^2$ or the use of an imaginary value of ϑ in $g_{\mu\nu}$.

In the following this concept is illustrated as a formal, general approach, where the Ricci scalar will be required to be:

23 The term for gravitational attraction, $F_{m,n;r}$ between two particles, m and n at a distance r, would be obtained by using $1/b_0$ as proportionality constant: $F_{m,n;r} \approx \frac{1}{b_0}\ W_{VS(m,r)}W_{VS(n,r)} \approx b_0\ \frac{\Pi_m\Pi_n}{r^2}\ \alpha_0^2$

24 coordinates t, r, ϑ, φ = x_0, x_1, x_2, x_3; only diagonal elements considered, μ=ν;

$R = -2/r^2$ (38)

and an exponential ansatz will be used for $g_{00,11}$ i.e.:

$g_{\mu\nu} = (+\exp(a\, v(r)), -\exp(b\, v(r)), +r^2, +r^2 \sin^2\vartheta)$

This will result in the following Ricci scalar (with the components belonging to ct, r and ϑ, φ still separated), (see [A6]):

$$R = \left(e^{-bv}\left[-a v'' - \frac{a^2 v'^2}{2} + \frac{ab\, v'^2}{2} - \frac{(a-b) v'}{r} \right]_{00,11} + e^{-bv}\left[\frac{(b-a) v'}{r} - \frac{2}{r^2} \right]_{22,33} \right) - 2/r^2 \tag{39}$$

To get $R = -2/r^2$ one has to set the term in curved brackets to zero.

The equation (39) refers to local coordinates and has to be solved for these or transformed to flat coordinates. The latter will be attempted by transforming the spherical object of a particle back into a photon of appropriate wavelength, assuming that

1.) for r ->0 the angular coordinates have to reflect the expansion ~ $1/r^2$, while

2.) the energy-space-time relation of a photon, i.e. W_{ph} ~1/r, ~1/T reflects a contraction of spacetime linear in coordinates ct, r .

A coefficient ρ [m] will be needed to obtain dimensionless terms [25]. This gives:

$$R = \left(e^{-bv}\left[-a v'' - \frac{a^2 v'^2}{2} + \frac{ab\, v'^2}{2} - \frac{(a-b) v'}{r} \right]_{00,11} \frac{\boldsymbol{r}}{\boldsymbol{\rho}} + e^{-bv}\left[\frac{(b-a) v'}{r} - \frac{2}{r^2} \right]_{22,33} \frac{\boldsymbol{\rho}^2}{\boldsymbol{r}^2} \right) - \frac{2\boldsymbol{\rho}^2}{r^2 \boldsymbol{r}^2} \tag{40}$$

and

$$\left[-a v'' - \frac{a^2 v'^2}{2} + \frac{ab\, v'^2}{2} - \frac{(a-b) v'}{r} \right]_{00,11} \frac{\boldsymbol{r}}{\boldsymbol{\rho}} + \left[\frac{(b-a) v'}{r} - \frac{2}{r^2} \right]_{22,33} \frac{\boldsymbol{\rho}^2}{\boldsymbol{r}^2} = 0 \tag{41}$$

(In a 5D approach the ansatz for the Kaluza term Φ discussed in 4.3 requires terms $(\rho/r)^n$ to be included in x_4)

An equation of type (41) will in general feature solutions of type $\exp(v) = \exp(-x/r^3)$, which is a sufficient criterion to obtain equations (11), (14)ff i.e. the numerical expression for α and the quantization of particle energies.

Setting e.g. a = b gives:

$$-a v'' \frac{r}{\rho} - \frac{2}{r^2}\frac{\rho^2}{r^2} = 0 \qquad => \qquad a v'' = -\frac{2\rho^3}{r^5} \tag{42}$$

and corresponds to equ. (4) if choosing an appropriate value for a. Using polar coordinates in flat space and setting a = b = 1/3 and $v = (-\rho/r)^3$ gives:

$$e^{v/3} = \Psi(r) = \exp\left(\frac{-\rho^3}{3r^3} \right) \qquad => \qquad \frac{-4\rho^3}{r^5} + \frac{2\rho^3}{r^5} = -\frac{2\rho^3}{r^5} \tag{43}$$

The Einstein tensor component G_{00} will be:

$G_{00} = [-v''/6 - v'/(3r)] + e^{v/3}\rho^2/r^4 = e^{v/3}\rho^2/r^4$ (44)

Equating with the component of the stress-energy tensor, $G_{00} = T_{00}$, and using the coefficient given in (37) will give (w = energy density):

$$e^{v/3}\frac{\rho^2}{r^4} \approx \frac{4\pi w}{\varepsilon_c} \qquad => \qquad \frac{\varepsilon_c e^{v/3}\rho^2}{4\pi r^4} \approx w \tag{45}$$

The volume integral over (45)f gives the particle energy according to

$$W_n = \frac{\varepsilon_c\ \rho^2}{4\pi}\int_0^{r_n} \frac{e^{v/3}}{r^4} d^3 r = 3^{1/3}\frac{\Gamma_{1/3}}{3}\varepsilon_c \rho \tag{46}$$

25 i.e. the factor r^2 in the angular terms will be canceled by ρ^2/r^2, restoring $C_{\infty,v}$ symmetry, while using factor r /ρ in the ct, r terms would give $W_{ph} \sim 1/\rho$.

To recover (8), (18) for the electron, ρ in (45)f has to be given by

$$\rho^3 = \frac{8}{3\sigma^* \alpha_0 (4\pi)} \left(\frac{e_c}{\varepsilon_c} \right)^3 \quad [26] \qquad (47)$$

i.e. a derivation from the EFE with $4\pi/\varepsilon_c$ in place of $8\pi G/c_0^4$ reproduces the basic equation (4) with essentially the same set of coefficients as used in chapter 2.

4 Discussion

4.1 Relationship to the standard model of particle physics

The standard model of particle physics is not particularly efficient in quantitative calculation of particle mass / energy. Lepton masses are treated as parameters of the model while calculation of hadron masses [8], [9] with lattice QCD methods uses quark masses, coupling constant and a reference particle for the absolute energy scale, i.e. typically about 4 parameters are required as input parameters to calculate mass of ~12 particles with an accuracy in the range of 1%. The model presented here achieves comparable results "ab initio" and includes both leptons and hadrons. The standard model distinguishes quite rigidly between both types, a major distinctive observable for both particle groups is assumed to be the strong force which is postulated to be zero for leptons.

According to this model it is suggestive to interpret strong interaction as evidenced in scattering events to be due to wave function overlap depending on [10]:

1) comparable size and energy of wave functions,

2) sufficient net overlap: If regions with same and opposite sign balance to give zero net overlap, no interaction occurs.

From condition 1) it is obvious that the wave functions of electron and muon can not be expected to exhibit effective interaction with hadrons [27]. In the case of the tauon the second rule is crucial. In this model the tauon is at the end of the partial product series for y_1^0 and should exhibit a high, potentially infinite number of nodes, separating densely spaced volume elements of alternating wave function sign prohibiting net overlap and effective interaction with hadrons of higher symmetry, such as the proton.

In the standard model mass of elementary particles is generated by the "Higgs mechanism". The model presented here may be interpreted in terms of connecting the groups U(1) -photon- with SO(3) / SU(2) objects -particles and obviously the energy of the Higgs boson and the VEV have an outstanding position as highest energy terms.

The most obvious symmetry breaking associated with the creation of a "localized photon" in this model is the generation of +/- charge due to the persistent orientation of the E-vector towards the origin. However, the implicitly assumed sign of the wave function Ψ might be an additional candidate for symmetry breaking which in turn might be related to orientation in a 5D space, see 4.3.

4.2 Relation to General Relativity

The relation with GR is inherent in the complete model for calculating particle energies and its parameters, in particular:

- the possibility to derive its basic equations - yielding quantized particle energies - from the Einstein field equation,
- obtaining absolute values for particle energies by replacing G/c_0^4 in the EFE by $1/\varepsilon_c$ = 2.998E+8[m/J]
- α being an electromagnetic as well as a geometric coefficient,
- the possibility to obtain a quantitative term for gravitational interaction from the expansion of the energy equation, implying curvature of spacetime to be in general identical to (the presence of) energy, and spatial coordinate and energy to be intertwined inextricably,
- suggesting a close relationship of several mass/energy related phenomena - particle energy, elements of the Higgs mechanism, Planck energy - with GR.

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26 The term $\sigma^* \alpha_0$ has to appear in the denominator since ρ^2 appears in the nominator of equ (45), not affecting the validity of the equations of this model.

27 See table 1; As for energy density ~ W_m/W_n^4 : e/p ~ E-13, μ/p ~ 6E-4; μ/π ~ 1/3, i.e. in case of μ/π some measurable effect should be expected; different symmetry may play an additional role.

Last not least GR is a general concept connecting geometry with energy and related phenomena and its applicability in the subatomic range would drastically underscore its universal validity.

The focus of this work has been more on the lower side of the energy series, featuring the electron. However, the upper side, with the limit given by a particle featuring the energy of the Higgs boson, might be an interesting starting point as well. According to 2.5 such a state should represent an archetype for a "rotating localized E-vector". Defining this object not by its energy but via possessing some well defined "maximum curvature of spacetime", or a property ascribed to a 4th spatial dimension (related to energy), this curvature may be spread over a larger volume resulting in particles of less average curvature and thus less energy - as seen from flat space. In complete spherical symmetry "curvature" is spread out most evenly corresponding to the lowest energy. The coefficient $|\Gamma_{-1/3}|$, attributed to integrals over Ψ(r) dr to yield lengths, should appear as term $4\pi|\Gamma_{-1/3}|^3/3 = \sigma^{1/3}/2$ in the denominator of the expressions as described by equ. (8), (10) for a spherical symmetric object. [28]

4.3 Outlook - 5D

Several aspects of this model hint at a possible 5D background, a Kaluza type model [11], [12] that combines GTR with electromagnetism being a particularly evident choice. The main problem addressed in Kaluza's original work was that mass to charge ratio of e.g. the electron resulted in values for dx_4/ds of an excessive order of magnitude, a problem which may be solved by using the substitution (37) in the electromagnetic coupling constant, κ, of Kaluza:

$$\kappa_G = \left(\frac{16\pi G \varepsilon_c}{c_0^2}\right)^{0.5} \quad => \quad \kappa_c \approx c_0 \tag{48}$$

resulting in physically valid values for dx_4/ds:

$$\frac{dx^5}{ds} = \frac{e_c}{m_e \kappa_G} \approx 3\text{E+29}[\text{m/s}] \quad >> \quad c_0 \quad => \quad \frac{dx^5}{ds} = \frac{e_c}{m_e \kappa_c} \approx 1\text{E+4}[\text{m/s}] \tag{49}$$

Components of a 4D-spatial extension of Ψ,

$$\Psi_4(r) = \exp(-(\rho/r)^4) \tag{50}$$

might provide appropriate candidates for the scalar field, Φ, in Kaluza's model which has a source in the electromagnetic field: $g^{\alpha\beta}\nabla_\alpha\nabla_\beta\Phi = -1/4\,\kappa^2\Phi^3 F^{\alpha\beta}F_{\alpha\beta}$ with $F^{\alpha\beta}$ being the electromagnetic tensor. Considering only the v'^2 derivative of $\Psi_4(r)$ in the following ansatz for Φ:

$$\Phi(r) \sim \left(\frac{e_c}{\varepsilon_c r}\right)^3 \Psi_4(r)^2 = \left(\frac{e_c}{\varepsilon_c r}\right)^3 \exp\left(-\left(\frac{e_c}{\varepsilon_c r}\right)^4\right) \tag{51}$$

would give:

$$\nabla_r \nabla_r \mathbf{\Phi}(r) \approx \left(\frac{e_c}{\varepsilon_c r}\right)^{11}\left(\frac{1}{r^2}\right)\Psi_4(r)^2 + \ldots \approx \left(\frac{e_c}{\varepsilon_c r}\right)^9\left(\frac{e_c}{\varepsilon_c r^2}\right)^2 \Psi_4(r)^2 + \ldots \approx \Phi(r)^3 \; E(r)^2 \; \Psi_4(r)^2 \; +\ldots. \tag{52}$$

Using Φ in the metric for an ansatz such as given in chpt.3 directly introduces terms of $\rho/r \sim e_c/(\varepsilon_c r)$ in the equations, rendering obsolete assumptions 1.) and 2.) for transformation to flat space.

In this respect it should be noted that Γ-functions, which are central in the equations of this model, represent a generalized factorial function and are known to provide a link between objects in different dimensions, such as spheres [13]. For the 3D and 4D integrals over space the following relation holds:

$$\int_0^r \Psi_3^2 dr \left(\int_0^r \Psi_3^2 r^{-2} dr\right)^2 \left(\int_0^r \Psi_4^2 dr\right)^{-1} = \frac{\Gamma_{+1/3}|\Gamma_{-1/3}|^2}{9\;|\Gamma_{-1/4}|} \approx 1 \qquad \left(\text{for } \lim_{x\to 0} \Gamma(n,x), \text{ see e.g. A[3]}\right) \tag{53}$$

which might be interpreted as the product of 3D curvature of space as represented by the ρ/r components of (40) times the inverse of the integral over the 4th spatial coordinate, which itself might be related to energy, to be a constant.

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28 The same reasoning of "spreading curvature" may apply to the radial coordinate of spherical symmetric states, resulting in the electron as the Higgs state spread over the largest volume available in flat space. Since $(W_{Higgs}/W_e)(W_{Higgs}/W_e)^3 \approx W_{Pl}$ it might be speculated that the Planck state represents the 3D equivalent to the Higgs as "maximum curvature of spacetime".

Within this model the coupling constant α is considered to be definable as geometric constant in 3D space which in principal should have a formal extension to 4D space. α may be expressed directly via the volume integral over the square of $1/r^2$ representing a point source in 3D times the corresponding integral symmetric in the Γ-function [29]:

$$\int_0^r \Psi_3(r)^2 dr \int_0^r \Psi_3(r)^2 r^{-4} d^3 r = \int_0^r \Psi_3(r)^2 r\, dr \int_0^r \Psi_3(r)^2 r^{-3} \mathbf{4\pi}\, dr = \frac{4\pi \Gamma_{1/3}\Gamma_{-1/3}}{9} = \frac{\alpha^{-1}}{9} \tag{54}$$

The result has to be corrected by the square of the argument of the Γ-function, in case of 3D, $\Gamma_{+/-1/3}$: 3^2 [30]
Extending this to the 4D case with surface area $S_4 = 2\pi^2 R^3$ and replacing r^{-4} by the equivalent r^{-6} gives:

$$\int_0^r \Psi_4(r)^2 r\, dr \int_0^r \Psi_4(r)^2 r^{-6} d^4 r = \int_0^r \Psi_4(r)^2 r\, dr \int_0^r \Psi_4(r)^2 r^{-3} \mathbf{2\pi^2}\, dr = \frac{2\pi^2 \Gamma_{1/2}\Gamma_{-1/2}}{16} = \frac{\pi^3}{4} = \frac{\alpha_{weak}^{-1}}{4} \tag{55}$$

Again the result has to be corrected by the square of the argument of the Γ-function, in case of 4D, $\Gamma_{+/-1/2}$: 2^2.
Both coupling constants may be expressed as:

$$\alpha_n = \frac{1}{arg(\Gamma(n))^2} \int_0^r \Psi_n(r)^2 r^{(n-3)} dr \int_0^r \Psi_n(r)^2 r^{-(n-1)^2} d^n r \qquad \Psi_n(r) = \exp(-(\rho/r)^n) \tag{56}$$

This gives a Weinberg angle of [31]:

$$\sin^2\theta_W = \frac{\alpha_{weak}}{\alpha} = \frac{\pi}{8\sqrt{3}} = 0.2267 \quad {}^{32} \tag{57}$$

and cos $\theta_W = m_W/m_Z = 0.8794 = 0.998\ (m_W/m_Z)_{exp}$ [16].

Conclusion

This article suggests a consistent and coherent model quantitatively connecting the concepts of general relativity with the properties of subatomic particles giving in particular the following results:

- the fine-structure constant, α, being given as a geometric coefficient defined by the product of the Γ- functions in the integrals over Ψ(r) related to photon and point charge symmetry, $4\pi\ \Gamma_{+1/3}\ |\Gamma_{-1/3}| \approx \alpha^{-1}$,
- a quantization of energy levels with terms α^(-1/3^n)
- electron and the Higgs boson energy as lower and upper limit for particle energy,
- additional information about particle properties e.g. the lepton character of the tauon,
- a series expansion for particle energy, including terms for rest energy, electromagnetic interaction and a 3rd term which at short range yields effects associated with strong interaction, at long range gives a quantitative term for gravitational interaction.

The basic terms of the model may be derived directly from the framework of the Einstein field equations and can be expressed without use of free parameters.

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29 Integration limit in both cases $r = \beta^{1/x}$;
30 At least one of these factors may be interpreted as being due to the limit of the lower integration limit of the Euler integral, see e.g. [A3];.
31 Using: $\Gamma(+x)\,\Gamma(-x) = \pi/(x \sin(\pi x))$
32 Experimental values: PDG [14]: $\sin^2\theta_W = 0.2312$, CODATA [15]: $\sin^2\theta_W = 0.2223$.

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Appendix

[A1] Differential equation

The approximation $\Psi(r<r_n)$ of equation (4) provides a solution to a differential equation of type

$$-\frac{r}{6}\frac{d^2\Psi(r)}{dr^2} + \frac{\beta_n/2}{2r^3}\frac{d\Psi(r)}{dr} - \frac{\beta_n/2}{r^4}\Psi(r) = 0 \tag{58}$$

which corresponds approximately to the limit l -> ∞ while has to be amended by σ in the denominator of the last term for the general case.
With the 3rd term in (30) used for potential energy, V:

$$V(r) = b_0\,\beta_{GS}/(2\,r^4) = b_0\,[\,\sigma^*\,\alpha_0\,(e_c/\varepsilon_c)^3/(4\pi)^2]\,/(2r^4) \tag{59}$$

and a corresponding expansion by $(\hbar c_0)^2\alpha^{-2}/b_0^2$ for the first term, the approximate differential equation for this model may be given as:

$$-\frac{(\hbar c_0)^2 r}{\alpha^{-2}b_0}\,\frac{d^2\Psi(r)}{dr^2} + r\,V(r)\,\frac{d\Psi(r)}{dr} - \frac{V(r)}{\sigma}\Psi(r) = 0 \tag{60}$$

Equations (58)ff give a satisfactory description for spherical symmetric states only. An improved approach, including non-spherical symmetric states and a more stringent condition for quantization may be derived from the EFE.

[A2] Coefficient 1.51

Factor 1.5088 of the ratio W_μ/W_e is subject to a 3rd power relationship of the same kind as the α coefficients:

$$\left(\frac{1.5133}{1.5088}\right) = \left(\frac{1.5133}{1.5}\right)^{1/3} \tag{61}$$

indicating that the radial terms of $\Pi_{\beta,n}$ in β_n and the angular components of σ are not correctly separated yet or may not be separable even in the case of spherical symmetric states.
The limit of a corresponding partial product in the energy expression is given by 1.5133 Π_0^∞ (1.5/1.533)^1/3^k ≈ 1.5066.
The necessary term in β will be: 1.5133^{-3} Π_0^n (1.533/1.5)^3/3^k , n={1;2;...}, for particles above the electron, see [A4].
The following relations hold:

$$1.5133 = 0.998\,|\Gamma_{-1/3}|/\Gamma_{1/3} = 4\pi\,\Gamma^2_{-1/3}\;\alpha \tag{62}$$

[A3] Calculation of α^{-1}

Using the Euler integrals with the minimal limit for the lower bound of integration, i.e. β/r^3 ->0, yields $\Gamma_{1/3}$ for $\Gamma(_{1/3,}\,\beta_n/r_n^3)$ and since

$$\Gamma(_{-1/3,}\,\beta_n/r_x^{\,3}) = \int_{\beta_n/r_x^3}^{\infty} t^{-4/3}e^{-t}\,dt \approx 3\,(\beta_n/r_x^3)^{-1/3} \tag{63}$$

holds $\Gamma(_{-1/3,}\,\beta_n/\lambda_{C,n}^{\,3}) = 3\,\lambda_{C,n}/\beta_n^{1/3}$ which inserted in (9) gives the identity $\lambda_{C,n} = \lambda_{C,n}$, however, may be used in this form or the related term $\lambda_{C,n}/r_n\;\sigma^{1/3}/2$ ($\lambda_{C,n}/r_n \sim 1.717$) to calculate α with (12) and an appropriate term for β_n.

A similar relation as (63) holds for the other incomplete Γ-functions with negative argument.

[A4] Particle parameter β

A more detailed expression for β than given in (18) will be attempted in the following.
The term (61) will be used within the particle specific factor (square brackets), thus coefficient 1.5133 of σ will be placed there, giving for the general term (i.e. excluding the electron):

$$\beta_n = \sigma * \frac{1}{(4\pi)^2}\left(\frac{e_c}{\varepsilon_c}\right)^3 \frac{2}{(2\pi)^3}\; 1.5133^{-3}\Pi^{n}_{k=0}\left[\alpha^3\left(\frac{1.5133}{1.5}\right)\right]^\wedge\left(\frac{3}{3^k}\right) \qquad n = \{0;1;2;...\} \tag{64}$$

factor 1.5133^{-3} represents ≈3/2 for the ratio of W_μ/W_e (see [A2]), omitted in the term for the electron:

$$\beta_e = \sigma * \frac{1}{(4\pi)^2}\left(\frac{e_c}{\varepsilon_c}\right)^3 \boldsymbol{\frac{2}{(2\pi)^3}\left[\alpha^3\left(\frac{1.5133}{1.5}\right)\right]^3} \approx \sigma * \frac{1}{(4\pi)^2}\left(\frac{e_c}{\varepsilon_c}\right)^3 \boldsymbol{\alpha_0} \tag{65}$$ [33]

the particle specific factor is given in square brackets (α_0 in bold). The other factors are due to
- factor 2: Ψ appearing squared in the integrals,
- factor $1/(2\pi)^3$: representing 2π of the integral limit in (22) ,
- factor 1.5133^{-3}: due to anomalous factor 2/3 in W_e/W_μ,
- $1/(4\pi)^2$: the power of 2 instead of the power of 3 as for the other components might be due to b_0 appearing squared in (60) and its analog in the asymmetry of the ρ/r components of (41)

Using (65) W_e may be given as:

$$W_e = 2b_0\frac{\Gamma_{+1/3}}{3}\left(\frac{9\pi^{5/3}\alpha}{|\Gamma_{-1/3}|}\left(\frac{\varepsilon_c}{e_c}\right)\left[\frac{\alpha^{-3}}{1.5133}\right]\right) = \frac{1.5\pi^{2/3}}{1.5133}\frac{\Gamma_{+1/3}}{|\Gamma_{-1/3}|}\frac{e_c}{\alpha^2} = 1.0001\ W_{e,exp} \tag{66}$$

[A5] Additional particle states

Assignment of more particle states will be not obvious. The following gives some possible approaches.

[A5.1] Partial products

Additional partial product series will have to start with higher exponents n in α^$(-1/3^n)$ giving smaller differences in energy while density of experimentally detected states is high. There might be a tendency of particles to exhibit a lower mean lifetime (MLT), making experimental detection of particles difficult [34]. To determine the factor y_l^m requires an appropriate ansatz for the differential equation yet to be found.
One more partial product might be inferred from considering d-like-orbital equivalents with a factor of $5^{1/3}$ as energy ratio relative to η giving the start of an additional partial product series at $5^{1/3}$ W(η) = 937MeV = 0.98 W(η'), i.e. close to energy values of the first particles available as starting point, η', Φ^0. However, in general it is not expected that partial products can explain all values of particle energies.

[A5.2] Linear combinations

The first particle family that does not fit to the partial product series scheme are the kaons at ~ 495MeV. They might be considered to be linear combination states of π-states. The π-states of the y_1^0 series are assumed to exhibit one angular node, giving a charge distribution of +|+, -|- and +|-. A linear combination of two π-states would yield the basic symmetry properties of the 4 kaons as:

K^+		K^-		K_S^o		K_L^o		
	+		-		-		+	
	+ +		- -		+ +		+ -	(+/- = charge)
	+		-		-		-	

providing two neutral kaons of different structure and parity, implying a decay with different parity and MLT values. For the charged Kaons, K^+, K^-, a configuration for wave function sign equal to the configuration for charge of K_S^o and K_L^o might be possible, giving two variants of P+ and P- parity of otherwise identical particles and corresponding decay modes not violating parity conservation.

$K^{+/-}$		$K^{+/-}$		
	-		+	
	+ +		+ -	(+/- = wave function sign)
	-		-	

[A6] Metric

Coordinate variables: x0=t, x1=r, x2=θ, x3=φ

$g_{\mu\nu} = (+\exp(a\, v(r)),\ -\exp(b\, v(r)),\ +r^2,\ +r^2 \sin^2\theta)$

$g^{\mu\nu} = (+1/\exp(av(r)),\ -1/\exp(bv(r)),\ +1/r^2,\ +1/r^2 \sin^2\theta)$

$\Gamma_{01}^{0} = \Gamma_{10}^{0} = a\, v'/2$ $\qquad \Gamma_{00}^{1} = a\, v'\, e^{(a-b)v}/2$ $\qquad \Gamma_{11}^{1} = b\, v'/2$

$\Gamma_{12}^{2} = \Gamma_{21}^{2} = \Gamma_{13}^{3} = \Gamma_{31}^{3} = 1/r$ $\qquad \Gamma_{22}^{1} = +\, r\, e^{-bv}$ $\qquad \Gamma_{33}^{1} = \Gamma_{22}^{1}\sin^2\theta$

33 Note: $2\,(2/3)^3/(2\pi)^3 \approx (1.5133\ \alpha^{-1}\ 2)^{-1}$;
34 Which might explain missing particles of higher n in the y_0^0 and y_1^0 series as well.

$\Gamma_{23}{}^3 = \Gamma_{32}{}^3 = \cot\theta$ $\Gamma_{33}{}^2 = - \sin\theta \cos\theta$

$R_{00} = e^{(a-b)v} [- a v''/2 - a^2v'^2/4 + ab v'^2/4 - a v'/r]$ $R_{11} = [+ a v''/2 + a^2v'^2/4 - ab v'^2/4 - b v'/r]$

$R_{22} = e^{-bv} [(b-a) v'r /2 - 1] -1$ $R_{33} = R_{22} \sin^2\theta$

$g^{00}R_{00} + g^{11}R_{11} = e^{-bv} [- a v'' - a^2v'^2/2 + ab v'^2/2 - (a-b) v'/r]$

$g^{22}R_{22} + g^{33}R_{33} = e^{-bv} [(b-a) v'/r - 2/r^2] -2/r^2$