

II.II. ROTATION CURVES OF INDIVIDUAL GALAXIES

Yes – movement patterns of visible objects may serve as signature of the presence of hidden or obscure matter. Visible scale movement in objects comes only from (i) inertia, (ii) physical impact, magnetic or electric force and; (iii) gravitational influence of nearby large matter. Here inertia is intrinsic movement; how it was originally induced in the moving object – that is not relevant here. On astronomical scales, physical impacts are rare while electric or magnetic influence can be supposed to be almost ineffective thus after assuming the insignificance of electric or magnetic influence we can further assume or accept that main source of astronomical level motion of objects is the influence of gravity and the rare impacts are also, off course, caused by matter. Therefore, we acknowledge that non-inertial patterns of visible motion come from the influence of other matter and for the cases of unusual patterns of non-inertial motion that apparently do not tally with the configuration of available matter; it is reasonable to investigate the presence of hidden or non-observable matter. By no means, however, it becomes justified to insist on the presence of mysterious type of matter once the existence of real matter is completely ruled out through the application of all the possible means. The claim of the existence of magical form of matter comes from denial to review the theory that tells the patterns of movement because that theory is regarded as long-established or even final truth. Improper application of the theory coupled with non-realization of possibility of error either in theory or application thereof may result in perplexing situation that could seem suggestive of ghostly or unreal solutions to the problem. The unanticipated rotation patterns of galaxies were already known in preliminary form; Edwin Hubble discussed this problem in year 1936¹ in such mode and shape as it existed by that time. Fritz Zwicky was noting these developments because he already had floated the proposal of unusually high proportional existence of dark matter and now he wanted to set out criteria for the determination of correct mass of galaxies. In his 1937 paper, he presented and evaluated a number of methods that could be employed to estimate correct mass of galaxies. In this paper, not only he presented revised calculations relating to Coma Cluster; among other things, he also evaluated rotation of galaxies as well as likelihood of gravitational lensing as possible methods for the determination of total mass of galaxies, though he had reservations for using galactic rotations for this task. Gianfranco Bertone and

Dan Hooper sum up the stance of Zwicky regarding the use of rotations of galaxies as a possible means to determine mass in following words².

Fritz Zwicky, in his famous 1937 article on galaxy clusters, discussed the possibility of using the rotation curves of galaxies to infer their mass distribution, concluding that:

“It is not possible to derive the masses of [galaxies] from observed rotations, without the use of additional information.”

Beside the lack of information on the ellipticity of orbits, one of Zwicky’s main concerns was the possible internal “viscosity” resulting from the mutual interactions of stars. Only four years later, Chandrasekhar would demonstrate in his classic paper, “The Time of Relaxation of Stellar Systems”, that these interactions are completely negligible, allowing one to reliably describe galaxies as systems of non-interacting stars.

This paper is telling that Zwicky had reservations in using galactic rotations as a means to determine total mass of galaxies but those reservations were removed few years later mainly through the classical work of Chandrasekhar. Another source³ tells us following important developments that surfaced in year 1939 and continued to proceed till year 1975:

Six years after Zwicky’s paper Babcock (1939) obtained long-slit spectra of the Andromeda galaxy, which showed that the outer regions of M 31 were rotating with an unexpectedly high velocity, indicating either (1) a high outer mass-to-light ratio or (2) strong dust absorption. Babcock wrote: “[T]he great range in the calculated ratio of mass to luminosity in proceeding outward from the nucleus suggests that absorption plays a very important role in the outer portions of the spiral, or, perhaps, that new dynamical considerations are required, which will permit of a smaller relative mass in the outer parts”. Subsequently Babcock’s optical rotation curve, and that of Rubin & Ford (1970), was extended to even larger radii by Roberts and Whitehurst (1975) using 21-cm line observations that reached a radial distance of ~ 30 kpc. These observations clearly showed that the rotation curve of M 31 did not exhibit a Keplerian drop-off. In fact, its rotational velocity remained constant over radial distances of 16 - 30 kpc. These observations indicated that the mass in the outer regions of the Andromeda galaxy increased with galactocentric distance, even though the optical luminosity of M 31 did not.

There were three important aspects of Babcock’s (1939) and later findings that were (i) Outer regions of Andromeda (M31) were rotating at speeds higher than expected; Babcock, though he noted sort of anomaly in this context but he did not commit mistake of straight

calling it 'Keplerian drop-off' – at a later stage (1975), this mistake would come from or get the confirmed shape out of the findings of Roberts and Whitehurst. (ii) Rotation speed was derived from study of spectral lines thus Babcock tried to justify the 'anomaly' by attributing it to possible more 'absorption' at outer regions, (iii) if 'absorption' had no important role then according to Babcock, 'new dynamical considerations' were required. And despite they treated it like 'Keplerian drop-off not observed', Roberts and Whitehurst noted another very important point which was the **indication that the mass in the outer regions of Andromeda galaxy 'increased' with galactocentric distance**, even though the optical luminosity of M31 did not.

These were very important developments. Babcock presented only careful assertions but by year 1975, scientists reached to the careless confirmation that rotation of galaxy M31 was not following 3rd law of Kepler. Babcock had not attributed it towards non-observed matter rather he said that role of absorption in outer regions should be checked and if this factor had no important role then perhaps 'new dynamical considerations were required'. So accurate he was. But with Roberts and Whitehurst (1975), all this was concluded with the affirmation of the possible existence of extra non-luminous matter at outer regions of spiral. In this way, Babcock presented the true facts and also outlined possible reasons that could possibly account for the observed anomaly that outer parts of M31 galaxy were rotating at higher than expected velocity. At the end, scientists altogether ignored one genuine possibility that 'new' dynamical considerations were required; rather with same but extended, refined and 'better quality' observations of 1970 and 1975, they adopted more exotic, better to say erroneous conclusion that rotation of M31 galaxy was not following 'Keplerian drop-off' and thus explicit voices in favor of the presence of extra mass at the outer regions of galaxies began to emerge⁴ within scientific writings and circles. Then onwards, gradually those explicit voices have assumed the form of dominant scientific point of view; like a hard scientific 'fact'; now mysterious form of dark matter is regarded as viable scientific interpretation of apparently anomalous galactic rotations on account of the 'fact' that a number of other 'scientific' observations also point towards the existence of (almost) same quantitative ratio of 'dark matter'. The task of this book is to show that none of those scientific observations actually point towards the existence of dark matter while the

obligation of this section is to deal with the specific problem of galactic rotations to show that this main problem also has nothing to do with dark matter.

II.II.I. WHY SHOULD GALACTIC ROTATIONS FOLLOW 3RD LAW OF KEPLER?

We identified in the previous section that during 1970s, scientists had reached to a careless 'confirmation' that **rotation curve of M 31 (Andromeda) did not exhibit a Keplerian drop-off**. In non-technical terms, Kepler's 3rd law says that a planet farther away from sun revolves slowly in orbit in comparison with the planet whose orbit is closer to sun. It means that orbital speed for closer orbit is fast and there is 'drop-off' in speed with distance of orbit from sun.

Mathematically, Kepler's 3rd law is: $P^2 = a^3$

An online source⁵ describes this law in following simple words:

Kepler's 3rd law is a mathematical formula. It means that if you know the period of a planet's orbit (P = how long it takes the planet to go around the Sun), then you can determine that planet's distance from the Sun (a = the semimajor axis of the planet's orbit).

It also tells us that planets that are far away from the Sun have longer periods than those close to the Sun. They move more slowly around the Sun.

According to another online source⁶, almost following is the overall scope of this law:

Kepler's third law (in fact, all three) works not only for the planets in our solar system, but also for the moons of all planets, dwarf planets and asteroids, satellites going round the Earth, etc. Well, not quite; if the secondary body – a planet, say – has a mass that's a significant fraction of the primary one (the Sun, say), then the law needs a small tweak.

Please note that scope of Kepler's 3rd law (in fact, all three) **does not cover 'galaxy'**. Actually Kepler (1571-1630) had discovered his three laws out of study of planetary motion data of our own Solar System. Rather than 'general laws', essentially these are descriptions of systematic orbital motion behavior of planets of our own specific Solar System. The main characteristic of our Solar System is that more than 99% of the mass is concentrated at central location i.e. Sun. At the most, laws of Kepler could be generally applied to any Solar or Planet-Moon system where central mass is far superior to orbiting bodies and any

secondary orbiting body does not possess mass which is significant fraction of the central body's mass

The irrelevancy of Kepler's 3rd law for the orbital motion of stars around galaxy is not disputed. Mr. Erik Anson⁷, Physics/Cosmology PhD Student at University of Washington and a famous Internet Physics writer replies to a question⁸ (asked by Mr.Damien Giraud⁹) regarding applicability of this solar system specific Law to whole Galaxy in following words:

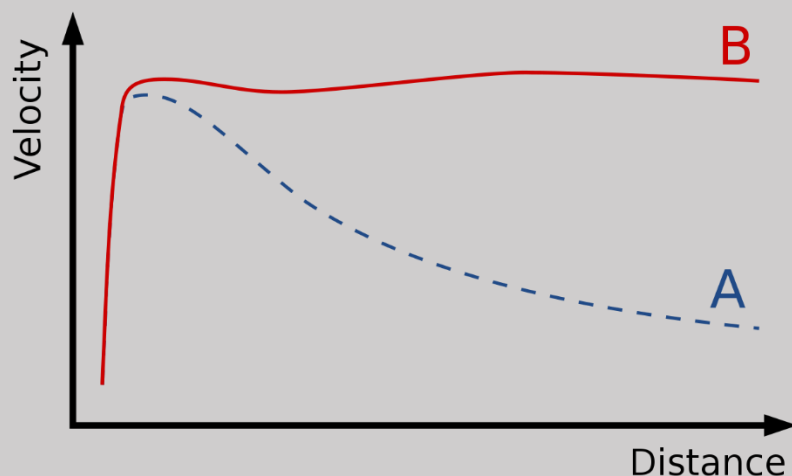
You're partially right, but you're also missing something huge.

The thing you're right about: Kepler's 3rd Law indeed doesn't apply to the orbits of the stars within the Milky Way. K3 only works in the special case where the system is almost completely dominated by a single mass (e.g., the Sun for our solar system). The mass of the Milky Way is much more spread out, and so Kepler can't tell us anything.

The thing you're apparently missing: *Physicists and astronomers aren't totally incompetent.* The evidence for dark matter that comes from the rotation curves of galaxies (which, by the way, is *far* from the only evidence there is), is not based on assuming that K3 holds. The "expected" orbital speeds, given the matter that we can see, are based on Newtonian gravity (with perhaps some small corrections from Einstein), not Kepler's Laws.

Here Mr. Erik Anson, thanks to him, accepted that Kepler's Third Law indeed doesn't apply to the orbits of the stars within galaxy. But he is not right that evidence for dark matter that comes from the rotation curves of galaxies is not based on assuming that Kepler's 3rd law holds as we have seen in the previous section that Scientists, by 1975, did reach to the careless confirmation that rotation curves of M31 (Andromeda) did not exhibit 'Keplerian drop-off'. Mr. Erik Anson has tried to justify that anticipated orbital speed was based on Newtonian Gravity with 'perhaps' some small corrections from Einstein. However, point is that even if so then it means that Newton's Gravity and General Relativity (Einstein) were giving approximately the same results for galaxy as could be expected by applying Kepler's 3rd law. But we have seen earlier that Kepler's 3rd law is not applicable to galactic dynamics. Scientists should not have expected to get 'Keplerian drop-off' by applying general theories like 'Newton's Gravity' and 'General Relativity'. Somehow they were getting same results from a particular law i.e. Kepler's 3rd law and General Theories i.e. Newton's Gravity and Einstein's General Relativity. Either the particular law had been elevated to the level of

general theory or scientists were really missing something with regards to the application of general theories. If flat rotation curves of galaxies indicate the presence of dark matter then it is possible only when Kepler's 3rd law has been elevated to the level of general theory. Affirmation of dark matter has come from out of scope expectations from Kepler's 3rd law. Additional problem was that general theories were also apparently giving results similar to Kepler's 3rd law and theoretical results did not tally with the actual observations.



Expected (A) and observed (B) star velocities as a function of distance from the galactic center. (Credit: Wikimedia: Commons)

In the above graph, actual observations are represented by line 'B' whereas line 'A' represents what results we should expect from Kepler's 3rd law. Keeping in view that Kepler's 3rd law is applicable where mass is concentrated at the central point, we must conclude that for the case of galaxies where mass is distributed and spread out, the same line 'A' should not have been expected by applying general theories like Newton's theory of gravity and Einstein's General Relativity. But we see that scientists are actually expecting line 'A' from the application of general theories as well and exact this is the problematic point. By the time of publishing my first book against the Big Bang Theory, my general take on the topic of Dark Matter was that there must be something missing in equations rather than something missing in observations. By that time, admittedly, I had not reached to the actual point of the problem. But soon after I realized that scientists have based their theory of Dark Matter on non-observance of 'Keplerian drop-off' which should not have been the case due to different dynamics of galaxies than solar system. Afterwards but before reaching

to the correct relevant points of the already existing theory, I was thinking that within galaxy, stars belonging to outer parts of galaxy are not in fact directly obeying the gravitational commands of galactic center. Those stars are basically drifted towards immediate next stars who are far nearer to them than the center and due to short distance, those nearer stars exert far greater gravitational pull that could have arrived from far-off central point. For the stars belonging to the outer edges of galaxy, the gravitation pull is coming from entire inward disk such that nearer stars have more influence than those who are at other side of the disk. The stars who are located at inner part of the disk are subject to more gravitational influence from one side than from the other and essentially experience the same gravity as if they are also located at the outer edge. In simple words, outer edge stars and inner disk stars should be subject to same gravity. I was thinking on these lines in a commonsense mode and it happened that eventually I reached to the conclusion that **while calculating the theoretical rotation behavior of galaxies, scientists have missed to include implications of Newton's Shell Theorem in their equations and that's why they are treating absence of Keplerian drop-off as an anomaly**. At this point, though I had no positive proof that Shell Theorem was actually skipped during the official determination of theoretical rotation of galaxies, I undertook to start writing this book with the intention to debunk prevailing theory of dark matter. Afterwards, I came to know that some other people are also thinking on same lines. For example, Nikolay Sones¹⁰ asked a question¹¹ on questioning website quora.com that when we have shell theorem then what the need of dark matter is. To this, I replied at that time, that they are strict mathematicians. They have shell theorem for a sphere therefore they did not apply the same on a disk structure. I also stated that if we apply the main theme of shell theorem on galactic disk then absolutely there is no need of dark matter in galactic rotations.

By that time I had made up mind to do something to explain applicability of shell theorem for the dynamics of galactic disks. But official mistake was not that simple. I found out finally that **they did incorporate Shell Theorem in their formula** through which they determined theoretical rotation of galaxies. **But they have wrongfully applied Shell Theorem in their formulation**.

II.II.II. WRONG APPLICATION OF SHELL THEOREM IN THE OFFICIAL THEORY

The main question is that why are scientists getting same result about galactic rotations from general theories (Newton's gravity and Einstein's GR) as they expect from applying a particular law i.e. Kepler's 3rd law? Clearly, their anticipation of applying Kepler law to the problem of galactic rotations was misleading and the results taken from applying general theories are incorrect because correct result from applying general theories should not tally with the one expected from applying Solar System specific law of Kepler. There is a definite mistake in the official application of general theories in this matter. And following quote from the "Galaxy Rotation Curves" section of the Wikipedia article¹² titled "Dark Matter" gives the true hint about the actual point of mistake.

The arms of [spiral galaxies](#) rotate around the galactic center. The luminous mass density of a spiral galaxy decreases as one goes from the center to the outskirts. If luminous mass were all the matter, then we can model the galaxy as a point mass in the center and test masses orbiting around it, similar to the [Solar System](#).^[d] From [Kepler's Second Law](#), it is expected that the rotation velocities will decrease with distance from the center, similar to the Solar System. This is not observed.^[48] Instead, the galaxy rotation curve remains flat as distance from the center increases.

If Kepler's laws are correct, then the obvious way to resolve this discrepancy is to conclude the mass distribution in spiral galaxies is not similar to that of the Solar System. In particular, there is a lot of non-luminous matter (dark matter) in the outskirts of the galaxy.

In the above given quote, after the sentence "then we can model the galaxy as a point mass in the center and test masses orbiting around it, similar to the Solar System", there is a footnote "[d]" which reads as "This is a consequence of the Shell Theorem and the observation that spiral galaxies are spherically symmetric to a large extent (in 2D)".

First thing we note here is the information that general theory (GR) has modelled gravity similar to Solar System. The general theory assumes that all the luminous matter of galaxy is located at center and the basis for this assumption is the shell theorem. It also has been explained that application of Shell Theorem on disk structure of galaxy is justified because spiral galaxies are spherically symmetric to a large extent.

Thus, apparently or on the face value, our main question regarding specific law (Kepler's 3rd law) and general theories giving same result about galactic rotations has been responded.

But – my reaction is that it is complete incorrect application of Shell Theorem. Yes Shell Theorem is applicable to disk structure of galaxy but it is applicable in a whole different way.

Before explaining the faulty application of Shell Theorem in the official theory, it is important to note second thing that applicability of Shell Theorem upon disk structure of galaxy has been admitted and explained by the official theory which means that now we are in no need to explain whether correct application of Shell Theorem is applicable to disk structure of galaxy or not and that for the forthcoming proceedings of this section, it will be taken for granted that Shell Theorem is applicable to disk of galaxy and that this issue is not disputed and thus term 'sphere' shall be treated as equivalent to 'disk' for the practical reasons.

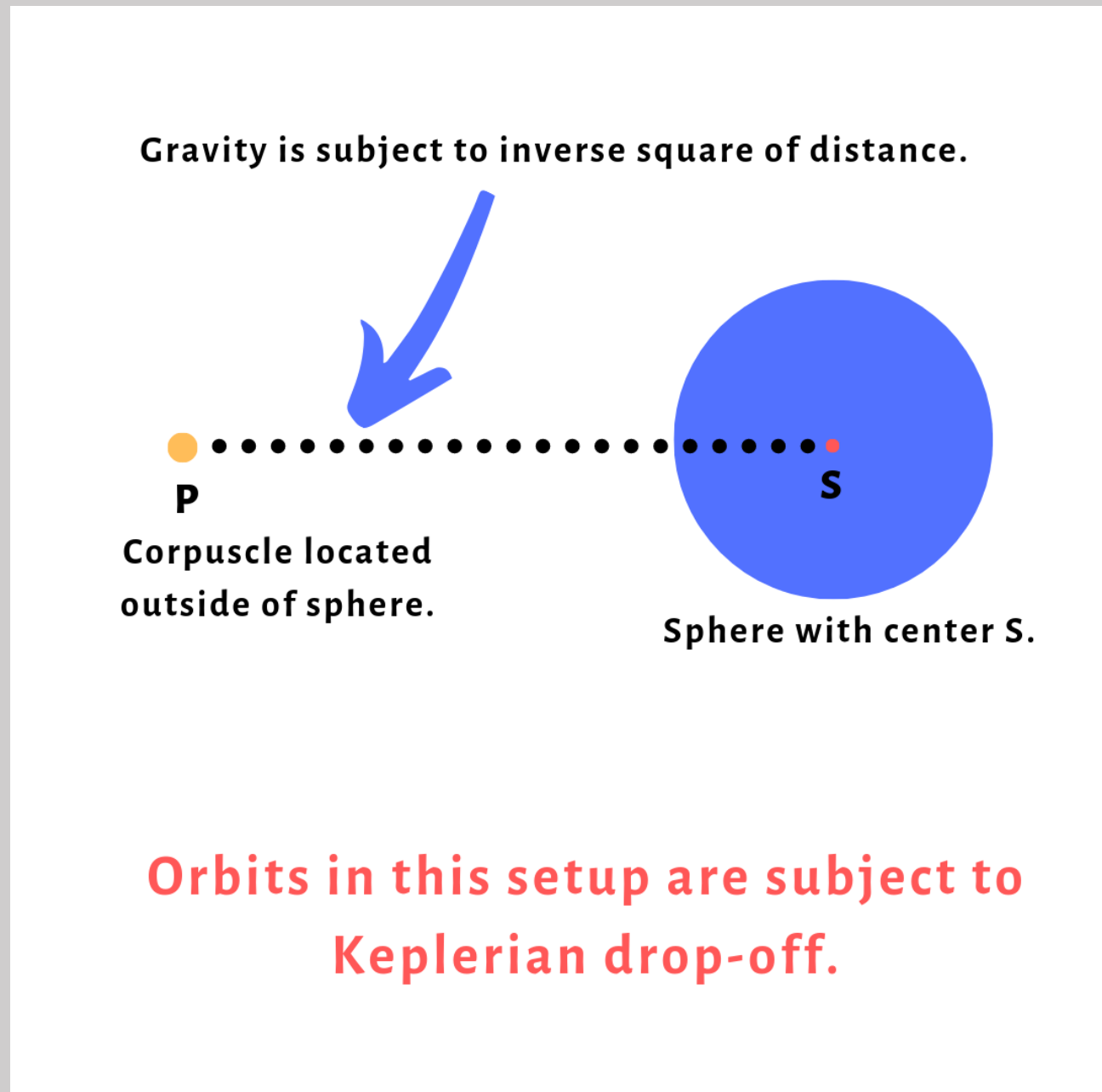
Newton's Shell Theorem does not simply state that the test particle will be attracted towards center of the spherical source of gravity by such and such force. In fact, it is not a single Theorem. In Principia Mathematica, Newton has presented more than dozen Theorems that all deal with gravitational effects of spherical bodies under different conditions. Basically, some of these Theorems are known as Shell Theorem such that the title 'Shell' is not assigned to them in the Principia.

Without going into the irrelevant details, we identify that it is Theorem XXXI of Principia which has been officially applied while determining the motion of stars within the disk of galaxy. This Theorem is actually applicable to a test particle which is located outside of the sphere i.e. like in Solar System. The Theorem XXXI says that gravitational attraction on test particle will be inversely proportional to the square of distance (of test particle) from center (of spherical body). And let me now assert that **this Theorem is not applicable to rotation of stars within galaxy because stars are located inside of the disk.**

Following is operative part of the Theorem XXXI from the English Translation (American Edition: 2007) of the Principia:

Theorem XXXI: A corpuscle placed without the spherical superficies is attracted towards the center of the sphere with a force reciprocally proportional to the square of its distance from that center.

This Theorem describes typical cases of Solar System and Planet-Moon systems etc. where principal gravitating mass is concentrated at the center and ‘test particles’ i.e. planets and moons are subject to gravity with a force reciprocally proportional to the square of the distance from the center of central mass. The following diagram depicts the situation where this Theorem XXXI is applicable and also explains implications thereof.

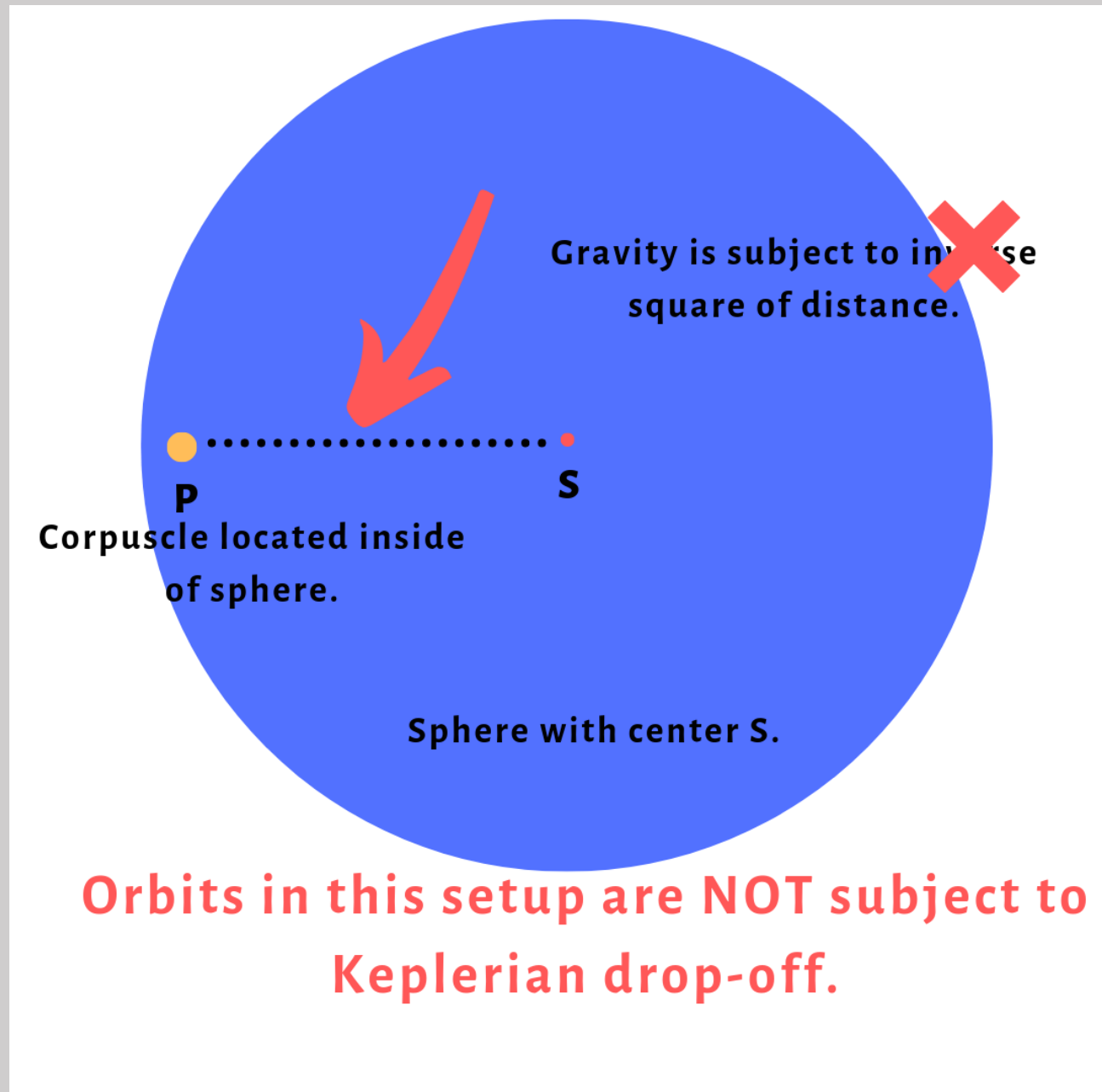


Above is the usual case of gravity which is applicable to solar system as well as Earth-Moon and other like systems. Due to the presence of inverse square distance law, orbits under

these systems are subject to Keplerian drop-off. However, we have seen already that galaxy is a different kind of system where rotation of galaxy is the rotation of stars within the disk of the galaxy. To such a system where test particle is located within sphere (or disk), Theorem XXXIII is applicable which states following:

If to the several points of a given sphere there tend equal centripetal forces decreasing in a duplicate ratio¹³ of the distances from the points; I say, that a corpuscle placed within the sphere is attracted by a force proportional to its distance from the center.

This Theorem is telling that for a test particle located within a sphere, inverse square distance with center law vanishes and instead, inverse distance with center (linear) law prevails. The following diagram explains the meaning and implications of this Theorem.



This Theorem also explains that if test particle is located at a particular depth within the sphere, the repercussion will be that the complete upper portion layer of the sphere will have no gravitational effect at all. Thus we see that for a test particle, movement from surface to the center of the sphere, the total mass will keep reducing in a linear mode such that the effect of gravity will reduce to zero at the point of center i.e. movement from surface to center will cause reduction of gravitating mass in a linear mode. At the same time, same movement from surface to center will cause linear increase of gravitational effects of inner available

(though reduced) mass i.e. mass of only the inner layers is exerting total gravity from the center and that total gravity effect (of reducing available mass) is linearly increasing because distance from center is reducing and the result will be that any depth will be subject to almost same gravity that was available at the outer surface. The overall effect will be that at every point, orbiting will be subject to almost same orbital velocity i.e. neither there will be Keplerian drop-off from center towards surface nor there will be Keplerian increase in the orbital velocity at points closer to the center. In our Solar System, orbital velocity of Mercury is far greater than that of Pluto. It is so because more than 99% of the mass of Solar system is located at the central point. In case our Solar System disk had uniform distribution of mass, then Mercury and Pluto would be having almost identical orbital velocity. Mercury would have been subjected to very low gravity and thus despite being close to the central point, it's orbital velocity would be almost as slow as that of Pluto. Therefore, rather than the case of the absence of Keplerian drop-off, in the galaxies, actually we are noticing the absence of Keplerian increase in orbital velocity near the center. Moreover, it is not the case of increase in mass as we move from center to the edges of the galactic disk, as we noted in a previous section and we copy here also:

Roberts and Whitehurst (1975) noted another very important point which was the indication that the mass in the outer regions of Andromeda galaxy 'increased' with galactocentric distance, even though the optical luminosity of M31 did not

Because now we are discerning that rather than the case of 'increase' in 'mass' in the outer regions of the galaxy, more appropriately it is the case of 'decrease' in 'gravitational mass' (i.e. mass having positive gravitational effect) towards the central regions of the galaxy. At the same time, it is also equally correct that mass of outer regions 'increases' with galactocentric distance. Because if 'gravitational mass' is decreasing from surface to center then it equally means that 'gravitational mass' is increasing from center to surface. But more appropriate, as we noted earlier, is the case of 'decrease' in 'gravitational mass' in the central regions of the galaxy because the actual galaxies near to central regions depict lowest point of rotational curve of velocity.

II.II.III. IMPLICATIONS OF THEOREM XXXIII ARE OFFICIALLY RECOGNIZED BUT SOMEHOW THEY WERE NOT INCORPORATED WITHIN THE STUDY OF GALACTIC ROTATIONS

Official science accepts that within a sphere, the gravity is subject to inverse distance from center law and that the fact of vanishing of the inverse square distance law within the sphere is not disputed. Wikipedia's article titled "Shell Theorem"¹⁴ states following:

[Isaac Newton](#) proved the shell theorem^[1] and stated that:

A [spherically symmetric](#) body affects external objects gravitationally as though all of its [mass](#) were concentrated at a [point](#) at its centre.

If the body is a spherically symmetric shell (i.e., a hollow ball), no net [gravitational force](#) is exerted by the shell on any object inside, regardless of the object's location within the shell.

A corollary is that inside a solid sphere of constant density, the gravitational force within the object varies linearly with distance from the center, becoming zero by symmetry at the center of mass.

Following stackexchange.com page¹⁵ explains some relevant points about effects of gravity within Earth:

Assuming spherically symmetric mass distribution within Earth, one can compute gravitational field inside the planet using Gauss' law for gravity. One consequence of the law is that while computing the gravitational field at a distance $r < R$ (with R being the radius of the Earth), one can ignore all the mass outside the radius r from the center

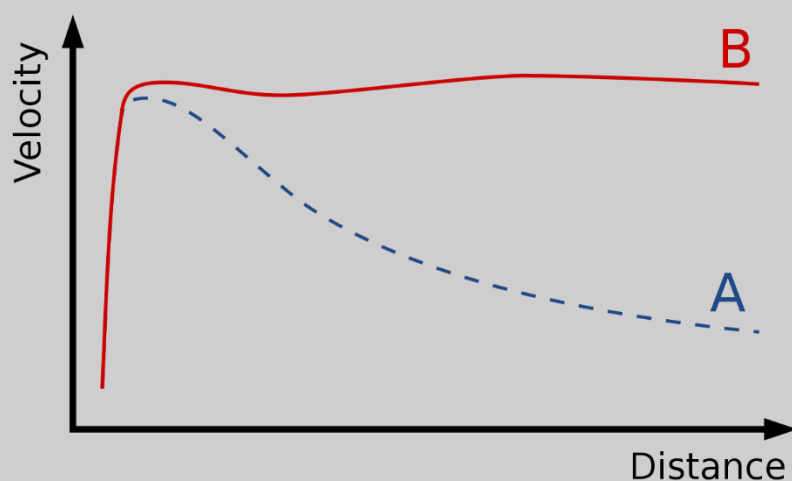
Actually Gauss' law of gravity is essentially equivalent to Newton's theory and we have already seen that [Newton's Theorem XXXIII](#) also had explained that at a depth r , the mass above r is to be ignored as it will have no gravitational effect.

The above-referred stackexchange.com page also refers to a graph taken from Wikipedia¹⁶ which is clearly showing that from the surface of Earth towards center, for the case of constant density, the gravity drops linearly and becomes zero at the central point.

Thus replacement of square distance law with linear distance law and the reduction of gravitational mass as one goes deeper inside the sphere are officially accepted stances. Official theory also accepts that galactic disk is spherically symmetric thus Shell Theorem is applicable to the disk. However, somehow, during the study of galactic rotations, official theory never realized that rotation of galactic disk is in fact the rotation of stars within disk and that the applicable Theorem was XXXIII according to which galactic disk should have depicted **flat rotation curves** and thus no discrepancy with the theory would have surfaced. Not only that there would have been no need of dark matter, the velocity meaning of redshift also might have been discarded by now due to the failure of having found out such huge quantities of normal kind of dark matter that was pointed out by Fritz Zwicky out of the study of Coma Cluster.

II.II.IV. FLAT ROTATION CURVES OF GALAXIES – PROPER INTERPRETATION

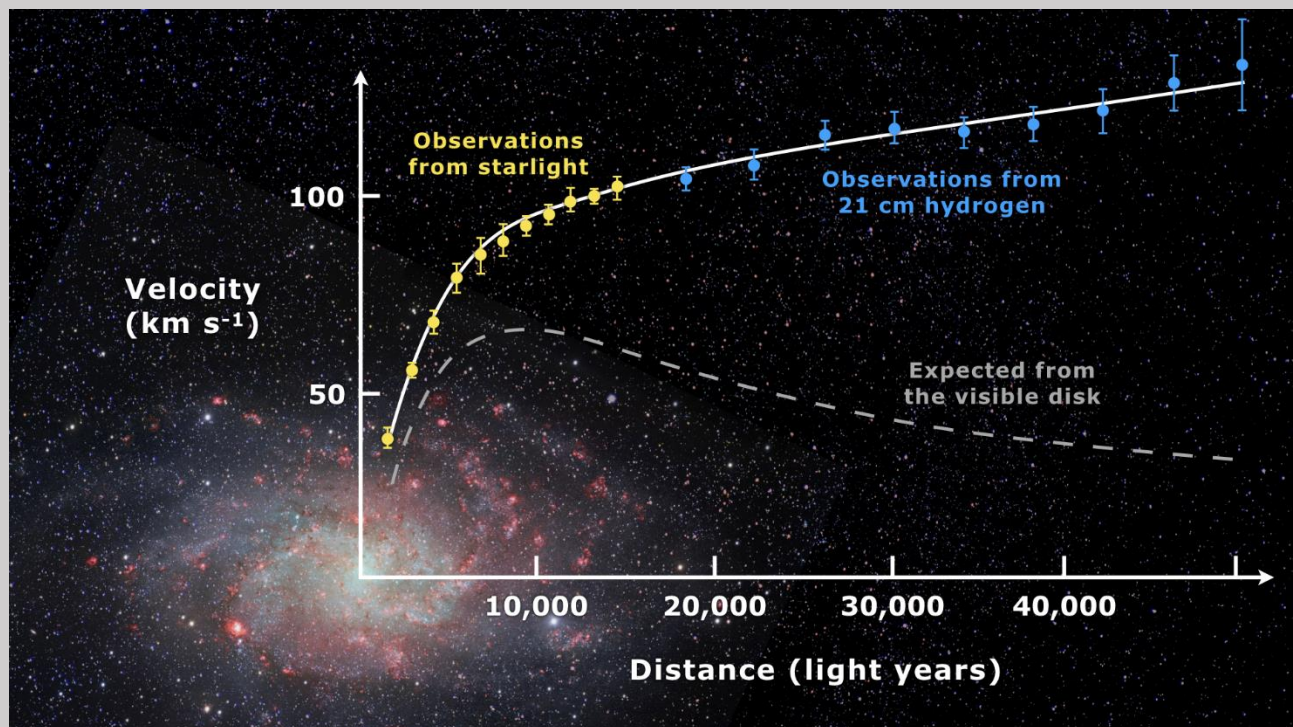
After having seen that galactic rotations should have been described in the light of Theorem XXXIII of Newton's Principia Mathematica, let us therefore try to do it now.



Expected (A) and observed (B) star velocities as a function of distance from the galactic center. (Credit: Wikimedia: Commons)

The prerequisite here is that we must completely forget the so-called 'Expected' line (A) because within the right context of Theorem XXXIII, we simply do not expect line (A). Line (B) is the actually observed line and the same is anticipated by applying Theorem XXXIII.

Regardless of what official theory tells us about the existence of super massive black hole at the center of galaxy, this graph is actually telling that closer to the center, orbital speed is lowest. Within a disk of uniform density of mass, we should expect zero orbital velocity at the center of the disk. The lowest orbital velocity at point close to the center is consistent with this theory which means that law of inside of sphere (or disk) is being demonstrated. Non-zero but lowest orbital velocity near the center of disk may or may not indicate the presence of super massive black hole at the center. Afterwards, over a very short distance, there is substantial increase in the orbital velocity as the velocity curve moves up quite sharply. Our interpretation is that this area is the central bulge of the galactic disk and over this short distance, actual mass is substantially increasing layer upon layer such that density of each layer almost remains the same. Following actual graph confirms the idea that area of sudden increase of orbital velocity approximately relates to central bulge of the galaxy M33.



M-33 Galaxy Rotation Curve, Credit: Wikimedia Common – Source link¹⁷

M33 is not very large galaxy as the diameter of galactic disk is only about 60000 light years. We see (or assume) in this picture that radius of the central bulge of the disk spans about

5000 light years and within this distance of 5000 light years, there is sharp jump in the velocity curve. This actual graph is showing gradual upward movement of velocity curve even beyond this point but for the sake of simplicity, we shall assume that after this point, velocity curve becomes flat.

Basically there are two distinct portions of the Rotation Curve of Galaxy. Up to the distance from center towards the edge of the central bulge, there is sharp increase in orbital velocity of stars within disk. The lowest orbital speed is found in the area closer to the center of the disk. It means that area close to the center is subject to lowest gravity and this thing is in harmony with the Shell Theorem as applicable within the sphere (or disk). In the example of galaxy M33, we see that radius of central bulge is almost 5000 light years. For the sake of our analysis regarding why orbital velocity is increasing very sharply over this distance, we suppose that there are 5 layers within the radius of central bulge and the width of each layer is 1000 light years. Our interpretation will not depend on the existence or absence of super massive black hole at the center of galactic disk. So the interpretation goes that for the five layers of central bulge, a huge quantity of mass, let's say 1 billion solar masses, is concentrated in the innermost layer that may or may not include super massive black hole. The second layer is orbiting around inner most layer with the lowest velocity. The second layer has same width of 1000 light years but due to being outer layer of the circle, the area is far greater than the innermost layer. The second layer has almost equal density of mass which means that total mass of the second layer may be around, let's say, 8 billion solar masses i.e. just approximate number only to explain the point.

Now the third layer is orbiting a total mass of 9 billion solar masses. Therefore, within the third layer, orbital velocity has increased quite sharply. Width of third layer is also same 1000 light years but area is still far larger than that of second layer. And again, the density of mass remains the same and thus total mass of this layer may be let's say 16 billion solar masses.

Now this setup repeats up to the fifth layer which is subject to the highest orbital velocity of stars within the disk so far and also marks the boundary of the central bulge of the galactic disk. The central bulge area is therefore the first portion of the Rotation Curve of Galaxies. The important thing of the first portion is that mass is considerably increasing layer upon

layer and reaches to, let's say, 32 billion solar masses for the fifth and outermost layer of the first portion.

The central bulge area was characterized by layer upon layer successive and substantial increase in mass such that overall density of the bulge remained uniform. The outermost layer of the central bulge contains greatest quantity of mass so far which is 32 billion solar masses (i.e. approximate number just to explain the point). Next to the central bulge area, the second 'flat' portion of the Rotation Curve of Galaxies begins.

If the radius of M33 galaxy is 30000 light years wide then this second portion starts from 5000 light years from center of the disk and ends at 30000 light years from the center of the disk. For the sake of simplicity, here again, we divide this second portion into 25 layers each having width of 1000 light years.

We know that outer layer of central bulge had mass of 32 billion solar masses. Now we interpret the start of flat curve portion by saying that inner layer of this portion contains almost same mass i.e. 32 billion solar masses. In this way, the innermost layer of the second portion is having same mass as the outer layer of the central bulge had. However due to larger area, the density and luminosity (per unit area) of this layer is lower than that of central bulge. Due to the fact that previous layer i.e. the outer layer of the central bulge had the greatest mass, our present layer i.e. the inner layer of outer area has the greatest orbital velocity and the rotation curve moves still higher. Therefore, flat portion of curve has not actually started yet.

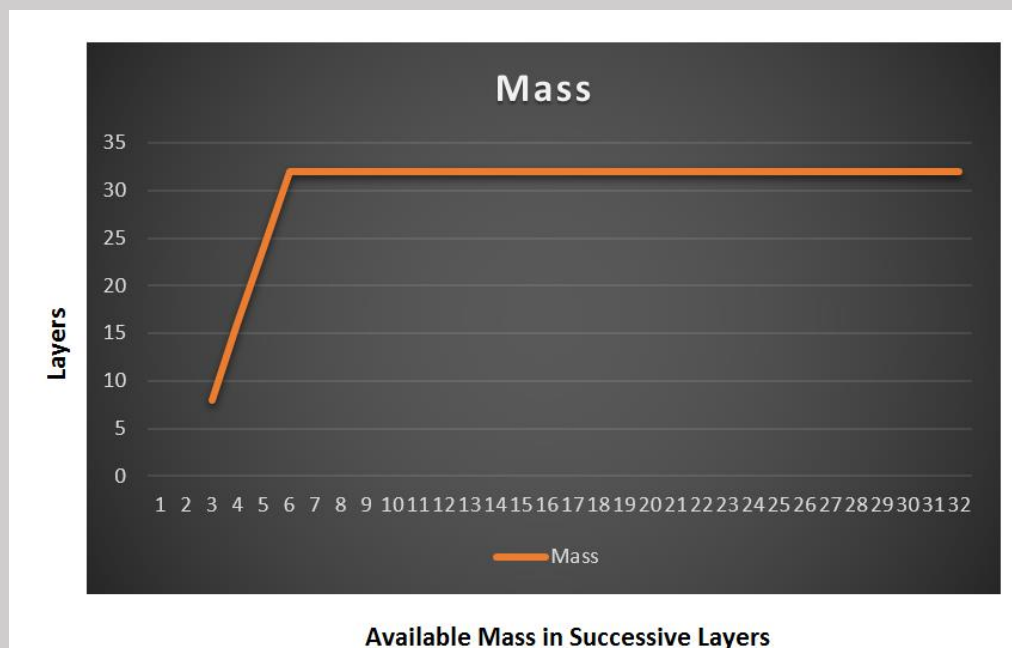
Now comes the second layer of the outer portion of galactic disk. Again mass will remain the same i.e. 32 billion solar masses and due to larger area, there will be slight reduction in the density and luminosity (per unit area) across this layer. Because previous layer had augmented a constant mass, therefore, keeping in view the applicable inverse distance law of gravity, orbital velocity curve will remain horizontally flat across the current layer.

If this pattern repeats up till 25th outermost layer, each successive layer will get equal quantity of mass however slightly lesser and lesser density and (per unit area) luminosity will be added and the overall galactic rotation, keeping in view the simplified assumptions, should show up as a flat curve on graph. It is possible that same pattern of successive

layers, up to few more, may continue even after 25th layer but that outer portion of galaxy may remain invisible or normally undetectable due to low density and (per unit area) luminosity over there.

An important thing to be noticed is that let's say when an object moves from 10th layer to 11th one, the object will be subject to gravity of the mass available in all the inner layers including central bulge and up to 10th layer (or even 11th layer). Objects placed in 11th layer will not be subject to gravitational effects of still outer layers i.e. 12th and rest of the outer layers because according to Theorem XXXIII, an object placed at certain depth within sphere (or disk) will not be affected by the gravity of outer surface area. With this setup, availability of constant mass in each successive outer layer will give the result of flat rotation curve because law of inverse square distance is also replaced with the law of linear inverse distance within the sphere (or disk).

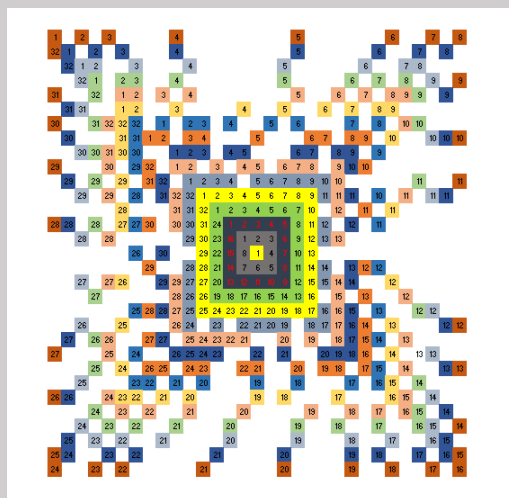
The following is the graph of mass available in successive layers and it is similar to the rotation curve graph of galaxies.



The above scheme of the things is actually based on oversimplified assumption of two dimensional setup of mass. In reality, galactic disk has thickness that is usually more or less or almost 1000 light years. Thus within central bulge, in reality, there should be far greater

increment of available mass than by the factor of just 8 which is being presented in this 2D scheme. Moreover, onward from central bulge, the quantity of mass may get slightly increased layer upon layer i.e. only as much that density of the layer should remain lower than that of previous layer and the net effect may be slightly upward velocity curve which is the case we have seen in the diagram of M33.

However, for the purpose of our analysis, we carry on with the simplistic two dimensional assumption and constant increase of mass for area onward from the central bulge. Following schematic diagram with inner five layers of central bulge with uniform density and outer (only) eleven layers each having mass equal to the outermost layer of central bulge shows that such a structure not only explains the observed flat rotation curves of galaxies, it also develops the spiral structure of galaxies.



Blocks placed in successive layers

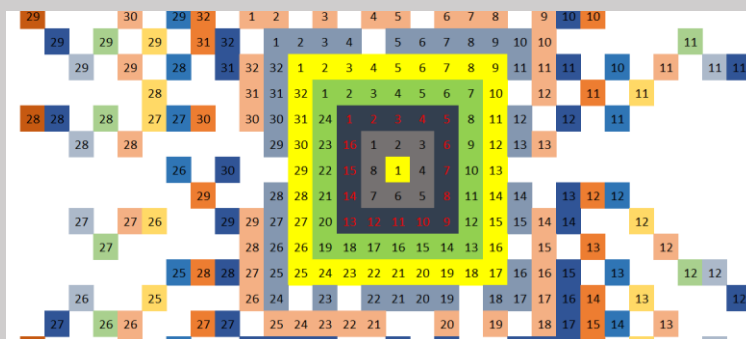
The above diagram is made up of equal size squares or blocks. The central yellow mark is the innermost layer of the central bulge and the other prominent yellow square is the outermost layer of the central bulge such that this layer consists of 32 small blocks which means that outer layer of the central bulge is 32 times massive than the innermost layer. Following is close up view of above diagram up to only the fifth layer and covers the complete central bulge area.

1	2	3	4	5	6	7	8	9
32	1	2	3	4	5	6	7	10
31	24	1	2	3	4	5	8	11
30	23	16	1	2	3	6	9	12
29	22	15	8	1	4	7	10	13
28	21	14	7	6	5	8	11	14
27	20	13	12	11	10	9	12	15
26	19	18	17	16	15	14	13	16
25	24	23	22	21	20	19	18	17

Blocks placed in successive layers – Bulge area close up. Outermost layer is 32 times massive than central layer.

In this schematic diagram, each small square represents equal quantity of mass let's say 1 billion solar masses. If there is mass of 1 billion solar masses in the innermost layer, then second layer contains 8 billion solar masses and overall density remains the same. The fifth layer is the outermost layer of the central bulge.

Following close up shows what would eventually look like spiral structure from a far-view:

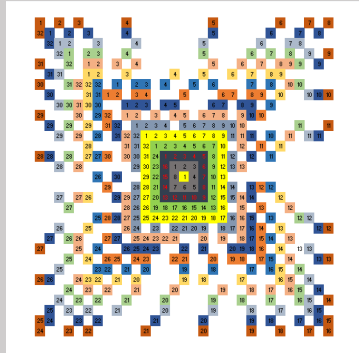


Blocks placed in successive layers – Outside of Bulge area close up

Here we see that outer layer of central bulge had mass of 32 billion solar masses whereas the total mass of the central bulge was $(1+8+16+24+32) = 81$ billion solar masses.

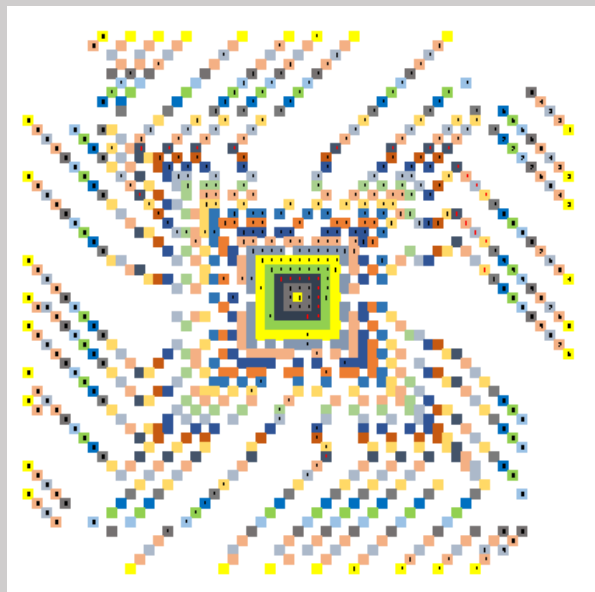
Next to the yellow layer starts the second portion of galaxy whose just eleven layers are shown in the image that starts looking like a spiral galaxy. In this portion, each layer contains 32 billion solar masses. While density remained uniform throughout the central bulge but beyond the central bulge, now mass is constant per layer and density per layer is getting

reduced layer upon layer. A random placement of 32 blocks in each successive layer would give the overall shape of a spiral structure.



Blocks placed in successive layers – total 11 layers after Central Bulge

Note that this schematic diagram is based on square blocks and yet the basic shape of spiral has been achieved. Here, equal number of blocks have been randomly placed in each successive layer of the second portion of galaxy which is outside of the central bulge and the result is a crude or basic shape of galaxy. In a real galaxy, matter is not randomly arranged as the actual shape is determined by the overall scheme of the larger structure as well as quantity and placement of nearby mass or the availability of local structures. After eleventh layer, if we add next layers up to 25th layer by placing the blocks in accordance with the already emerging shape, the following final shape is achieved.



Blocks placed in successive layers – total 25 layers after Central Bulge

The real galaxies are often arranged in spiral shapes such as following.



Pinwheel Galaxy. Image also displayed in cover of this book. Image Credit: NASA/ESA

In real galaxy, there is no empty space between spiral arms. But it does not mean that Spirals are merely illusions. In the schematic diagram, one billion solar masses was represented by just one square box. But in a real galaxy, mass of one billion solar masses is spread out in the form of fog of stars. Secondly, one box actually represents the compacted mass of central bulge area. For the outward area, mass should remain the same layer upon layer but one billion solar masses, being non-compact area, actually takes space of more than one box and this would be the reason why in-between spiral arms areas are not empty for the real galaxies. The in-between spaces of spiral arms are not empty or devoid of matter but however spiral arms are the places where greater mass is concentrated and thus spiral arms are real (i.e. not illusion) and assume their shape due to slightly greater mass but overall reduced density of the successive outer layers of galaxy. Within an actual galaxy, each successive layer may get more than slightly greater mass which seems to be the case with M33 galaxy where flat rotation curve is actually a slightly upward curve. It is also possible that in any galaxy, each successive layer may get slightly reduced mass than the previous layer and dark matter regime 'scientists' may identify such a galaxy as 'dark matter free' galaxy. Scientists do have identified two such galaxies so far but firstly they

have not measured the rotation speed of stars within galaxies rather they have taken the velocity dispersions of globular clusters around them therefore inside of sphere or disk rule does not apply. Secondly, they also assert that these are not the confirmed cases of dark matter free galaxies as with 'latest' observations, they have considerably reduced the distance of those galaxies¹⁸ and have started saying that these are not dark matter free galaxies. Therefore it seems appropriate to not discuss this issue here at length.

As for as mainstream Astrophysics goes, standard interpretation accepts that there seems to be increase of available mass as one moves from inner parts of galaxy towards the outer ones. But within the standard interpretation, the total mass of galaxy is theorized to be concentrated at the center and test particles (stars) are orbiting around the center. Test particles are facing full gravity subject to inverse square distance law while the source of gravity is the central point of galaxy and there is no distinction between inner or outer layers and also it is not deliberated that mass belonging to outer layers has no actual gravitational bearing on this setup and thus, due to non-consideration of important factors, Keplerian drop-off is expected for this system. But since actually observed rotation curve is flat therefore they theorize (or hypothesize) that extra mass, over and above the total mass of galaxy is increasing with increase of distance from the center and to this supposed extra mass they assign the name 'Dark Matter'.

II.II.V. CASE OF DWARF GALAXIES

According to the standard interpretations, observations have shown that dwarf galaxies are rich in dark matter. To the question, "Why are dwarf galaxies dark matter rich?"¹⁹ Mr. Stephen Perrenod²⁰, PhD Astronomy from Harvard, provided the following answer:

Dwarf galaxies are more representative of the first, smaller galaxies to form, as large galaxies represent those that cannibalized their neighbors (other dwarf galaxies) most effectively. Galaxies formed initially as concentrations of dark matter since dark matter is 5 times more abundant by mass, and they also pulled in some ordinary matter.

The first stars formed in dwarf galaxies tended to be quite massive, evolve very rapidly and go supernova, throwing off lots of material (ordinary matter)

into the intragalactic media at high speeds. Some of this would have cooled down and remained in the dwarf galaxy, however...

Since the dwarf galaxies have weaker gravitational fields, it was easier for much of that matter to escape the galaxies in question and such ordinary matter might still be in intergalactic space, or have been pulled into a larger galaxy.

This meant that dwarf galaxies have been less efficient at holding onto intragalactic gas that can be used for formation of subsequent generations of stars, and that helps to explain their low luminosities.

There may be additional reasons, but this is one generally favored scenario, a lot of research is going into detecting more dwarf galaxies and modeling their evolution.

Mr. Stephen Perrenod is a staunch supporter of the standard Lambda CDM model. We see that standard interpretations are based on false confidence that birth as well as conditions, shape, form and state of evolution of the early Universe are exactly known. They know such astounding things as initially galaxies formed around clumps or concentrations of dark matter and those early galaxies were dwarf galaxies which later on merged to form larger galaxies. The standard interpretations, we see, are coming from outside of the world. In the real world, the existence of dark matter is not even confirmed; there is no proof of dark matter in large galaxies and richness of dark matter in dwarf galaxies is also like a conceptual illusion. We have seen that within central bulge area of large galaxies, there are sharp upward rotation curves. Therefore, based on sharp upward rotation curves, within standard interpretations, there should be high concentration of dark matter within central bulge areas of larger galaxies as well. In other words, central bulge areas of large spiral galaxies should be rich in dark matter. Simple fact however is, that there is just layer upon layer substantial increase in available mass such that overall density across the whole area of the central bulge remains the same.

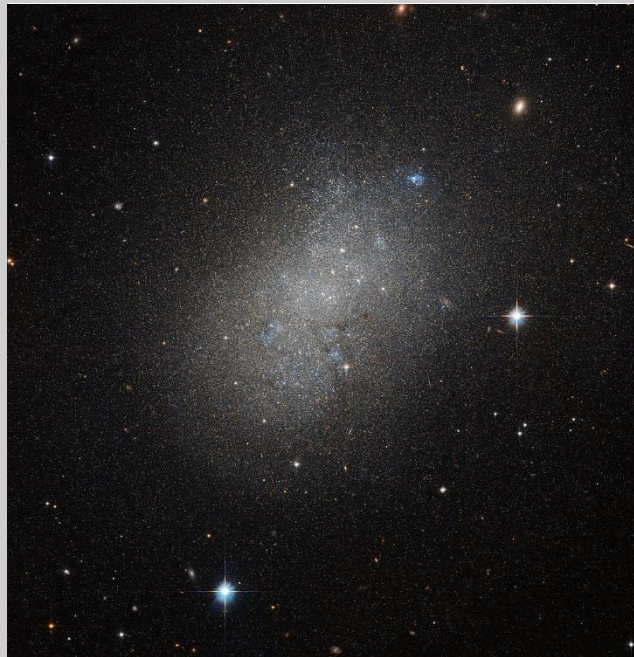
Actually almost same is the case with dwarf galaxies. Some of the dwarf galaxies seem like remnants of large galaxies where spiral arm area seem to have been disbursed due to greater gravitational influence by nearby large galaxy which appear to be the case with Large Magellanic Cloud. For the case of some other dwarf galaxies, it appears that only

central bulge area was formed that was not powerful enough to attract outer spiral layers of stars or star forming gas which is likely the case with Small Magellanic Cloud.



Large and Small Magellanic Clouds over Paranal Observatory. Image Credit: ESO²¹

Following further examples show that dwarf galaxies usually only lack high luminosity and great density however they are similar in structure and overall uniformity in density with central bulge areas of the large spiral galaxies.



NGC 5264 – dwarf galaxy. Image Credit: NASA/ESA



A dwarf galaxy. Image Credit: NASA/ESA

The typical structure of dwarf galaxies is only telling that mass is considerably increasing for the outer layers but not increasing as much to give perfect uniform density and luminosity for the whole of the structure. This structure will give slightly less sharp upward rotation curve than the central bulge areas of large spiral galaxies and thus for the standard model supporters, it will be the case of greater quantity of dark matter.

II.II.VI. IS DARK MATTER THE FAILURE OF THEORY?

We conclude that Newton's Theory, subject to correct application, would have rightly described the rotation pattern of galaxies. Accurate theory already existed but problem of rotation curves of galaxies was never interpreted in the light of relevant part of the available theory. By 1920, when on the basis of famous 1919 solar eclipse experiment, Arthur Eddington and co-authors wrote in their paper that Einstein's General Theory of Relativity was found superior theory of gravity to Newton's theory, at that point in time, Relativity Theory did not even have Shell Theorem. Relativistic Shell Theorem was presented in year 1923 or as early as 1921²². Yes – it should mean that relativistic shell theorem was available at the time when scientists were dealing with the problem of dark matter. But it seems like the Birkhoff's Theorem i.e. the Relativistic Shell Theorem does not consider the specific case of gravity field experienced by a test particle which is placed inside a sphere having uniform density which mean that till date relativistic counterpart of Newton's Theorem XXXIII does not exist. But overall implication of this Birkhoff's Theorem is that general relativity reduces to Newtonian gravitation in the Newtonian limit²³.

The problem of rotation curves was within the Newtonian limit and the theory to be applied was Newton's Theory thus we can accept that, in principle, theory was complete; rotation patterns could have been rightfully interpreted without invoking the need of dark matter. But – it did not happen; rotation curves were not rightfully interpreted. Theoretical Physicists did apply Newton's theory but missed an important aspect i.e. Theorem XXXIII of the theory. Instead, they applied irrelevant Theorem XXXI. The wrong application of theory was dubbed as incredible discovery of 'dark matter' which was basically a ghost object; an unprovable

hypothesis that was also found out to be seemingly supportive of few other unprovable conjectures relating to the Big Bang Cosmology and credit of those farfetched findings was assigned to the 'more accurate' theory of General Relativity. In this way, Theoretical Physicists extended the wrong application of (Newton's) simple theory to their so-called 'precise' theory (GR) without realizing that they merely interpolated the results of incorrect application of simple theory to their 'precise' theory and this thing casts serious doubt on their claim that they do understand their counterintuitive theories.

Dark Matter was thus not the failure of theory. Precisely, it was the failure of correct application of the theory whereas the theory itself was capable for the task. What happened was like that while first time noting the rotational pattern of galaxies, scientists were naturally anticipating Keplerian drop-off in the rotation curves because by that time, it was the only observed pattern. But deviation of actual finding from the expectations did not spark the willingness to review the dynamical considerations even though Babcock (1939) had pointed out the need for the same. Scientists focused their attention towards getting better accuracy of observed data regarding rotation of galaxies but no one questioned in official papers concerning why Keplerian drop-off should be expected at all when galaxy is a whole different structure than solar system. Experimental Scientists were doing their job well as their task was really to gather correct observational data. But Theoretical Physicists were not using their commonsense because commonsense is a despised thing which they officially do not use. At least they should have seriously reviewed the relevancy of Keplerian drop-off for the dynamics of the galaxy.

Experimental scientists were doing their job well and they were presenting their findings along with judgments regarding what they had observed. In 1939, Horace Babcock reported in his PhD thesis that measurements of the rotation curve for Andromeda suggested that the mass-to-luminosity ratio increased radially²⁴. Yes – it was accurate judgment because at least gravitational mass does increase radially in terms of Theorem XXXIII. Babcock was accurate also because he pointed out that new dynamical considerations were required; a right proposal that was not taken seriously. Off course, whole new theory was not required; only requisite thing was to get rid of the Keplerial drop-off anticipations and to reach to the relevant Theorem XXXIII of the already available theory. Likewise, following quote out of

Wikipedia article titled “Galaxy Rotation Curve”²⁵ also informs that MS Vera Rubin (1970) not only reported her observations but also came up with accurate judgment that observations had the implication that galaxy masses grow approximately linearly with radius well beyond the location of most of the stars.

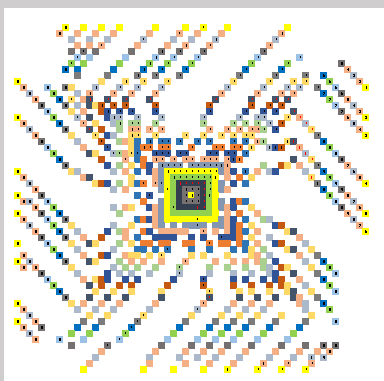
In the late 1960s and early 1970s, Vera Rubin, an astronomer at the Department of Terrestrial Magnetism at the Carnegie Institution of Washington, worked with a new sensitive spectrograph that could measure the velocity curve of edge-on spiral galaxies to a greater degree of accuracy than had ever before been achieved.[13] Together with fellow staff-member Kent Ford, Rubin announced at a 1975 meeting of the American Astronomical Society the discovery that most stars in spiral galaxies orbit at roughly the same speed,[14]and that this implied that galaxy masses grow approximately linearly with radius well beyond the location of most of the stars (the galactic bulge). Rubin presented her results in an influential paper in 1980.[15] These results suggested that either Newtonian gravity does not apply universally or that, conservatively, upwards of 50% of the mass of galaxies was contained in the relatively dark galactic halo. Although initially met with skepticism, Rubin's results have been confirmed over the subsequent decades.[16]

Here we note that MS Vera Rubin said in year 1970 that **galaxy masses grow approximately linearly with radius well beyond the location of most of the stars.**

We know that according to Theorem XXXIII, a test particle placed at a particular depth within a sphere of uniform density will not be gravitationally affected by the outer layers of the sphere (or disk). It means that ‘gravitational mass’ of outer layers can be regarded as non-existent. Now suppose that test particle was placed at the edge of the galactic bulge and then starts moving towards outer area of the disk. This movement towards outer surface will cause regular ‘growth’ in the gravitational mass which according to MS Rubin, will be approximately linear with increase in radius. And yes, MS Rubin was talking about regular linear growth in mass over and above the total luminous mass that, for the purpose of determining the influence of gravity, was already theorized to be located at center. Theorem XXXIII, on the other hand, have the implication of regular linear growth in gravitational mass such that at every depth, the available (gravitational) mass is exerting full gravity from the center. This gravitational mass is not over and above the luminous (observable) mass. One thing Experimental Scientists missed was that they only radially determined the luminosity

of disk. Yes radially the luminosity decreases over large distances but great distance with low (per unit distance) luminosity when projected in complete circumference of the outer belt, band or layer then 'total' luminosity also should remain the same layer upon layer just like total mass also remains the same layer upon layer. For example, Roberts and Whitehurst (1975)²⁶ also concluded the same that mass increases linearly towards the outer edge of the M31 galaxy. They had studied southern end of M31 and observed rotation and luminosity, off course, relating to only that southern end and observed, for that part of the galaxy, that luminosity decreases with no decrease of rotational velocity. The Astrophysical Journal (Aug:2011) has published a paper titled "The Luminosity Profile and Structural Parameters of The Andromeda Galaxy"²⁷. This paper presents bell shaped graphs of luminosity of Andromeda as recorded along major and minor axis of the disk. Thus luminosity is decreasing only along the line of diameter and so far there is no realization that total luminosity of the outer bands or layers should be almost equal to total luminosity of inner bands or layers. Therefore, there may actually be no increase of mass to luminosity ratio taking place for the outer parts of galactic disks.

Now we can recall our schematic diagram where mass increased linearly with radius well beyond the central bulge.



Outer edge of central bulge have 32 square boxes (representing mass). Each succeeding outer layer also has 32 boxes which means that mass is increasing linearly with radius i.e. exact wording of MS Vera Rubin.

This schematic diagram is based on idea that in accordance with Theorem XXXIII, after central bulge, mass should linearly increase so as to give flat rotation curve like graph. The outer layer of the central bulge consists of 32 equal size boxes. Now onward mass should increase linearly therefore each succeeding layer also consists of exact 32 boxes. By

random placement of boxes in succeeding outer layers up to 11th layer (after bulge), the basic shape of spiral started to emerge. Rest of the layers, up to 25th, were arranged by placing the boxes in accordance with already emerging shape of spiral.

Here basic spiral shape was achieved but actual spirals of real galaxies are denser and in-between spiral areas are also not empty. Therefore, in real galaxies, mass increases more than linearly and 'flat rotation curves' may actually be slightly upward curves throughout most of the disk as we see in the case of M33 which seems to be usual case and these curves are accomplished due to offsetting caused by the **inverse distance (from center)** law of gravity as applicable within the sphere (or disk). The galaxy rotation is actually an excellent confirmation of the astonishing accuracy of Newton's Theory. Here we are dealing with the inside of sphere or disk scenario and if we wrongfully consider inverse **square** distance law, we shall get Keplerian drop-off even though gravitational mass grows linearly. The flat or slightly higher rotation curves and usual spiral structures of galaxies are in great harmony with Theorem XXXIII of Newton's Principia.

'Dark Matter' is thus not the failure of the Theory but can be regarded as failure of counterintuitive regime. It is failure of overrated understanding level of the theory and it is the failure of the idea that counterintuitive ideas are correct and are actually understood when they, intrinsically being 'counterintuitive' were not actually comprehensible. Failure was in the unscientific method that assigns reality status to ghost objects. For example, following paragraph from Wikipedia article titled 'Dark Matter' shows that they do not treat this ghost object just as a placeholder only to denote a shortage of proper explanation but they take it for a real object that cannot be traced in the real world:

Dark matter is a form of matter thought to account for approximately 85% of the matter in the universe and about a quarter of its total energy density. The majority of dark matter is thought to be non-baryonic in nature, possibly being composed of some as-yet undiscovered subatomic particles.[a] Its presence is implied in a variety of astrophysical observations, including gravitational effects which cannot be explained by accepted theories of gravity unless more matter is present than can be seen. For this reason, most experts think dark matter to be abundant in the universe and to have had a strong influence on its structure and evolution. Dark matter is called dark because it does not appear to interact with observable electromagnetic radiation, such as light, and is thus invisible to

the entire electromagnetic spectrum, making it undetectable using existing astronomical instruments.[1]

¹ Edwin Hubble “The Realm of the Nebulae”, (1936) P:179

² Gainfranco Berton and Dan Hooper “A History of Dark Matter” – Page 19 (<https://arxiv.org/pdf/1605.04909.pdf>)

³ SIDNEY VAN DEN BERGH “The Early History of Dark Matter” – Page 4-5 (<https://arxiv.org/pdf/astro-ph/9904251.pdf>)

⁴ Gainfranco Berton and Dan Hooper “A History of Dark Matter” – Page 21 (<https://arxiv.org/pdf/1605.04909.pdf>)

⁵ https://www.windows2universe.org/the_universe/uts/kepler3.html

⁶ <https://www.universetoday.com/41021/keplers-third-law/>

⁷ <https://www.quora.com/profile/Erik-Anson>

⁸ <https://www.quora.com/How-does-gravity-work-at-galactic-level-Am-I-wrong-to-feel-that-Kepler-s-3rd-law-is-totally-irrelevant-in-such-complex-system-and-therefore-that-there-is-maybe-nothing-wrong-with-the-way-galaxies-are-orbiting-and>

⁹ <https://www.quora.com/profile/Damien-Giraud-1/questions>

¹⁰ <https://www.quora.com/profile/Nikolay-Sones>

¹¹ <https://www.quora.com/If-we-have-Newtons-shell-theorem-then-why-do-we-need-dark-matter-to-explain-why-galaxies-stay-together>

¹² https://en.wikipedia.org/wiki/Dark_matter

¹³ <https://www.quora.com/What-is-a-duplicate-ratio/answer/Carl-Bryan>

¹⁴ https://en.wikipedia.org/wiki/Shell_theorem

¹⁵ <https://physics.stackexchange.com/questions/18446/how-does-gravity-work-underground?noredirect=1&lq=1>

¹⁶ https://en.wikipedia.org/wiki/Gravity_of_Earth#Depth

¹⁷

[https://en.wikipedia.org/wiki/Galaxy_rotation_curve#/media/File:Rotation_curve_of_spiral_galaxy_Messier_33 \(Triton\).png](https://en.wikipedia.org/wiki/Galaxy_rotation_curve#/media/File:Rotation_curve_of_spiral_galaxy_Messier_33_(Triton).png)

¹⁸ https://en.wikipedia.org/wiki/NGC_1052-DF2 --- “A more recent study on NGC 1052-DF2 suggests the previously reported distance of the galaxy was greatly exaggerated. Consequently, the galaxy now looks “normal” in every way. Using five independent methods to estimate distances of heavenly bodies, a team of researchers from the Instituto de Astrofísica de Canarias (IAC) found the correct distance of NGC 1052-DF2 to be 42 million light years (13 Mpc), not some 64 million light years (19 Mpc) from the Earth.^[10] The total mass of the galaxy is around one-half of the mass estimated previously, but the mass

of its stars is only about one-quarter of the previously estimated mass. This implies a significant part of NGC 1052-DF2 could be made up of dark matter, like any other galaxies.”

¹⁹ <https://www.quora.com/Why-are-dwarf-galaxies-dark-matter-rich>

²⁰ <https://www.quora.com/profile/Stephen-Perrenod>

²¹ <https://www.eso.org/public/images/potw1511a/>

²² [https://en.wikipedia.org/wiki/Birkhoff%27s_theorem_\(relativity\)](https://en.wikipedia.org/wiki/Birkhoff%27s_theorem_(relativity)) – This Wikipedia article about Relativistic Shell Theorem says following:

“The theorem was proven in 1923 by [G. D. Birkhoff](#) (author of another famous *Birkhoff theorem*, the *pointwise ergodic theorem* which lies at the foundation of [ergodic theory](#)). However, [Stanley Deser](#) recently pointed out that it was published two years earlier by a little-known Norwegian physicist, [Jørg Tofte Jebsen](#).”

²³ [https://en.wikipedia.org/wiki/Birkhoff%27s_theorem_\(relativity\)](https://en.wikipedia.org/wiki/Birkhoff%27s_theorem_(relativity))

“Thus, this part of the theorem is just what we would expect from the fact that general relativity reduces to [Newtonian gravitation](#) in the [Newtonian limit](#).”

²⁴ Babcock, H. W. (1939). "The rotation of the Andromeda Nebula".

²⁵ https://en.wikipedia.org/wiki/Galaxy_rotation_curve

²⁶ <http://adsabs.harvard.edu/full/1975ApJ...201..327R>

²⁷ <https://iopscience.iop.org/article/10.1088/0004-637X/739/1/20/meta>