LESSON PLAY: SUPPORTING PRE-SERVICE TEACHERS TO ENVISAGE PUPILS' SENSE-MAKING IN MATHEMATICS LESSONS

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In this paper the potential of Lesson Play in mathematics teacher education is explored. Through the process of script writing in Lesson Play, teachers imagine their own responses to classroom situations. We describe how script writing has the potential to help pre-service teachers envisage ways in which pupils make sense of mathematics, and become more aware of the teacher moves that allow pupils to articulate and modify ideas in mathematics lessons. We analyse the lesson script of one pre-service teacher with reference to Grice's Conversational Maxims, and discuss ways in which Lesson Play can be developed to further enhance pre-service teachers' ability to facilitate classroom discussions.

INTRODUCTION

Socio-constructivist perspectives on mathematics teaching and learning have gained considerable traction in recent years. From these perspectives, the learning of mathematics is seen as a social process in which the teacher and students co-construct ideas within the domain through talk and argumentation. While the relationship between mathematics and language has various interpretations in the research literature, the position we take is that "doing mathematics essentially entails speaking mathematically" (Morgan, Craig, Schuette & Wagner, 2014, p.846). As elaborated by Rowland (2000), this is strongly linked with a view of mathematics as the product of human activity and interpersonal dialogue, leading to classroom practices where pupils are encouraged to articulate ideas and modify them as necessary in order to make sense of mathematics.

The importance of discussion and communication in mathematics lessons is emphasized in the 1999 Irish Primary School Mathematics Curriculum (Gov. of Ire., 1999). However, there is considerable evidence that teachers continue to control much of the talking that occurs. For example, one of the findings of TIMSS 2015, in which fourth class children's mathematical performance was assessed, was that 73% of pupils in Ireland were asked to listen to their teacher explaining new content in 'every or almost every lesson' (Clerkin, Perkins, & Chubb, 2017). In contrast with this, 34% of pupils work on problems together in the whole class with direct guidance from the teacher in most or all lessons. While the orchestration of mathematical discussion is challenging for teachers, it is particularly so for pre-service teachers (PTs) who are often uncomfortable in a classroom environment where they cannot take complete control of the direction of a lesson (e.g., McGlynn-Stewart, 2010).

In this paper we explore how Lesson Play (LP) offers a means of helping PTs to envisage the ways in which pupils use language to make sense of new mathematical ideas and, moreover, the moves a teacher might make to facilitate the development of this sense-making. LP allows teachers to imagine their own responses to particular classroom situations, and envisage how the conversation between the learner and a teacher might proceed. We use an LP script created by a PT to demonstrate this and consider the implications for further development of this approach.

CONVERSATIONAL MAXIMS

Taking the position on language and mathematics outlined in the introduction, we argue that pupils make sense of mathematics by articulating their ideas and modifying them as necessary. This suggests that mathematics lessons in which sense-making is at the core are characterised by a 'to-ing and fro-ing of ideas' such as applies in a conversation. If this is the case, it could be expected that conversational maxims would apply. The philosopher, Paul Grice, proposed that normal conversation is based on co-operative principles, meaning for which can be found in 'maxims' of conversation that specify what the participants have to do to ensure that their conversation is co-operative and rational (Grice, as cited by Rowland, 2000, p.81-82):

- Quality: Let your contribution be truthful; do not say what you believe to be false.
- Quantity: Let your contribution be as informative as required (for the current purposes) and not be more informative than is required.
- Manner: Let your contribution be clearly expressed, e.g., be brief, orderly, unambiguous.
- Relevance: let your contribution be relevant to the matter in hand.

The maxims are supposed to apply both to the delivery and the interpretation of messages but it is not the case that they are always observed. Grice maintains, however, that participants of a conversation behave as if cooperative principles are being upheld. The following interaction is a case in point:

Teacher: Where is your home exercise?

Student: My aunty called last night

Although it might seem that the student is not addressing the teacher's question, the teacher might infer that she did not do her home exercise because her aunt called on the previous evening. In other words, the student's input is interpreted by the teacher as if there is conformance to the maxims at least at some level.

Rowland (2000) reminds us that 'co-operative' in the Gricean sense is not necessarily associated with pleasantness but has more to do with the 'sense-making' of spoken interactions of the participants of a conversation. He also contends that Grice's Cooperative Principles, can account for many of the vague features of conversation. For example, citing Brockway (1981), Rowland describes the word 'well' as a maxim hedge - it is often used by speakers to notify the hearer that a contribution will in some respect fall short of one or more of Grice's maxims. For example, in calculating the sum of two numbers, say 25 + 27, a pupil might make the following contribution in whole-class conversation:

Áine: Well, I got 52.

Here Áine uses 'well' to indicate that her input might not meet the requirement of the maxim of quality, that is, she is not entirely sure that her contribution is truthful. There are other ways that speakers might convey to their audience the awareness that they are violating the Gricean principles, for example, pausing, giving hints and clues, under- and over-elaborating statements, being ironic and using rhetorical questions (Bills, 2000; Rowland, 2000).

Teacher moves can also be described in terms of the Gricean principles (Forman and Larreamendy-Joerns, 1998). Among teacher moves associated with sense-making mathematics lessons are those of 'press' and 'revoicing' (Brodie, 2011). A press move occurs when a teacher asks a learner to elaborate, clarify, justify or explain an idea while a revoicing move is seen when a teacher repeats or rephrases a student's idea. Forman and Larreamendy-Joerns (1998) contend that there is often a discrepancy between what students take for granted as understood and what teachers are willing to accept as explicit information. While everyday and mathematical conversations both depend on the co-operative principle, the degree of accountability differs in the case of each. The degree of accountability is concerned with the level of explanation that participants are expected to make. Everyday explanations are highly condensed because of familiarity, shared history, trust etc. More extensive explanations are required in the sciences. In the mathematics classroom, requests by the teacher for further explanation serve in general to develop appropriate socio-mathematical norms. These are norms that pertain to normative aspects of students' mathematical activity, for example, what counts as a different solution, a sophisticated solution, an efficient solution, and an acceptable explanation as constituted in classroom interaction (Cobb and Yackel, 1998). Teachers' conversational meta-messages, of which revoicing and requests for explanation are examples, invoke the Gricean maxims by conveying to students the need to provide explanations that are 'explicit, relevant, orderly, precise and informative' (Forman and Larreamendy-Joerns, 1998, p.111) and thus help to build a bridge between every day and mathematical explanations. It would seem then that PTs should be aware of these maxims. LP, described next, is a context where this awareness might be developed.

FICTIONAL DIALOGUES AND LESSON PLAY

The use of fictional dialogues in mathematics education has had many different purposes over the past number of years (see Crespo, Oslan & Parks, 2011). In mathematics teacher education, one approach in which fictional dialogues are utilised is LP. Here, teachers write a script of an imagined dialogue between the teacher and students or between a group of students (Zazkis, Liljedhal & Sinclair, 2009). It was first introduced as an alternative way to allow teachers to anticipate students' ideas, providing "an opportunity to imagine the future, being informed by the past" (p. 46). It usually follows a prompt, e.g., the beginning of a dialogue in which there is a misconception or gap in a learner's understanding. Following this prompt a script is written, usually involving an interaction between teacher and pupil. The script is informed by the writers' (PTs') own learning, teaching and research experience. LP as presented in this paper did not begin with a prompt. The reason for this was that we believed it would allow the PTs to draw on their own experience to produce the script, and not focus only on addressing the issue pertaining to the prompt. The PTs' experience encompassed both a practicum (school placement) and a literature review conducted as part of the LP process.

LESSON PLAY: AN EXAMPLE

The lesson script outlined in this paper was written by a PT, Sara, in the 4th year of a Bachelor of Education programme. At this stage, PTs have undergone a number of weeks of school placement. The PTs in this programme complete a final year undergraduate research project in a subject area of their choice. The grade awarded for this project contributes to the final marks they receive for their degree. Sara was one of a group who chose to conduct research in mathematics education using LP. As part of this, PTs had to reflect on a previously taught lesson, then design a new lesson plan based on this reflection and a literature review. Finally, they engaged in LP. PTs were asked to imagine a scene (or several scenes) that might occur in the lesson, and to write and analyse a script for the interaction between students in the class and the teacher during that scene. We chose Sara's script because it exemplified, more than scripts written by other PTs in the group, some of the conversational maxims described above.

Sara explored the idea of differentiation in multi-grade classroom consisting of 3rd and 4th class children (aged 8-10 years). The focus of the lesson in 3rd class was 'regular tessellations', while 4th class children considered 'semi-regular tessellations' [1]. For her LP, Sara wrote a script for a scene that involved the teacher and six children (three from 3rd class and three from 4th class). Sara analysed this lesson script with reference to her own research question. However, for the purpose of this paper we are focussing not on her analysis but on the script itself, in particular, the ways in which she presented the classroom interactions. We analyse her script from the perspective of conversational maxims and teacher moves, although these were not explicitly taught to PTs as part of the undergraduate research module.

Analysis of the script

In Sara's script (see Appendix) we can see some examples of her use of the 'press' move. For example, in the interchange:

Teacher:	Good. Now that we know that squares tessellate. What do we know about tessellation?
Shane (3rd):	It means that when you make a pattern, the shapes fit together perfectly.
Teacher:	Exactly Shane. But what do we need to be careful about when making patterns that tessellate?
Kevin (3rd):	Shapes don't tessellate if there are any gaps... or overlapping shapes in the pattern.
Teacher:	That's correct.

In everyday conversation about, say, tiling the explanation given by Shane that tessellation means that '…shapes fit together perfectly' would be adequate. The meaning of 'perfectly' could well be inferred by the other party in the conversation to mean 'without gaps'. It seems that the teacher is happy that Shane has an adequate understanding of the concept ('Exactly') but her follow-up question ('What do we need to be careful about?') suggests that she feels some duty to the other pupils in the setting. Here she is pressing them for an explanation that fulfils the Gricean maxim of quantity (i.e., 'Let your contribution be as informative as required for the current purposes'). Kevin does exactly that when he proposes that 'Shapes don't tessellate if there are any gaps... or overlapping shapes in the pattern'. This explanation is sufficient for this group of children since the topic of tessellation is first introduced in 3rd class (Gov. of Ire., 1999).

There are other reasons that the teacher may have been happy with Shane's description of tessellation. His use of the pronoun 'you' tells something of his understanding. Rowland (1999) suggests that pupils seldom use the term 'you' to address a teacher in classroom because of asymmetrical power relationship in adult-child mathematical conversations. However, pupils often use the pronoun 'you' in such conversations. He contends that 'you' in such instances tends to refer to something rather than someone – that is, it can function as a 'generaliser' pointing to what happens 'every time'. It can be inferred from Shane's use of the word 'you' that he had generalised his understanding of tessellation.

The follow-up conversation on tessellation with the 3rd class pupils is characterised by greater certainty on the part of the pupils. There is very little hesitation in their deliberations and in general they use declarative sentences, that is, sentences that assert how things are (Vanderveken, 1990). For example, when asked to identify shapes that form a regular tessellation, Ciara says, 'And triangles! Because equilateral triangles have the same length of side and their angles are the same size too so that means they tessellate'. Although further press on the matter of equal angles might have injected more vagueness into the pupils' input, the next example of a violation of the maxims of conversation occurs when Sara introduces semi-regular tessellation to the older pupils. For example, Anna uses the maxim hedge 'Well' in the following exchange:

Teacher:	Well done. Now, 4th class, watch carefully as to how I make this pattern (pause). How is it different to the last pattern?
Anna (4th):	Well, you used more than one shape.

It seems that Anna understands that her suggestion might fall short of the maxim of quality and her 'Well' serves to give notice of this. While it is true that more than one shape has been used in the pattern, Anna is probably aware that this response will not satisfy this classrooms norms for a satisfactory explanation – as has already been displayed in the conversation with the third-class children. In fact, Sara demonstrates in her script that the description of a semi-regular tessellation might prove difficult for these children as Lucy's contribution is laced with hesitation:

Lucy:	Isn't it that all the corners in the pattern have to be the same? So for that pattern with hexagons and squares, if you picked one corner at the top of the square, each square would have to always have two hexagons touching it... is that right Ms.?

In Sara's LP, the pattern shown to the children consisted of one made by regular octagons and squares (see Figure 1). This is significant since a semi-regular tessellation with hexagons and squares also includes equilateral triangles. Moreover, in the semi-regular tessellation of regular octagons and squares, the 'corner' of each square does have two regular octagons touching it.

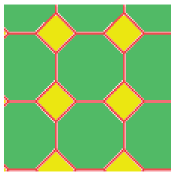

Figure 1: Semi-regular tessellation created using regular octagons and squares

Lucy's explanation is correct in terms of the tessellation presented in the lesson (Fig. 1) and it can be assumed that her use of the word, 'hexagons' is a slip occasioned by Sara in her writing of the lesson script. However, she prefaces her input with a question which we can assume to be rhetorical since, as evidenced in the transcript, she does not appear to pause for a response. This shows that she is aware that her input might not comply with the maxim of quality. Her next sentence is more convincing. Like Shane from 3rd class, her use of the pronoun 'you' indicates her belief that the polygons in question are arranged the same at every vertex, a definition that is key to semi-regular tessellation. Her generalisation of this can also be inferred by her use of the word 'always' later in the sentence. Her question - 'is that right, Ms?' serves a different purpose to that at the beginning of this turn. It reveals her awareness (and Sara's) that the teacher has asked a question to which she knows the answer - a common trait of classroom discussion. Sara's affirmation of Lucy's input and what seems to be her oversight of the slip ('hexagons') is also consistent with classroom practice. As described by O'Connor (2001), at any one moment there are several demands competing for the teacher's attention – the alignment of students with each other, sensitivity to individual students, the maintenance of mutual respect and trust, the development of social norms and socio-mathematical norms, the coordination of a student's own ideas with those of the class and with the accepted mathematical practices of the school and wider community. In a real life sense-making lesson, it is very likely that a teacher would be impressed by the sophistication of Lucy's understanding of semi-regular tessellation and consequently might not notice the slip. While Sara may not have deliberately planned this error in her script, it represents a reflection of actual talk in a sense-making classroom. In the development of LP with PTs, an example such as this could serve as an important reflective piece - reminding PTs that conversation in a sense-making classroom can have many twists and turns.

CONCLUSION

In this paper we explored the potential of LP to support PTs' engagement in sense-making conversations with pupils in mathematics lessons. It is important to note we did not provide PTs with an opening prompt, e.g. a classroom scenario where there was either a misconception or an alternative understanding on the part of a pupil. It would seem that in not providing a prompt, Sara was encouraged to focus the discussion not on ways to correct student misconceptions, but rather on how she could facilitate a sense-making discussion in the classroom. In her script Sara, showed an awareness of (a) the ways in which pupils 'try out' new ideas in sense-making mathematical conversations and (b) the teacher moves that prompt the accountability that is necessary for development of disciplinary understanding. We believe that this indicates her engagement in a fictional dialogue, that is, she entered into the classroom as if it were real. It is reasonable to expect that she will carry some of these teacher moves into her mathematics lessons in the future. It is also reasonable to suggest that LP offers a realistic way in which PTs begin to give careful consideration to how children make sense of new mathematical ideas. While Sara analysed her LP from a different perspective, consideration should be given to introducing PTs to conversational maxims in future courses. This might enable greater focus by PTs on sense-making mathematical discussions but this warrants further investigation.

NOTES

1. For the purpose of these lessons, regular tessellations were defined as tessellations made using a single regular polygon, and semi-regular tessellations were defined as tessellations made using a combination of two or more regular polygons.

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APPENDIX

T = Teacher, K = Kevin, L = Lucy, S = Shane, C = Ciara, M = Max, A = Anna

T: So, boys and girls, how do we identify squares?

K (3rd): Squares have sides that are the same length.

T: Yes. Can anyone help him out further?

L (4th): Squares also have equal angles that are all 90 degrees.

T: Indeed. So is a square a regular shape or an irregular shape? Yes Lucy?

L (4th): It's a regular shape because all sides are the same length and all angles are the same size.

T: Good. Now we know that squares tessellate. What do we know about tessellation?

S (3rd): It means that when you make a pattern, the shapes fit together perfectly.

T: Exactly Shane. But what do we need to be careful about when making patterns that tessellate?

K (3rd): Shapes don't tessellate if there are any gaps... or overlapping shapes in the pattern.

T: That's correct.

L (4th): There are other shapes that tessellate though, not just squares!

T: And you say so because?

L (4th): The honeycomb cells make up lots of hexagons stuck together and they don't overlap either.

T: Great observation Lucy.

C (3rd): And triangles! Because equilateral triangles have the same length of sides and their angles are the same size too so that means they tessellate.

T: Excellent Ciara. Equilateral triangles are one of the three 2D shapes that make up regular tessellations. Now, I will make a pattern on the board using the tangrams (pause). Does my pattern tessellate?

S (3rd): Yes, because you used squares and squares have the same length of sides and the same angles and they don't overlap.

M (4th): There are no gaps either!

T: Well done. Now, 4th class, watch carefully as to how I make this pattern (pause). How is it different to the last pattern?

A (4th): Well, you used more than one shape.

T: Yes and what name is given to a tessellation pattern with more than one shape?

L (4th): Semi-regular tessellation.

M (4th): But how do you actually know it is semi-regular?

T: How could we help Max?

L (4th): Isn't it that all the corners in the pattern have to be the same? So for that pattern with hexagons and squares, if you picked one corner at the top of the square, each square would have to always have two hexagons touching it... is that right Ms.?

T: Yes Lucy, that is correct. Do you understand now Max?

M (4th): Yes.