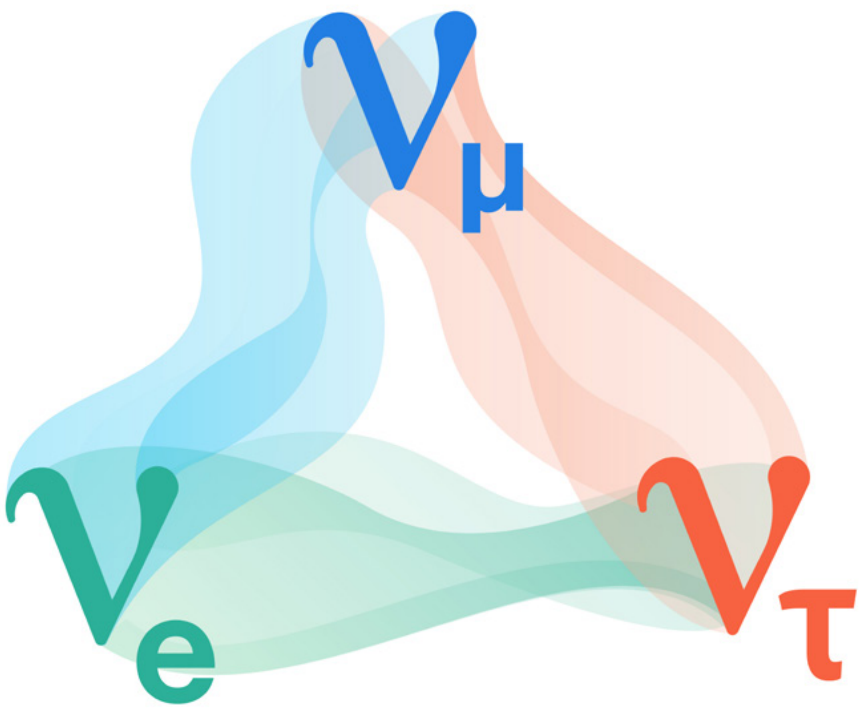
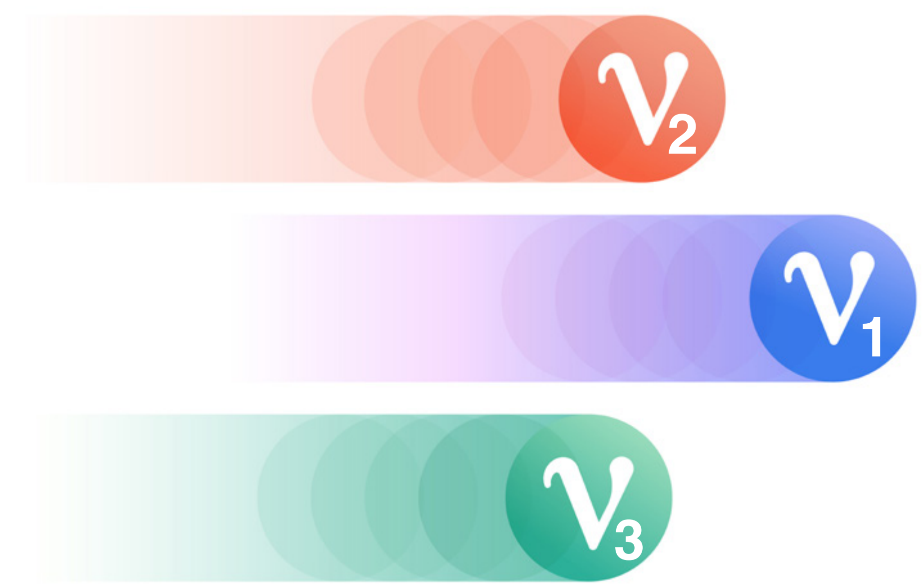
Neutrino Propagation in Matter: 3 flavors & beyond



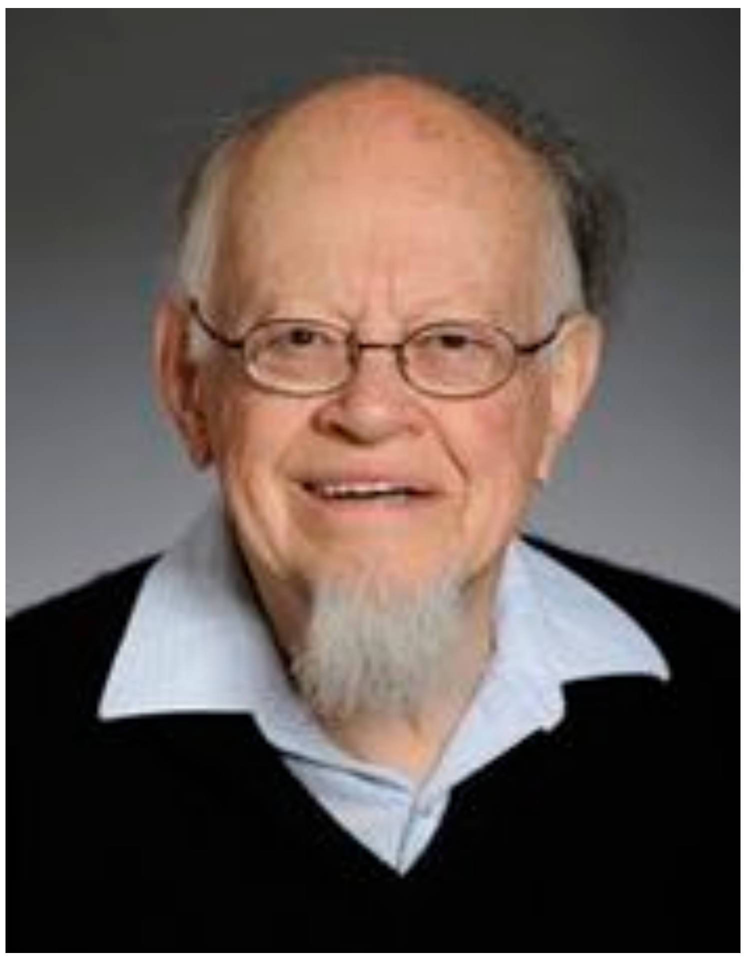
Stephen Parke
Fermilab
https://orcid.org/0000-0003-2028-6782



with Peter Denton, Xining Zhang + other collaborators

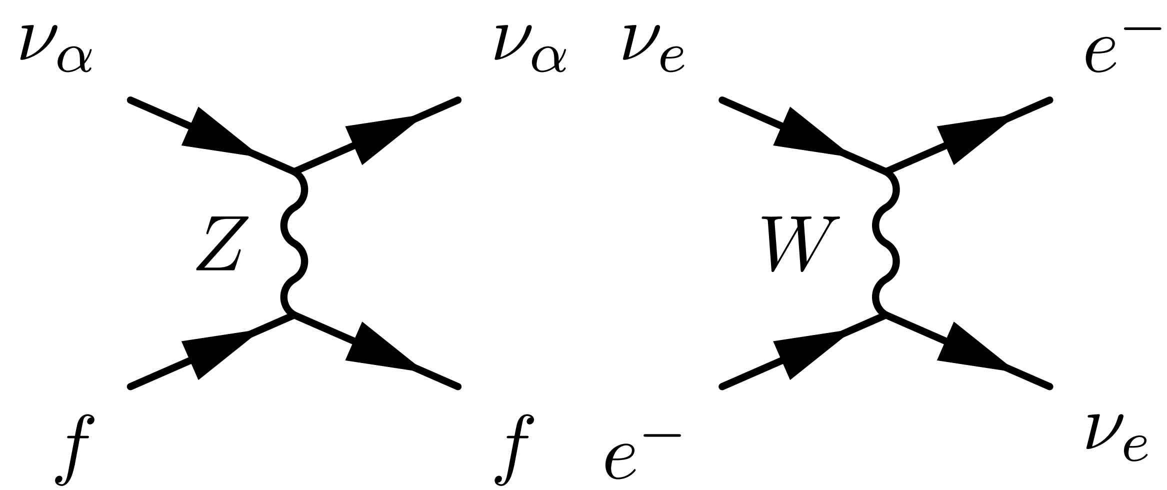
Wolfenstein Matter Effect:

— coherent forward scattering



Z: same for all flavors

W: ν_e only



L. Wolfenstein, PRD 17 (1978)

$$\sqrt{2}G_F N_e \sim \frac{\Delta m^2}{2E_\nu}$$

N_e is # density of e's

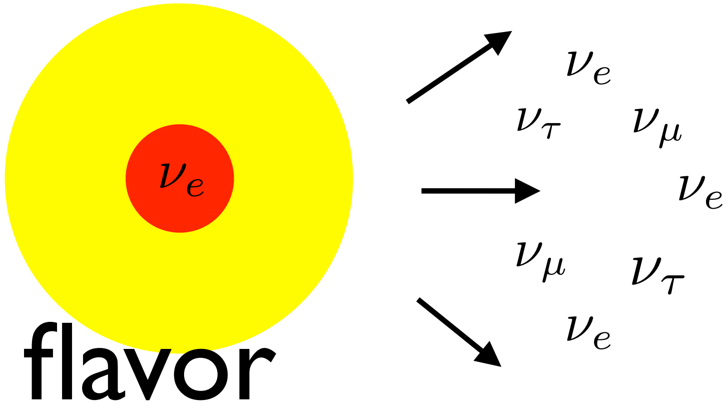
- Solar neutrinos $E_\nu \sim 10$ MeV

$\Delta m^2_\odot \sim 7.5 \times 10^{-5}$ eV^2 and $\rho \sim 150$ g.cm^{-3}

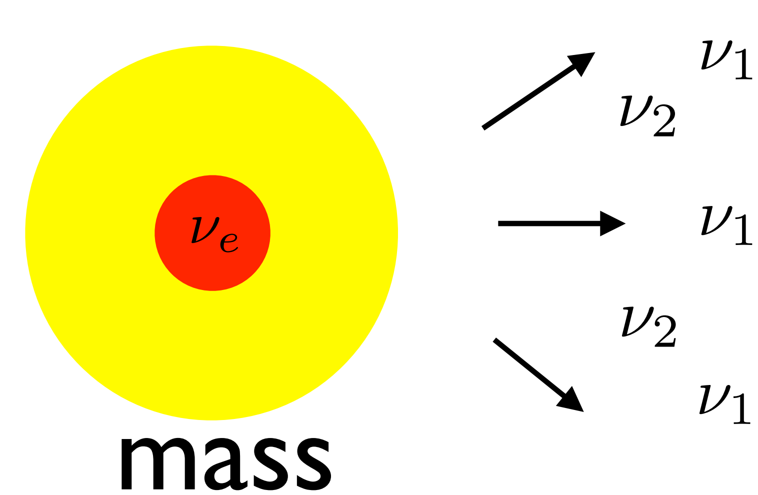
- Accelerator neutrinos $E_\nu \sim 10$ GeV

$\Delta m^2_{atm} \sim 2.5 \times 10^{-3}$ eV^2 and $\rho \sim 3$ g.cm^{-3}

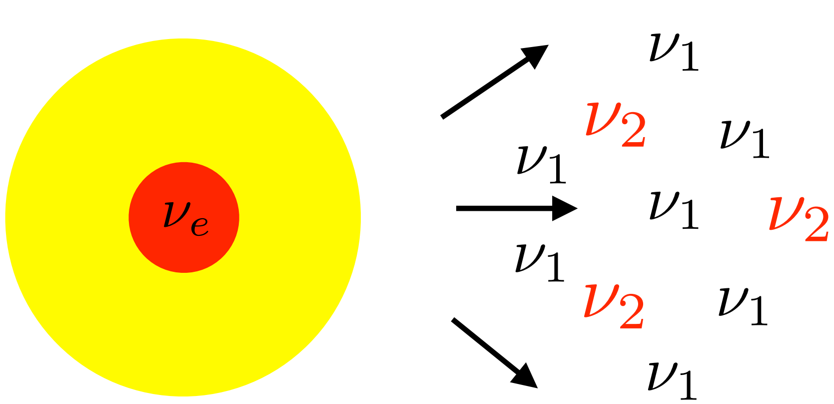
Solar Neutrinos:



or

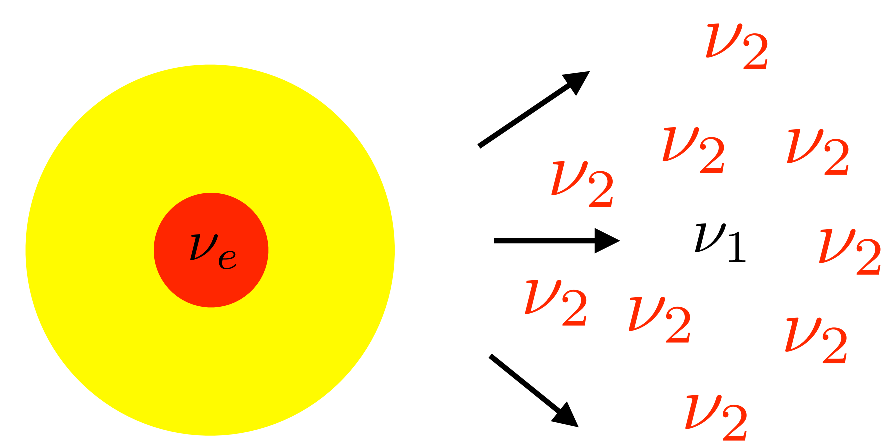


$E_\nu < 1$ MeV (pp & ^{7}Be)



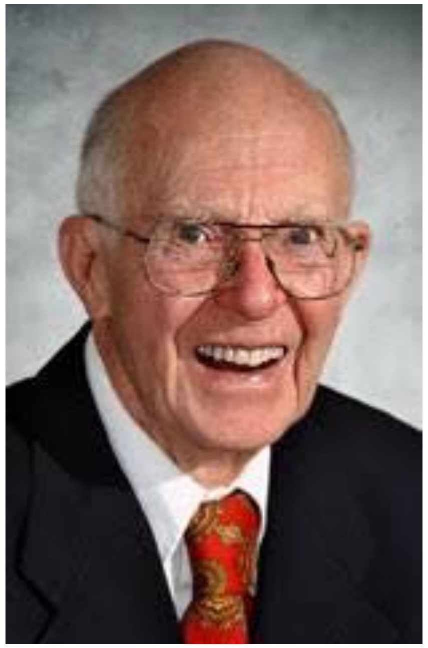
$\sqrt{2} G_F N_e \sim \frac{\Delta m^2}{E_\nu}$

$E_\nu > 10$ MeV (^{8}B & hep)



$< P_{ee} > \sim 0.58$

Vac. Osc.



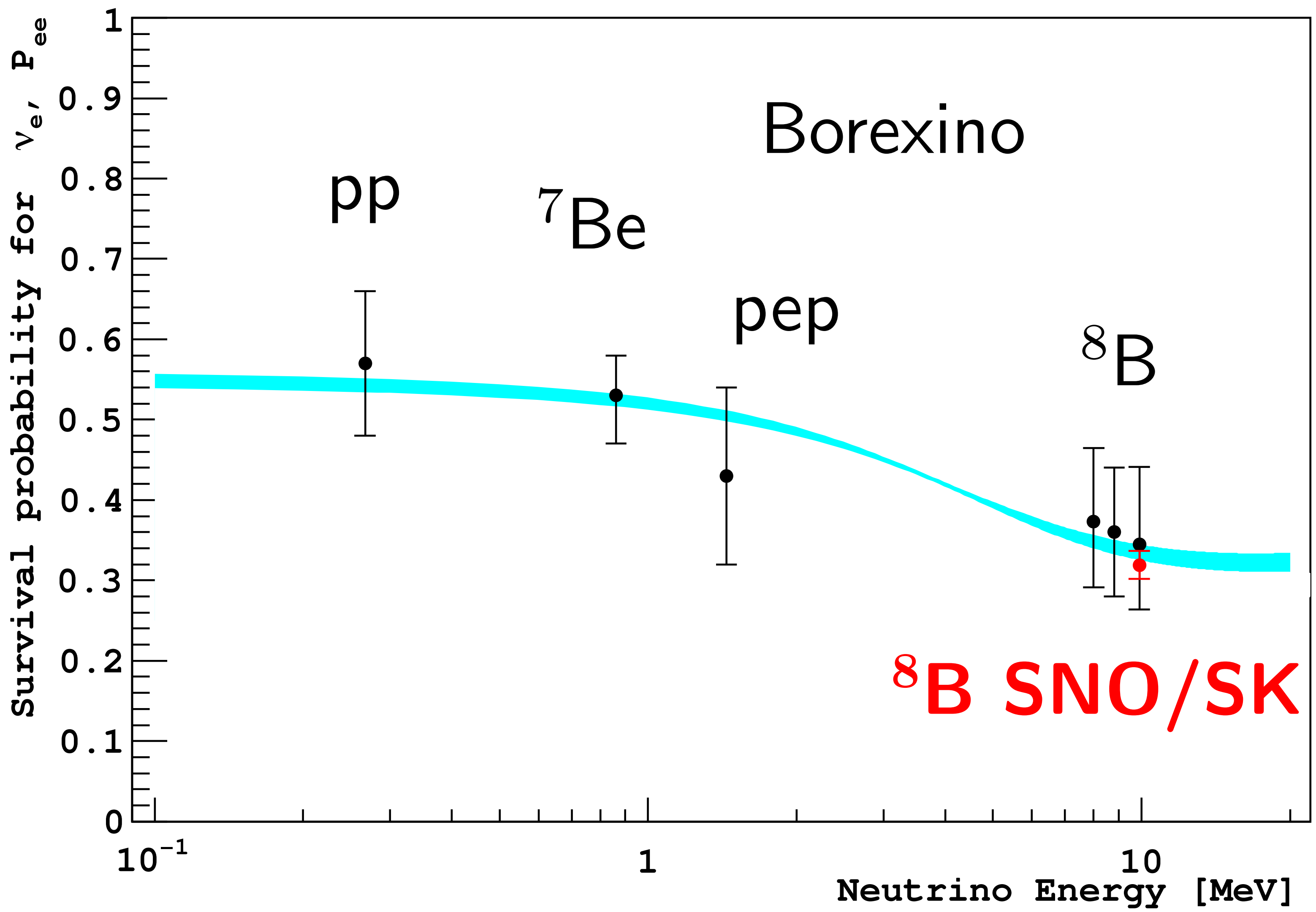
Ray Davis

$< P_{ee} > \sim 0.34$

Matter Dominate
Flavor Transformations

MSW mechanism
Parke PRL (1986)

Solar Neutrinos:



Vacuum:
averaged osc

$\sim 68\% \ \nu_1$
$\sim 30\% \ \nu_2$
$\sim \ 2\% \ \nu_3$

MSW:

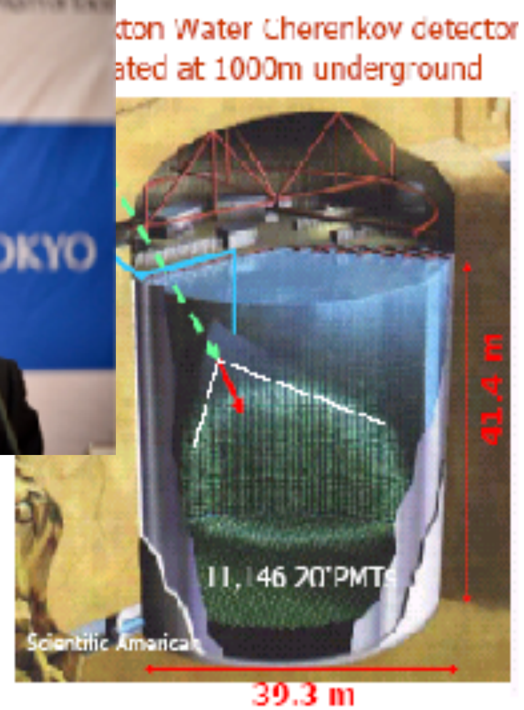
$> 90\% \ \nu_2$

matter effect

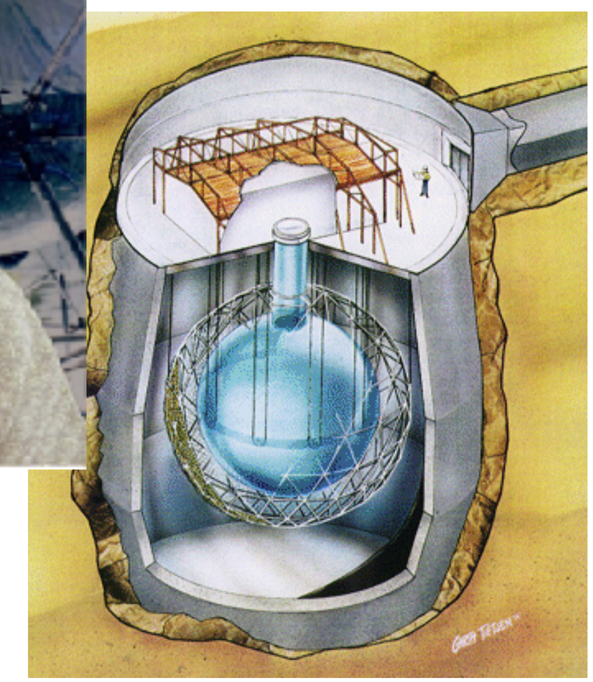
Nunokawa, SP, Zukanovich-Funchal
arXiv:hep-ph/0601198

NOBEL 2015

"for the discovery of neutrino oscillations, which shows that neutrinos have mass"



Takaaki Kajita
SuperKamiokaNDE



Art McDonald
SNO

"for the discovery of neutrino flavor transformations, which shows that neutrinos have mass"

~ vacuum oscillations

Wolfenstein matter effects dominant flavor transformations

See Smirnov arXiv:1609.02386

Terrestrial Experiments:
where Matter Effects are Important

Terrestrial Experiments: with Matter Effects

Accelerator: $\nu_\mu \to \nu_e$

- T2K, T2HK (295 km)
- NOvA (810 km)
- T2HKK (1100 km)
- DUNE (1300 km)

Reactor: $\bar{\nu}_e \to \bar{\nu}_e$

- JUNO (52 km)

Atmospheric: $\nu_\mu \to \nu_e$

- ICECUBE .... (13,000 km)

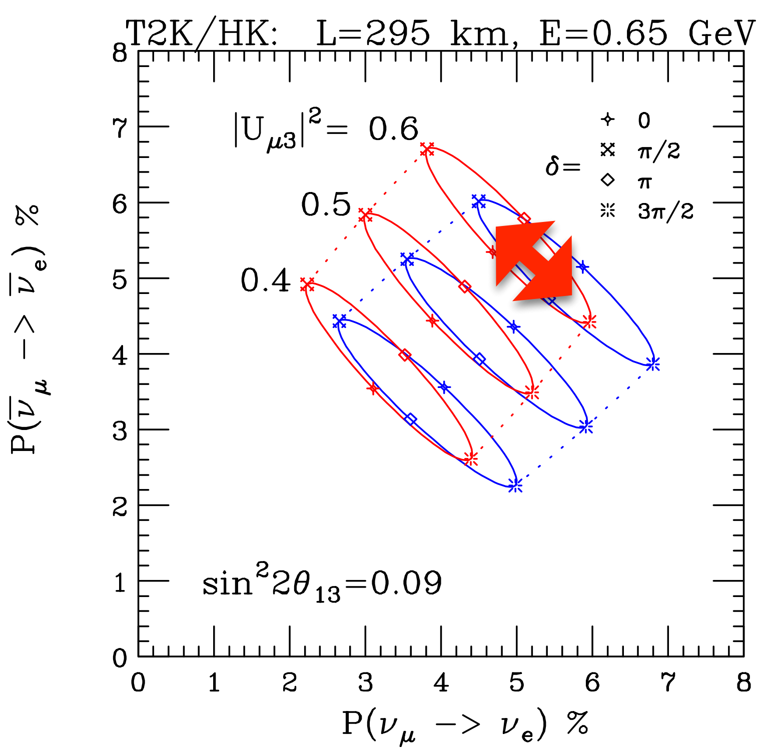
At 1st Osc. Peak $E_\nu \sim$ (L /500 km) GeV

Correlations between

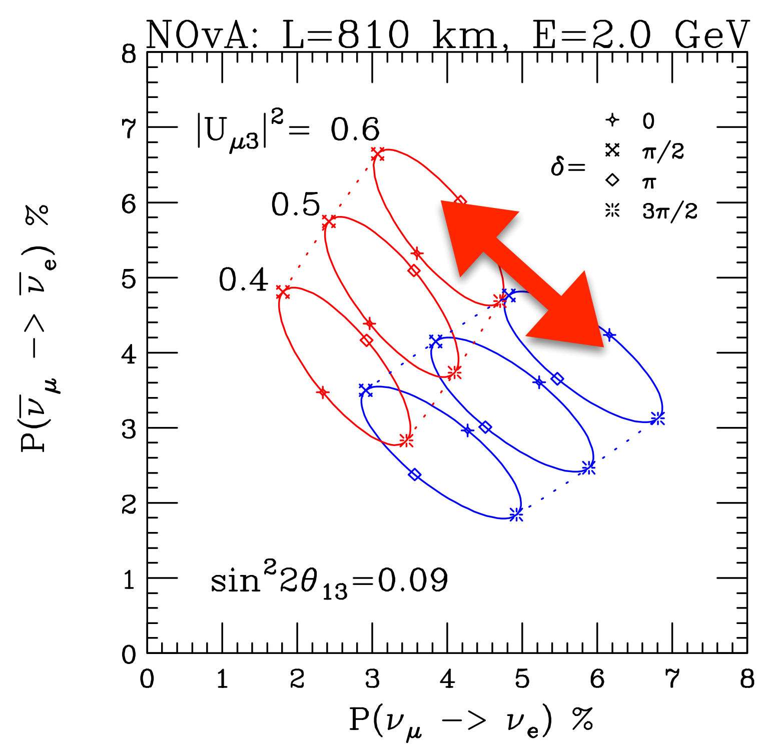
$$\nu_\mu \rightarrow \nu_e \quad \bar{\nu}_\mu \rightarrow \bar{\nu}_e$$

Normal Ordering — **Inverted Ordering**

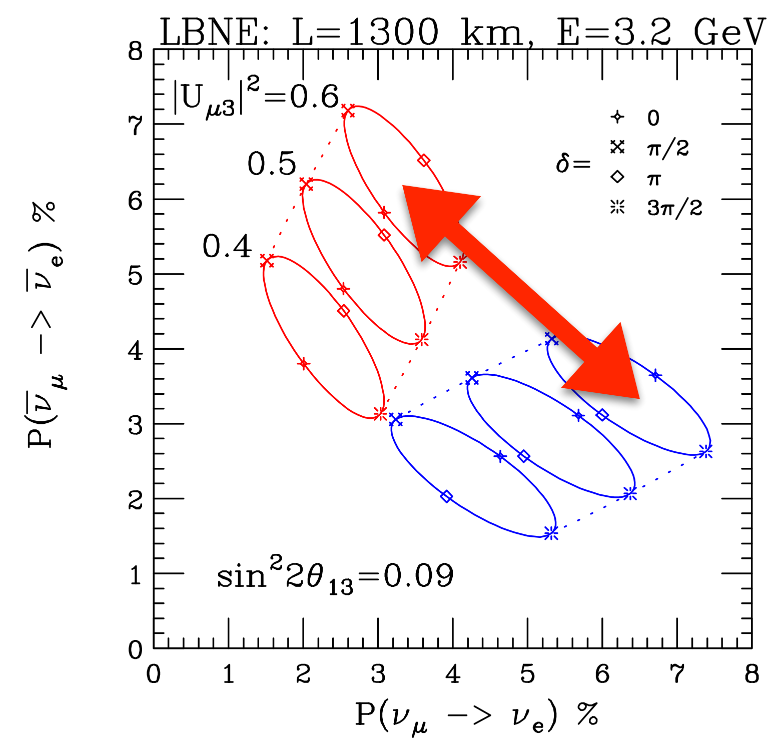
T2K/HK



NOvA



DUNE Same L/E as NOνA

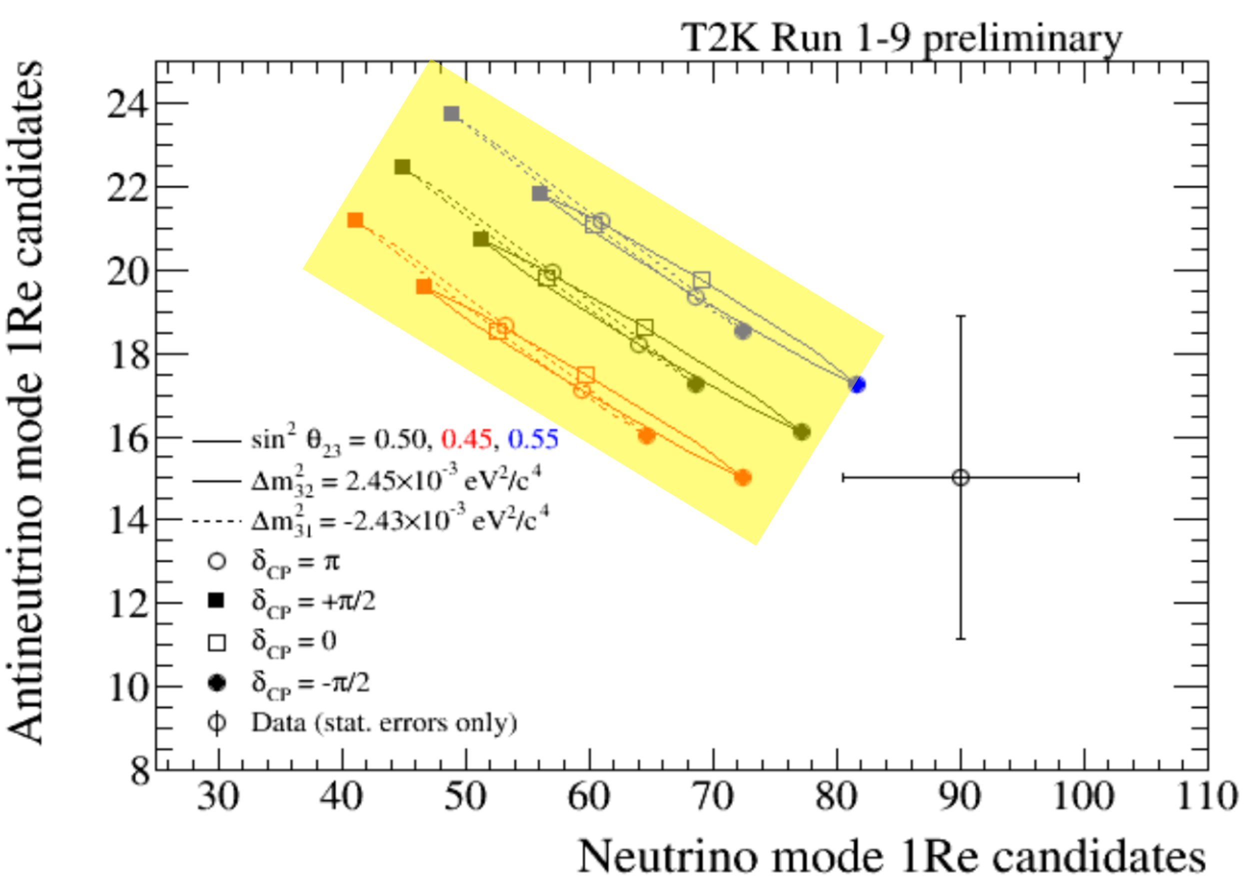


$\propto \rho L \, \sin^2 \theta_{23}$

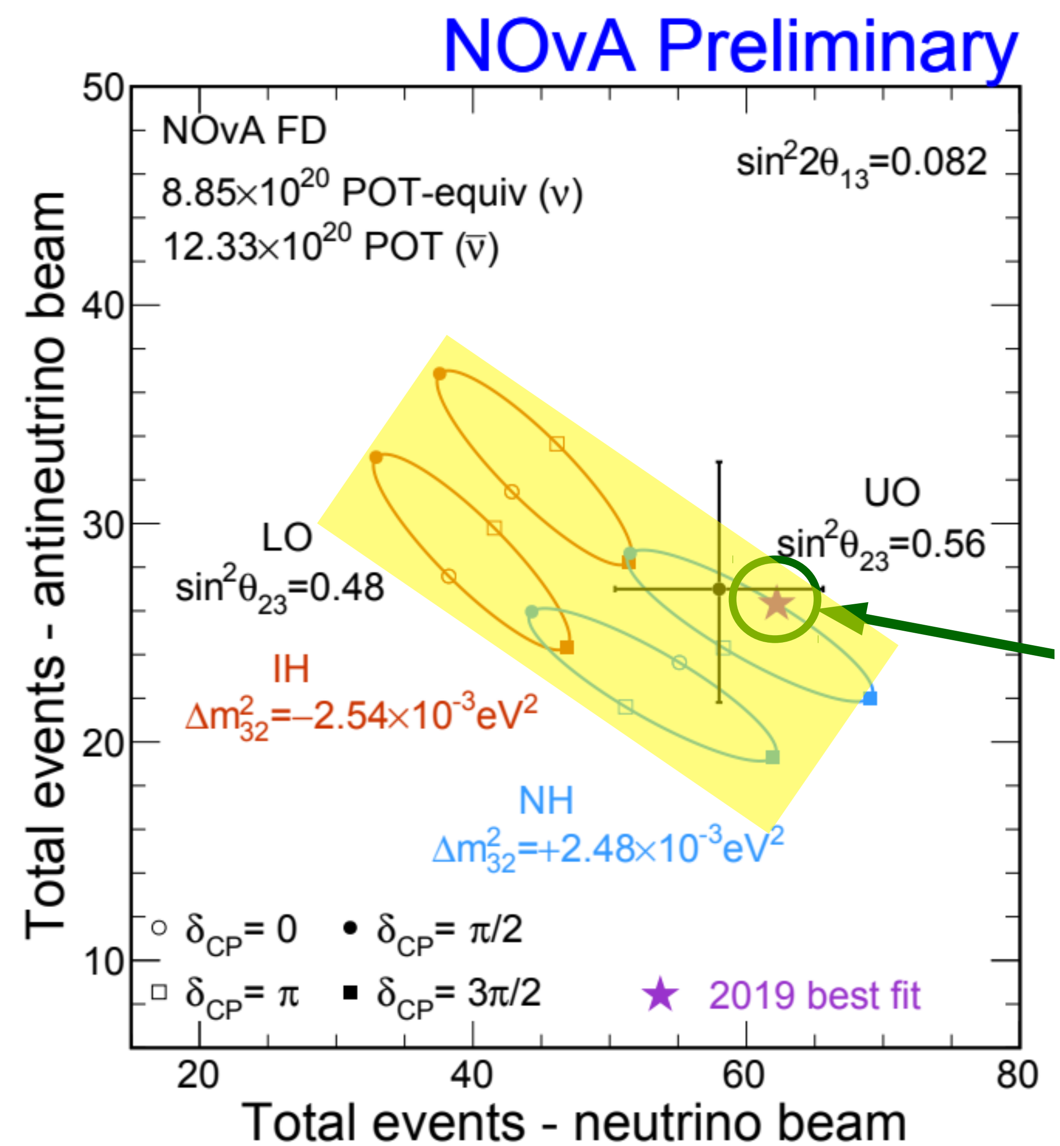
O. Mena & SP hep-ph/0408070

T2K & NOvA:

T2K

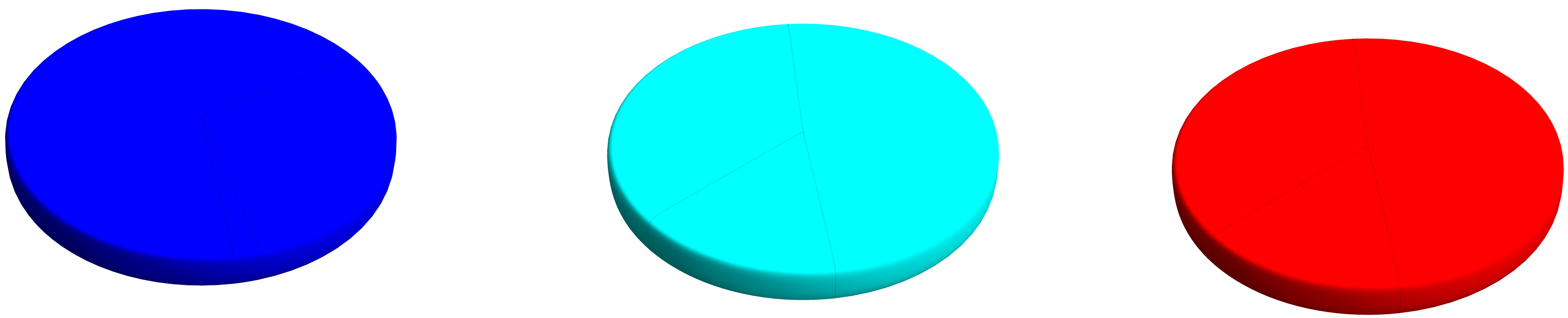


NOvA



Neutrino Flavor or Interaction States:

$W^+ \rightarrow e^+ \nu_e$ $\quad$ $W^+ \rightarrow \mu^+ \nu_\mu$ $\quad$ $W^+ \rightarrow \tau^+ \nu_\tau$



ν_e $\quad$ ν_μ $\quad$ ν_τ

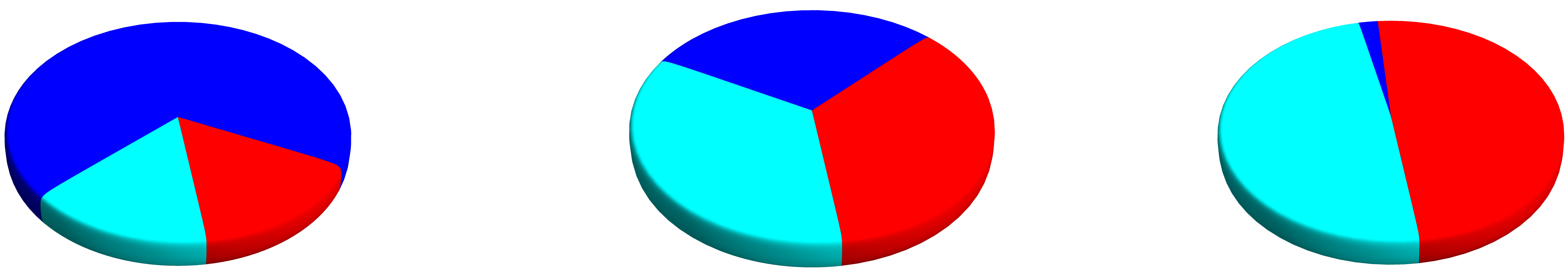
provided $L/E \ll \mathbf{0.5}$ km/MeV $= \mathbf{500}$ km/GeV !!!

$\sim$ 1 picosecond in Neutrino rest frame !!!

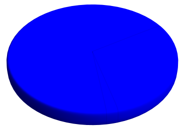
Neutrino Mass EigenStates or Propagation States:

$$\text{Propagator } \nu_j \to \nu_k \ = \delta_{jk}\, e^{-i\left(\frac{m_j^2 L}{2E_\nu}\right)}$$

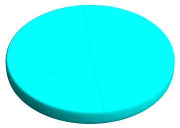
ν_1 most ν_e | ν_2 | ν_3 least ν_e



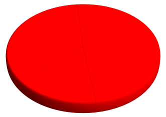
δ, θ_{23} | δ, θ_{23} | θ_{23}

$\nu_e =$ 

Solar Exp, SNO
KamiLAND
Daya Bay, RENO, …

$\nu_\mu =$ 

SuperK, K2K, T2K
MINOS, NOvA
ICECUBE

$\nu_\tau =$ 

Unitarity
SK, Opera
ICECUBE ?

unitary matrix

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

by defn $|U_{e1}|^2 > |U_{e2}|^2 > |U_{e3}|^2$

$$U_{PMNS} = U_{23}(\theta_{23}, \delta)\ U_{13}(\theta_{13}, 0)\ U_{12}(\theta_{12}, 0)$$

Why this order ???

$$= \begin{pmatrix} 1 & & \\ & c_{23} & s_{23} e^{+i\delta} \\ & -s_{23} e^{-i\delta} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & & s_{13} \\ & 1 & \\ -s_{13} & & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & \\ -s_{12} & c_{12} & \\ & & 1 \end{pmatrix}$$

$s_{ij} = \sin\theta_{ij},\ c_{ij} = \cos\theta_{ij}$

$$\times\, \mathrm{diag}(1,\ e^{i\frac{\alpha_{21}}{2}},\ e^{i\frac{\alpha_{31}}{2}})$$

Towards a better understanding of Osc. Prob.

Globes,
while a very useful tool,
is not enough !

Hamiltonian:

flavor/interaction basis:

$$\frac{1}{2E}\left\{ U \begin{pmatrix} 0 & & \\ & \Delta m^2_{21} & \\ & & \Delta m^2_{31} \end{pmatrix} U^\dagger + \begin{pmatrix} a & & \\ & 0 & \\ & & 0 \end{pmatrix} \right\}$$

vac. mass eigenstate basis

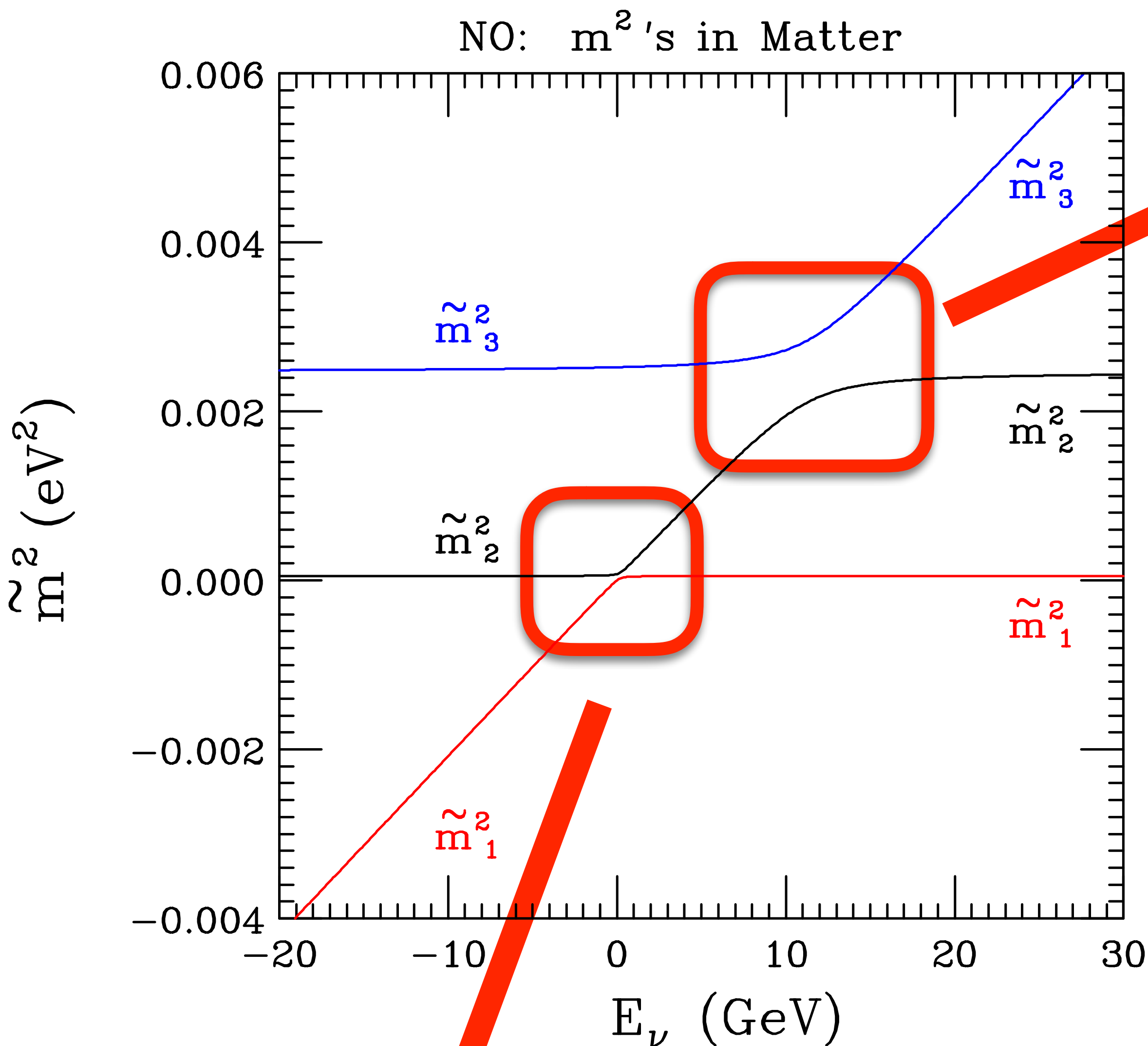
$$\frac{1}{2E}\left\{ \begin{pmatrix} 0 & & \\ & \Delta m^2_{21} & \\ & & \Delta m^2_{31} \end{pmatrix} + U^\dagger \begin{pmatrix} a & & \\ & 0 & \\ & & 0 \end{pmatrix} U \right\}$$

$$a \equiv 2\sqrt{2} G_F N_e E$$

$$U = U_{23} U_{13} U_{12}$$

arbitrary "a"

Eigenvalues: Analytically or Numerically



$$a \equiv 2\sqrt{2}G_F N_e E$$

Occurs at $a = \Delta m^2_{21}\cos 2\theta_{12}/c^2_{13}$

with Minimum $\Delta\widehat{m^2}_{21} = \Delta m^2_{21}\sin 2\theta_{12}$

Occurs at $a = \Delta m^2_{ee}\cos 2\theta_{13}$

with Minimum $\Delta\widehat{m^2}_{32} = \Delta m^2_{ee}\sin 2\theta_{13}$

$$\Delta m^2_{ee} \equiv c^2_{12}\Delta m^2_{31} + s^2_{12}\Delta m^2_{32}$$

ν_e average

directly measured
by Daya Bay/RENO

flavor basis:

$$\frac{1}{2E}\left\{U\begin{pmatrix}0 & & \\ & \Delta m^2_{21} & \\ & & \Delta m^2_{31}\end{pmatrix}U^\dagger + \begin{pmatrix}a & & \\ & 0 & \\ & & 0\end{pmatrix}\right\}$$

2 flavor mixing in matter

$$ax^2 + bx + c = 0$$

simple, intuitive, useful

3 flavor mixing in matter

$$ax^3 + bx^2 + cx + d = 0$$

complicated, counter intuitive, ...

- Solve Cubic Characteristic Eqn.

$$\lambda^3 - \left(a + \Delta m^2_{21} + \Delta m^2_{31}\right) \lambda^2$$
$$+[\Delta m^2_{21}\Delta m^2_{31} + a \left\{(c^2_{12} + s^2_{12}s^2_{13})\Delta m^2_{21} + c^2_{13}\Delta m^2_{31}\right\}] \lambda$$
$$- c^2_{12}c^2_{13}a\Delta m^2_{21}\Delta m^2_{31} \qquad = 0$$

See Zaglauer & Schwarzer, Z. Phys. C 1988

$$\lambda_1 = \frac{1}{3}s - \frac{1}{3}\sqrt{s^2-3t}[u+\sqrt{3(1-u^2)}],$$

$$\lambda_2 = \frac{1}{3}s - \frac{1}{3}\sqrt{s^2-3t}[u-\sqrt{3(1-u^2)}],$$

$$\lambda_3 = \frac{1}{3}s + \frac{2}{3}u\sqrt{s^2-3t},$$

$$s = \Delta_{21}+\Delta_{31}+a,$$

$$t = \Delta_{21}\Delta_{31}+a[\Delta_{21}(1-s^2_{12}c^2_{13})+\Delta_{31}(1-s^2_{13})],$$

$$u = \cos\left[\frac{1}{3}\cos^{-1}\left(\frac{2s^3-9st+27a\Delta_{21}\Delta_{31}c^2_{12}c^2_{13}}{2(s^2-3t)^{3/2}}\right)\right],$$

here $\Delta_{ij} \equiv \Delta m^2_{ij}$

- then calculate mixing angles in matter or mixing matrix, V:
  eg Kimura Takamura & Yokomakura PLB, PRD 2002

both analytic & numerical are black boxes

Hamiltonian:

H. Minakata + SP arXiv:1505.01826
P. Denton + H. Minakata + SP arXiv:1604.08167

$$H = \frac{1}{2E}\left\{U\begin{bmatrix} 0 & 0 & 0 \\ 0 & \Delta m^2_{21} & 0 \\ 0 & 0 & \Delta m^2_{31} \end{bmatrix}U^\dagger + \begin{bmatrix} a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\right\}$$

Rewrite as $\boldsymbol{H} = \boldsymbol{H_0} + \boldsymbol{H_1}$

solvable perturbation

where H_0 is diagonal

and H_1 is off-diagonal.

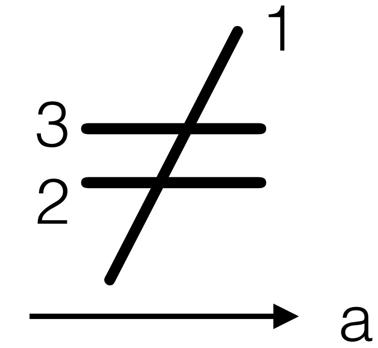
small #'s $\epsilon \equiv \frac{\Delta m^2_{21}}{\Delta m^2_{ee}} \sim 0.03$ $\sin^2\theta_{13} \sim 0.02$

Neutrino Evolution in Matter:

$$U_{23}^{\dagger}(\theta_{23},\delta)\,H\,U_{23}(\theta_{23},\delta) = H_D + H_{OD}$$

D=diagonal   OD= off-diagonal

$$(2E)\,H_D = \begin{bmatrix} a + s_{13}^2 \Delta m_{ee}^2 & & \\ & (c_{12}^2 - s_{12}^2)\Delta m_{21}^2 & \\ & & c_{13}^2 \Delta m_{ee}^2 \end{bmatrix}$$



naturally appears: $\Delta m_{ee}^2 \equiv c_{12}^2 \Delta m_{31}^2 + s_{12}^2 \Delta m_{32}^2$

!!! level crossing !!!

$$(2E)\,H_{OD}/\Delta m_{ee}^2 = s_{13}c_{13} \begin{bmatrix} & & 1 \\ & 0 & \\ 1 & & \end{bmatrix}$$

$$+\; c_{13}\; s_{12}c_{12} \left(\frac{\Delta m_{21}^2}{\Delta m_{ee}^2}\right) \begin{bmatrix} & 1 & \\ 1 & & 0 \\ & 0 & \end{bmatrix}$$

$$-\; s_{13}\; s_{12}c_{12} \left(\frac{\Delta m_{21}^2}{\Delta m_{ee}^2}\right) \begin{bmatrix} & 0 & \\ 0 & & 1 \\ & 1 & \end{bmatrix}$$

0.15

0.015

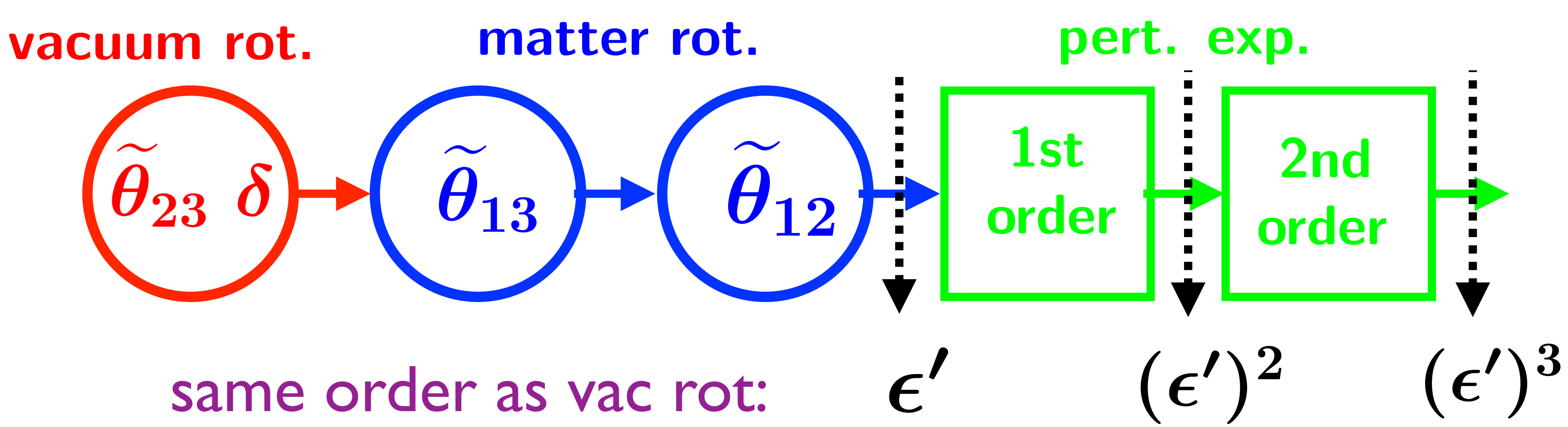
0.002

The Method:

- Rotation in 1-3 sector to diagonalize that sector: $\widehat{\theta}_{13}$
  – removes 1-3 level crossing

- Rotation in 1-2 sector to diagonalize that sector: $\widehat{\theta}_{12}$
  – removes 1-2 level crossing

- Perform Perturbation expansion in H_{OD} after these two rotations:



$$\epsilon' \equiv \sin(\widetilde{\theta}_{13} - \theta_{13}) \, (s_{12}c_{12}) \, \left(\frac{\Delta m^2_{21}}{\Delta m^2_{ee}}\right) < 0.015$$

The Method (conti):

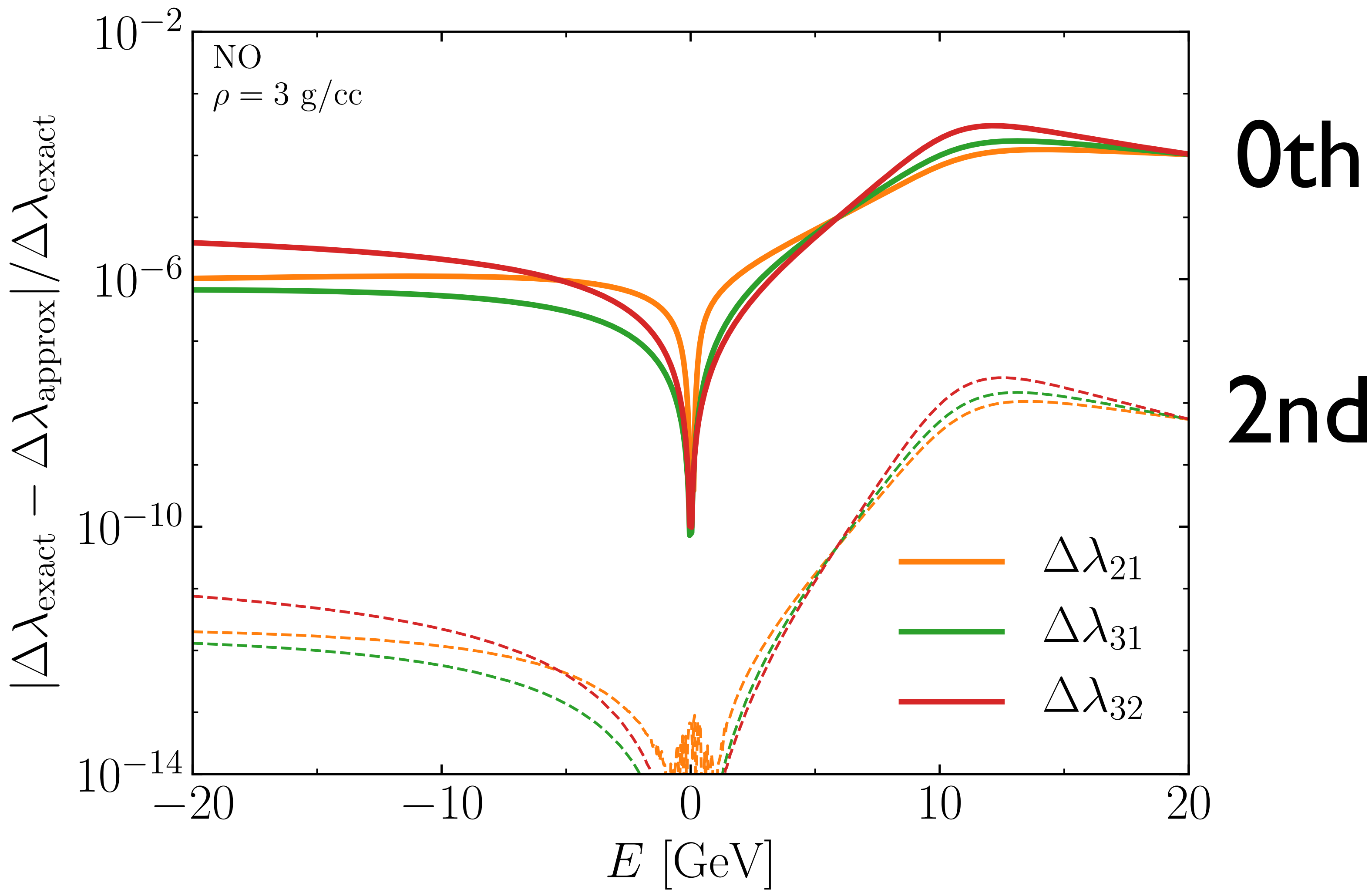
$$\frac{(2E)\ H_{OD}}{\Delta m^2_{ee}} = \sin(\widehat{\theta}_{13} - \theta_{13})\ s_{12}c_{12} \left(\frac{\Delta m^2_{21}}{\Delta m^2_{ee}}\right) \begin{pmatrix} & & -\sin\widehat{\theta}_{12} \\ & & \cos\widehat{\theta}_{12} \\ -\sin\widehat{\theta}_{12} & \cos\widehat{\theta}_{12} & \end{pmatrix}$$

- Vanishes in vacuum, as $\widehat{\theta}_{13} = \theta_{13}$ (same order important)
- Is small, ≤ 0.015

(arXiv:1907.02534)

- And diagonal (by construction) plus one other element is zero :
  $\Longrightarrow$ Implies ALL odd perturbative corrections to eigenvalues are zero !!!
  Therefore for eigenvalues expansion parameter is $\leq 2 \times 10^{-4}$

Corrections to Eigenvalues:



Denton, SP, Xining Zhang: arXiv:1907.02534

Mixing Angles (eigenvectors) from Eigenvalues:

$$\begin{pmatrix} \lambda_\sigma & \\ & \lambda_\rho \end{pmatrix} = U(\phi)^\dagger \begin{pmatrix} \lambda_a & \lambda_x \\ \lambda_x & \lambda_c \end{pmatrix} U(\phi)$$

$$U(\phi) \equiv \begin{pmatrix} c_\phi & s_\phi \\ -s_\phi & c_\phi \end{pmatrix} .$$

$$|U_{11}|^2 = \cos^2\phi = \frac{\lambda_\sigma - \lambda_c}{\lambda_\sigma - \lambda_\rho} \qquad |U_{12}|^2 = \sin^2\phi = \frac{\lambda_\sigma - \lambda_a}{\lambda_\sigma - \lambda_\rho}$$

arXiv:1604.08167: Appendix A

Can this be generalized to 3x3 ?

Generalization to 3x3:

$$|\widehat{U}_{\alpha i}|^2 = \frac{(\lambda_i - \xi_\alpha)(\lambda_i - \chi_\alpha)}{(\lambda_i - \lambda_j)(\lambda_i - \lambda_k)}$$

- λ_i are eigenvalues of original matrix, 3x3
- ξ_α and χ_α are eigenvalues of original matrix with α row & α column removed, 2x2

Can this be generalized to nxn ?

Denton, SP, Xining Zhang: arXiv:1907.02534

Generalization to nxn:

- Let H be an nxn Hermitian matrix
  with eigenvalues $\lambda_i(H)$ and eigenvectors v_i

- Let h_j be the (n-1)x(n-1) Hermitian matrix from H
  with j-th row and j-th column deleted with eigenvalues $\lambda_i(h_j)$

$$|v_{i,j}|^2 = \frac{\prod_{k=1}^{n-1} \left(\lambda_i(H) - \lambda_k(h_j)\right)}{\prod_{k=1,k\neq i}^{n} \left(\lambda_i(H) - \lambda_k(H)\right)}$$

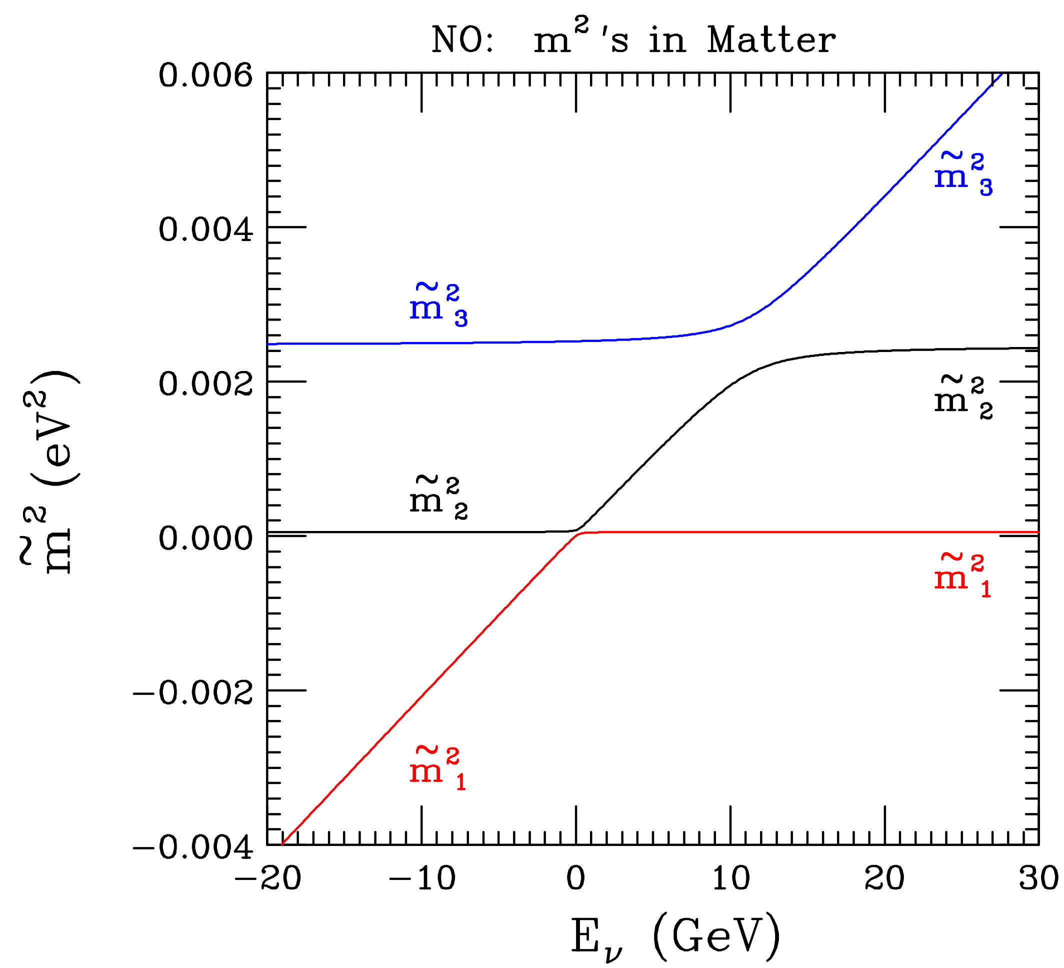
- Phase information is a more complicated expression.

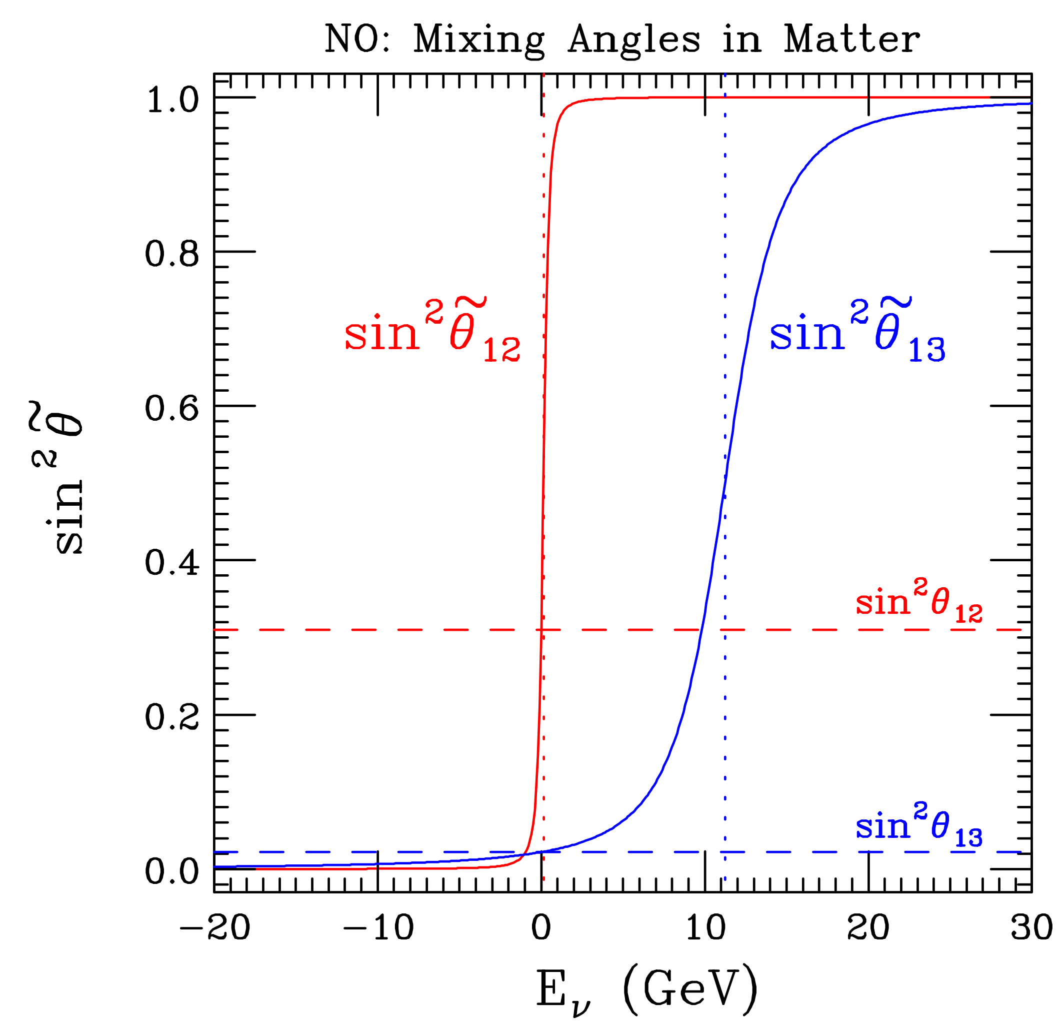
- Numerator is characteristic function for h_j evaluated at $\lambda_i(H)$

- Normalized $\sum_i |v_{i,j}|^2 = 1 = \sum_j |v_{i,j}|^2$

Denton, SP, Terrence Tao, Xining Zhang: arXiv:1908.03759 [math.RA]

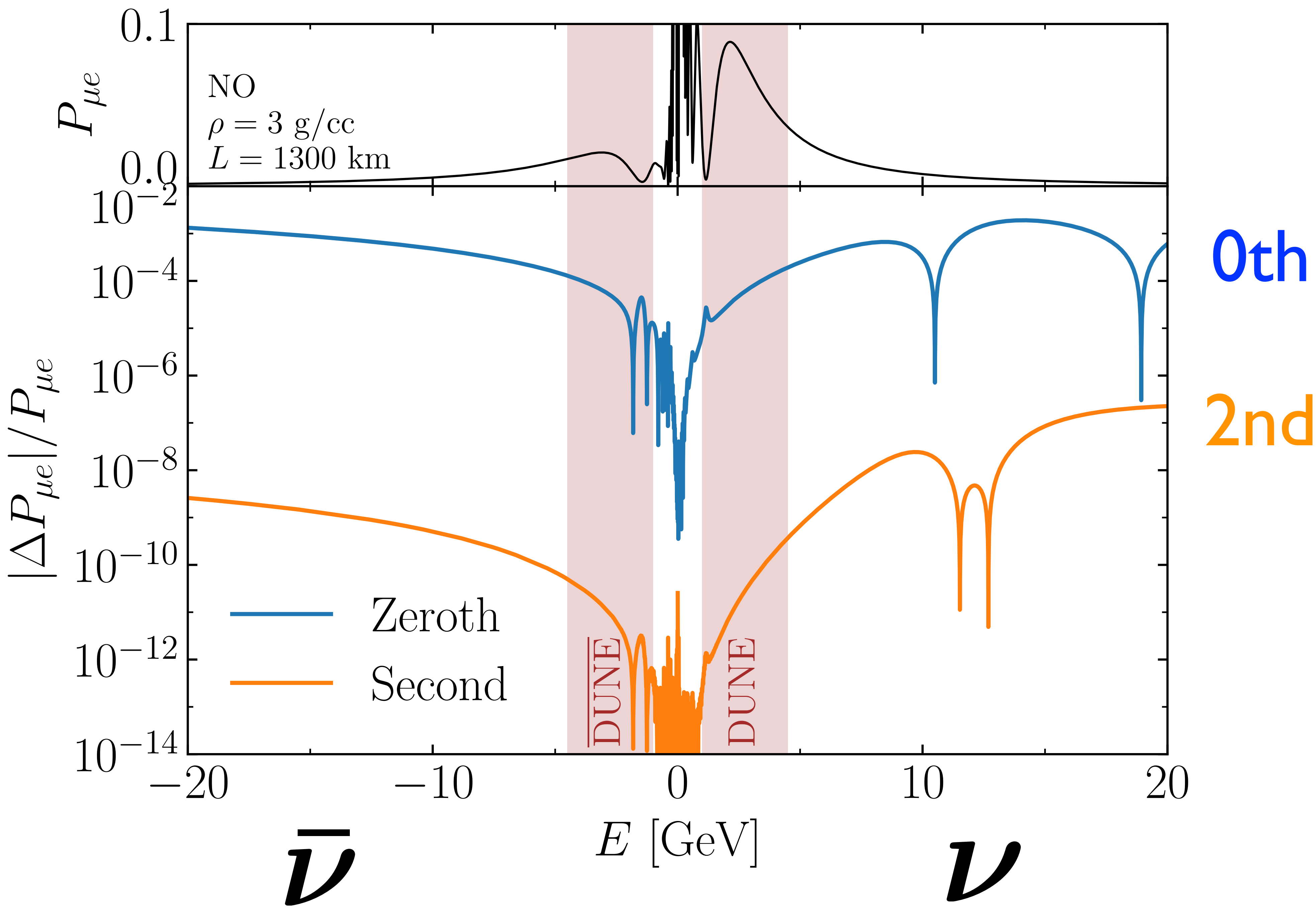
The eigenvalues then give us the eigenvectors (mixing angles)



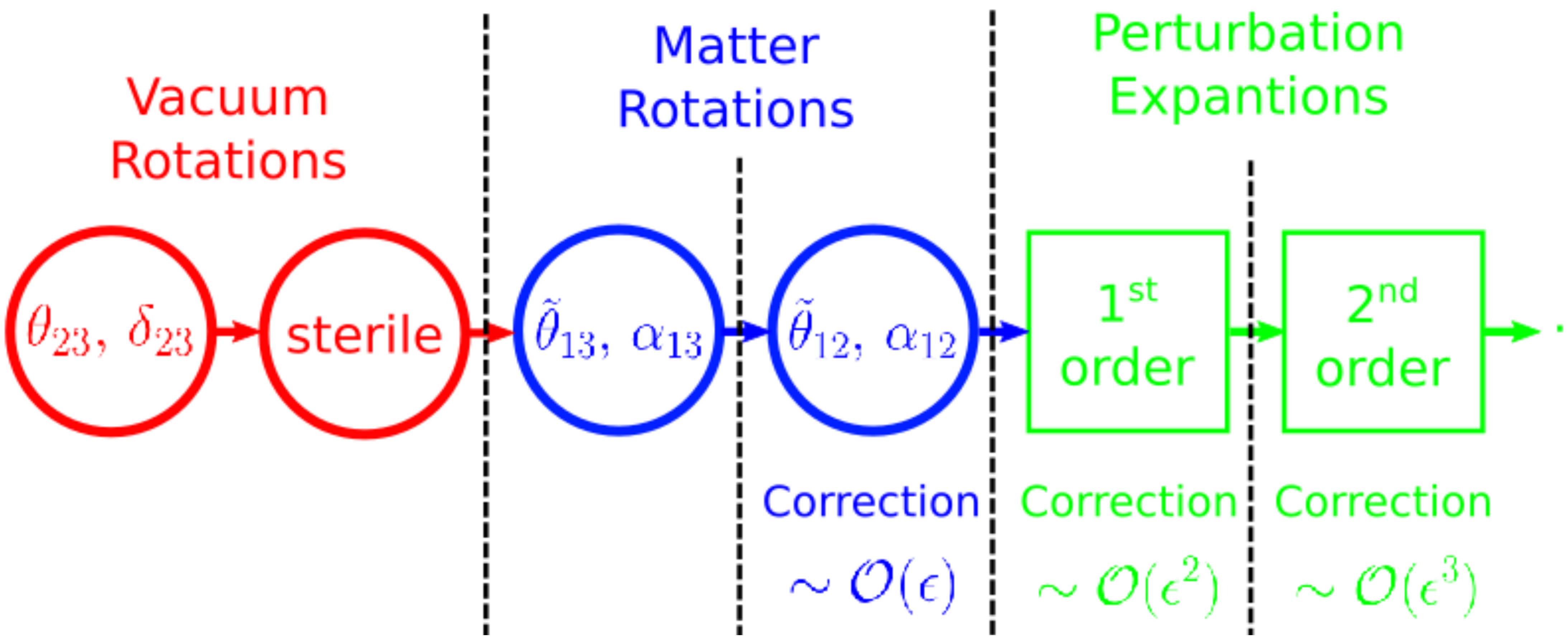


$$\tilde{\theta}_{13}, \tilde{\theta}_{12}$$

Oscillation Probabilities:



Adding Sterile Neutrinos: 3+1



SP, Xining Zhang: arXiv:1905.01356

Intrinsic CP Violation:

Jarlskog Invariant in Matter:

$$J \equiv s_{23}c_{23}s_{13}c_{13}^2 s_{12}c_{12}\sin\delta$$

CPV: $$\boldsymbol{\nu_\mu \to \nu_e}$$

$$\Delta_{jk} \equiv \frac{\Delta m_{jk}^2 L}{4E}$$

in Vacuum: $$8J\sin\Delta_{31}\sin\Delta_{32}\sin\Delta_{21}$$

in Matter: $$8\widehat{J}\sin\widehat{\Delta}_{31}\sin\widehat{\Delta}_{32}\sin\widehat{\Delta}_{21}$$

$$\widehat{J} \approx \frac{J}{S_{\odot}(a)\ S_{atm}(a)}$$

S's are two flavor resonance factors: a Solar and an Atmospheric one

$$S_{\odot}(a) = \sqrt{1 - 2\cos 2\theta_{12}\left(\frac{c_{13}^2 a}{\Delta m_{21}^2}\right) + \left(\frac{c_{13}^2 a}{\Delta m_{21}^2}\right)^2} = \left| \, 1 - \left(\frac{c_{13}^2 a}{\Delta m_{21}^2}\right) \, e^{i2\theta_{12}} \, \right|$$

c_{13}^2 correction to matter potential is important

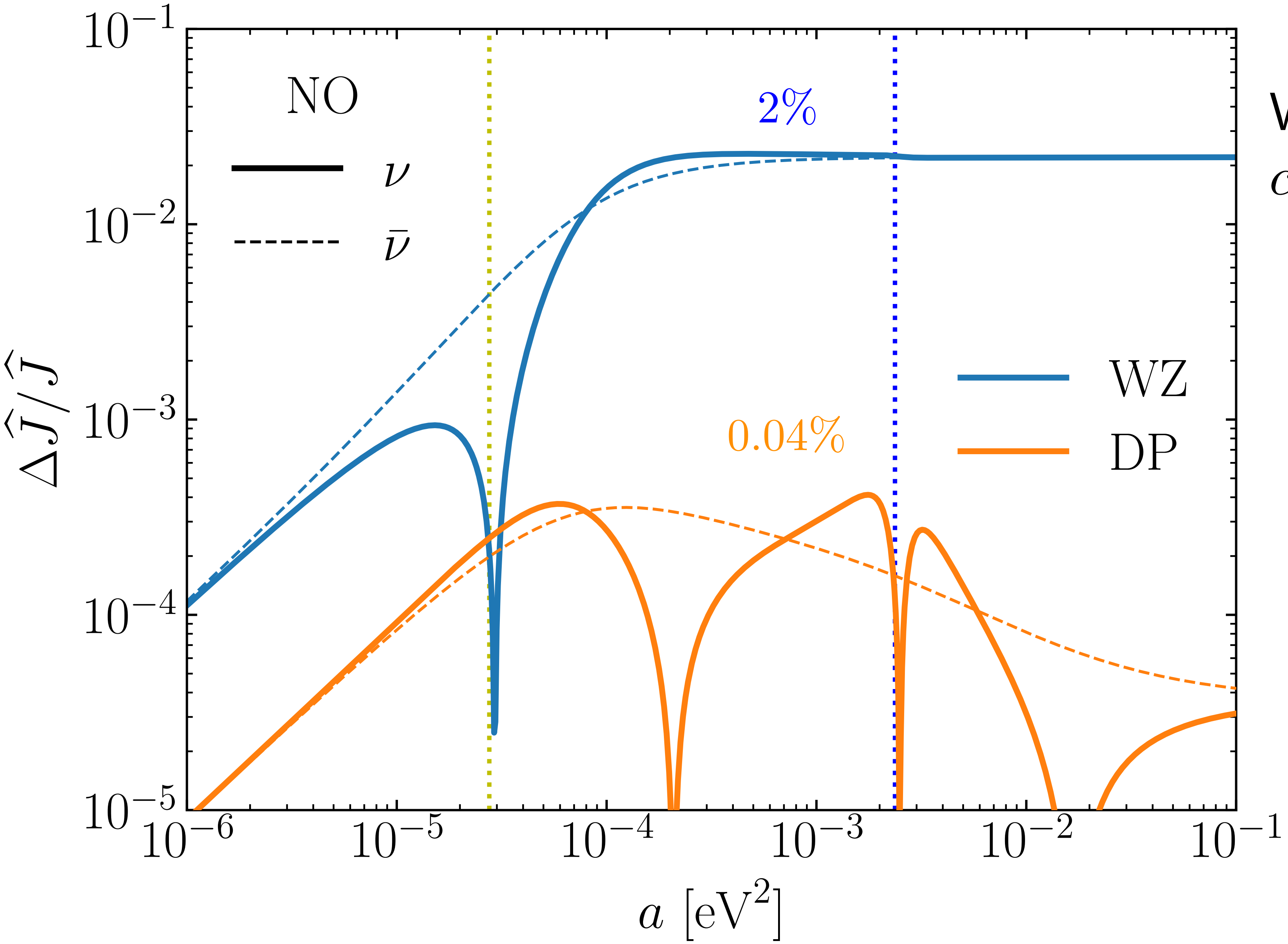
$$S_{atm}(a) = \sqrt{1 - 2\cos 2\theta_{13}\left(\frac{a}{\Delta m_{ee}^2}\right) + \left(\frac{a}{\Delta m_{ee}^2}\right)^2} = \left| \, 1 - \left(\frac{a}{\Delta m_{ee}^2}\right) \, e^{i2\theta_{13}} \, \right|$$

Δm_{ee}^2 not $\Delta m_{31/32}^2$ for position of the resonance is important

Denton + SP arXiv: 1902.07185

Fractional error for this approximation is 0.04%:

$s_{13}^2(\Delta m_{21}^2/\Delta m_{ee}^2)$ and $(\Delta m_{21}^2/\Delta m_{ee}^2)^2$



WZ: without both c_{13}^2 and Δm_{ee}^2

In Terms of Angles

$$J \equiv s_{23}c_{23}s_{13}c_{13}^2 s_{12}c_{12}\sin\delta$$

$$\sin 2\widehat{\theta}_{23}\sin\widehat{\delta} = \sin 2\theta_{23}\sin\delta\,.$$

Toshev ID exact

$$s_{\widehat{13}}c_{\widehat{13}} \approx s_{13}c_{13}\,/\,\mathcal{S}_{\rm atm}$$

DMP2016: +0.4%

$$c_{\widehat{13}}s_{\widehat{12}}c_{\widehat{12}} \approx c_{13}s_{12}c_{12}\,/\,\mathcal{S}_{\odot}$$

DP2019: -0.4%

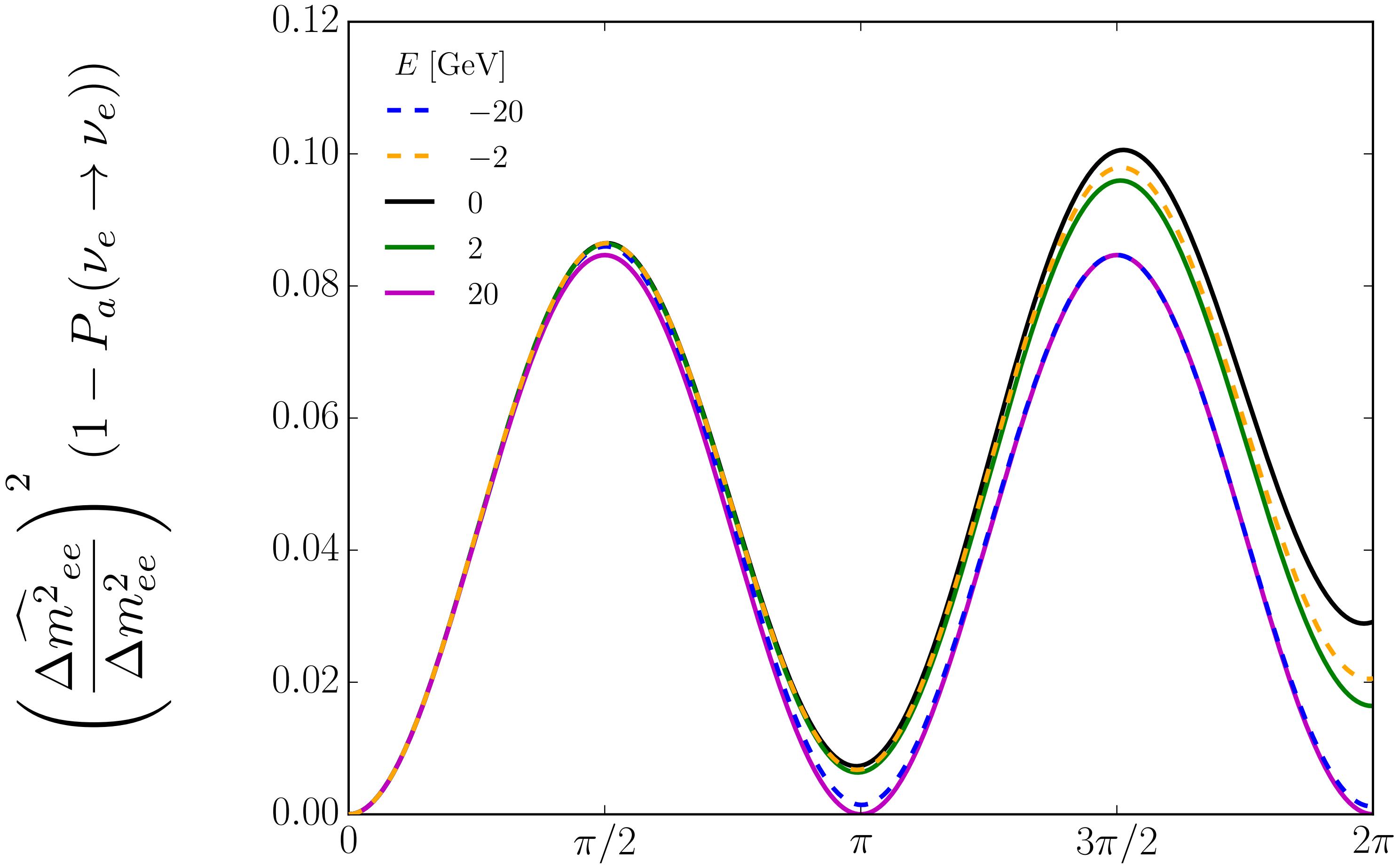
Combined 0.04%

(cancellation)

$\nu_e \rightarrow \nu_e$

$$P_a(\nu_e \to \nu_e) \approx 1 - \sin^2 2\theta_{13} \left(\frac{\Delta m^2_{ee}}{\widehat{\Delta m^2}_{ee}} \right)^2 \sin^2 \widehat{\Delta}_{ee} \,, \qquad \widehat{\Delta}_{ee} \equiv \widehat{\Delta m^2}_{ee} L/(4E) \,,$$

$$\widehat{\Delta m^2}_{ee} \approx \Delta m^2_{ee} \sqrt{(\cos 2\theta_{13} - a/\Delta m^2_{ee})^2 + \sin^2 2\theta_{13}} \,,$$

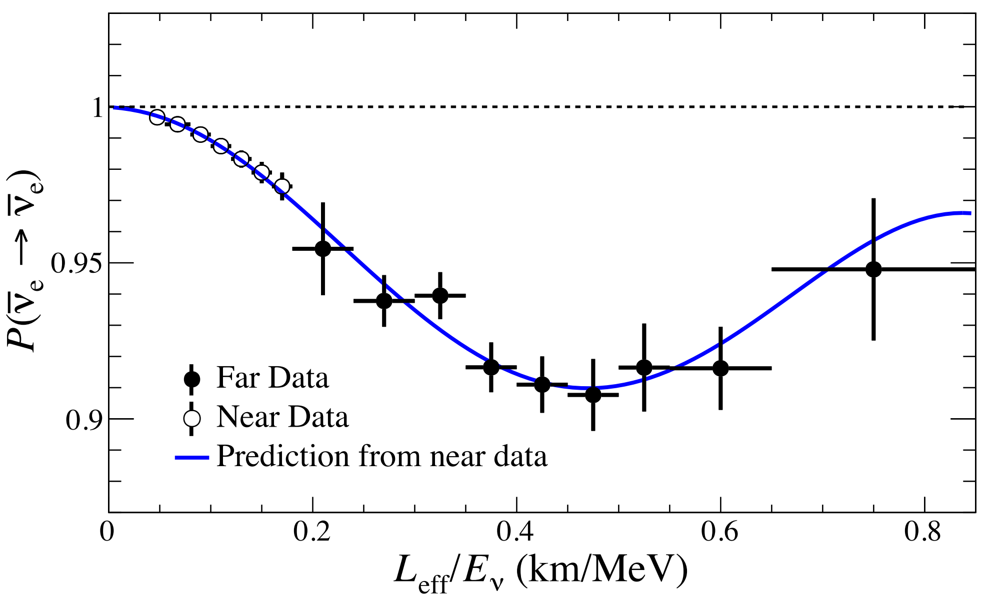


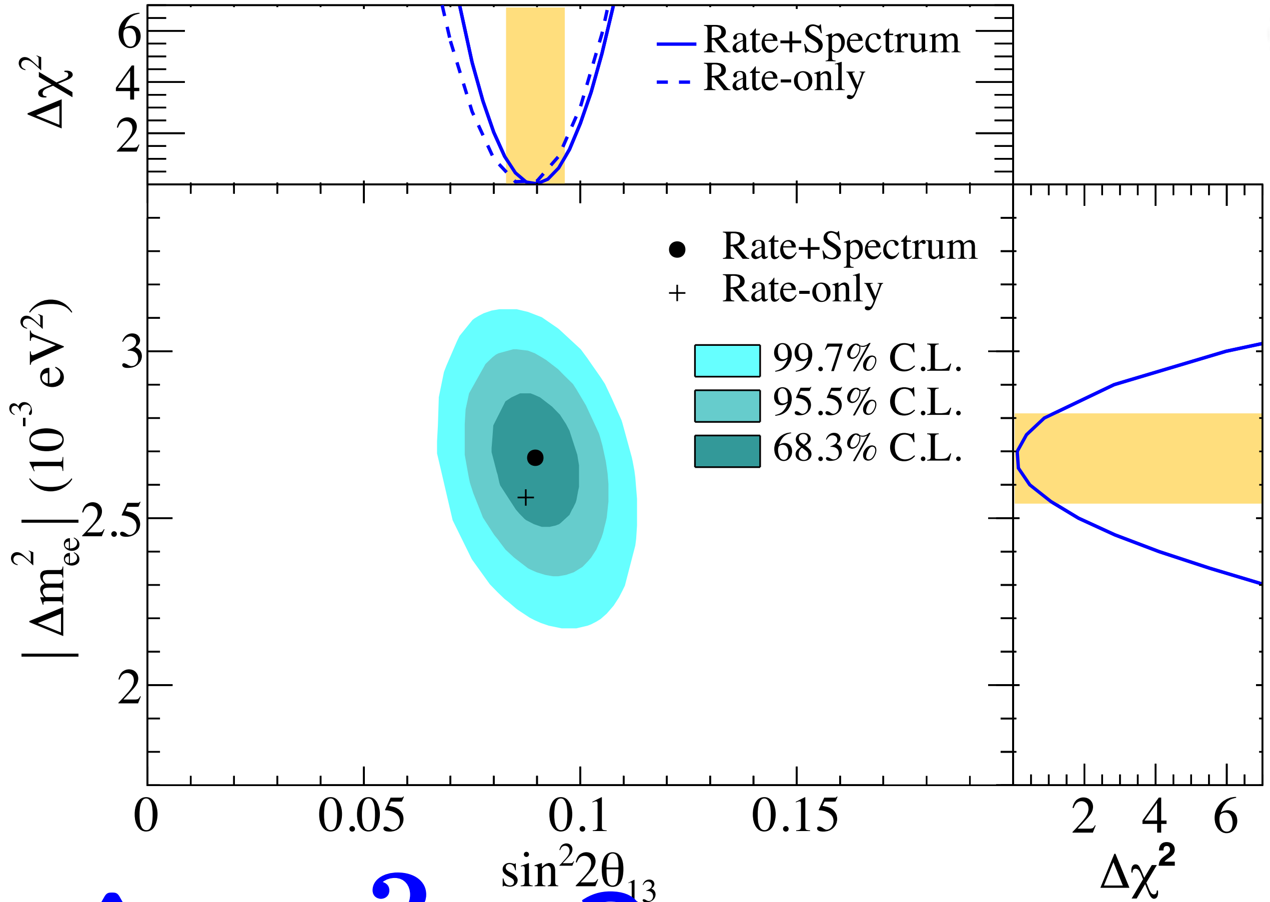
$|\widehat{\Delta}_{ee}|$

Denton, SP 1808.09453

Δm^2_{ee} and Daya Bay

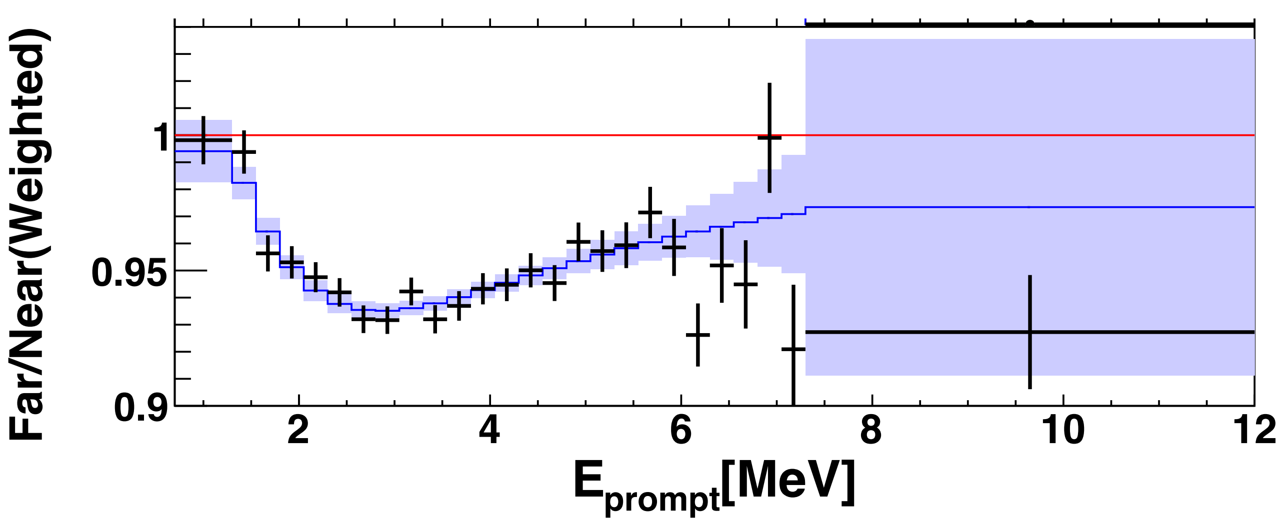
RENO 2200 days

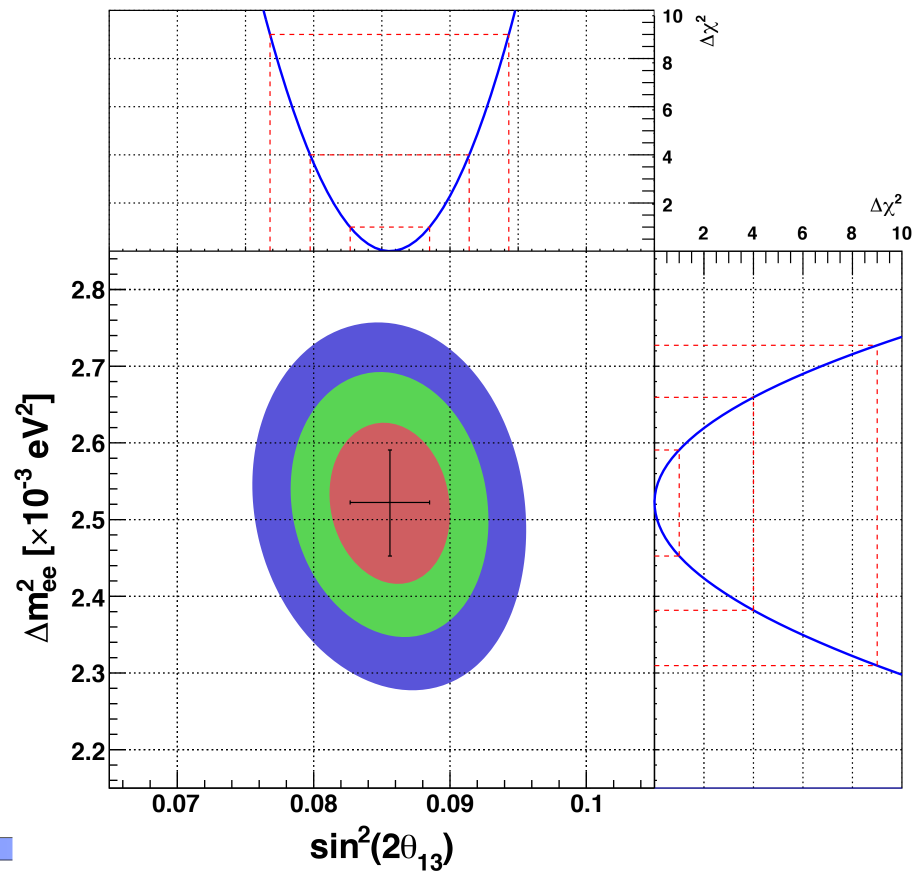




What is Δm^2_{ee} ?

Daya Bay 1958 days





~3%

$$P_x(\bar\nu_e \to \bar\nu_e) = 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \sin^2 2\theta_{13}\,(\cos^2\theta_{12}\sin^2\Delta_{31} + \sin^2\theta_{12}\sin^2\Delta_{32})$$

$\boldsymbol{\Delta m^2_{31} = \Delta m^2_{ee} + s^2_{12}\Delta m^2_{21}}$ and $\boldsymbol{\Delta m^2_{32} = \Delta m^2_{ee} - c^2_{12}\Delta m^2_{21}}$

where $\boldsymbol{\Delta m^2_{ee} = c^2_{12}\Delta m^2_{31} + s^2_{12}\Delta m^2_{32}}$

DB: $\Delta_{ee} \sim \pi/2$ and $\frac{\Delta m^2_{21}}{\Delta m^2_{ee}} \sim 0.03$, then $\Delta_{21} \sim \pi/60$

perform Taylor Series expansion:

$$P_{\text{xshort}}(\bar\nu_e \to \bar\nu_e) = 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \sin^2 2\theta_{13}\,[\,\sin^2|\Delta_{ee}| + \sin^2\theta_{12}\cos^2\theta_{12}\Delta^2_{21}\cos(2|\Delta_{ee}|)\,]$$

10^{-3}

No linear term in Δ_{21}

Mass Ordering comes in at Δ^3_{21}

- $P_{ee} \approx 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \sin^2 2\theta_{13}\sin^2\Delta_{ee}$

where $\Delta m^2_{ee} = c^2_{12}\Delta m^2_{31} + s^2_{12}\Delta m^2_{32}$

$\frac{\Delta P}{P} \sim 10^{-4}$ for Daya Bay and RENO

H. Nunokawa, S. J. Parke and R. Zukanovich Funchal,
"Another possible way to determine the neutrino mass hierarchy,"
Phys. Rev. D **72**, 013009 (2005), hep-ph/0503283

SP arXiv:1601.07464

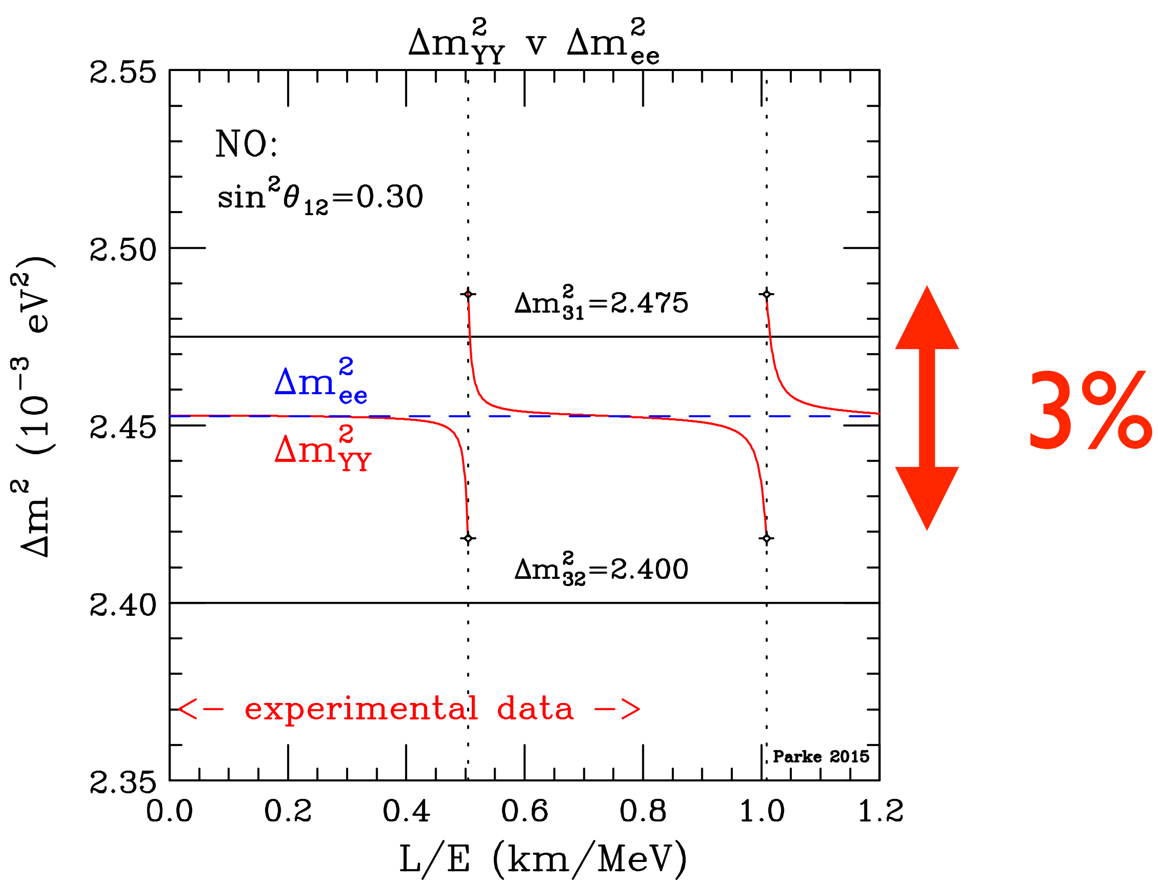
Daya Bay I:

- $P_{ee} \approx 1 - \cos^4\theta_{13}\sin^2 2\theta_{12}\sin^2\Delta_{21} - \sin^2 2\theta_{13}\sin^2\Delta_{YY}$

fit with constant Δm^2_{YY}

defn $$\Delta m^2_{YY} \equiv \left(\frac{4E}{L}\right)\arcsin\left[\sqrt{(\cos^2\theta_{12}\sin^2\Delta_{31} + \sin^2\theta_{12}\sin^2\Delta_{32})}\right]$$

(or $\sin^2\Delta_{YY} \equiv \cos^2\theta_{12}\sin^2\Delta_{31} + \sin^2\theta_{12}\sin^2\Delta_{32}$)



arXiv:1310.6732
+ 1505.03456v1

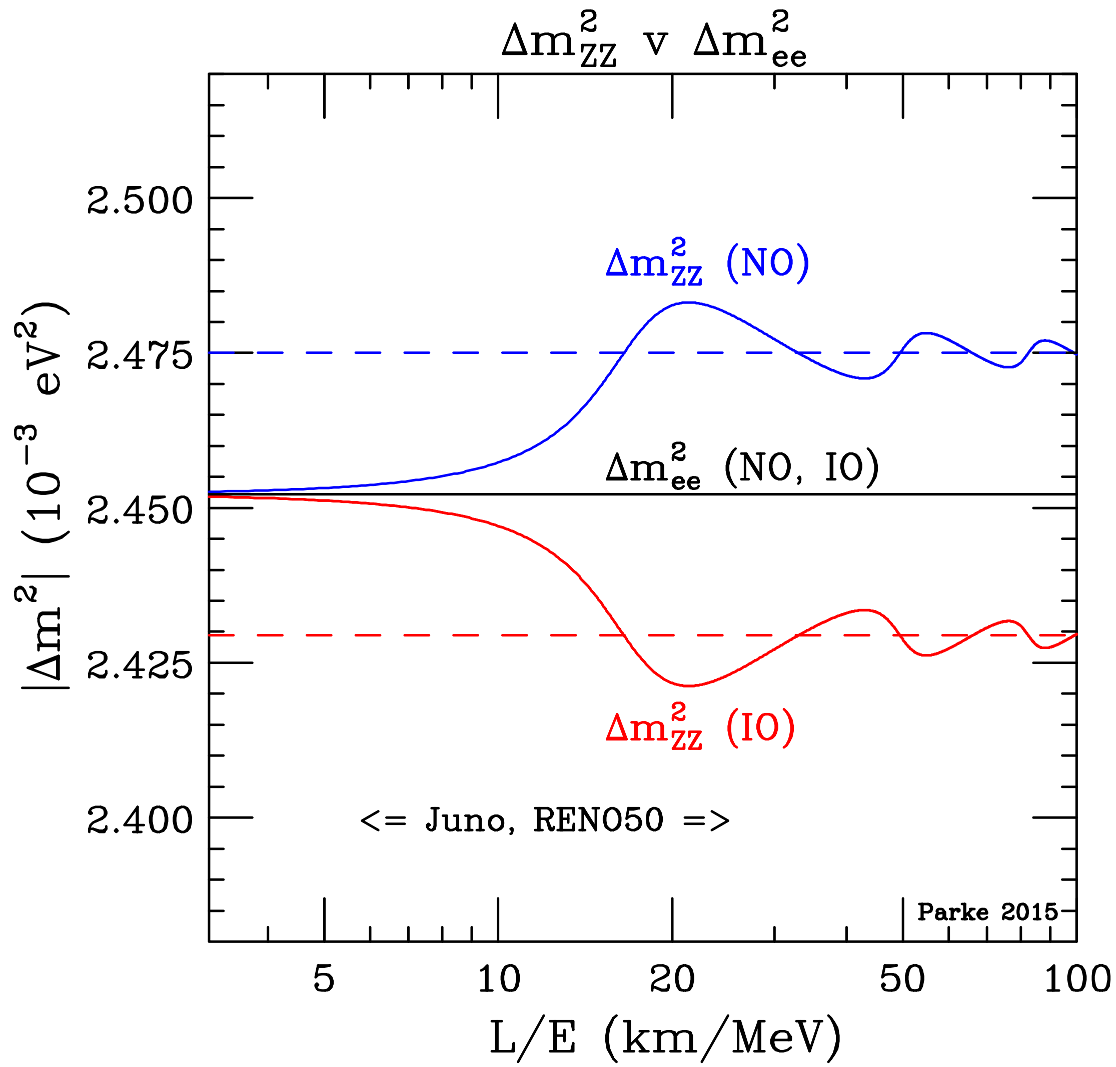
$$\Delta m^2_{ee}(\mathrm{DB2}) \equiv \Delta m^2_{32} + \frac{2E}{L}\arctan\left(\frac{\sin 2\Delta_{21}}{\cos 2\Delta_{21} + \tan^2\theta_{12}}\right)$$

Still L/E dependent:

OK for Daya Bay

but not useful JUNO

$$\Delta m^2_{ee}(\mathrm{DB2}) = \Delta m^2_{ee}(\mathrm{NPZ}) + \Delta m^2_{21}\left(\cos 2\theta_{12}\sin^2 2\theta_{12}/6\right)\Delta^2_{21} + \ldots$$



Daya Bay has three independent Δm^2 analyses/measurements:

$$\boldsymbol{\Delta m^2_{ee}},\ \ |\boldsymbol{\Delta m^2_{32}}|_{NO},\ \ |\boldsymbol{\Delta m^2_{32}}|_{IO}$$

if $\Delta m^2_{ee} \equiv c^2_{12}\Delta m^2_{31} + s^2_{12}\Delta m^2_{32}$ then

$$\frac{1}{2}(|\Delta m^2_{32}|_{IO} + |\Delta m^2_{32}|_{NO}) \quad = \quad \Delta m^2_{ee}$$

$$(DB)\ 2.523 \pm 0.069 \quad = \quad 2.522 \pm 0.069\ (DB), \quad \text{in units of } 10^{-3}\ \text{eV}^2$$

$$|\Delta m^2_{32}|_{IO} - |\Delta m^2_{32}|_{NO} \quad = \quad 2\cos^2\theta_{12}\ \Delta m^2_{21}$$

$$(DB)\ 0.104 \pm 0.097 \quad = \quad 0.104 \pm 0.004\ (PDG'18), \quad \text{in units of } 10^{-3}\ \text{eV}^2$$

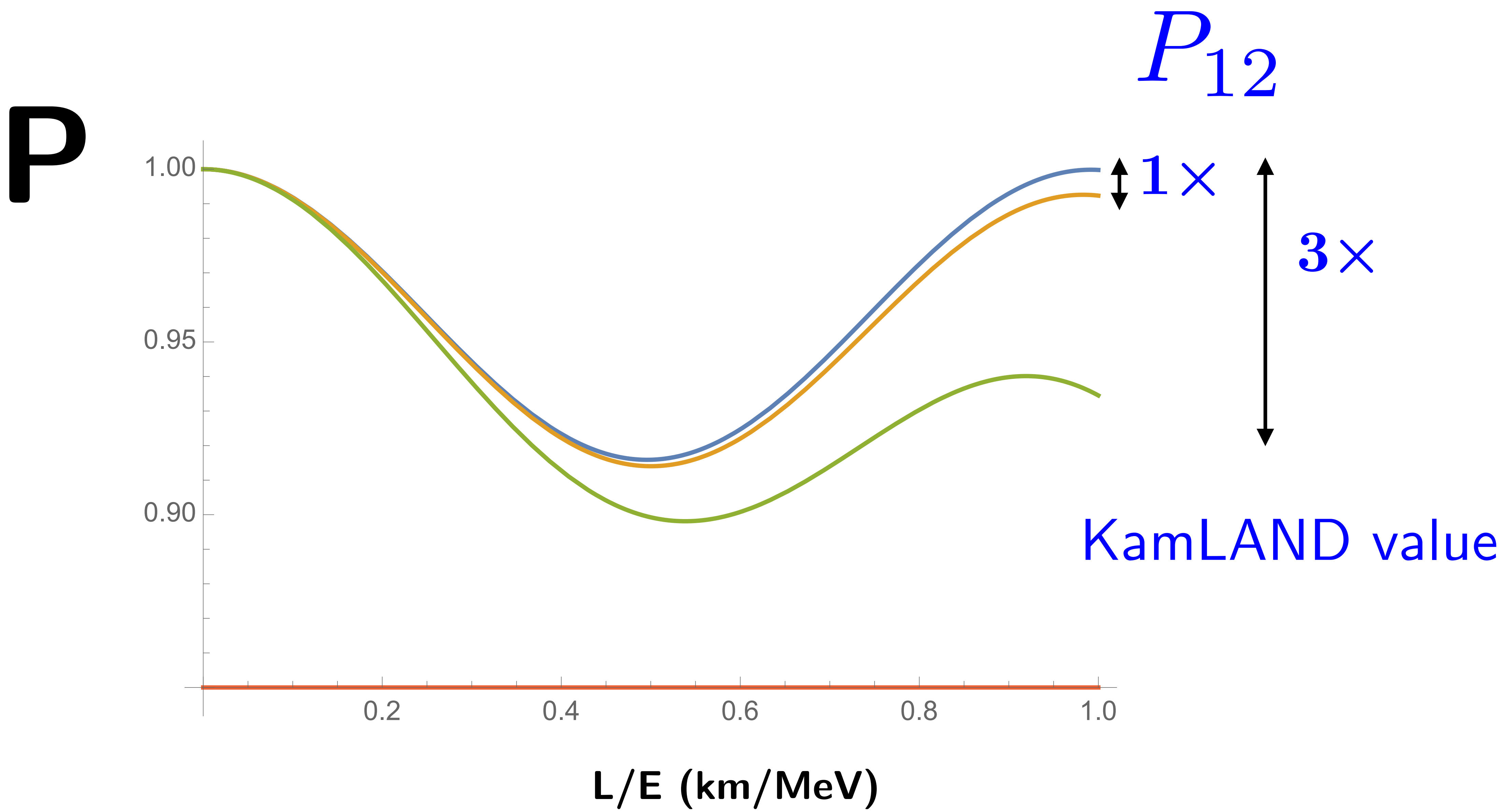
~93% ~4%

Not independent !

Δm^2_{21} and Daya Bay/RENO

Dependence on Solar Parameters: (monte carlo)

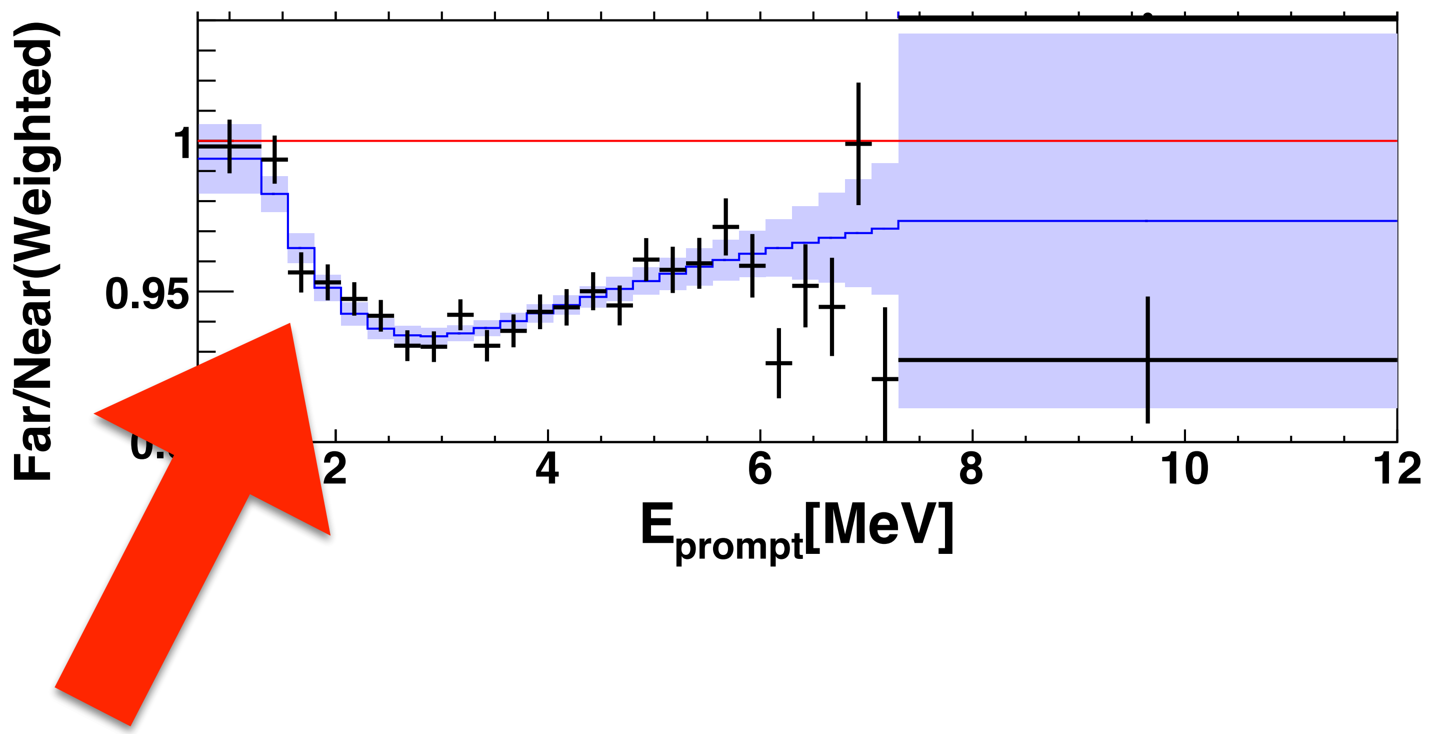
S.H. Seo and SP arXiv:1808.09150



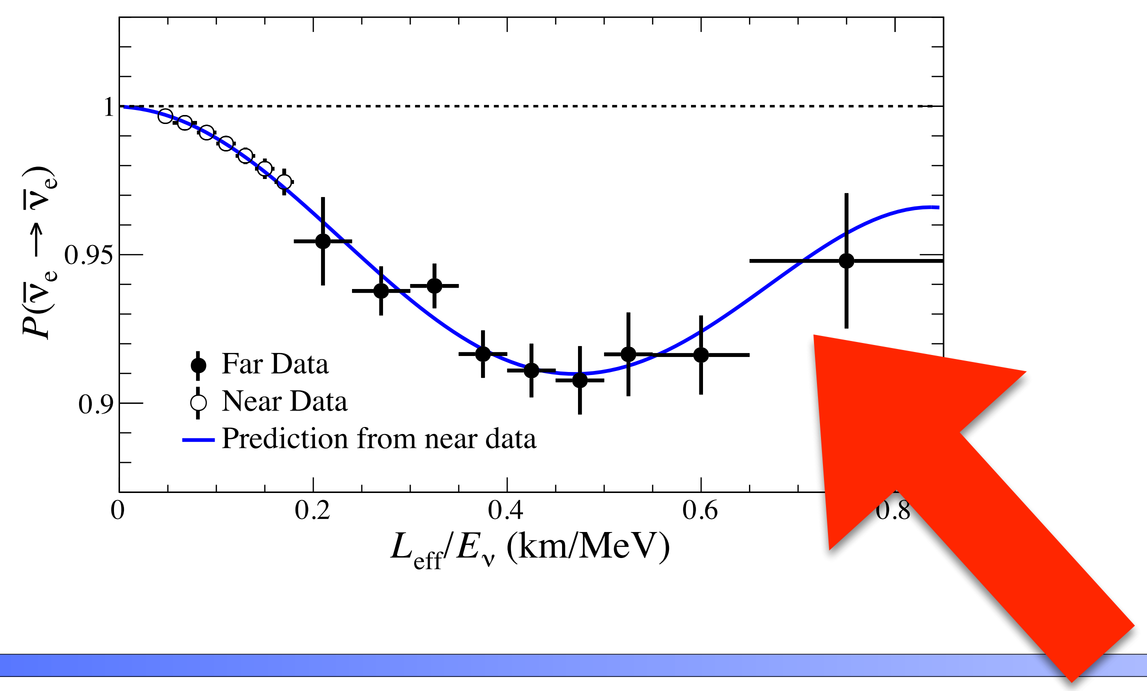
If Δm^2_{21} is 3 times bigger, P_{12} is 9 times larger !

dependence is on $\sin 2\theta_{12} \Delta m^2_{21}$

Daya Bay 1958 days

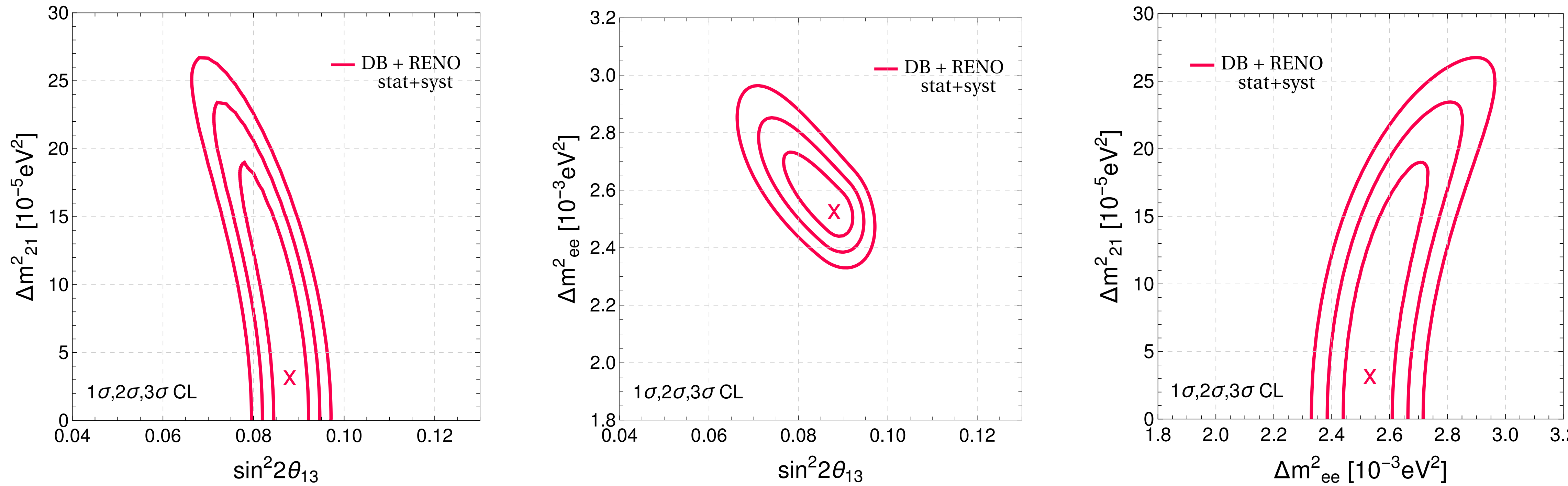


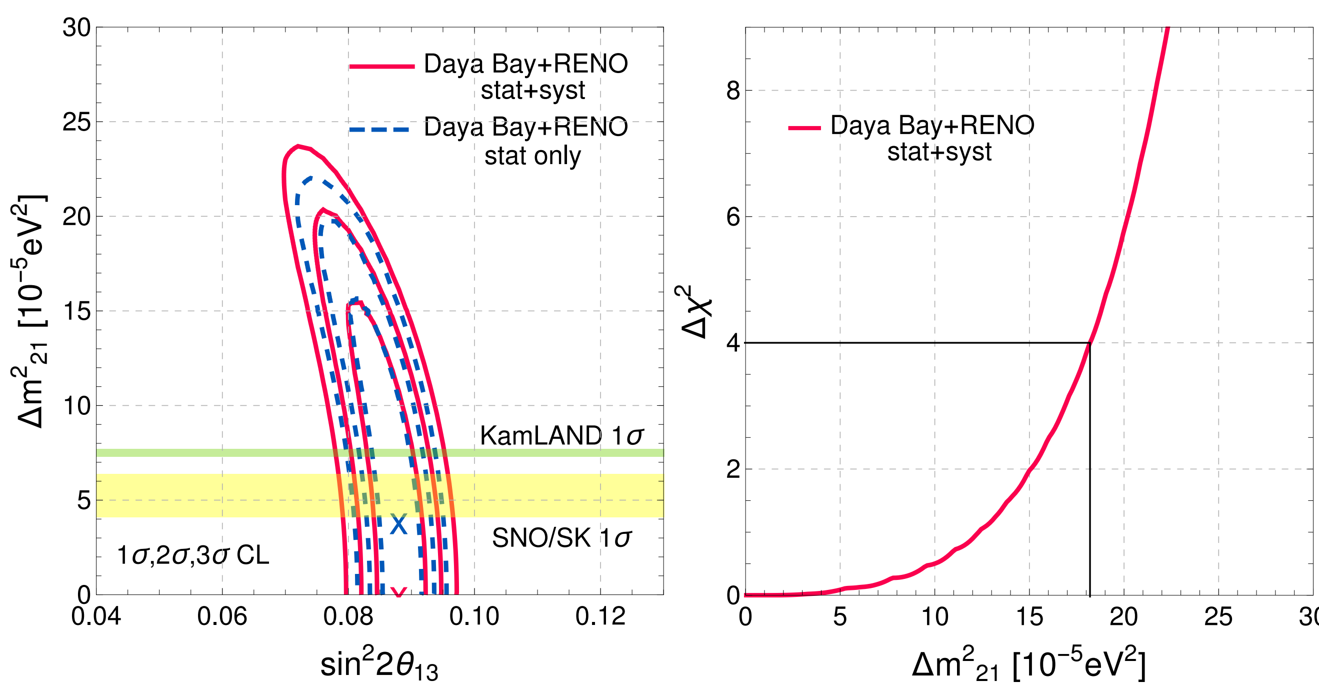
RENO 2200



Simultaneous Fit: $\sin^2\theta_{13}$, Δm^2_{ee} and Δm^2_{21}

Alvaro Hernandez-Cabezudo, SP, and Seon-Hee Seo arXiv:1905.09479





2.3 x KamLAND at 95% CL

Summary:

- from Nu1998 to now, tremendous progress on nuSM
- Wolfenstein matter effects play an extremely important role in Neutrino Flavor Transformation Physics
- 3 flavor mixing in Matter (and vacuum) needs better understanding as we enter the precision era
- to discover New Physics we need to understand and stress test the nuSM with superb precision