Efficient boundary condition estimation for continuous casting machinery

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Reduced Order Modelling, Simulation and Optimization of Coupled Systems (ROMSOC)

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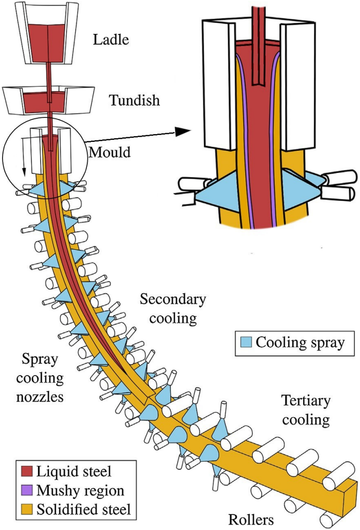
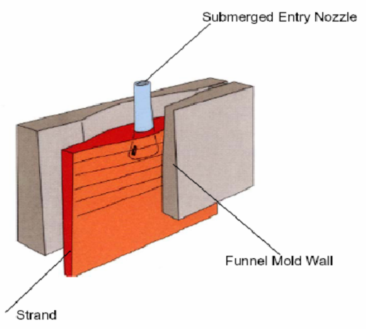
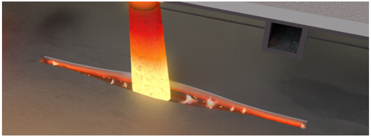
https://www.danieli.com/en/flat_products_43_2.htm

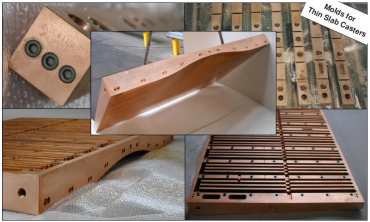
A caster



Credits: Klimes et al.

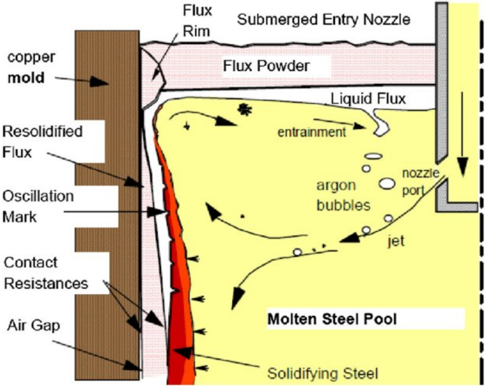






- Copper plates
- Cooled by water flowing in channels

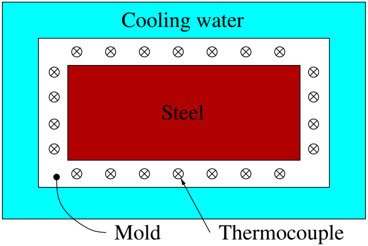
Credits: Carl Schreiber GmbH Neunkirchen, Pprime



Credits: Zhou et al.

Mold issues:

- Steel sticking to the mold
- Solid skin braking
- Cracks



Section of the mold

The casting is mainly controlled by changing the casting speed

Information available:

- Temperature measurements inside the mold
- Cooling water temperature increase
- Liquid steel level

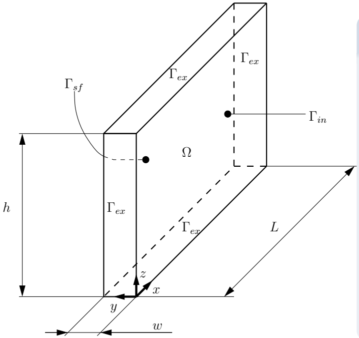
Temperatures measurements

$\Downarrow$

Computation of heat flux at the steel-mold interface in real-time

1. **Full Order Inverse Problem**
   - Alifanov's Regularization
   - Levenberg-Marquardt method

2. Reduced Order Inverse Problem



Mold model

Given $k \in \mathbb{R}^+$, $h \in \mathbb{R}^+$, $g \in L^2(\Gamma_{in})$ and $T_f \in L^2(\Gamma_{in})$. Find T such that

$$-k\Delta T(\mathbf{x}) = 0 \text{ on } \mathbf{x} \in \Omega,$$

$$\begin{cases} -k\nabla T(\mathbf{x}) \cdot \mathbf{n} = g(\mathbf{x}) & \text{in } \mathbf{x} \in \Gamma_{in}, \\ -k\nabla T(\mathbf{x}) \cdot \mathbf{n} = 0 & \text{in } \mathbf{x} \in \Gamma_{ex}, \\ -k\nabla T(\mathbf{x}) \cdot \mathbf{n} = h(T(\mathbf{x}) - T_f(\mathbf{x})) & \text{in } \mathbf{x} \in \Gamma_{sf}. \end{cases}$$

- T - Mold temperature
- T_f - Cooling water temperature
- h - Heat transfer coefficient
- k - Copper thermal conductivity
- g - Steel-mold heat flux (unknown)

Temperatures measurements

$\Downarrow$

Computation of heat flux at the steel-mold interface

Inverse problem

Given the temperature measurements $\tilde{T}(\mathbf{x}_i) \in \mathbb{R}^+$, $i = 1, 2, \ldots, M$, find $g(\mathbf{x}) \in L^2(\Gamma_{in})$ which minimizes the functional

$$J[g] = \frac{1}{2} \sum_{i=1}^{M} [T[g](\mathbf{x}_i) - \tilde{T}(\mathbf{x}_i)]^2,$$

where $T[g](\mathbf{x})$ is solution of the direct problem.

Temperatures measurements

$\Downarrow$

Computation of heat flux at the steel-mold interface

Inverse problem

Given the temperature measurements $\tilde{T}(\mathbf{x}_i) \in \mathbb{R}^+$, $i = 1, 2, \ldots, M$, find $g(\mathbf{x}) \in L^2(\Gamma_{in})$ which minimizes the functional

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where $T[g](\mathbf{x})$ is solution of the direct problem.

Ill-posed problem $\rightarrow$ Requires regularization

Regularization techniques:

- **Alifanov's regularization**
  - Conjugate gradient method applied to the adjoint problem
- Levenberg-Marquardt method
  - Parameterization of the heat flux $g(\mathbf{x}) = \sum_{i=1}^{N} w_i \gamma_i(\mathbf{x})$

Alifanov's Regularization

Set $g^0(\mathbf{x})$;
while $n \leq nMax$ **do**
    Solve direct problem;
    Compute J;
    if *convergence* **then**
        Stop;
    Solve adjoint problem $\rightarrow$ gradient of J, J' ;
    $\gamma^n = \frac{\int_{\Gamma_{s_{in}}} [J'_{g^n}(\mathbf{x})]^2 d\mathbf{x}}{\int_{\Gamma_{s_{in}}} [J'_{g^{n-1}}(\mathbf{x})]^2 d\mathbf{x}}$;
    Search direction, $P^n(\mathbf{x}) = J'_{g^n}(\mathbf{x}) + \gamma^n P^{n-1}(\mathbf{x})$;
    Solve sensitivity problem ;
    $\beta^n = \arg\min_\beta J[g^n - \beta P^n] = \frac{\sum_{i=1}^M \{T[g^n](\mathbf{x}_i) - \tilde{T}(\mathbf{x}_i)\}\delta T[P^n](\mathbf{x}_i)}{\sum_{i=1}^M (\delta T[P^n](\mathbf{x}_i))^2}$;
    $g^{n+1} = g^n - \beta^n P^n$;
    $n = n + 1$;

Adjoint problem

$$\frac{1}{k}\Delta\lambda(\mathbf{x}) + \sum_{i=1}^{M}(T[g](\mathbf{x}_i) - \tilde{T}(\mathbf{x}_i))\delta(\mathbf{x} - \mathbf{x}_i) = 0, \text{ on } \Omega,$$

$$\begin{cases} \frac{1}{k}\nabla\lambda(\mathbf{x})\cdot\mathbf{n} = 0 & \text{in } \Gamma_{in}\cup\Gamma_{ex}, \\ \frac{1}{k}\nabla\lambda(\mathbf{x})\cdot\mathbf{n} + \frac{1}{k^2}h\lambda(\mathbf{x}) = 0 & \text{in } \Gamma_{sf}. \end{cases}$$

Sensitivity problem

$$-k\Delta\delta T(\mathbf{x}) = 0, \text{ on } \Omega,$$

$$\begin{cases} -k\nabla\delta T(\mathbf{x})\cdot\mathbf{n} = P^n(\mathbf{x}) & \text{in } \Gamma_{in}, \\ -k\nabla\delta T(\mathbf{x})\cdot\mathbf{n} = 0 & \text{in } \Gamma_{ex}, \\ -k\nabla\delta T(\mathbf{x})\cdot\mathbf{n} = h(\delta T(\mathbf{x})) & \text{in } \Gamma_{sf}. \end{cases}$$

$$g(\mathbf{x}) = -x \cdot z \cdot 10^5 \, \frac{\text{W}}{\text{m}^2}$$

$$\Downarrow$$

Finite Volume discretization

$$A\mathbf{T} = \mathbf{b}_g + \mathbf{b}$$

$$\Downarrow$$

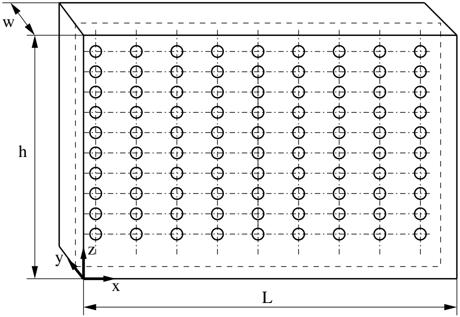
Temperature measurements, $\tilde{T}$

$$\Downarrow$$

Inverse problem solver

$$\Downarrow$$

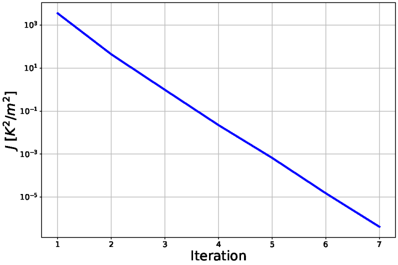
Estimated heat flux, $g(\mathbf{x})$



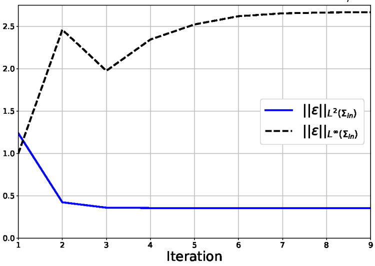
Positions of the thermocouples

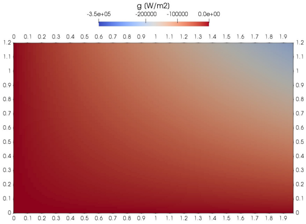
Results

$J = \frac{1}{2}\sum_{i=1}^{M}[T[g](\mathbf{x}_i) - \tilde{T}(\mathbf{x}_i)]^2$



L^2 and L^∞ norm of the relative error, ϵ





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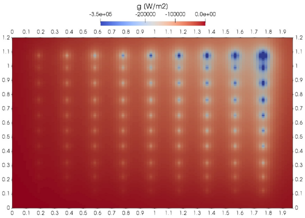
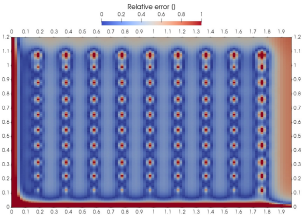


Reconstructed heat flux



Relative error, ϵ

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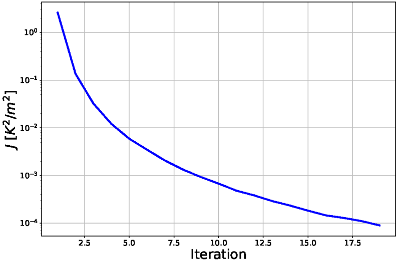
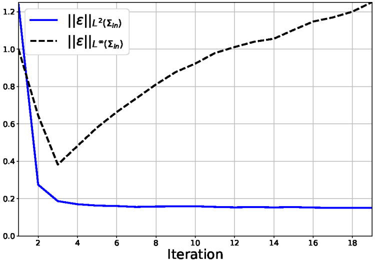
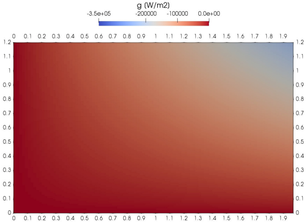


L^2 and L^∞ norm of the relative error, ϵ





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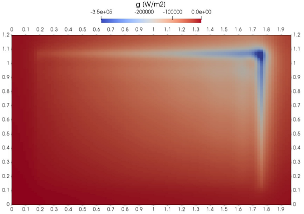

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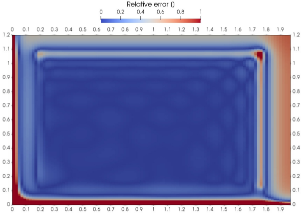

Relative error, ϵ

Regularization techniques:

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- **Levenberg-Marquardt method**
  - Parameterization of the heat flux $g(\mathbf{x}) = \sum_{i=1}^{N} w_i \gamma_i(\mathbf{x})$

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Inverse problem

Given the temperature measurements $\tilde{T}(\mathbf{x}_i) \in \mathbb{R}^+$, $i = 1, 2, \dots, M$, find $\mathbf{w} \in \mathbb{R}^N$ which minimizes the functional

$$J[g] = \frac{1}{2} \sum_{i=1}^{M} [T[g](\mathbf{x}_i) - \tilde{T}(\mathbf{x}_i)]^2,$$

where $T[g](\mathbf{x})$ is solution of

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Levenberg-Marquardt Algorithm

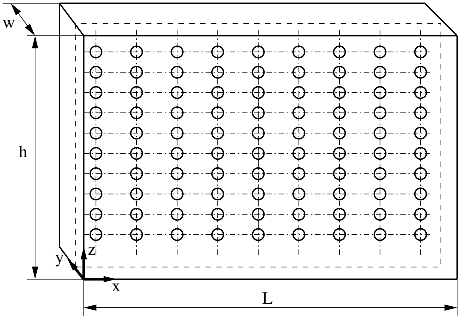
```
Set w^0;
while n ≤ nMax do
    Solve direct problem;
    Compute J;
    if convergence then
        Stop;
    Compute the Jacobian, 𝒥 ;
    Solve [(𝒥^n)^T 𝒥^n − s^n I]δw^n = −(𝒥^n)^T R^n ;
    Update weights w^{n+1} = w^n + δw^n ;
    n = n + 1;
```

- $\mathcal{J}_{ij} = \frac{\partial T_i[\mathbf{w}]}{\partial w_j}$
- s - Regularization factor

Numerical Test

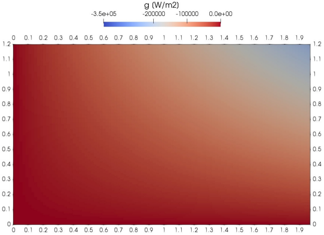
Basis functions $\gamma_i(\mathbf{x})$ are Gaussian Radial Basis Functions centered at the projection of the thermocouples on the boundary Γ_{in}

$$\gamma_i(\mathbf{x}) = e^{-\alpha^2 r_i(\mathbf{x})^2}$$



Positions of the thermocouples

Results



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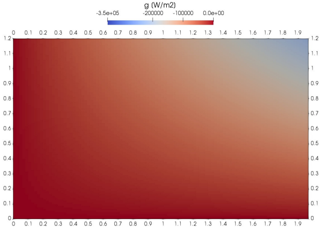
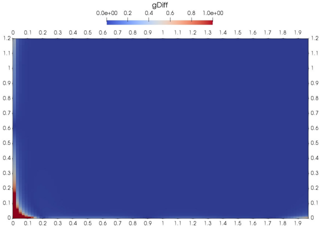


Reconstructed heat flux



Relative error, ϵ

Real Time Computation

- Approx. **40 seconds** required for the solution
- For **real time computation** the we have to reduce the computation time to less than 1 second
- We use Reduced Basis Method to reduced the cost of solving the direct problem

Parameterized PDE

Direct problem

$$-k\Delta T(\mathbf{x}) = 0 \text{ on } \mathbf{x} \in \Omega,$$

$$\begin{cases} -\,k\nabla T(\mathbf{x})\cdot\mathbf{n} = \sum_{i=1}^{N} w_i\gamma_i(\mathbf{x}) & \text{in } \mathbf{x}\in\Gamma_{in}, \\ -k\nabla T(\mathbf{x})\cdot\mathbf{n} = 0 & \text{in } \mathbf{x}\in\Gamma_{ex}, \\ -k\nabla T(\mathbf{x})\cdot\mathbf{n} = h(T(\mathbf{x}) - T_f(\mathbf{x})) & \text{in } \mathbf{x}\in\Gamma_{sf}. \end{cases}$$

The parameters are the weights **w**

POD-Galerkin approach

$\Downarrow$

we have to sample the parameter space

$\Downarrow$

Reduction of the parameter space, i.e. the dimension of **w**

POD for Heat Flux Basis

Experimental measurements from a real mold, $\tilde{T}_i, i = 1, 2, \ldots, M$

$\Downarrow$

Solve inverse problem, obtain $g(\mathbf{x})$ for each set of measurements

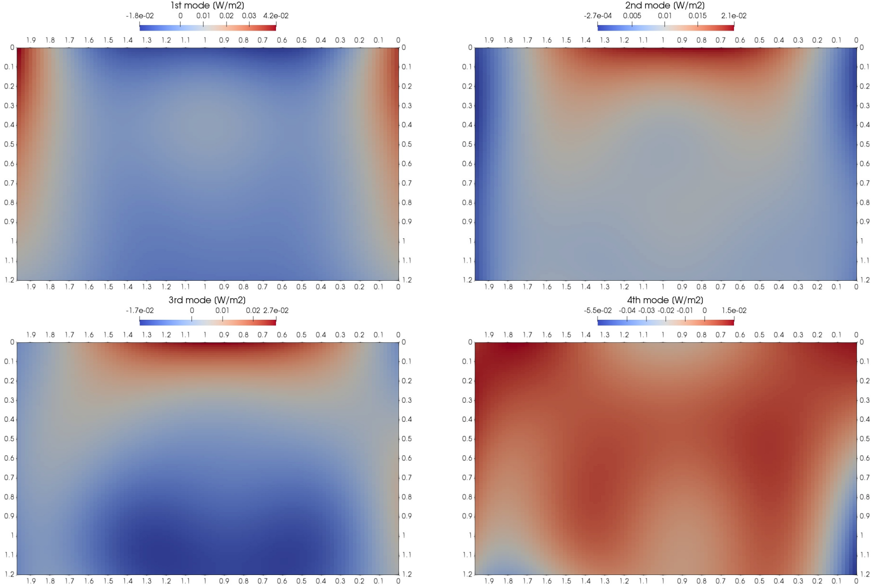
$\Downarrow$

Perform a Proper Orthogonal Decomposition (POD) on the obtained set of heat flux, $g(\mathbf{x})$

$\Downarrow$

Use the first few modes, $\gamma_r(\mathbf{x})$, $r = 1, 2, \ldots, R$, as basis for the heat flux, $g(\mathbf{x}) = \sum_{r=1}^{R} w_r \gamma_r(\mathbf{x})$

Heat Flux POD Modes



Reduced Basis for Direct Problem

- Having reduced the number of parameters, we can sample the parameter space and obtain a set of solutions of the direct problem (snapshots)

$$\mathbb{V}_T = \text{span}(\boldsymbol{T}_1, \boldsymbol{T}_2, \ldots, \boldsymbol{T}_S)$$

- POD on solution space to obtain orthonormal basis $\boldsymbol{\phi}$

$$\mathbb{V}_T = \text{span}(\boldsymbol{\phi}_1, \boldsymbol{\phi}_2, \ldots, \boldsymbol{\phi}_S),$$

- Select the first few modes to have a reduced basis spaces $\mathbb{V}_{T_{RB}}$

$$\mathbb{V}_{T_{RB}} = \text{span}(\boldsymbol{\phi}_1, \boldsymbol{\phi}_2, \ldots, \boldsymbol{\phi}_{N_r}), N_r << N_h$$

- Approximation of full order fields by linear combinations of the modes

$$T \approx \sum_{i=1}^{N_r} T_{r_i} \phi_i$$

Reduced Basis for Direct Problem

– Galerkin projection of the full order model on the reduced basis

$$L := \begin{bmatrix} | & & | \\ \phi_1 & \dots & \phi_{N_r} \\ | & & | \end{bmatrix} \in \mathbb{R}^{N_h \times N_r}, \mathbf{T} = L\mathbf{T}_r$$

Full order, N_h unknowns

$$A\boldsymbol{T} = \boldsymbol{b}_g + \boldsymbol{b}$$

$$\Downarrow$$

$$L^T A L \boldsymbol{T}_r = L^T \boldsymbol{b}_g + L^T \boldsymbol{b}_T$$

$$\Downarrow$$

Reduced order, N_r unknowns

$$A_r \boldsymbol{T}_r = L^T G \boldsymbol{w} + \boldsymbol{b}_r = G_R \boldsymbol{w} + \boldsymbol{b}_r$$

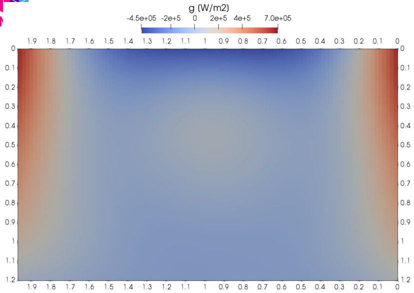
Reduced Order Levenberg-Marquardt Regularization

Offline

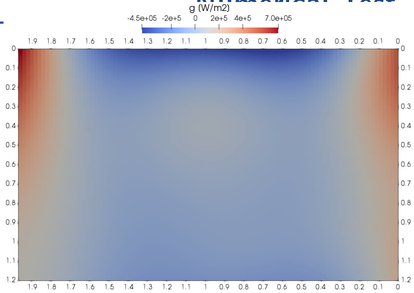
- Solve full order inverse problem with meaningful set of experimental measurements
- Perform POD on heat flux samples
- Compute snapshot for direct problem
- POD on snapshot set
- Assemble $A_r, G_r, \mathbf{b}_r$

Online

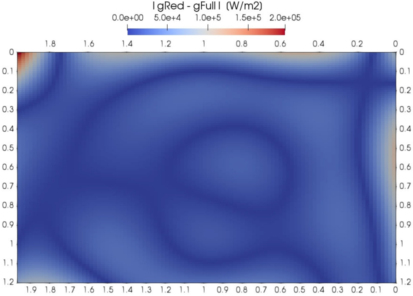
- Use Levenberg-Marquardt Regularization



Full order



Reduced order



Error

Computational cost of the online phase

$\approx$ 2 seconds

Conclusions

- Implemented full order methodology
- Developed reduced method for inverse problem

Future Work

- Error estimate
- Move to Bayesian approach
- Study noise on input

Thank you