CONNECTIVITY MODELLING OF HETEROGENEOUS SYSTEMS: ANALYSIS AND FIELD STUDY

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ABSTRACT

A statistical approach is proposed and validated against a realistic field dataset to model the connectivity of heterogeneous systems. An object based technique is used to model the spatial distribution of facies bodies. The connectivity of the model is estimated using percolation theory. The approach is then developed to be applicable for variable body sizes as well as a system with oriented bodies. Finally, a conventional facies model of a layered reservoir is used to be compared with the results of the proposed method. The comparisons were in good agreement.

1. INTRODUCTION

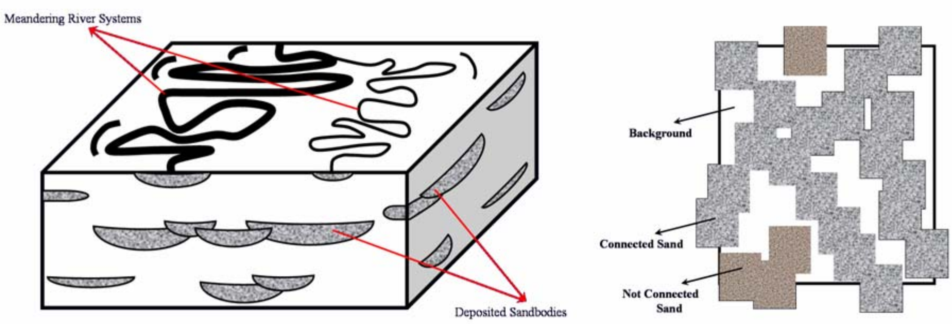
Hydrocarbon reservoirs are highly heterogeneous and have a very complicated geometry. This is usually because of the complex sedimentary processes deposited them over the years. These reservoirs are mainly a mixture of good sandstone (*i.e.* with high permeability) and poor siltstones, mudstones and shales (*i.e.* with low permeability). Good sandstones with high permeability and porosity are the main body containing oil within their pores. Therefore to be able to determine the amount of the recoverable oil from a reservoir, it is important to know what fraction of good oil bearing sands are connected. This connectivity between a pair of wells becomes significant parameter as a potential for the recovery of oil. It is the main concern in low to intermediate net to gross reservoirs. The knowledge of the connectivity of the facies and specifically the sandbodies across the reservoir not only helps to find out the potential oil recovery within a particular well configuration but also affects the other reservoir engineering decisions such as infill drilling. The overall shape of the connected sands moreover affects the flow rates of oil recovery from a reservoir *e.g.* through tortuousity effects. Connectivity also controls the swept fraction of the hydrocarbon in place in secondary displacement other than the recoverable rates of the hydrocarbon.

The usual approach to find the connectivity, oil recovery, sweep efficiency and other information for decision-making is to build detailed geological models, which are then up-scaled to a coarser grid where flow simulations are run. The whole process is repeated for other possible stochastic realisations of the reservoir and further repeated for alternative development scenarios. All these are generally far too time consuming to be carried out practically. The result is that usually a very small number of realisations are considered and the true uncertainty in the results is very poorly estimated.

An alternative approach [*King,* 1990] is to use a simple model of reservoir permeability (such as sand *vs.* shale, shale barriers, faults or fractures) and then model facies (*e.g.* sandbodies) as simple geometrical objects located in space with simple statistics. The main assumption is that the connectivity of sand controls flow. Using percolation theory we can evaluate the connectivity of low to intermediate net to gross reservoirs and predict the uncertainty of estimated production based on typical reservoir data. The main advantage is that it is very fast and can be done in a fraction of seconds on a spreadsheet.

2. OVERLAPPING SANDBODY MODEL

Reservoirs typically consist of geometrically complex connected and disconnected sandbodies [*Haldorsen et al.,* 1988; *King,* 1990]. Sandbodies within a reservoir are assumed to be flow or hydraulic units complicatedly connected [*Bridge and Leeder,* 1979]. For example a meandering river deposits layers of sand over the time in its flowing bed as represented in FIGURE 1*(a)*. The deposited sand shapes a sandbody which covers the meander belt. Due to an event on the upstream, river changes its path and deposits a new sandbody that may overlap a previous body. The process continues and forms a system of embedded sandbodies in an impermeable background.



(a) *(b)*

FIGURE 1. *(a)* A meandering river system deposits sandbodies over the ages, which overlap each other and form connectivity, where they can be modelled as bodies in space *(b)*.

This is a simple representation of the complicated process of sedimentation over millions of years. There are other several depositional and post-depositional events such as crevasse splays, mud drapes, shale layers, faults and fractures which may alter this simple model.

3. ISOTROPIC SANDBODY MODEL

The simple sandbody model of the overlapping sandbodies as shown in FIGURE 1*(b)* is an isotopic sandbody model in *2D* in which all bodies are represented by squares of the same size. A complete discussion of this simple model is given by Nurafza *et al.* [2006]. This isotropic sandbody model, like all percolation systems, assumes that the bodies are distributed independently in space and the statistics of this distribution are uniform in space (stationarity). Furthermore, the bodies are assumed to be entirely permeable, also it is assumed that there is a perfect hydraulic contact between them, and there is no contribution from the impermeable background. The problem of what fraction of sand connects left boundary of the system to right boundary, which may represent a pair of wells, is identical to a continuum percolation. Here the connectivity of the basic model shown by P, is a function of both net to gross ratio, p (the total area of sand/the total area of the region) and the system dimensionless size, L (size of the system/size of the bodies). Also called the connected sand fraction, $P\ (p, L)$ is defined as the probability that a point on the placed sandbodies in the area of the region, belongs to the percolating cluster. As the percolation theory is true for infinite systems, it has been shown by Stauffer and Aharony [1994] that the finite size scaling law for the connectivity and its standard deviation, in the overlapping sandbodies can be written as:

$$P\ (p, L) = L^{-\beta/\nu}\ F\ [(p - p_c)\ L^{1/\nu}] \qquad , \qquad \Delta\ (p, L) = L^{-\beta/\nu}\ D\ [(p - p_c)\ L^{1/\nu}]$$

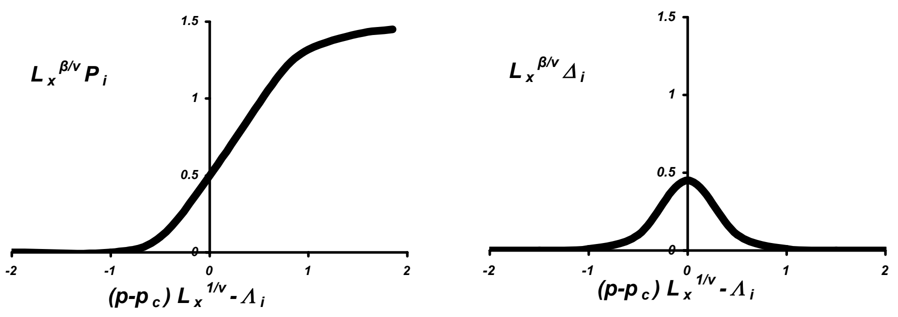
where $\Delta(p, L)$ is the standard deviation in connectivity, $F[z]$ and $D[z]$ are universal functions and β and ν are universal exponents. Universality here means that the function and the exponents are independent of the size and shape of the system and only depend on the number of space dimensions. The percolation threshold, p_c is defined as the threshold of the infinite system, as bellows:

$$p < p_c \Rightarrow P_\infty = 0 \qquad , \qquad p < p_c \Rightarrow P_\infty\ \alpha\ (p - p_c)^\beta$$

It is not universal and depends on the system size, the shape of distributed objects and the dimensions of space. The values for $\nu = 1.62 \pm 0.07$, $p_c = 66.74\ \% \pm 0.1$ and $\beta = 0.14 \pm 0.01$ are computed and their values are in good agreement with the values in the literature [*King,* 1990; *Sahimi,* 1994; *Stauffer and Aharony,* 1994; *Baker et al.,* 2002].

4. ANISOTROPIC SANDBODY MODEL

For the anisotropic models as shown in FIGURE 3*(a)* an arbitrary external rectangle region with size $A \times B$ is assumed and a number of rectangular bodies of size $a \times b$ are placed independently and uniformly in the region. The size of the system is defined by two dimensionless lengths in X and Y directions, $L_x = A/a$ and $L_y = B/b$ respectively. Here another parameter called aspect ratio, $\omega = L_x/L_y$, is also defined. For finite size systems, the connected sand fraction, P, not only is a function of the system size, L_x (or L_y), and p, but also a function of aspect ratio, ω, *i.e.:* $\quad P = P_i\ (\omega, L_x, p) \quad i \in \{x, y\}$.



(a) *(b)*

FIGURE 2. *(a)* The universal curve for finite size scaled connectivity, F and *(b)* its standard deviation, D

If finite size scaling (at a fixed aspect ratio) as already described is performed, it is found that there is no longer a single universal curve but rather two, in either of the coordinate directions. *King* [1990] suggests that these universal curves for P_x and P_y have the same shape as of the universal curve for the isotropic bodies, but for $\omega < 1$, P_x curve is elevated and P_y curve is depressed, while for $\omega > 1$, P_x curve is depressed and P_y curve is elevated. Further, as connectivity in the x-direction with aspect ratio ω is equivalent to connectivity in y-direction with the reciprocal aspect ratio, therefore using a constant of proportionality, $\Lambda_i(\omega)=c(\omega^{1/v}-1)$, and its universal coefficient, c (≈ 0.41) the horizontal and vertical curves can be shifted forward and backward respectively and lie on top of the universal curve, and hence [*King,* 1990]:

$$P_i\,(p, L_x, \omega) = L_x^{-\beta/v}\,F\,[(p-p_c)\,L_x^{1/v} - \Lambda_i]$$
$$\Delta_i\,(p, L_x, \omega) = \omega^{1/2} L_x^{-\beta/v}\,F\,[(p-p_c)\,L_x^{1/v} - \Lambda_i]$$

An illustration of the universal curve for finite size scaled connectivity, F and its standard deviation, D is shown in FIGURE 2. As for the isotropic case, where $\omega=1$ and hence $\Lambda_i(\omega=1)=0$, therefore the same curve can be used for both isotropic and anisotropic cases.

5. ORIENTED SANDBODY MODEL

The above model is still incomplete, as in reality the bodies will not all be aligned in one direction. The orientation of sandbodies in space is undoubtedly related to the depositional system and its sedimentation environment. Here we briefly explain the main idea behind this extension to the basic model. The effect of orientation distribution in sandbodies is a little synthetic for the cross sections (*i.e. 2D*) of the systems we are considering and is more relevant in *3D*. However this may help to improve the idea in three dimensions. It is clear that the orientational disorder of the bodies will greatly enhance the connectivity of the system, particularly for the systems with long thin bodies. This becomes more important in very long and thin objects, such as fractures, where without orientation the problem becomes one dimensional with the threshold of unity, and with a small amount of angular dispersion, they will start to intersect and connect two opposite sides at very low fractional concentration. The extension of this work for fracture networks (line segments) is under a current research and achieved good results [*Masihi et al.,* 2005*,* 2006].

For systems with oriented bodies, there are two possible cases, one is when there is a fixed orientation, and the other is when there is a distribution of orientation of bodies. In the first case where bodies are oriented all with a fixed angle, θ, the only effect is that the bodies appear a bit larger and a bit less elongated, and therefore this effect can be evaluated as shown in FIGURE 3*(b)* by replacing the bodies with effective ones given by the extent of a body in each direction, ℓ_x and ℓ_y, *i.e.*:

$$\ell_x = a\cos\theta + b\sin|\theta| \qquad , \qquad \ell_y = a\sin|\theta| + b\cos\theta$$

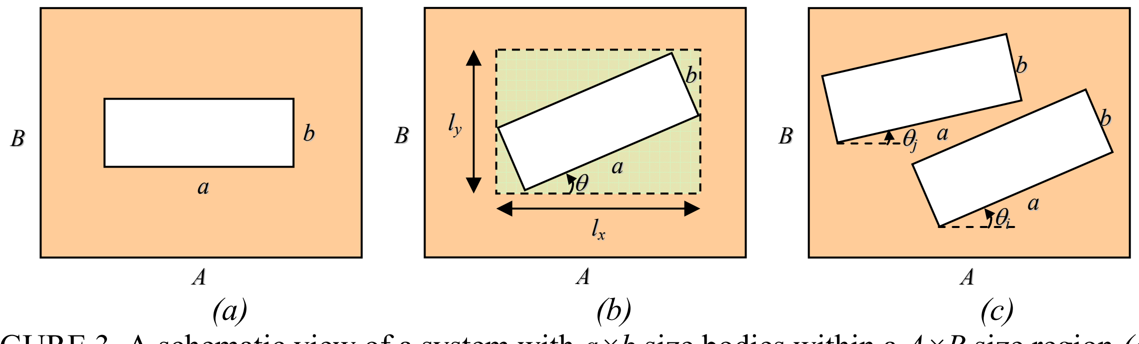


FIGURE 3. A schematic view of a system with $a\times b$ size bodies within a $A\times B$ size region *(a)* aligned, *(b)* with fixed orientation θ, and the size of bodies' accessible extents of $\ell_x\times\ell_y$, *(c)* and a distribution of orientation $\theta_i,\theta_j\in[\theta_1,\ \theta_2]=\theta_s\pm\theta_0$

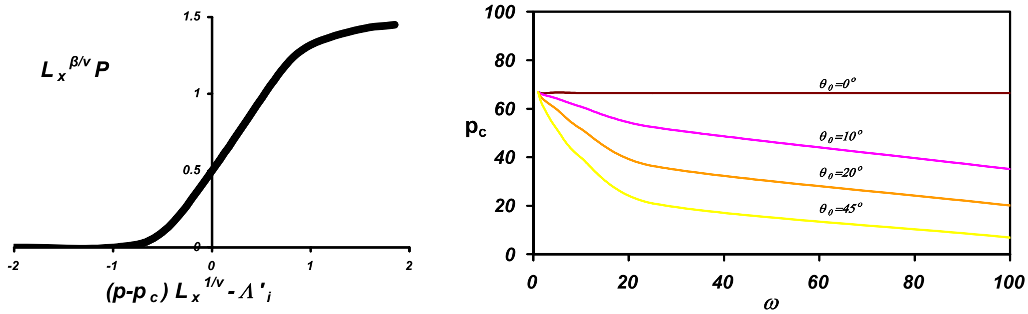
Then the new aspect ratio will be defined as $\omega'=L'_x/L'_y$, where the new system dimensionless sizes are $L'_x=A/\ell_x$ and $L'_y=B/\ell_y$. There would be no change in the percolation threshold value, in this case and so the connectivity and its standard deviation can be evaluated by the same universal curves and exponents, and by replacing $\Lambda'_i(\omega)=c(\omega'^{1/\nu}-1)$, instead of the $\Lambda_i(\omega)$ as shown in FIGURE 4*(a)*.

In the case of having a distribution of orientation *i.e.* $\theta_i\in[\theta_1, \theta_2]=\theta_s\pm\theta_0$ as shown in FIGURE 3*(c)*, there would be an average effective size in each direction, $\langle\ell_x\rangle$ and $\langle\ell_y\rangle$, for number of n bodies:

$$\langle \ell_x \rangle = \frac{\sum_{\theta_i=\theta_1}^{\theta_2} a\cos\theta_i + b\sin|\theta_i|}{n} \quad , \quad \langle \ell_y \rangle = \frac{\sum_{\theta_i=\theta_1}^{\theta_2} a\sin|\theta_i| + b\cos\theta_i}{n}$$

Thereafter using average system dimensionless sizes, which are $\langle L'_x\rangle=A/\langle\ell_x\rangle$ and $\langle L'_y\rangle=B/\langle\ell_y\rangle$, it can be concluded that the aspect ratio is: $\omega'=\langle L'_x\rangle/\langle L'_y\rangle$.
As a test case, we shall assume a uniform distribution for the angular orientation of each of the bodies, and then ω' can be derived to be as below:

$$if\, 0<\theta_1<\theta_2: \quad \omega' = \frac{\sin\theta_s + \omega\cos\theta_s}{\omega\sin\theta_s + \cos\theta_s}, \qquad if\, \theta_1<0<\theta_2: \quad \omega' = \frac{(\sec\theta_s - \cos\theta_0) + \omega\sin\theta_0}{\omega(\sec\theta_s - \cos\theta_0) + \sin\theta_0}$$



(a) *(b)*

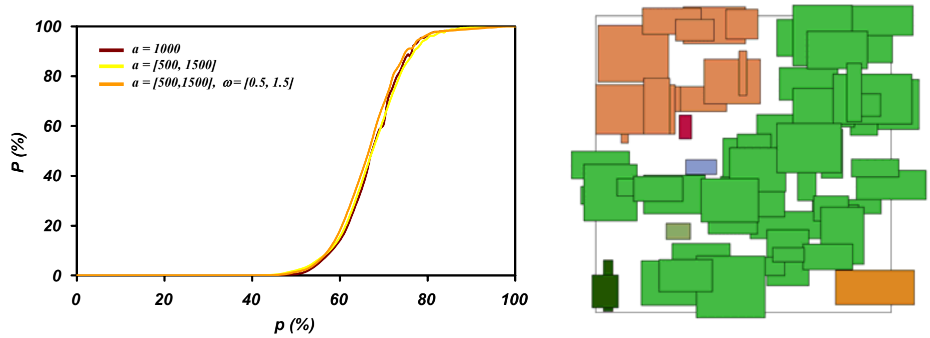
FIGURE 4. *(a)* The universal curve of connectivity of oriented systems by using Λ'_i as a function of ω' and new p_c values from the graph *(b) vs.* ω and θ_0 for uniform distribution

Using computational methods and running a large number of realizations determines that there would also be a shift in the percolation threshold, p_c. The p_c values as a function of both ω and the orientation distribution range θ_0, are computed and shown in FIGURE 4*(b)*.

6. SIZE DISTRIBUTED SANDBODY MODEL

The distribution of sandbody sizes introduces another complexity to the problem. Two possible cases for the distribution can be considered, continuous or discontinuous. If the distribution of sandbody sizes is a continuous distribution, *e.g.* a Uniform or a Gaussian (Normal) distribution, then it has been suggested [*Roach,* 1968] to replace the sands with bodies all with the same effective size. The hypothesis is that the connectivity of sandbodies of variable size is identical to the connectivity of sandbodies of the same size. Subsequently an effective size based on the square root of the average area of bodies can be used to represent the distribution of body sizes.

Here we investigate the uniform distribution for both squares and rectangles. In the first case, the sandbodies are represented as squares of size $a \times a$, where $a \in [u, v]$ has a uniform probability distribution. With this distribution the mean length $<a>$ is $(u+v)/2$ and the effective length $\tilde{a}$ (square root of the mean area) is $\sqrt{(u^2+uv+v^2)/3}$. If we consider a set of rectangular sandbodies of variable size we may consider two cases. One is where the aspect ratios of the sands are all the same. In this case we may rescale in the same way as for square bodies, and taking into account the aspect ratio shift described in section *4*. Of more interest is the case where the aspect ratio is also variable. That is the length of the sand is described by one random variable a and the aspect ratio by another ω. The height of the body is then given by $b=a\omega$. FIGURE 5 demonstrates both square and rectangular systems and compares the results with the isotropic case.



(a) *(b)*

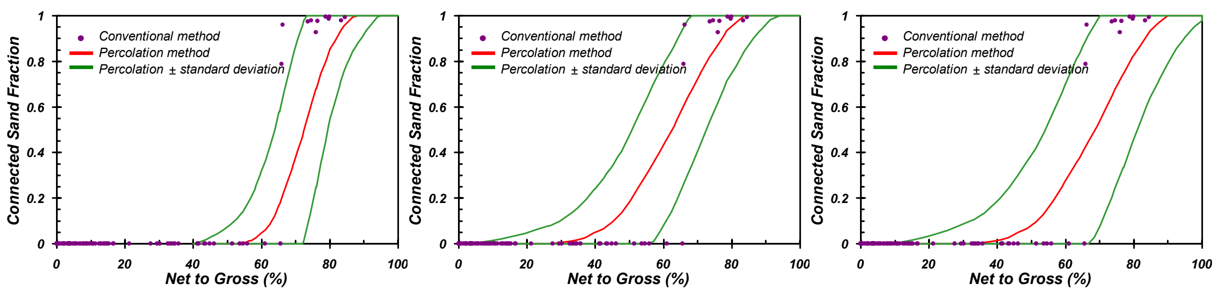
FIGURE 5. *(a)* Connectivity *vs.* net to gross ratio for three different systems: isotropic system with a fixed size of $a=1000$, a system with a uniformly distributed square body sizes of $a=[500, 1500]$ and a system with a uniform distribution for both sizes $a=[500, 1500]$ and aspect ratios $\omega=[0.5, 1.5]$, and one of its realizations *(b)* with $p=70\%$ & $P=71\%$.

Alternatively if the distribution of the sandbody sizes is a discontinuous range of length, *e.g.* a bimodal distribution, where no single body size represents the behaviour of the whole system. In this case, actually there are two systems, one made up of the small bodies and another one of the large bodies. If the difference between the number of small and large bodies is large, then most of the sand area is covered by the large bodies and the small ones are almost irrelevant, which will convert the problem to the continuous distribution. However if the number of small bodies is large enough to be comparable with the large ones, then they will slightly improve the effective size of the large bodies by a correlation length of the smaller bodies. The results show that by inclusion of the effective system size, the universal curves are still applicable for variable size systems.

7. COMPARISON WITH REAL DATA

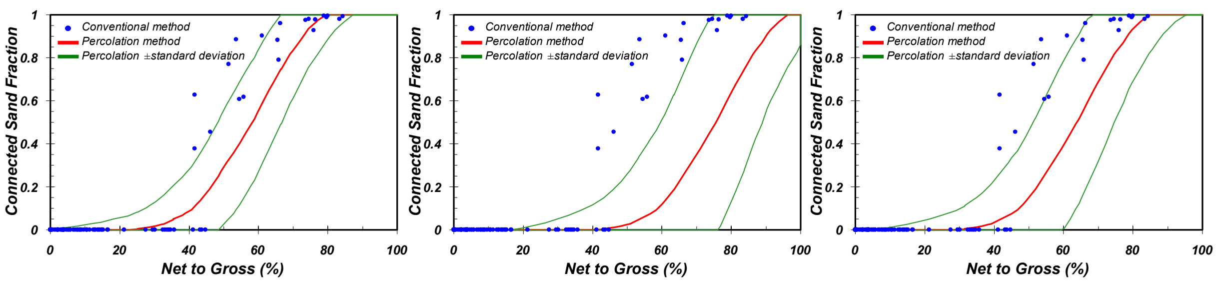
So far we have described a method based on percolation theory to predict the connectivity within a reservoir system based on reservoir and sandbody dimensions. In this section, the above procedure will be followed to evaluate the connectivity within a real reservoir by the percolation approach, and compare the results with the connectivity of the same reservoir obtained from conventional modelling. We use this comparison as a validation procedure, to find P, as a function of p, by both the percolation method and conventional modelling. Permeability map of a fine grid model is used to evaluate the connectivity. Then based on a threshold determined from permeability distribution histogram for each cross section of the map, the permeabilities are set to be zero (non-permeable) and one (permeable). Afterwards the net to gross ratio, p, is computed as the ratio of number of permeable cells to the total number of cells, for each cross section. The connected sand fraction, P_x and P_y, defined as the ratio of number of permeable cells connecting one side to the other in either X or Y directions, to the number of all permeable cells is then computed by a modified Hoshen-Kopelman multiple-labelling technique [*Hoshen and Kopleman,* 1976; *Babalievski,* 1998]. The outcome of this part based on the conventional modeling system is a graph of scatter points of computed P_x and P_y, *vs.* p. Each point on this graph represents a specific cross section of the permeability map. There are three sets of this graph; one for aerial cross sections, and two more for vertical and horizontal cross sections.

Two examples of the comparison results shown in FIGURE 6 and FIGURE 7, for the aerial connectivity of two different facies type with average sizes of $1000\times3000m^2$ and $4900\times1000m^2$ in the X and Y directions of a $9000\times18500m^2$ size system respectively, confirm nearly good matches. It can be deducted that the inclusion of the complexity of the system, *e.g.* fixed orientation and orientation distribution enhances the connectivity predictions. Although the fit may not be perfect, but the *CPU* time required for percolation is performed in a fraction of seconds on spreadsheet whereas conventional method is very computationally intensive so a good engineering approach is appropriate.



(a) *(b)* *(c)*

FIGURE 6. Aerial connected sand fraction *vs.* Net to gross ratio graphs from two different conventional and percolation methods for the a facies body of size $1000\times3000m^2$ in the X direction of a $9000\times18500m^2$ size system *(a)* with aligned bodies *(b)* with a fixed orientation of 330^o *(c)* with an orientation distribution of $330^o\pm30^o$



(a) *(b)* *(c)*

FIGURE 7. Aerial connected sand fraction *vs.* Net to gross ratio graphs from two different conventional and percolation methods for the a facies body of size $4900\times1000m^2$ in the Y

direction of a $18500 \times 9000m^2$ size system *(a)* with aligned bodies *(b)* with a fixed orientation of 330^o *(c)* with an orientation distribution of $330^o \pm 30^o$

8. CONCLUSIONS

We briefly described and evaluated a theoretical method for connectivity prediction of heterogeneous systems. The facies bodies modelled by rectangles located in space and the connectivity in the reservoir region (or between a pair wells) modelled by a cluster of connected bodies. The rectangles allocated a variety of characteristics to mimic real bodies.

The percolation nature of this model is observed and the previous results validated by developed *C++* code. Also further enhancements such as anisotropy, size and orientation distribution is programmed, analyzed and validated. Using universal values and simple algebraic manipulation, the mean connectivity and its related uncertainties are predicted in a very small amount of computational time, not comparable with the huge amount of human and *CPU* time for the conventional methods. Therefore a large number of explicit realizations of facies configurations, which are usually necessary to evaluate connectivity between wells can be avoided by a simple algebraic method. Finally a validation procedure is performed on the real field data to compare the results for the connectivity of a real system from both the conventional and percolation methods. The results revealed promising and good matches.

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