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Comparative Evaluation of Virtual and Augmented Reality for Teaching Mathematics in Primary Education

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Keywords Virtual Reality · Augmented Reality · Mathematics · Geometric Solids

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Abstract Primary school students often find it difficult to understand the differences between two dimensional and three-dimensional geometric shapes. Taking advantage of the ability of virtual and augmented reality to visualize 3D objects, we investigate the potential of using virtual and augmented reality technologies for teaching the lesson of geometric solids to primary school children. As part of the study 30 fourth, fifth and sixth class primary school students were divided into three groups that include a control group and two experimental groups. The first and second experimental groups used dedicated virtual and augmented reality applications to learn about geometric solids, while students from the control group used traditional printed material as part of the learning process. The results indicate that the implementation of new technologies in education of virtual and augmented reality improve interactivity and student interest in mathematics education, contributing to more efficient learning and understanding of mathematical concepts when compared to traditional teaching methods. No significant difference was found between virtual and augmented reality technologies with regards to the efficiency of the methods that contribute to the learning of mathematics, suggesting that both virtual and augmented reality display similar potential for educational activities in Mathematics.

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1 Introduction

The rapid pace of technological progress has a significant impact on education leading to its transformation and modernization. The role of computing and ICT

[1] Institution 1
[2] Institution 2

in today's education is becoming more diverse, since new technological methods can be used within the classroom as cognitive-exploratory tools, as a means of searching and collecting information, and as a means for communication and interaction among the students. Furthermore computer-based visualizations can be very important in different disciplines as they can be used for overcoming limitations of traditional teaching practices.

In terms of mathematics and geometry, many scientists argue that students often fail to make the connection between objects in a real-life three-dimensional world, with those of the two-dimensional space, and as a result, they have difficulties in distinguishing geometric solids from flat shapes [11]. Modern technologies such as virtual and augmented reality can provide a solution to the above problem. Kaufmann and Schmalstieg [16], suggest that the use of Virtual Reality (VR) allows the students to enter the virtual world, actively participate and interact with the various objects enhancing their spatial abilities. Similarly, via using Augmented Reality (AR), the user is not totally disconnected from reality, but the user can add or remove objects from the real world, facilitating in that way the process of learning about shapes or figures [11].

Nevertheless, there is a lack of research concerning the systematic development of virtual and augmented reality applications for practical training purposes and enhancement of students' spatial abilities in the field of mathematics and particularly in geometry. To address the gap in the interface between virtual and augmented reality with mathematics and geometry training, specialists have developed three-dimensional geometric tools, like Construct3D, and Handwaver. Construct3D uses augmented reality to provide a natural environment to enhance the collaboration between teachers and students. Students using this tool can observe real three-dimensional objects, which until recently had to be computed and designed using traditional pen-based methods [16]. The HandWaver environment, on the other hand, is a gesture-based mathematical construction environment that uses virtual reality technologies to offer a dynamic geometry experience, allowing the users to design, modify, measure and explore mathematical objects within a VR space using repetitions of gestures [7]. In considering the aforementioned studies it is important to note that they mainly focus on the use of either VR or AR technologies. Thus, there is a lack of evidence in the literature regarding a comparison between the two technologies.

This paper aims to present a comparison between the use of VR and AR technologies in the field of mathematics and more specifically in the delivery of a course on solids (Cube, Sphere, Cylinder, Cone, Pyramid, Rectangle) in primary education. Another objective of the research is to provide a more accurate picture of the extent to which AR and VR technologies contribute to the learning of mathematics. To the best of our knowledge there are no previous cases in the literature describing the comparative evaluation of VR and AR technologies in education, and more specifically in the field of mathematics for primary school children. For this reason, the findings of our research can provide valuable feedback to educators and developers who plan to introduce or develop VR or AR technologies for educational activities.

Previous efforts in comparing VR and AR technologies include the work of Krichenbauer et al.[20] who investigated user performance for 3D object manipulation. Dedicated VR or AR headsets were used for the visualization and a 3D input device and a mouse was used for the interaction. The results of a user evaluation revealed that users performed the object manipulation task faster in the AR rather than the VR environment, but no significant differences in the user comfort were detected. Unlike the work of Krichenbauer et al.[20], in our work, the VR and AR systems use simple mobile phone-based or tablet based equipment rather than dedicated headsets. Furthermore, the key factor investigated in our experiments is the learning outcome and user engagement rather than response time for object manipulation.

Recently Huang et al.[13] compared the influence of mobile-phone-based VR and AR in educational activities. As part of the experimental evaluation, volunteers had the chance to use two mobile applications, namely the Solar System and Space Museum on a smartphone using either VR or AR. Participants were provided with questionnaires before (pre-test) and after the test (post-test) to measure factors such as user attention, presence, enjoyment, science knowledge, auditory knowledge, and visual knowledge. The most important conclusion from this study is that VR is more suited for visual content whereas AR for auditory content. While this study is closely related to our work, in our study we focus our attention on delivering a course in solids for primary school pupils rather than using general knowledge applications. Furthermore, unlike our work that considers primary school children, Huang et al. [13] focuses their attention on college students.

In the remainder of the paper we present a literature review on the topics of using VR and AR for mathematics training and present the methodology adopted in our study. In Section 4 we present the results of the experimental evaluation followed by a discussion, plans for future work and concluding comments.

2 Literature Review

In this section, a brief literature review of the use of VR and AR in Mathematics education is presented. As part of the discussion, specific tools used for educational activities in Mathematics are discussed.

2.1 Virtual and augmented reality in mathematics

Virtual Reality (VR) is defined as "an interactive three-dimensional environment that is created on the computer and in which the user can be immersed" [6]. Latest developments in VR technology resulted in low-cost hardware and software tools [5] that enable the application of VR applications in different application domains, including educational applications [10].

Several research studies indicate that VR technologies seem to affect the academic performance and motivation of students positively [14,22,23,31]. Additionally, Hampel & Dancshzy [12] state that creating a virtual learning environment can be particularly useful for students, as students acquire the ability to "conquer" knowledge on their own. Furthermore, there are indications that VR technologies promote students' communicative and collaborative skills as well as their psychomotor and cognitive skills [39,16,23] while VR technologies can also be utilized for teacher training [32].

Apart from VR, Augmented Reality (AR) also has the ability to provide users an interactive experience, by adding virtual information to the user's physical environment and allowing the user to use his/her entire body to interact with both real and virtual content [1]. Moreover, experts point out that there are many potential benefits that AR can offer in children's lives, while at the same time, the use of AR in the field of education has been linked to specific learning benefits [27]. Chen et al. [4] conducted a review of the use of AR in education from 2011 to 2016, considering several factors, uses, benefits, characteristics, and effectiveness of AR in educational environments. The main outcome of their research was that scientific research on AR applications increased significantly from 2013 onwards. According to Torok et al. [34], augmented reality in the future will become a very important educational tool, allowing the students to understand the facts better through illustrative demonstrations, which are invisible to everyday life.

In the field of mathematics, experts point out that imaging and projection have significant benefits for the students in terms of understanding mathematical concepts suggesting that it is necessary to change introduce contemporary visualization methods in the teaching material for those subjects [2]. Siegler & Ramani [29] emphasized the importance for students to acquire numerical knowledge through linear representations and visual aids and stimuli. Yeh [38] states that a VR learning environment called VRMath2 has been developed to link mathematical thinking to programming in virtual reality microworlds. Additionally, the use of AR applications in teaching mathematics provide the opportunity to enhance students motivation and reinforce both applied and theoretical concepts in mathematics [9]. Furthermore, it has been reported in the literature that the use of AR can result in better cooperative teamwork and better problem-solving abilities [30].

2.2 Virtual and augmented reality in geometry

In the field of geometry, little research has been conducted towards the systematic development of virtual reality applications for practical training purposes and enhancement of pupils spatial abilities. "Construct3D" is a three dimensional geometric construction tool designed for mathematical and geometric education for the fields of geometry research, pedagogy, psychology and augmented reality [16]. The goal of Construct3D is to create a natural environment that aims to promote the collaboration between teachers and students, improve student's spatial competencies and maximize the process of learning and the transmission of knowledge [16]. Additionally, another interesting tool is the "SketchUp", that utilizes augmented reality to provide a new way of presenting 3D models that can be uploaded directly to the web within the same program and stored directly in the database [11].

“CyberMath”, is another virtual educational environment, developed for interactive mathematical exploration, consisting of four large exhibition halls, each of which contains a collection of mathematical constructions expressing a common theme. It is worth noting that students using “CyberMath” and the teacher are in different physical locations introducing the idea of a telepresence system in virtual educational environments [19].

“VRMath” is an interactive VR/AR educational application that helps students understand, through the use of virtual and augmented reality technologies, various fields of mathematics, such as 3D geometry, graphics and vectors. “VRMath” offers students the ability to move within the specific environment, manipulate objects, and build programs for the creation of objects in a three-dimensional(3D) environment, through the use of Logo programming language, but also with the Internet so as to facilitate cooperative learning of 3D geometric concepts and processes [37]. Finally, the “HandWaver ” application is a mathematical construction environment based on gestures that make use of virtual reality technologies allowing users to construct mathematical objects of one, two or three dimensions, by a repetition of gestures. The goal of “HandWaver” was to exploit the modes of representation and interaction that are available in virtual environments and can be used to create experiences where trainees use their hands to construct and modify mathematical objects [7].

3 Methodology

The goal of the current research is to compare the use of VR and AR technologies against traditional teaching methods, in the field of mathematics and more specifically for teaching geometric solids (Cube, Sphere, Cylinder, Cone, Pyramid, Rectangle) to primary school pupils. The research hypothesis under investigation are:

(i) **Virtual and augmented reality applications can make the teaching of mathematics more interactive and interesting and can contribute to more efficient learning and understanding of mathematical concepts.**

(ii) **The implementation of virtual reality technologies for teaching mathematics is more effective compared to augmented reality technologies.**

As part of the experimental evaluation teaching material related to the delivery of a lecture in geometrical solids was developed using traditional printed material, VR and AR technologies. Students had the chance to try these methods, and a questionnaire-based method was used to determine the merits of each approach. All the steps of the methodology adopted are presented in the remainder of this section.

3.1 Sample

The size of the sample for the main research consisted of thirty primary education pupils aged 9 to 11 years. The sample was divided equally into the Control, VR and AR groups. Pupils in the three groups used traditional, VRbased and AR-based teaching material respectively.

Within the sample, there was an absolute balance of the gender and age distribution with fifteen boys and fifteen girls, and equal distribution in the ages of 9, 10 and 11 years. Concerning ethics, students participating in the research, as well as their parents were informed regarding the purpose of the research both orally and in written form to obtain their informed consent. Moreover, the participants were given assurances of confidentiality and anonymity with regards to their personal data, while they were informed that they could withdraw at any time without providing explanations.

3.2 Teaching Materials

For the design of the applications, a lesson plan was initially created, that comprised of three activities:

(a) Classification of shapes into solid or plane shapes.
(b) Identification of solid shapes appearing in a typical city environment.
(c) Identification of solid shapes.

For the purpose of this study, the lesson plan was implemented based on printed material (traditional method), three related virtual reality and three augmented reality applications. The lesson plan used in the experiment and the relevant teaching material were designed by considering the actual curriculum and the advice of active mathematics educators.

The worksheets of the control group and the images that were used in the applications of the augmented reality had been created through the software Adobe Illustrator, and the actual applications were developed using the software ENTiTi Creator 2.805. The solid and plane shapes were created with the software Autodesk Maya, whereas the rest of the models used were downloaded from Unity Asset store and the websites turbosquid.com, cgrader.com, and free3d.com. Sound information was recorded through the programme icecream screen-recorder and processed using Adobe Premiere.

The developed VR and AR applications for the current research do not require specialized equipment. For the AR applications, the users only need to use their mobile device or tablet that provides them the ability to use the applications developed anywhere and anytime. The same applies to the VR applications since the equipment that is needed is limited only to a mobile phone and cheap virtual reality glasses suitable for mobile devices. Hence, one of the benefits of the designed applications is that they can be integrated within the classroom environment but also can be used by the students at home. In the remainder of the paper, a description of the teaching material is presented.

3.2.1 Teaching Materials used by the Control Group

The worksheets of the control group were created using Adobe Illustrator software. Sketches and shapes from the eBook of the fourth class of Primary School were used to coincide with the curriculum. The first worksheet provides information about the names of any types of the solids in question while the second worksheet relates to the localization of solids in an urban environment. The participants of the control group had 6 minutes to read the two pages about solids, and then, they had to hand them into the researcher (see figure 1).

Fig. 1 Worksheets that were given in the control group. (a) Naming of the geometric solids. (b) Localization of solids in an urban environment.

3.2.2 Teaching Materials used by the VR Group

Subjects from the first experimental group used the following three first-person VR applications:

First VR Activity - Identifying three-dimensional shapes: The user is placed in a childs bedroom, where toys depicting three-dimensional and two-dimensional shapes are located on the floor (see figure 2). The user is asked to recognize which of the toys correspond to geometrical solids. The student can choose a shape by focusing his gaze on it, and when he/she selects a solid, it disappears. The activity ends, when the pupil selects all the solid items. In case the pupil selects a 2-dimensional shape an error message comes up asking the student to retry.

Fig. 2 Environment of first activity in Virtual Reality.

Second VR Activity - Identifying three-dimensional shapes in a city: The user is asked to identify objects in a city view that resemble geometrical solids (see figure 3). For example, the user selects bushes, cypress trees, the block of flats/truck trailers that resemble spheres, cones, and rectangular parallelepipeds respectively. During the process, audible and visual information provides guidance and rewards users for correct identification. The ultimate aim of this application is to allow pupils to realize that solid geometrical objects are frequently encountered in their daily life.

Fig. 3 Environment of second activity in Virtual Reality.

Third VR Activity - Recognizing geometrical Solids: The third activity allows users to practice in identifying the name of different solids. In this activity, the names of the solids appear as text labels, and in every twenty seconds, a solid appears in the scene. Using gaze tracking the user must choose the correct name for the given solid (see figure 4). Appropriate feedback for correct/wrong responses is given to the user.

Fig. 4 Environment of third activity in Virtual Reality in which the solid of the cylinder is demonstrated.

3.2.3 Teaching Materials used by the AR Group

The second experimental group participated in three activities with the use of AR technology. These applications are simple in use, demanding only the use of a personal tablet/mobile and specific images that need to be printed and serve as targets on which the virtual content is displayed when the targets are detected in the field of view of the camera. The functionality of the three AR applications is similar to the corresponding VR applications after the necessary adjustments are made in the interaction mode. In summary, the three AR applications are:

First AR Activity - Identifying three-dimensional shapes: The user is asked to identify items with solid shapes among a group of two- and three-dimensional objects (see figure 5). Within this context, objects are selected by touching the appropriate area occupied by a shape on the screen of the mobile device. Correctly identified shapes disappear, and the activity carries on until all solids are identified. In case the participant has not selected a solid, then he/she has the possibility to retry.

Fig. 5 Environment of first activity in Augmented Reality.

Second AR Activity - Identifying three-dimensional shapes in a city: The user is asked to select, by touching the touch screen, objects in the city view that resemble geometrical solids (see Figure 6). Once all solids are located, the activity is completed.

Fig. 6 Environment of second activity in Augmented Reality.

Third AR Activity - Recognizing geometrical Solids: The user is asked to match a random solid that appears in the view, with a label showing the name of the solid, among a set of labels depicting names of standard geometrical shapes (see figure 7). The user is rewarded for correct answers and gets feedback for wrong answers.

Fig. 7 Environment of third activity in Augmented Reality in which the solid, sphere is demonstrated.

3.3 Data Collection Instruments

Questionnaires were used for collecting data, because it is easy to design and use [21], while they provide the opportunity to record in detail the views of the participants [15]. Two questionnaires were given to the participants:

The first questionnaire was administered to all experimental groups at the beginning of the experiment. It consisted of three parts, with mainly closed-ended questions. The first part concerned mainly demographic data (country, gender, age, and class). The second part, in the VR and AR groups, referred to the experience and knowledge of the participants regarding computers, internet, and virtual/augmented reality. The second part of the questionnaire was different for the control group, as no questions regarding the participants experience and knowledge concerning VR and AR were included. Moreover, the second part, consisted of 5 - Likert scale questions (1 - not at all, 5 - very much, 1 definitely not - 5 definitely yes) and multiple-choice questions. The third part of the questionnaire consisted of questions related to participants' knowledge regarding geometrical solids. In this part, there were three exercises in which the pupils were called to circle, match or even write the correct answer.

The second questionnaire was given to the VR and AR groups, after the use of the VR and AR applications. For the control group, the second questionnaire was administered after the pupils had the chance to go through learning material for solids using traditionally printed worksheets. The second questionnaire constituted of two sections with both open and closed-ended questions. The first part consisted of questions regarding the participants knowledge for geometrical solids. This questionnaire was identical to the one administered at the beginning of the experiment, so that it was possible to register possible improvements in the knowledge of students in relation to geometrical solids. The second part was different for the participants of the three groups and consisted of questions regarding the evaluation of the VR/AR experience that was administered to the VR and AR groups and questions related to the evaluation of the printed material administered to the control group. Additionally, the second part of the questionnaire consisted of 5-Likert scale questions (1 not at all, 5 - very much, 1 definitely not - 5 definitely yes), as well as multiple choice questions. Furthermore, at the end of part one, there was an open-ended question, which was included to allow and encourage the participants to express their feelings and understanding, providing significant feedback and data regarding their experience. The reliability of the questionnaires has been tested with the Cronbach's alpha reliability index and, according to the results the index varied between 0.739 ≤ a ≤ 0.792. These values prove the reliability of the questionnaires used in the experiment [24].

During the experiment, pupils answered the first questionnaire, and then they were given a chance to learn information about geometric solids using a book-based approach or VR applications or AR applications respectively. Pupils who participated in the control group had six minutes to read the worksheets about solids before they return the worksheets. The pupils from the AR and VR groups used for about six minutes the dedicated AR and VR applications developed for this study (see Figure 8). Once the learning activities were completed pupils completed the second questionnaire.

Fig. 8 Our study compared VR, AR and Control group in exercises related to geometric solids. (a)Student used VR application with virtual reality glasses suitable for a mobile device.(b)Student used AR application with tablet. (c)Student read worksheets related to geometric solids.

For the data analysis, the package SPSS was used. In the analysis both Descriptive and Inferential Statistics were applied for exploring the differences amongst the groups and utilizing parameter tests, paired sample t-test and independent samples t-test. The level of importance for these tests was set at 5%.

4 Experimental Results

Table 1 shows the results of the paired sample t-test for the control, VR and AR group of the differentiation of performance of the pupils before and after the interference in their knowledge of the geometrical shapes. This test is used because the samples are depended since it has to do with the same persons in different circumstances (before and after the interference). The null hypothesis (H_0) is that there is no significant statistical difference regarding the knowledge of the students before and after the interference and the alternative (H_1) is that there is a statistically important difference with regards to the knowledge of the students before and after the interference.

Table 1 Mean, Standard Deviations of the mean number of correct answers among each test group(M) and the results of the paired sample t-test with regards to the participants knowledge gained (N= 30).

	Control Group		VR Group		AR Group	
Activities	M(*SD*)	p-Value	M(*SD*)	p-Value	M(*SD*)	p-Value
Circle the geometric solids (pre-test)	4,9(.88)	**,037***	4,8(1.14)	**,009***	4,9(.88)	**,004***
Circle the geometric solids (post-test)	5,3(.82)		6(,00)		5,8(.42)	
Match the solids with their names(pre-test)	4,1(1.45)	**,037***	4,1(1.10)	**,001***	3,8(1.40)	**,002***
Match the solids with their names(post-test)	4,5(1.1)		5,8(.63)		5,4(.97)	
Solid-town(pre-test)	3,2(.63)	**,025***	3,2(.63)	**,000***	3,2(.63)	**,000***
Solid-town(post-test)	3,9(.74)		4,8(.42)		4,6(.52)	

SD, standard deviation
*** Statistically significant change (p-value<0,05)**

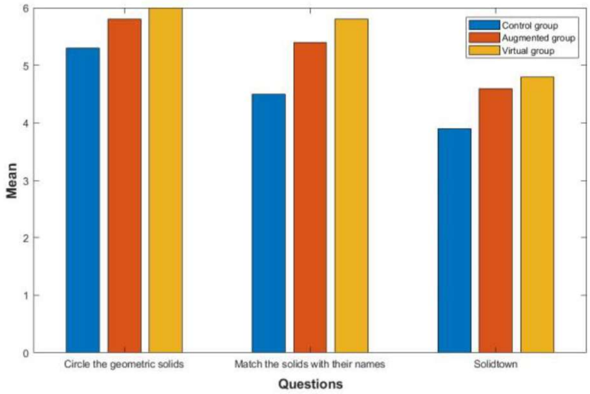

Fig. 9 A comparison of the three groups related to their knowledge of mathematical solids after the experiment. Blue, orange and yellow bars indicate the mean number of correct answers for different exercises for the Control, AR and VR groups respectively.

The null hypothesis is accepted when p-value≥ 0,05 and is rejected when p-value < 0, 05. The results presented in table 1 indicate that since p-value is less than 0,05, there is a statistically significant difference in all three groups with regards to the new knowledge of solids gained after the completion of the training. The results indicate that the students showed improvement with regards to their knowledge of solids with the differences being more intense in the groups of virtual and augmented reality than the control group. All the performances of all groups before the intervention can be characterized as medium to good. The performance of the pupils following the interference, for the group of virtual and augmented reality, can be described as excellent.
Therefore, we reject the null hypothesis and accept H_1. We can also observe that, in the virtual group, the means after the intervention are a bit higher than the means of other groups.

Subsequently, Table 2 and Figure 10, present the results related to the comparisons of the middle values amongst the groups in the variables "Liking", "I would like the programme that I followed to be applied to the subject of Mathematics" and "I would like the application of worksheets instead of technology" through the independent samples t-test. This test was used in this case because the samples are independent (different pupils). Assumptions were examined according to the satisfaction of the pupils for what they did the context of the research and for the inclusion of the applications to the lesson of Mathematics, as well for their preference for worksheets rather than of technology.

In each one of the situations, the null hypothesis (H_0) is that there is no major difference statistically among the groups and the alternative hypothesis (H_1) is that a statistically significant difference exists amongst the groups. Results indicate that the VR and AR group would prefer the use of VR and AR technologies to be used at a greater extent to the subject of Mathematics in contrast to the students of the control group who were taught the lesson through worksheets. No substantial statistical difference was found at the variable "To what extent you liked this experience" between the control group and the VR and AR groups, suggesting that for all three groups the overall experience was pleasant to the pupils.

Table 3 below presents the results of independent sample t-test related to the differentiation of the performance of students after the use of VR and AR regarding the knowledge of geometrical solids. This test is used because the samples are independent (different individuals). The null hypothesis (H_0) is that there is no statistically significant as far as the effectiveness of technologies for the learning of Mathematics and the alternative (H_1) is that there is a statistically significant difference concerning the effectiveness of technologies for the learning of mathematics. We observe from Table 3 that significant differences are not detected (p-value>0,05) between the groups of virtual and augmented reality, as far as the effectiveness of methods for the learning of Mathematics. Consequently, we do not reject H_0 for H_1.

Table 2 Mean, Standard Deviations, and the results of the independent samples t-test regarding the impressions of the participants (N= 30).

Questions	Groups	M(*SD*)	p-Value
(a) To what extent you liked this experience*	Control VR	4,1(1.2) 4,8(.42)	,109
	Control AR	4,1 4,8(.42)	,109
(b) I would like the programme that I followed to be applied to the subject of Mathematics*	Control VR	1,9(1.2) 4,9(.32)	**,000****
	Control AR	1,9 4,7(.48)	**,000****
(c) I would like the application of worksheets instead of technology*	Control VR	1,3(.48) 1(.00)	,081
	Control AR	1,3 (.48) 1(.00)	,081

SD, standard deviation
* The Likert scale for question (a) was:
1-Not at all,2-Little,3-Moderately,4-Very,5-Very much and for questions (b) and (c) was:
1-Definitely not,2-Probably not,3-Unsure,4-Probably yes, 5-Definitely yes
****Statistically significant change (p-value<0,05)**

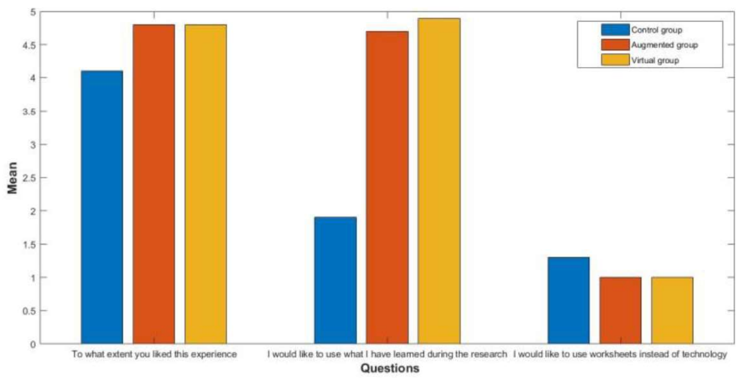

Fig. 10 A comparison of the three groups regarding their impressions for using traditional, VR and AR teaching materials (See the note of table 2 about the Likert scale used for each question)**.** Blue, orange and yellow bars indicate the mean values for the impressions of the Control, AR and VR groups respectively**.**

Table 3 Mean and the results of the independent samples t-test regarding the knowledge gained by the participants that belonged in the virtual and augmented groups. The numbers in the table correspond to the mean number of correct answers among each test group.

	VR Group	AR Group	
Activities	M	M	p-Value
Circle the geometric solids (post-test)	6	5,8	,168
Match the solids with their names(post-test)	5,8	5,4	,290
Solid-town (post-test)	4,8	4,6	,355

5 Discussion

This research aimed to determine experimentally if Virtual and Augmented Reality applications can make the teaching of mathematics more interactive and interesting and can contribute to more efficient learning and understanding of mathematical concepts. Furthermore, the study aimed to compare the effectiveness and suitability of VR against AR technologies about the delivery of a course in geometrical solids for primary school students.

The participants in the VR group stated that they enjoyed the experience in the virtual world. After their experience, they wanted to have similar applications as part of the mathematics course. This is in line with the views of many scientists who have found that virtual reality technologies offer a unique experience to students and make them feel more devoted, but at the same time they enable them to acquire the necessary tools needed for learning [3, 18,33]. The findings indicate that student's performance in the virtual reality group was improved. The performance of the students can be characterized as good before the intervention and excellent after the intervention. The results confirm previous scientific studies associate virtual reality technologies with improvements in academic performance [14,23,31]. Some studies also conclude that spatial abilities can be improved through virtual reality [16,25,28]. Many scientists point out that VR technologies promote the communicative and collaborative skills of the students as well as their psychomotor and cognitive skills [39,16,23]. The findings of the present study also indicate that the participants in the virtual group would not prefer the solids lesson to be done with worksheets but through technology. The students would prefer the implementation of the VR application to the lessons of mathematics to a greater extent compared to the control group that used printed worksheets. Some scientists point out that, activities in virtual environments attract more the interest of the students making the learning process interesting and entertaining [17].

Students after their experience with AR applications showed a positive attitude as they wanted to have similar applications as part of the mathematics course in relation to the students in the control group. This is also consistent with many studies that indicate that the use of AR increases the motivation and interest of pupils [8,9,27,30,36]. Furthermore, pupils' performance improvement after the implementation of the AR applications was more intense when compared to the performance improvement for control group students. Moreover, after the intervention with the AR applications, the students' performance as far as their solid knowledge is concerned was excellent. Many scientists have argued that AR environments help students develop skills and knowledge more effectively than traditional teaching [8,9,27,30,34,36]. According to research results reported in the literature, AR may help to make the process of modeling a shape or a figure more understandable. AR is characterized as the technology that creates a reality that improves and grows [11, 36]. One of the surveys in which the use of AR has had positive results for students is that of Sollerval [30], where students who used augmented reality technology demonstrated better cooperative ability, teamwork, and better problem-solving.

No statistically significant difference was found between the control group and the VR and AR groups regarding the knowledge gained as part of the training activities. This is justified because the training methods used for all three groups were carefully prepared to provide the necessary knowledge. In the case of VR and AR, this information was provided interactively based on the use of new technological tools, and this made the experience more attractive. In line with findings that claim that young learners can acquire a positive attitude towards geometry only through aesthetics [35], the nice and engaging graphics incorporated in the printed material used by the control group also provided effectively the necessary information for the pupils. However, although effective in terms of learning outcomes, the overall experience was not highly rated by the pupils when compared to the results for the VR and AR groups.

Although the positive outcomes of the research further investigation is needed to confirm the accuracy of the findings of the present study. Nevertheless, the discussion revealed that the bibliographic review is in agreement with the results obtained from the inspection of the first research case. Consequently, improving pupils' performance after intervention proves that students have been able to understand the geometric solids fully, as well as to perceive the difference between the objects of the three-dimensional space and those of the two-dimensional area. Throughout the discussion, students also develop skills from the use of VR and AR technologies. These technologies promote pupils' communicative and collaborative skills as well as psychomotor skills.

With regards to the comparison between AR and VR, our results indicate that virtual and augmented reality technologies are equally effective for learning mathematics, something that highlights the originality and importance of this work.

6 Limitations and Future Work

The current research is essential due to the fact that a comparison of VR and AR applications as part of teaching methodology has been conducted, in the field of Mathematics training in primary school, adding value in the use of those technologies in the field of education in general. Additionally, the research aimed to fill in the lack of research in the field of Mathematics, as the use of such kind of technologies has been inadequately investigated so far.

Nevertheless, there are several limitations concerning the current research. First of all, it is possible that the results of the survey have been influenced by the limited time available to conduct the survey. The total time that the students participated in the survey was only twenty minutes. Consequently, the time factor may have influenced the results of the research, which may have been different if the students were engaged for more time with the applications.

In addition, only 30 pupils of the fourth, fifth and sixth grade of Primary School were selected for the research by random sampling. However, since the non-representative sample of convenience of the 30 individuals was used in this research, the sample is not sufficient to generalize the results. Hence, future research is required using a larger sample and representative of the population for more objective results. Moreover, probability sampling and a larger number of participants with representative characteristics of the wider population. In this way, sampling error will be avoided, the validity and reliability of research will be improved, and generalized results will be extracted [26]. Furthermore, surveys with pupils from different regions, as well as for a longer period, can allow for comparisons and further inductive analysis of the factors that affect the effectiveness of applications for learning mathematics.

It should be noted that in the existing literature, no similar research has been spotted to compare technologies for teaching mathematics in a Primary school. In the bibliography, there are not enough articles that compare technologies of virtual and augmented reality in education. Some characteristic examples are the studies of Krichenbauer, M., et al. [20] and Huang, K. T., et al. [13], in which there is a comparison of virtual and augmented reality. Krichenbauer et al.[20] present a comparison of the two technologies regarding 3D object manipulation, whereas in the research of Huang, K. T., et al.[13], there is a comparison of the two technologies in education related to the solar system. However, there are no studies comparing virtual and augmented reality in Mathematics. Hence, the results of this study cannot be compared with other studies to provide a clearer indication of the most efficient technology that is appropriate for learning Mathematics.

What is more, this research indicated that the use of new technologies in teaching mathematics had increased the interest of the students, making the course of Mathematics more understandable and engaging. In future research, it would be important to develop cooperative activities using virtual and augmented reality technologies to develop the collaborative skills of the students. In addition, in future research, it would be useful to make virtual and augmented real-world applications to teach more complex topics, such as the calculation of the area and volume of solids, so that students can understand the geometric interpretation of the area and volume of solids, something which was suggested by the participants. Taking into account the feedback from the participants, new applications of augmented reality regarding the area of solids are under development.

7 Conclusion

The current paper presented the development and implementation of AR and VR applications in the field of Mathematics. The implementation of the applications was simple, as non-specialized equipment was used, including participants personal mobile devices and low-cost virtual reality glasses for mobile devices. The use of non-specialized equipment allowed users to have access to applications anytime and anyplace. This fact allowed the implementation of the applications not only in the classroom during the lesson but also allowed the students to use them at home. Moreover, the applications are provided free of charge through the ENTiTi application on a mobile or a tablet.

The results of the research indicated that the use of AR and VR applications had a higher impact on student's learning and understanding of mathematical concepts compared to traditional teaching approaches. Furthermore, the results revealed that these technologies are more interactive and interesting for the students than the use of printed material. Regarding the comparison of the two technologies and their effectiveness in learning mathematics, according to the results, there are no significant differences between AR and VR. Additionally it is worth mentioning, is that many of the students expressed their interest to download these applications in their mobile devices and use them at home in their leisure time.

This research has highlighted that the use of AR and VR applications in the classroom affected student engagement and their learning performance concerning maths positively. Nevertheless, further research is required focusing on and investigating the use of AR and VR technologies in teaching mathematics, as well as ways to integrate them into the curriculum and the classrooms.

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