The equation of canonical quantum cosmology

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Modern quantum cosmology has difficulty formulating the Schrödinger equation for the entire universe. Wheeler's equation has no workaround. In this paper, we use the full formulation of the equation for the wave function of the Universe. For this, the Schwinger equation is used for the evolution of the wave function through the hyper-surface of space-time.

Consider the time equation of Schrödinger.

$$ih\frac{\partial \Psi}{\partial t} = E\,\Psi$$

The Schwinger equation generalizes the evolution of the wave function through the hyper-surface of space-time. Where the Hamiltonian is the energy density operator.

$$ih\frac{\partial \Psi(V,t)}{\partial \Omega} = \frac{dE}{dV}\,\Psi(V,t)$$

$$d\Omega = dt\,dV$$

This equation is incomplete. It is necessary to find the energy density operator through derivatives with respect to the volume of space.

You can use dimensional analysis for this. For example, in a system of units, multiplying energy by the volume of space gives a combination of fundamental constants (G, h, c).

$$[E][V] = \left[\frac{Gh^2}{c^2}\right]$$

For the energy density and squared volume it turns out

$$\left[\frac{E}{V}\right][V^2] = \left[\frac{Gh^2}{c^2}\right]$$

Using this last relation, we can assume that the energy density and the square of the volume of space establish a new uncertainty relation.

$$\Delta\left(\frac{E}{V}\right)(\Delta V)^2 \geq \frac{Gh^2}{c^2}$$

Hence one can define the operator of the energy density through derivatives with respect to the volume of space.

$$\frac{Gh^2}{c^2}\frac{\partial^2 \Psi}{\partial V^2} + \frac{dE}{dV}\,\Psi = 0$$

This allows us to formulate a general equation for the evolution of the wave function through the hyper-surface of space-time. Where the wave function itself is a function of the volume of space and time.

$$ih\frac{\partial\Psi}{\partial\Omega} = -\frac{Gh^2}{c^2}\frac{\partial^2\Psi}{\partial V^2}$$

This is a canonical equation in the form similar to the Schrödinger equation. Most likely, this equation will be applied in quantum cosmology, taking into account the definition of boundary conditions and the form of potential energy density.

In the time part, the equation takes the integral form.

$$ih\frac{\partial\Psi}{\partial t} = -\frac{Gh^2}{c^2}\int\frac{\partial^2\Psi}{\partial V^2}\,dV$$

Obviously, the full energy of the universe can definitely be in the formulation of the question for an open or closed universe.

In this paper, we studied the problem of applying the Schrödinger equation for the entire Universe. In fact, the evolution of the wave function of the Universe takes place even for the time coordinate. It is important that what form of the Hamiltonian operator can be chosen to formulate the Schrödinger equation of the entire Universe.

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