

## **Proof of light speed invariance and relative velocity invariance**

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### ABSTRACT

In this paper, we will explain and prove in detail Einstein's postulate with respect to the invariance of light speed. The "mysterious" invariance of light speed is a very normal natural phenomenon, but it is impossible to dominate time and space. The two observers on two relative motion reference frames have opposite views regarding which reference frame is stationary. We also give the precise definition of invariance of light speed, and explain the postulate of maximum speed.

This paper also derives the principle of invariance of relative velocity, which is the physical reason of the principle of relativity. In fact, Newtonian mechanics and Maxwell's equations are based on relative motion. We give the precise definitions of relative velocity

Distinguishing physical time and mathematical time, and distinguishing physical space and mathematical space are the key to understand the reason of invariance of light speed. All motions are relative. All motion velocities are based on a reference frame we define. We don't have an absolutely stationary reference frame in the universe.

### KEYWORD

special relativity; space time; covariance; Galilean; physics; velocity; invariance; invariant; light speed.

Referring the paper "Physical meaning of transformation equations between two reference frames", Section 2 [1], the principle of relativity were proposed by Galileo and by Einstein. This principle became a physical law. It states "the laws of physics are invariant (i.e. identical) in all inertial frames of reference" [3]. Why this principle is correct? The reason is that Newtonian mechanics and Maxwell's equations are based on relative motion or relative action, which is independent of reference frames, and which is symmetry. In fact, the principle of relativity is based on the principle of invariance of relative velocity.

### 1. Principle of invariance of relative velocity

What is relative velocity? "A particle is moving in a reference frame" means that a particle and a reference frame is in relative motion, or a particle and the original point of a reference frame is in relative motion. Thus, we give following definition.

**The definition of relative velocity:** The relative velocity between two particles in any reference frame is equal to the following operation: we set one of the particles as the origin of a moving frame  $S'$  to measure another particle velocity in that frame  $S'$ .

The Eq.(19) and (39) of paper [2] give speed transformation of particle  $Q_2$  from frame  $S$  to frame  $S'$ :

$$u' = (u - v)/(1 - vu/c^2) \text{ when } u \text{ direction is the same as } v \quad (1)$$

$$u' = (u + v)/(1 + vu/c^2) \text{ when } u \text{ direction is opposite to } v \quad (2)$$

Where  $u$  is the speed of particle  $Q_2$  in frame  $S$ .

As shown in Fig. 1,  $u_1$  and  $u_2$  represent the moving speeds of particle  $Q_1$  and  $Q_2$  in reference frame  $S$ , and we set the origin of frame  $S'$  at point  $Q_1$ . As per Eq.(1), due to  $v = u_1$ , the moving speed of particle  $Q_2$  in frame  $S'$  is:

$$u'_2 = (u_2 - u_1)/(1 - u_1u_2/c^2) \quad (3)$$

If the particle  $Q_2$  moving direction is opposite to  $x$ -axis direction, the moving speed of particle  $Q_2$  in frame  $S'$  is:

$$u'_2 = (u_2 + u_1)/(1 + u_1u_2/c^2) \quad (4)$$

Since the origin of frame  $S'$  is at point  $Q_1$ ,  $u'_2$  represents the relative velocity between particle  $Q_1$  and  $Q_2$ . We use symbol  $w$  to represent the relative speed between particle  $Q_1$  and  $Q_2$ . Thus,

$$w = (u_2 - u_1)/(1 - u_1u_2/c^2) \quad u_1 \text{ or } u_2 < c, \quad u_2 \text{ direction is the same as } u_1 \quad (5)$$

$$w = (u_2 + u_1)/(1 + u_1u_2/c^2) \quad u_2 \text{ direction is opposite to } u_1 \quad (6)$$

Eqs.(5) and (6) indicate the relative speed of particle  $Q_1$  and  $Q_2$  in frame  $S$  (viewed in frame  $S$ ). Please note that the equation  $w = u_2 - u_1$  or  $w = u_2 + u_1$  from Galilean transformation are not correct.

If the particle  $Q_1$  and  $Q_2$  move in a line which is not parallel to the  $x$ -axis, we use  $w$  and  $u_1, u_2$  to represent the velocity vectors. The direction of  $w$  and  $u_1, u_2$  are all along a line. The Eq.(5) and (6) are still valid. Please see paper [2], Section 8.

As shown in Fig.2, if we view the motions of particle  $Q_1$  and  $Q_2$  in frame  $S$ , we obtain the relative speed between particle  $Q_1$  and  $Q_2$  as per Eq. (6):

$$w = (u_2 + u_1)/(1 + u_1u_2/c^2) \quad (6)$$

Where  $u_1, u_2$  represent the speeds of particle  $Q_1$  and  $Q_2$  in frame  $S$ . Viewing in frame  $S$ , we can obtain the speeds of particle  $Q_1$  and  $Q_2$  in frame  $S'$

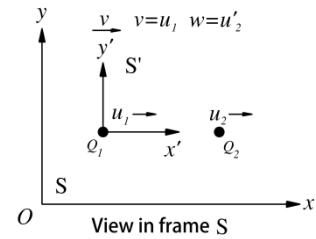


Fig. 1

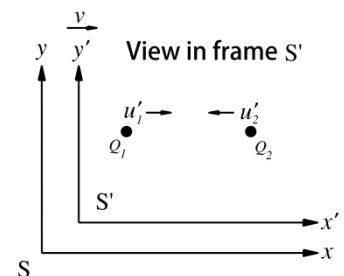


Fig. 2

as per Eqs.(1), (2):

$$u'_1 = (u_1 - v)/(1 - u_1v/c^2)$$

$$u'_2 = (u_2 + v)/(1 + u_2v/c^2)$$

Thus, we obtain relative speed between particle  $Q_1$  and  $Q_2$  in frame  $S'$  as per Eq. (6):

$$w' = (u'_2 + u'_1)/(1 + u'_1u'_2/c^2)$$

Substituting  $u'_1$  and  $u'_2$  into the above equation, we obtain:

$$w' = (u_2 + u_1)/(1 + u_1u_2/c^2) = w$$

Since  $w' = w$ , we obtain:

**The principle of invariance of relative velocity:** Relative velocity between two particles is independent of all inertial frames of reference.

The above derivation is based on one-dimensional setup of two inertia reference frames. The principle of invariance of relative velocity in three-dimensional space also holds good. Please read paper [2], Section 8.

Referring to Fig. 1,  $u'_2$  represents the relative velocity between particle  $Q_1$  and  $Q_2$  when we view them in frame  $S$ . Since the relative velocity is invariant,  $u'_2$  also represents the relative velocity between particle  $Q_1$  and  $Q_2$  in frame  $S'$ . Therefore, the concept of relative velocity can be extended to the velocity of an object relative to a stationary point in the physical space. The origin of any reference frame represents a stationary point in the physical space.

Since Newtonian mechanics and Maxwell's equations are based on relative motion or relative action, the principle of invariance of relative velocity verifies that the principle of relativity is correct.

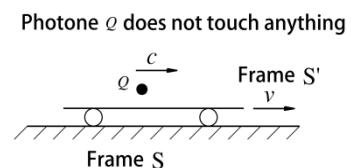
## 2. Proving the principle of invariance of light speed

Please refer to paper [1], Section 2. Einstein's postulate of the invariance of light speed is based on Maxwell's law and the experiment of Michelson and Morley. However, many people still don't understand why the invariance of light speed is correct since it seems counterintuitive. Why  $c' = c$  when frame  $S'$  is moving? Many people believe that the invariance of light speed leads to the Special relativity. In fact, the invariance of light speed is a fundamental nature phenomenon. For understand it, we must correctly interpret the Newtonian first law and correctly interpret the notion of physical space.

The principle of the invariance of light speed contains three concepts. First, the speed of light is always equal to a constant  $c$  in vacuum; second, the measurement of the speed of light in any reference frame is the same regardless of whether the light source is moving. i.e.  $c' = c$ ; Third, the frame of the light source can be a non-inertial frame.

Newtonian first law states: "In an inertial frame of reference, an object either remains at rest or continues to move at a constant velocity, unless acted upon by a force".

Referring to Fig.3, the word "move" or "rest" to describe the motion state of the particle  $Q$  must be related to a specified reference frame. Since no reference frame is absolutely stationary in the universe, the nature space is not an absolutely stationary frame. All motions are relative. The location of point  $Q$  in nature space is defined in a reference frame. If the particle  $Q$  is free and does not touch anything, the natural space point  $Q$  defined in the frame  $S'$  does not coincide with the natural space point  $Q$  defined in the frame  $S$ , due to the motion between the frames  $S'$  and  $S$ . That is why  $c' = c$  is true.



**Fig. 3**

Natural space does not contract or expand or move by itself. The particle  $Q$  moves in a space defined in frames  $S$  or  $S'$ . If we specify frame  $S$  as a reference frame, the space in frames  $S$  represents the nature space, i.e. physical space.

In addition, let us explain the invariant light speed from four aspects.

- See Fig.3. The relative speed  $v$  between frame  $S$  and  $S'$  does not affect the motion speed of the particle  $Q$  in reference frame  $S$  or  $S'$ . This is why the measurement of the speed of light is independent of the motion of the light source.
- The speed of sound in the air medium is also constant. If there is a relative motion  $v$  between an observer (stationary frame  $S$ ) and the air (moving frame  $S'$ ), the relative speed  $u$  between the observer and the sound will be changed. However, light propagation does not require a medium, so the "medium" is equal to zero. Thus,  $c'=c$ .  $c$  is a constant in vacuum.
- Bullets do not require media to fly in the air. However, when a moving pistol fires a bullet, the movement of the pistol will give the bullet an initial velocity. Thus, a bullet fired from a moving gun is faster than a bullet fired from a stationary gun. Physics defines the mass of a photon is zero, so the motion of light source does not cause photons have an initial velocity. Thus,  $c'=c$ .
- When measuring an object velocity, the observer always sets himself in a reference frame, which represents the physical space. The space in the reference frame is stationary, which means the physical space is stationary. Referring to Fig.4, when you view in the frame  $S$ , a point in physical space is a stationary point, and the frame  $S'$  is moving. When you view in the frame  $S'$ , the point in physical space is also a stationary point, and the frame  $S$  is moving. The "ether" is always stationary for earth observers or moon observers. Let's further explain this point of view as follows.

Fig.4 illustrates why the speed of light is invariant in both frames  $S$  and  $S'$ .

The photons  $P_1$  and  $P_2$  represent a light ray. When the moving  $y'$ -axis overlapped with the  $y$ -axis, a light source, which is fixed at the origin- $O'$  of the frame  $S'$ , emitted a light ray. Observing in frame  $S'$ , the instant point- $O'$  of the physical space and the light source as well as the observer himself are all stationary, while the frame  $S$  is moving to left side. The relative speed between photon  $P_1$  and the point- $O'$  is  $c'$ .

Observing in frame  $S$ , the instant point- $O$  of the physical space, at which the ray was emitted, and the observer himself are stationary, while the frame  $S'$  with the light source are moving to right side. The relative speed between photon  $P_1$  and the point- $O$  is  $c$ . We can see  $c'=c$  from Fig. 4.

Please note that the instant point- $O'$  and the instant point- $O$  are not a coincidence point in the physical space except the moment when the light ray was emitted. Now, it is necessary to give a clear definition for the principle of the invariance of light speed.

**The definition of principle of the invariance of light speed:** The relative speed between a light photon and an instant point of physical space, at which the light was emitted or reflected or refracted, is always invariant, and its value is equal to  $c$ . The instant point of physical space is always a stationary point in any reference frame we specify.

This definition contains two key points: The invariance of light speed is related to an instant point of PHYSICAL SPACE, and this point is always a stationary point in reference frame  $S$  or  $S'$ .

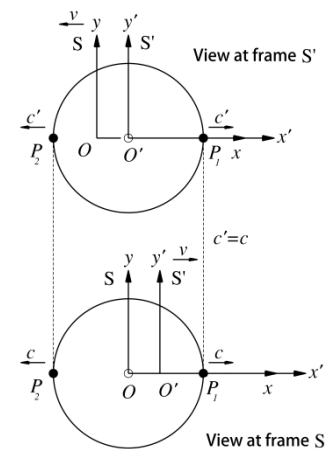


Fig. 4

**The notion of space and time:** All points in physical space are stationary in observer's frame of reference or in the frame which is defined as reference frame by an observer. Physical time is the time which is defined by an observer in reference frame. If there is relative motion between frame  $S'$  and  $S$ , a nature space point or an observer cannot be stationary in both frame  $S'$  and  $S$  simultaneously. Who is moving depends on where the reference frame is. Again, nature space is not absolutely stationary.

### 3. The application of definition of invariant light speed

According to the definition of invariant light speed, we can explain some nature phenomenon and result of physical experiments.

For example, if we are on earth to measure the speed of light from the Big Dipper, the physical space points at which the Big Dipper emitted light beam are stationary points in the earth's frame, and the speed of light is equal to  $c$ . If we are on the moon to measure the speed of light of the Big Dipper, the physical space points at which the Big Dipper emitted light beam are stationary points in the moon's frame, and the speed of light is also equal to  $c$ .

Fig.5 is a schematic of the Michael Morley experiment. A ray of light  $P$  is emitted from the light source  $S$ , and is split into rays  $P_1$  and  $P_2$  at the half-silvered mirror  $A$ .  $P_1$  is reflected at the mirror  $B$  as ray  $P_1'$ , and then it passes through  $A$  to reach the receiver  $R$ .  $P_2$  passes through the mirror  $A$  and reaches the mirror  $C$ , at which it is reflected as ray  $P_2'$ . Then,  $P_2'$  is reflected at  $A$  and reaches the receiver  $R$ .

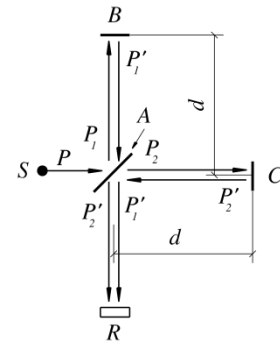


FIG 5

Since no interference is shown at the receiver  $R$ , it indicates  $P_1'$  and  $P_2'$  reaches the receiver  $R$  at the same time, and earth's motion does not affect the light travel. Why? According to the definition of invariant light speed, it is clear that  $S, A, B$  and  $C$  are all a physical space point, at which the ray is emitted or is reflected or is refracted. These instant points of the physical space are all a stationary origin point in reference frame  $S$  or  $A$  or  $B$  or  $C$ . Due to no relative motion between light source  $S, A, B$  and  $C$ , the frame  $S, A, B$  and  $C$  make up a big reference frame  $S$ . Since the speed of light is constant, so the time required for light travel is proportional to the travel distance. The distance  $AB = AC = d$ , so  $P_1'$  and  $P_2'$  will reach the receiver  $R$  at the same time.

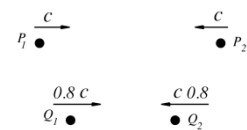
**Clarification:** Light sources always emit light photons intermittently at very fast frequencies. If a light source is moving, the time required for photons to reach the measuring instrument from the instant points of physical space is slightly different. The speed of light is obtained by the average distance divided by the average time. If the light source moves at a very fast speed and the frequency is very slow, the speed of light actually changes slightly.

### 4. The postulate of maximum speed

From Newtonian mechanics, we have  $F = ma = mv/t$ , that is  $v = Ft/m$ . If the mass  $m$  is small enough, a small force can generate a large speed.  $m \rightarrow 0, v \rightarrow \infty = v_{max}$ . Physics defines the photon mass  $m_0 = 0$ , so we guess  $c = v_{max}$ .

Referring to Fig.6, as per Eq.(6), we obtain the relative speed between particle  $Q_1$  and  $Q_2$  is

$$w = (0.8c + 0.8c)/(1 + 0.8c \times 0.8c/c^2) = 0.976c < c$$



View in frame S

Fig.6

Also, we obtain the relative speed between photon  $P_1$  and  $P_2$  is

$$w = (c + c)/(1 + c \times c/c^2) = c$$

It is clear that the relative velocity between the two particles is always not greater than  $c$ . Thus, the following postulate is understood.

**The postulate of maximum speed:** Particle velocity in any frame of reference or any relative velocity between any two particles is not greater than the speed of light.

Please note that above postulate discusses "velocity" rather than "action".

## 5. Conclusion

We don't have an absolutely stationary frame as reference frame in the universe, so there is no absolutely motion in the universe. All motions are relative. All motion velocities are based on a reference frame we define. The relative velocity between two particles (objects) is independent of reference frames. The measurement value of speed of light is independent of reference frames. Light speed is the maximum speed (not action) in the universe.

If there is relative motion between frame  $S'$  and  $S$ , a nature space point or an observer himself cannot be stationary in both frame  $S'$  and  $S$  simultaneously, so a natural space point defined in frames  $S'$  and  $S$  is not coincide.

## Reference:

- [1] Mingke Yuan. (2019, November 13). Physical meaning of transformation equations between two reference frames. Zenodo. <http://doi.org/10.5281/zenodo.3541096>
- [2] Mingke Yuan. (2019, November 22). Derivation and application of three-dimensional logical transformation.
- [3] The Free encyclopedia, Wikipedia.

## History Record:

- 1) This paper is the second part of a long paper. Original paper is divided into three papers.
- 2) Original paper is called "New theory of correcting Relativistic mechanics, Newton's second law and Newtonian mechanics". It was submitted to Nature Communication on 30th July 2019. Manuscript #NCOMMS-19-25599. File name: 218588\_0\_art\_file\_3974642\_pvfyh1.pdf.
- 3) Original paper was also submitted to Foundations of Physics. The title is "A new theory for modifying relativistic mechanics and relativistic quantum mechanics". First time submission is on 08th Aug. 2019, submission ID: FOOP-D-19-00436, File name: FOOP-D-19-00436. Second time submission is on 13th Sep. 2019, submission ID: FOOP-D-19-00488. File name: FOOP-S-19-00542.pdf.