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Neutrosophic Vague Binary Sets

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Abstract: Vague sets and neutrosophic sets play an inevitable role in the developing scenario of mathematical world. In this modern era of artificial intelligence most of the real life situations are found to be immersed with unclear data. Even the newly developed concepts are found to fail with such problems. So new sets like Plithogenic and new combinations like neutrosophic vague arose. Classical set theory dealt with single universe and can be studied by taking it's subsets. Situations demand two universes instead of a unique one in certain problems. In this paper two universes are introduced simultaneously and under consideration in a neutrosophic vague environment. It's basic operations, topology and continuity are also discussed with examples. A real life example is also discussed.

Keywords: binary set, fuzzy binary set, vague binary set, neutrosophic vague binary sets, neutrosophic vague binary topology, neutrosophic vague binary continuity

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1. Introduction

Functions are tightly packed but relations are not. They are more general than functions. Decimal system deals with ten digits while binary with two - only with 0 and 1. For detecting electrical signal's on or off state binary system can be used more effectively. It is the prime reason of selecting binary language in computers. Binary operations in algebra will give another idea! After a binary operation, 'operands' produce an element which is also a member of the parent set - means 'domain and co-domain' are in the same set. But binary relations are quite different from the ideas mentioned above. They are subsets of the cartesian product of the sets under consideration, taken in a special way. It is clear that binary stands for two. In point-set topology information from elements of topology will give information about subsets of the universal set under consideration. But real life can't be confined into a single universal set. It may be two or more than two. Being an extension of classical sets [George Cantor, 1874-1897] [27], fuzzy sets (FS's) [Zadeh, 1965] [29] can deal with partial membership. In intuitionistic fuzzy sets (IFS's) [Attanassov, 1986] [12] two membership grades are there - truth and false. As an extension of fuzzy sets Gau and Buehrer [9] introduced vague sets in 1993. Neutro-sophy means knowledge of neutral thought. It is a new branch of philosophy introduced by Florentin Smarandache [6] in 1995 - by giving an additional component - indeterminacy. Movement of paradoxism was set up by him in early 1980's. New concept dealt with the principle of using non-artistic elements to set artistic. Within no time so many hybrid structures developed by using the merits of the newly developed theory. In 2014, Alblowmi. S. A and Mohmed Eisa [1] gave some new concepts of neutrosophic sets. In 1996, Dontchev [5] developed Contra-continous functions and strongly s-closed spaces. In 2014, Salama A.A, Florentin Smarandache and Valeri Kromov [25] developed neutrosophic closed set and neutrosophic continous functions.

Shawkat Alkhazaleh [26] introduced the concept of neutrosophic vague in 2015. To loosen the hard structure of classical sets, Molodtsov [15] introduced soft set theory in 1999. In 2017, Gulfam Shahzadi, Muhammad Akram and Arsham Borumand Saeid [10] gave an application via 3 different methods of single-valued neutrosophic sets in medical field. Mai Mohamed et al., [19] developed a critical path problem in network diagrams under uncertain activity time. Later in 2018, Mohamed Abdel Basset et al., [23] developed a project selection method using TOPSIS and trapezoidal neutrosophic number. Mai Mohamed et al., [21] made a medical application to aid cancer patients based on neutrosophic set theory. As an extenstion to crisp, fuzzy, intuitionistic and neutrosophic sets, Florentin Smarandache [7] introduced plithogenic sets in 2018. In 2018, Mary Margaret A, Trinita Pricilla M [14] developed neutrosophic vague generalized Pre-continous and irresolute mappings. In 2018, Mohamed Abdel-Basset, Asmaa Atef, Florentin Smarandache [18] introduced a hybrid neutrosophic multiple criteria group decision making approach for project selection. In 2018, Vildan Cetkin and Halis Ayg$\ddot{u}n$ [28] developed an approach to neutrosophic ideals. Later in 2019, Mohamed Abdel-Basset et al., [22] applied plithogenic aggregation operators to a decision making method for projects viz., 'supply chain sustainability'. In 2019, Mohamed Abdel-Basset and Mai-Mohamed [20] introduced linear fractional programming based on triangular neutrosophic numbers. In 2019, Mohamed Abdel- Basset, Gunasekaran Manogaran et al., [17] developed a neutrosophic multi criteria decision making method for type 2 diabetic patients. In 2019, Hazwani Hashim, Lazim Abdullah and Asharaf Al-Quran [11] developed interval neutrosophic vague sets. In 2019, Mohamed Abdel-Basset, El-hosney, M., Gamal., & Smarandache.F [16] gave a new model for evaluation hospital medical care systems based on plithogenic sets. In 2019 Banu Priya et al., [2] investigated neutrosophic αgs continuity and neutrosophic αgs irresolute maps. In 2019, Dhavaseelan et al., [3] introduced neutrosophic α^m-closed setsand discussed it's continuity, strongly continuity and irresoluteness. In 2019 Dhavaseelan, Subash Moorthy and S. Jafari [4] introduced gNcompact open topology and discussed on generalized neutrosophic exponential map. In 2019, Muhammad Akram et al., [24] proposed the notion of neutrosophic Soft topological K-Algebras and discussed it's several terms like C_5- connectedness, super connectedness, compactness etc. In 2019, Mary Margaret A, Trinita Pricilla M and Shawkat Alkhazaleh [13] developed neutrosophic vague topological spaces. Vague binary soft set theory was developed by Dr. Francina Shalini. A [8] and Remya.P.B in 2018. In this paper a new concept neutrosophic vague binary set is developed by using two universes. It's topology, continuity and various types of continuities are also under concern.

2. Preliminaries

Definition 2.2. [26] (Neutrosophic vague set)

A neutrosophic vague set A_{NV} (NVS in short) on the universe of discourse X can be written as $A_{NV} = \{\langle x\,;\,\hat{T}_{AN}(X)\,;\,\hat{I}_{AN}(X), \hat{F}_{AN}(X)\rangle; x \in X\}$ whose truth-membership, indeterminacy-membership and falsity-membership functions are defined as

$\hat{T}_{A_{NV}}(x)$=$[T^-, T^+]$, $\hat{I}_{A_{NV}}(x)$= $[I^-, I^+]$ and $\hat{F}_{A_{NV}}(x)$= $[F^-, F^+]$

where (1) T^+= $1-F^-$; $F^+ = 1-T^-$ and

(2) $^-0 \leq T^- + I^- + F^- \leq 2^+$

$^-0 \leq T^+ + I^+ + F^+ \leq 2^+$

Definition 2.3. [26] (Unit Neutrosophic Vague Set)

Let Ψ_{NV} be a neutrosophic vague set (NVS in short) of the universe U where $\forall\ u_i \in U$, $\hat{T}_{\Psi_{NV}}(x) = [1, 1]$, $\hat{I}_{\Psi_{NV}}(x) = [0, 0]$, $\hat{F}_{\Psi_{NV}}(x) = [0, 0]$, then Ψ_{NV} is called a unit NVS, where $1 \leq i \leq n$

Definition 2.4. [26] (Zero Neutrosophic Vague Set)

Let Φ_{NV} be a neutrosophic vague set (NVS in short) of the universe U where $\forall\ u_i \in U$, $\hat{T}_{\Phi_{NV}}(x) = [0, 0]$, $\hat{I}_{\Phi_{NV}}(x) = [1, 1]$, $\hat{F}_{\Phi_{NV}}(x) = [1, 1]$, then Φ_{NV} is called a zero NVS, where $1 \leq i \leq n$

Definition 2.5. [26] (Neutrosophic vague subset)

Let A_{NV} and B_{NV} be two $NVS's$ of the universe U.

If $\forall\ u_i \in$; $[1 \leq i \leq n]$

1. $\hat{T}_{A_{NV}}(u_i) \leq \hat{T}_{B_{NV}}(u_i)$

2. $\hat{I}_{A_{NV}}\ (u_i) \geq \hat{I}_{B_{NV}}\ (u_i)$ and
3. $\hat{F}_{A_{NV}}\ (u_i) \geq \hat{F}_{B_{NV}}\ (u_i)$

then the $NVS\ A_{NV}$ are included by B_{NV} denoted by $A_{NV} \subseteq B_{NV}$

Definition 2.6. [26] (Complement of a Neutrosophic vague set)

The complement of a $NVS\ A_{NV}$ is denoted by $A^c{}_{NV}$ and is defined by

$\hat{T}^c{}_{A_{NV}}(x) = [1-T^+, 1-T^-]$, $\hat{I}^c A_{NV}\ (x) = [1-I^+,\ \ 1-I^-]$ and $\hat{F}^c A_{NV}\ (x) = [1-F^+, 1-F^-]$

Definition 2.7. [26] (Union of Neutrosophic vague sets)

Union of two $NVS's\ A_{NV}$ and B_{NV} is a $NVS\ C_{NV}$ written as $C_{NV} = A_{NV} \cup B_{NV}$ whose truth-membership, indeterminacy-membership and false-membership functions are related to those of A_{NV} and B_{NV} given by

$\hat{T}_{C_{NV}}(x) = [\max(T^- A_{NV}(x), T^- B_{NV}(x)), \max(T^+ A_{NV}(x), T^+ B_{NV}(x))]$

$\hat{I}_{C_{NV}}(x) = [\min(I^- A_{NV}(x),\ I^- B_{NV}(x)), \min(I^+ A_{NV}(x),\ \ I^+ B_{NV}(x))]$

$\hat{F}_{C_{NV}}(x) = [\min(F^- A_{NV}(x), F^- B_{NV}(x)), \min(F^+ A_{NV}(x), F^+ B_{NV}(x))]$

Definition 2.8. [26] (Intersection of Neutrosophic vague sets)

Intersection of two $NVS's\ A_{NV}$ and B_{NV} is a $NVS\ C_{NV}$ written as $D_{NV} = A_{NV} \cap B_{NV}$ whose truth-membership, indeterminacy-membership and false-membership functions are related to those of A_{NV} and B_{NV} given by

$\hat{T}_{D_{NV}}\ (x) = [\min(T^- A_{NV}(x), T^- B_{NV}(x)), \min(T^+ A_{NV}(x), T^+ B_{NV}(x))]$

$\hat{I}_{D_{NV}}\ (x) = [\max(I^- A_{NV}(x),\ I^- B_{NV}(x)), \max(I^+ A_{NV}(x),\ \ I^+ B_{NV}(x))]$

$\hat{F}_{D_{NV}}(x) = [\max(F^- A_{NV}(x), F^- B_{NV}(x)), \max(F^+ A_{NV}(x), F^+ B_{NV}(x))]$

Definition 2.9.[14]

Let (X, τ) be a topological space. A subset A of X is called:

(i) Semi-closed set if $int(cl(A)) \subseteq A$
(ii) Pre-closed set if $cl(int(A)) \subseteq A$
(iii) Semi-pre closed set if $int(cl(int(A))) \subseteq A$
(iv) Regular-closed set if $A = cl(int(A))$
(v) Generalized semi-closed set if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X

Definition 2.10. [4] (Image and Pre-image of neutrosophic vague sets)

Let X_{NV} and Y_{NV} be two non-empty neutrosophic vague sets and f : $X_{NV} \longrightarrow Y_{NV}$ be a function, then the following statements hold:

(1)If $B_{NV} = \{\langle x,\ \hat{T}_B(x);\ \hat{I}_B(x);\ \hat{F}_B(x)\rangle; x \in X_{NV}\}$ is a NVS in Y_{NV}, then the preimage of B_{NV} under f , denoted

by $f^{-1}(B_{NV})$, is the NVS in X_{NV} defined by

$$f^{-1}(B_{NV}) = \left\{\langle x,\ f^{-1}\left(\hat{T}_B(x)\right)\ ;\ f^{-1}\left(\hat{I}_B(x)\right);\ f^{-1}\left(\hat{F}_B(x)\right)\rangle\ ; x \in X_{NV}\right\}$$

(2)If $A_{NV} = \{\langle x,\ \hat{T}_A(x);\ \hat{I}_A(x);\ \hat{F}_A(x)\rangle; x \in X_{NV}\}$ is a NVS in X_{NV}, then the image of A_{NV} under f , denoted

by $f(A_{NV})$, is the NVS in Y_{NV} defined by

$$f(A_{NV}) = \left\{\langle y,\ f_{sup}\left(\hat{T}_A(y)\right)\ ;\ f_{inf}\left(\hat{I}_A(y)\right);\ f_{inf}\left(\hat{F}_A(y)\right)\rangle\ ; y \in Y_{NV}\right\}$$

where

$$f_{sup}\left(\hat{T}_A(y)\right) = \begin{cases} sup_{x \in f^{-1}(\mathrm{y})}\hat{T}_A(x), & if\ f^{-1}(\mathrm{y}) \neq \emptyset \\ 0, & otherwise \end{cases}$$

$$f_{inf}\left(\hat{I}_A(y)\right) = \begin{cases} sup_{x \in f^{-1}(\mathrm{y})}\hat{I}_A(x), & if\ f^{-1}(\mathrm{y}) \neq \emptyset \\ 0, & otherwise \end{cases}$$

$$f_{inf}\left(\hat{F}_A(y)\right) = \begin{cases} sup_{x \in f^{-1}(\mathrm{y})}\hat{T}_A(x), & if\ f^{-1}(\mathrm{y}) \neq \emptyset \\ 0, & otherwise \end{cases}$$

for each $y \in Y_{NV}$

Definition 2.11.[5] (Strongly continous functions)

A function f : (X, τ) $\rightarrow$(Y, σ) to be strongly continous if $f(\bar{A}) \subset f(A)$, $\forall$ subset A of X or equivalently, if the inverse image of every set in Y is clopen in X.

Definition 2.12. [14] (Neutrosophic Vague Continuous Mapping)

Let (X, τ) and (Y, σ) be any two neutrosophic vague topological spaces. A map f : (X, τ) $\rightarrow$(Y, σ) is said to be neutrosophic vague continuous (NV continuous) if $f^{-1}(V)$ is neutrosophic vague closed set in (X, τ) for every neutrosophic vague closed set V of (Y, σ)

Definition 2.13 [14] (Neutrosophic Vague semi-continous mapping)

Let (X, τ) and (Y, σ) be any two neutrosophic vague topological spaces. A map f : (X, τ) $\rightarrow$(Y, σ) is said to be neutrosophic vague semi-continous if $f^{-1}(V)$ is neutrosophic vague semi-closed set in (X, τ) for every neutrosophic vague closed set V of (Y, σ)

Definition 2.14 [14] (Neutrosophic Vague pre-continous mapping)

Let (X, τ) and (Y, σ) be any two neutrosophic vague topological spaces. A map f : (X, τ) $\rightarrow$(Y, σ) is said to be neutrosophic vague pre-continous if $f^{-1}(V)$ is neutrosophic vague pre-closed set in (X, τ) for every neutrosophic vague closed set V of (Y, σ)

Definition 2.15 [14] (Neutrosophic Vague regular continous mapping)

Let (X, τ) and (Y, σ) be any two neutrosophic vague topological spaces. A map f : (X, τ) $\rightarrow$(Y, σ) is said to be neutrosophic vague regular continous if $f^{-1}(V)$ is neutrosophic vague regular-closed set in (X, τ) for every neutrosophic vague closed set V of (Y, σ)

Definition 2.16 [14] (Neutrosophic Vague semi pre-continous mapping)

Let (X, τ) and (Y, σ) be any two neutrosophic vague topological spaces. A map f : (X, τ) $\rightarrow$(Y, σ) is said to be neutrosophic vague semi pre-continous if $f^{-1}(V)$ is neutrosophic vague semi pre-closed set in (X, τ) for every neutrosophic vague closed set V of (Y, σ)

3. Neutrosophic Vague Binary Sets

In this section neutrosophic vague binary sets are discussed with examples. For this as a preliminary tool fuzzy binary sets and vague binary sets are discussed as a general case by taking all members instead of taking a subset of cartesian product in a confined manner.

Definition 3.1. (Binary Set)

Binary set A over a common universe $\left\{U_1 = \{x_j/ 1 \le j \le n\}; U_2 = \{y_k/1 \le k \le p\}\right\}$ is an object of the form $\breve{A} = \{\langle x_j\rangle, \langle y_k\rangle\}$

Definition 3.2. (Fuzzy Binary Set)

Fuzzy binary set A over a common universe $\left\{U_1 = \{x_j/ 1 \le j \le n\}; U_2 = \{y_k/1 \le k \le p\}\right\}$ is an object of the form

$\breve{A}_F = \left\{\langle\frac{\mu_A(x_j)}{x_j}; \forall x_j \in U_1\rangle, \langle\frac{\mu_A(y_k)}{y_k}; \forall y_k \in U_2\rangle\right\}$ where $\mu_A(x_j)$: $U_1 \longrightarrow [0, 1]$ gives the truth membership value of the elements x_j in U_1; $\mu_A(y_k)$: $U_2 \longrightarrow [0, 1]$ gives the truth membership values of the elements y_k in U_2

Example 3.3.

$\breve{A}_F = \left\{\langle\frac{0.2}{h_1^N}, \frac{0.4}{h_2^N}, \frac{0.1}{h_3^N}\rangle, \langle\frac{0.6}{h_1^S}, \frac{0.3}{h_2^S}\rangle\right\}$ represents the fuzzy binary set

Definition 3.4. (Vague Binary Set)

Vague binary set A over a common universe $\left\{U_1 = \{x_j/ 1 \le j \le n\}; U_2 = \{y_k/1 \le k \le p\}\right\}$ is an object of the form

$\breve{A}_V = \left\{\langle\frac{V_A(x_j)}{x_j}; \forall x_j \in U_1\rangle, \langle\frac{V_A(y_k)}{y_k}; \forall y_k \in U_2\rangle\right\} = \left\{\langle\frac{[t_A(x_j), 1- f_A(x_j)]}{x_j}; \forall x_j \in U_1\rangle, \langle\frac{[t_A(y_k), 1- f_A(y_k)]}{y_k}; \forall y_k \in U_2\rangle\right\}$;

$V_A(x_j)$: $U_1 \longrightarrow [0, 1]$; $V_A(y_k)$: $U_2 \longrightarrow [0, 1]$

Example 3.5.

$\breve{A}_V = \left\{\langle\frac{[0.2,0.6]}{h_1^N}, \frac{[0.4,0.7]}{h_2^N}, \frac{[0.1,0.9]}{h_3^N}\rangle, \langle\frac{[0.6,0.9]}{h_1^S}, \frac{[0.3,0.4]}{h_2^S}\rangle\right\}$ is a vague binary set where $U_1 = \left\{h_1^N, h_2^N, h_3^N\right\}$, $U_2 = \left\{h_1^S, h_2^S\right\}$

Definition 3.6. (Neutrosophic binary set)

Neutrosophic binary set $\check{A}_N$ over a common universe $\{U_1 = \{x_j/\ 1 \le j \le n\}; U_2 = \{y_k/1 \le k \le p\}\}$ is an object of the form

$$\check{A}_N = \left\{\langle\frac{(T_A(x_j),\ I_A(x_j),\ F_A(x_j))}{x_j} /\ \forall\, x_j \in U_1\rangle, \langle\frac{(T_A(y_k),\ I_A(y_k),\ F_A(y_k))}{y_k} /\ \forall\, y_k \in U_2\rangle\right\}$$

$T_A(x_j),\ I_A(x_j),\ F_A(x_j)\ :\ U_1 \longrightarrow [0, 1]$ gives the 'truth, indeterminacy and false' membership values of the elements x_j in U_1 and $T_A(y_k), I_A(y_k), F_A(y_k): U_2 \longrightarrow [0, 1]$ gives the 'truth, indeterminacy and false' membership values of the elements y_k in U_2

Example 3.7.

$\check{A}_N = \left\{\langle\frac{(0.2,\ 0.3,\ 0.4\,)}{h_1^N}, \frac{(0.4,\ 0.1,\ 0.3\,)}{h_2^N}, \frac{(0.1,\ 0.3,\ 0.1\,)}{h_3^N}\rangle, \langle\frac{(0.6,\ 0.2,\ 0.1\,)}{h_1^S}, \frac{(0.3,\ 0.5,\ 0.6\,)}{h_1^S}\rangle\right\}$ is a neutrosophic binary set where $U_1 = \{h_1^N, h_2^N, h_3^N\}$, $U_2 = \{h_1^S, h_2^S\}$

Definition 3.8. (Neutrosophic vague binary set)

A neutrosophic vague binary set M_{NVB} ($NVBS$ in short) over a common universe $\{U_1 = \{x_j/\ 1 \le j \le n\}; U_2 = \{y_k/1 \le k \le p\}\}$ is an object of the form

$$M_{NVB} = \left\{\langle\frac{\hat{T}_{M_{NVB}}(x_j),\ \hat{I}_{M_{NVB}}(x_j),\ \hat{F}_{M_{NVB}}(x_j)}{x_j};\ \forall\, x_j \in U_1\rangle\, \langle\frac{\hat{T}_{M_{NVB}}(y_k),\ \hat{I}_{M_{NVB}}(y_k),\ \hat{F}_{M_{NVB}}(y_k)}{y_k};\ \forall\, y_k \in U_2\rangle\right\}$$

is defined as

$\hat{T}_{M_{NVB}}\ (x_j) = [T^-(x_j), T^+(x_j)]$, $\hat{I}_{M_{NVB}}(x_j) = [I^-(x_j),\ I^+(x_j)]$ and $\hat{F}_{M_{NVB}}(x_j) = [F^-(x_j),\ F^+(x_j)]$; $x_j \in U_1$ and $\hat{T}_{M_{NVB}}\ (y_k) = [T^-(y_k), T^+(y_k)]$, $\hat{I}_{M_{NVB}}(y_k) = [I^-(y_k),\ I^+(y_k)]$ and $\hat{F}_{M_{NVB}}(y_k) = [F^-(y_k),\ F^+(y_k)]$; $y_k \in U_2$

where (1) $T^+(x_j) = 1 - F^-(x_j);\ F^+(x_j) = 1 - T^-(x_j)\ ;\ \forall\, x_j \in U_1$ and
$T^+(y_k) = 1 - F^-(y_k);\ F^+(y_k) = 1 - T^-(y_k)\ ;\ \forall\, y_k \in U_2$

(2) $^-0 \le T^-(x_j) + I^-(x_j) + F^-(x_j) \le 2^+\ ;\ \ ^-0 \le T^-(y_k) + I^-(y_k) + F^-(y_k) \le 2^+$

or

$^-0 \le T^-(x_j) + I^-(x_j) + F^-(x_j) + T^-(y_k) + I^-(y_k) + F^-(y_k) \le 4^+$

and

$^-0 \le T^+(x_j) + I^+(x_j) + F^+(x_j) \le 2^+\ ;\ \ ^-0 \le T^+(y_k) + I^+(y_k) + F^+(y_k) \le 2^+$

or

$^-0 \le T^+(x_j) + I^+(x_j) + F^+(x_j) + T^+(y_k) + I^+(y_k) + F^+(y_k) \le 4^+$

(3) $T^-(x_j),\ I^-(x_j),\ F^-(x_j)\ :\ V(U_1) \longrightarrow [0,1]$ and $T^-(y_k),\ I^-(y_k),\ F^-(y_k)\ :\ V(U_2) \longrightarrow [0,1]$
$T^+(x_j),\ I^+(x_j),\ F^+(x_j)\ :\ V(U_1) \longrightarrow [0,1]$ and $T^+(y_k),\ I^+(y_k),\ F^+(y_k)\ :\ V(U_2) \longrightarrow [0,1]$

Here $V(U_1),\ V(U_2)$ denotes power set of vague sets on $U_1,\ U_2$ respectively.

Example 3.9.

Let $U_1 = \{x_1,\ x_2,\ x_3\}$, $U_2 = \{y_1,\ y_2\}$ be the common universe under consideration.

A $NVBS$ is given below:

$$M_{NVB} = \left\{\begin{array}{c}\langle\frac{[0.2,\ 0.3],\ [0.6, 0.7],\ [0.7, 0.8]}{x_1}; \frac{[0.3,\ 0.7],\ [0.5, 0.6],\ [0.3, 0.7]}{x_2}; \frac{[0.1,\ 0.9],\ [0.4, 0.8],\ [0.1, 0.9]}{x_3}\rangle \\ \langle\frac{[0.6,\ 0.8],\ [0.5, 0.7],\ [0.2, 0.4]}{y_1}; \frac{[0.2,\ 0.7],\ [0.6, 0.9],\ [0.3, 0.8]}{y_2}\rangle\end{array}\right\}$$

Definition 3.10. (Zero neutrosophic vague binary set and Unit Neutrosophic vague binary set)

Let $\{U_1 = \{x_j/\ 1 \le j \le n\}; U_2 = \{y_k/1 \le k \le p\}\}$ be two universes under consideration.

(i) A zero $NVBS$ denoted as Φ_{NVB} over this common universe is given by,

$$\Phi_{NVB} = \left\{\langle\frac{[0,\ 0],\ [1,\ 1],\ [1, 1]}{x_j};\ \forall\, x_j \in U_1\rangle, \langle\frac{[0,\ 0],\ [1,\ 1],\ [1, 1]}{y_k};\ \forall\, y_k \in U_2\rangle\right\}$$

(ii) A unit $NVBS$ denoted as Ψ_{NVB} over this common universe is given by,

$$\Psi_{NVB} = \left\{\langle\frac{[1,1],\ [0,0],\ [0,0]}{x_j};\ \forall\, x_j \in U_1\rangle, \langle\frac{[1,1],\ [0,0],\ [0,0]}{y_k};\ \forall\, y_k \in U_2\rangle\right\}$$

4. Operations on Neutrosophic Vague Binary sets

In this section some usual set theoretical operations are developed for $\boldsymbol{NVBS's}$

Definition 4.1. (Subset of Neutrosophic vague binary sets)

Let $\boldsymbol{M_{NVB}}$ and $\boldsymbol{P_{NVB}}$ be two $\boldsymbol{NVBS's}$ on a common universe $\boldsymbol{U_1}, \boldsymbol{U_2}$. Then $\boldsymbol{M_{NVB}}$ is included by $\boldsymbol{P_{NVB}}$ denoted by $\boldsymbol{M_{NVB}} \subseteq \boldsymbol{P_{NVB}}$ if the following conditions found true :

If $\forall\, \boldsymbol{x_j} \in \boldsymbol{U_1}$ and $1 \le j \le n$

(1) $\widehat{\boldsymbol{T}}_{\boldsymbol{M_{NVB}}}(\boldsymbol{x_j}) \le \widehat{\boldsymbol{T}}_{\boldsymbol{P_{NVB}}}(\boldsymbol{x_j})$ (2) $\widehat{\boldsymbol{I}}_{\boldsymbol{M_{NVB}}}(\boldsymbol{x_j}) \ge \widehat{\boldsymbol{I}}_{\boldsymbol{P_{NVB}}}(\boldsymbol{x_j})$ and (3) $\widehat{\boldsymbol{F}}_{\boldsymbol{M_{NVB}}}(\boldsymbol{x_j}) \ge \widehat{\boldsymbol{F}}_{\boldsymbol{P_{NVB}}}(\boldsymbol{x_j})$

and $\forall\, \boldsymbol{y_k} \in \boldsymbol{U_2}$ and $1 \le k \le p$

(1) $\widehat{\boldsymbol{T}}_{\boldsymbol{M_{NVB}}}(\boldsymbol{y_k}) \le \widehat{\boldsymbol{T}}_{\boldsymbol{P_{NVB}}}(\boldsymbol{y_k})$ (2) $\widehat{\boldsymbol{I}}_{\boldsymbol{M_{NVB}}}(\boldsymbol{y_k}) \ge \widehat{\boldsymbol{I}}_{\boldsymbol{P_{NVB}}}(\boldsymbol{y_k})$ and (3) $\widehat{\boldsymbol{F}}_{\boldsymbol{M_{NVB}}}(\boldsymbol{y_k}) \ge \widehat{\boldsymbol{F}}_{\boldsymbol{P_{NVB}}}(\boldsymbol{y_k})$

Example 4. 2.

Let $\boldsymbol{U_1} = \{\boldsymbol{x_1}, \boldsymbol{x_2}\}$, $\boldsymbol{U_2} = \{\boldsymbol{y_1}\}$ be a common universe. Let

$$M_{NVB} = \left\{ \langle \frac{[0.1,\ 0.2],\ [0.6, 0.7],\ [0.8, 0.9]}{x_1} ; \frac{[0.2,\ 0.6],\ [0.5, 0.6],\ [0.4, 0.8]}{x_2} \rangle \langle \frac{[0.1,\ 0.3],\ [0.6, 0.7],\ [0.7, 0.9]}{y_1} \rangle \right\}$$

$$P_{NVB} = \left\{ \langle \frac{[0.2,0.3],\ [0.5,0.6],\ [0.7,0.8]}{x_1} ; \frac{[0.3,0.7],\ [0.4,0.5],\ [0.3,0.7]}{x_2} \rangle \langle \frac{[0.2,0.4],\ [0.5,0.6],\ [0.6,0.8]}{y_1} \rangle \right\}.$$

Clearly, $M_{NVB} \subseteq P_{NVB}$

Definition 4.3. (Union of two neutrosophic vague binary sets)

Let M_{NVB} and P_{NVB} are two $NVBS's$

(i) Union of two $NVBS's$, M_{NVB} and P_{NVB} is a $NVBS$, given as

$$M_{NVB} \cup P_{NVB} = S_{NVB} = \left\{ \langle \frac{\hat{T}_{S_{NVB}}(x_j),\ \hat{I}_{S_{NVB}}(x_j),\ \hat{F}_{S_{NVB}}(x_j)}{x_j} ; \forall\, x_j \in U_1 \rangle \langle \frac{\hat{T}_{S_{NVB}}(y_k),\ \hat{I}_{S_{NVB}}(y_k),\ \hat{F}_{S_{NVB}}(y_k)}{y_k} ; \forall\, y_k \in U_2 \rangle \right\}$$

whose truth-membership, indeterminacy-membership and false-membership functions are related to those of M_{NVB} and P_{NVB} is given by

$$\hat{T}_{S_{NVB}}(x_j) = \left[\max\left(T^- M_{NVB}(x_j), T^- P_{NVB}(x_j)\right), \max\left(T^+ M_{NVB}(x_j), T^+ P_{NVB}(x_j)\right)\right]$$
$$\hat{I}_{S_{NVB}}(x_j) = \left[\min\left(I^- M_{NVB}(x_j), I^- P_{NVB}(x_j)\right), \min\left(I^+ M_{NVB}(x_j), I^+ P_{NVB}(x_j)\right)\right]$$
$$\hat{F}_{S_{NVB}}(x_j) = \left[\min\left(F^- M_{NVB}(x_j), F^- P_{NVB}(x_j)\right), \min\left(F^+ M_{NVB}(x_j), F^+ P_{NVB}(x_j)\right]\right.$$

and

$$\hat{T}_{S_{NVB}}(y_k) = [\max(T^- M_{NVB}(y_k), T^- P_{NVB}(y_k)), \max(T^+ M_{NVB}(y_k), T^+ P_{NVB}(y_k))]$$
$$\hat{I}_{S_{NVB}}(y_k) = [\min(I^- M_{NVB}(y_k), I^- P_{NVB}(y_k)), \min(I^+ M_{NVB}(y_k), I^+ P_{NVB}(y_k))]$$
$$\hat{F}_{S_{NVB}}(y_k) = [\min(F^- M_{NVB}(y_k), F^- P_{NVB}(y_k)), \min(F^+ M_{NVB}(y_k), F^+ P_{NVB}(y_k)]$$

Example 4. 4.

In example 4. 2.

$$S_{NVB} = \left\{ \langle \frac{[0.2,\ 0.3],\ [0.5, 0.6],\ [0.7, 0.8]}{x_1} ; \frac{[0.3,\ 0.7],\ [0.4, 0.5],\ [0.3, 0.7]}{x_2} \rangle \langle \frac{[0.2,\ 0.4],\ [0.5, 0.6],\ [0.6, 0.8]}{y_1} \rangle \right\}$$

Definition 4. 5. (Intersection of two neutrosophic vague binary sets)

Let M_{NVB} and P_{NVB} are two $NVBS's$

(i) Intersection of two $NVBS's$, M_{NVB} and P_{NVB} is a $NVBS$, given as

$$M_{NVB} \cap P_{NVB} = R_{NVB} = \left\{ \langle \frac{\hat{T}_{R_{NVB}}(x_j),\ \hat{I}_{R_{NVB}}(x_j),\ \hat{F}_{R_{NVB}}(x_j)}{x_j} ; \forall\, x_j \in U_1 \rangle \langle \frac{\hat{T}_{R_{NVB}}(y_k),\ \hat{I}_{R_{NVB}}(y_k),\ \hat{F}_{R_{NVB}}(y_k)}{y_k} ; \forall\, y_k \in U_2 \rangle \right\}$$

whose truth-membership, indeterminacy-membership and false-membership functions are related to those of M_{NVB} and P_{NVB} is given by

$$\hat{T}_{R_{NVB}}(x_j) = \left[\min\left(T^- M_{NVB}(x_j), T^- P_{NVB}(x_j)\right), \min\left(T^+ M_{NVB}(x_j), T^+ P_{NVB}(x_j)\right)\right]$$
$$\hat{I}_{R_{NVB}}(x_j) = \left[\max\left(I^- M_{NVB}(x_j), I^- P_{NVB}(x_j)\right), \max\left(I^+ M_{NVB}(x_j), I^+ P_{NVB}(x_j)\right)\right]$$
$$\hat{F}_{R_{NVB}}(x_j) = \left[\max\left(F^- M_{NVB}(x_j), F^- P_{NVB}(x_j)\right), \max\left(F^+ M_{NVB}(x_j), F^+ P_{NVB}(x_j)\right]\right.$$

and

$$\hat{T}_{R_{NVB}}(y_k) = [\min(T^- M_{NVB}(y_k), T^- P_{NVB}(y_k)), \min(T^+ M_{NVB}(y_k), T^+ P_{NVB}(y_k))]$$
$$\hat{I}_{R_{NVB}}(y_k) = [\max(I^- M_{NVB}(y_k), I^- P_{NVB}(y_k)), \max(I^+ M_{NVB}(y_k), I^+ P_{NVB}(y_k))]$$
$$\hat{F}_{R_{NVB}}(y_k) = [\max(F^- M_{NVB}(y_k), F^- P_{NVB}(y_k)), \max(F^+ M_{NVB}(y_k), F^+ P_{NVB}(y_k)]$$

Example 4. 6.

In example 4. 2. $R_{NVB} = \left\{ \langle \frac{[0.1,0.2],\ [0.6,0.7],\ [0.7,0.8]}{x_1} ; \frac{[0.2,0.6],\ [0.5,0.6],\ [0.4,0.8]}{x_2} \rangle \langle \frac{[0.1,0.3],\ [0.6,0.7],\ [0.7,0.9]}{y_1} \rangle \right\}$

Definition 4. 7. (Complement of a NVBS)

Let M_{NVB} is defined as in definition 3.1. It's complement is denoted by M_{NVB}^C and is given by

$M^c{}_{NVB} = \left\{ \langle \frac{\hat{T}^c{}_{M_{NVB}}(x_j),\ \hat{I}^c{}_{M_{NVB}}(x_j),\ \hat{F}^c{}_{M_{NVB}}(x_j)}{x_j} ; \forall\, x_j \in U_1 \rangle \langle \frac{\hat{T}^c{}_{M_{NVB}}(y_k),\ \hat{I}^c{}_{M_{NVB}}(y_k),\ \hat{F}^c{}_{M_{NVB}}(y_k)}{y_k} ; \forall\, y_k \in U_2 \rangle \right\}$

is defined as

$\hat{T}^c{}_{M_{NVB}}(x_j) = [\ 1 - T^+(x_j),\ 1 - T^-(x_j)]$,

$\hat{I}^c{}_{M_{NVB}}(x_j) = [1 - I^+(x_j),\ 1 - I^-(x_j)]$ and

$\hat{F}^c{}_{M_{NVB}}(x_j) = [1 - F^+(x_j),\ 1 - F^-(x_j)]\ ;\ \ \forall\, x_j \in U_1$

and

$\hat{T}^c{}_{M_{NVB}}(y_k) = [\ 1 - T^+(y_k),\ 1 - T^-(y_k)]$,

$\hat{I}^c{}_{M_{NVB}}(y_k) = [1 - I^+(y_k),\ 1 - I^-(y_k)]$ and

$\hat{F}^c{}_{M_{NVB}}(y_k) = [1 - F^+(y_k),\ 1 - F^-(y_k)]\ ;\ \forall\, y_k \in U_2$

Example 4. 8.

Let M_{NVB} is defined as in example 3.2. It's complement is given by,

$$M_{NVB}^c = \left\{ \begin{array}{c} \langle \frac{[0.7,\ 0.8],\ \ [0.3, 0.4],\ \ [0.2, 0.3]}{x_1} ; \frac{[0.3,\ 0.7],\ \ [0.4, 0.5],\ \ [0.3, 0.7]}{x_2} ; \frac{[0.1,\ 0.9],\ \ [0.2, 0.6],\ \ [0.1, 0.9]}{x_3} \rangle \\ \langle \frac{[0.2,\ 0.4],\ \ [0.3, 0.5],\ \ [0.6, 0.8]}{y_1} ; \frac{[0.3,\ 0.8],\ \ [0.1, 0.4],\ \ [0.2, 0.7]}{y_2} \rangle \end{array} \right\}$$

5. Neutrosophic vague binary topology

In this section neutrosophic vague binary topology ($NVBT$ in short) is developed for $NVBS's$. It's various concepts are also discussed.

Definition 5.1. (Neutrosophic vague binary topology)

A neutrosophic vague binary topology on a common universe U_1, U_2 is a family τ_Δ^{NVB} of neutrosophic vague binary sets in U_1, U_2 satisfying the following axioms:

(1) $\Phi_{NVB},\ \Psi_{NVB} \in \tau_\Delta^{NVB}$

(2) For any $M_{NVB},\ P_{NVB} \in \tau_\Delta^{NVB},\ M_{NVB} \cap P_{NVB} \in \tau_\Delta^{NVB}$

i.e., finite intersection of $NVBS's$ of τ_Δ^{NVB} is again a member of τ_Δ^{NVB}

(3) Let $\{M_{NVB}^i; i \in I\} \subseteq \tau_\Delta^{NVB}$ then $\cup_{i \in I} \tau_\Delta^{NVB} \subseteq \tau_\Delta^{NVB}$

i.e., arbitrary union of neutrosophic vague binary sets in τ_Δ^{NVB} is again a member of τ_Δ^{NVB}

Example 5.2.

Let $U_1 = \{x_1,\ x_2\}\ ;\ U_2 = \{y_1\}$. Following is a neutrosophic vague binary topology ;

$$\tau_\Delta^{NVB} = \left\{ \begin{array}{c} \Phi_{NVB} = \left(\left\{ \langle \frac{[0,0],[1,1],[1,1]]}{x_1} \rangle,\ \langle \frac{[0,0],[1,1],[1,1]}{x_2} \rangle \right\}, \left\{ \langle \frac{[0,0],[1,1],[1,1]}{y_1} \rangle \right\} \right), \\ M_{NVB} = \left(\left\{ \langle \frac{[0.2,0.4],[0.6,0.8],[0.6\ 0.8]]}{x_1} \rangle,\ \langle \frac{[0.3,0.6],[0.7,0.8],[0.4,0.7]}{x_2} \rangle \right\}, \left\{ \langle \frac{[0.6,0.8],[0.7.9],[0.2,0.4]}{y_1} \rangle \right\} \right), \\ P_{NVB} = \left(\left\{ \langle \frac{[0.6,0.7],[0.1,0.9],[0.3,0.4]]}{x_1} \rangle,\ \langle \frac{[0.7,0.8],[0.3,0.7],[0.2,0.3]}{x_2} \rangle \right\}, \left\{ \langle \frac{[0.6,0.7],[0.2,0.5],[0.3,0.4]}{y_1} \rangle \right\} \right), \\ K_{NVB} = M_{NVB} \cap P_{NVB} = \left(\left\{ \langle \frac{[0.2,0.4],[0.6,0.9],[0.6,0.8]]}{x_1} \rangle,\ \langle \frac{[0.3,0.6],[0.7,0.8],[0.4,0.7]}{x_2} \rangle \right\}, \left\{ \langle \frac{[0.6,0.7],[0.7,0.9],[0.3,0.4]}{y_1} \rangle \right\} \right), \\ H_{NVB} = M_{NVB} \cup P_{NVB} = \left(\left\{ \langle \frac{[0.6,0.7],[0.1,0.8],[0.3,0.4]]}{x_1} \rangle,\ \langle \frac{[0.7,0.8],[0.7,0.8],[0.2,0.3]}{x_2} \rangle \right\}, \left\{ \langle \frac{[0.6,0.8],[0.2,0.5],[0.2,0.4]}{y_1} \rangle \right\} \right), \\ \Psi_{NVB} = \left(\left\{ \langle \frac{[1,1],[0,0],[0,0]]}{x_1} \rangle,\ \langle \frac{[1,1],[0,0],[0,0]}{x_2} \rangle \right\}, \left\{ \langle \frac{[1,1],[0,0],[0,0]}{y_1} \rangle \right\} \right) \end{array} \right\}$$

Definition 5.3. (Neutrosophic vague binary open set)

Every elements of a $NVBT$ is known as a neutrosophic vague binary open set ($NVBOS$ in short)

Example 5.4.

In example 5.2. $\Phi_{NVB},\ M_{NVB},\ P_{NVB},\ K_{NVB},\ H_{NVB},\ \Psi_{NVB}$ are all $NVBOS'$s

Definition 5.5. (Neutrosophic vague binary closed set)

Complement of a $NVBOS$ is known as a neutrosophic vague binary closed set ($NVBCS$ in short)

Example 5.6.

In example 5.2. Φ^c_{NVB}, M^C_{NVB}, P^C_{NVB}, K^C_{NVB}, H^C_{NVB}, Ψ^c_{NVB} are all $NVBCS$'s, where

Φ^c_{NVB}= $\left(\left\{\langle\frac{[1,1],[0,0],[0,0]}{x_1}\rangle,\ \langle\frac{[1,1],[0,0],[0,0]}{x_2}\rangle\right\},\left\{\langle\frac{[1,1],[0,0],[0,0]}{y_1}\rangle\right\}\right) = \Psi_{NVB}$

M^C_{NVB}= $\left(\left\{\langle\frac{[0.6,0.8],[0.2,0.4],[0.2\ 0.4]}{x_1}\rangle,\ \langle\frac{[0.4,0.7],[0.2,0.3],[0.3,0.6]}{x_2}\rangle\right\},\left\{\langle\frac{[0.2,0.4],[0.1.3],[0.6,0.8]}{y_1}\rangle\right\}\right)$

P^C_{NVB}= $\left(\left\{\langle\frac{[0.3,0.4],[0.1,0.9],[0.6,0.7]}{x_1}\rangle,\ \langle\frac{[0.2,0.3],[0.3,0.7],[0.7,0.8]}{x_2}\rangle\right\},\left\{\langle\frac{[0.3,0.4],[0.5,0.8],[0.6,0.7]}{y_1}\rangle\right\}\right)$

K^C_{NVB}=$\left(\left\{\langle\frac{[0.6,0.8],[0.1,0.4],[0.2,0.4]}{x_1}\rangle,\ \langle\frac{[0.4,0.7],[0.2,0.3],[0.3,0.6]}{x_2}\rangle\right\},\left\{\langle\frac{[0.3,0.4],[0.1,0.3],[0.6,0.7]}{y_1}\rangle\right\}\right)$

H^C_{NVB}=$\left(\left\{\langle\frac{[0.3,0.4],[0.2,0.9],[0.6,0.7]}{x_1}\rangle,\ \langle\frac{[0.2,0.3],[0.2,0.3],[0.7,0.8]}{x_2}\rangle\right\},\left\{\langle\frac{[0.2,0.4],[0.5,0.8],[0.6,0.8]}{y_1}\rangle\right\}\right)$

Ψ^c_{NVB}=$\left(\left\{\langle\frac{[0,0],[1,1],[1,1]}{x_1}\rangle,\ \langle\frac{[0,0],[1,1],[1,1]}{x_2}\rangle\right\},\left\{\langle\frac{[0,0],[1,1],[1,1]}{y_1}\rangle\right\}\right) = \Phi_{NVB}$

Remark 5.7.

Φ_{NVB} and Ψ_{NVB} will both acts as $NVBOS$ and $NVBCS$

Definition 5.8. (Neutrosophic vague binary topological space)

The triplet $(U_1, U_2, \tau^{NVB}_\Delta)$ is known as a neutrosophic vague binary topological space ($NVBTS$ in short), where τ^{NVB}_Δ is a neutrosophic vague binary topology defined as in definition 5.1.

Example 5.9.

If $U_1 = \{x_1,\ x_2\}$; $U_2 = \{y_1\}$; τ^{NVB}_Δ= $\{\Phi_{NVB}, M_{NVB}, P_{NVB}, K_{NVB}, H_{NVB}, \Psi_{NVB}\}$ defined as in example 5.2. then the triplet $(U_1, U_2, \tau^{NVB}_\Delta)$ is clearly a $NVBTS$.

Definition 5.10. (Neutrosophic vague binary discrete topology and Neutrosophic vague binary discrete topological Space)

A topology consisting of only empty and unit $NVBS$'s is known as a neutrosophic vague binary discrete topology ($NVBDT$ in short) and the corresponding neutrosophic vague binary topological space is known as a neutrosophic vague binary discrete topological space ($NVBDTS$ in short).

i.e., τ^{NVB}_Δ= $\{\Phi_{NVB},\ \Psi_{NVB}\}$

Example 5.11.

In example 5.2.

$$\tau^{NVB}_\Delta= \left\{\begin{matrix}\Phi_{NVB} = \left(\left\{\langle\frac{[0,0],[1,1],[1,1]}{x_1}\rangle,\ \langle\frac{[0,0],[1,1],[1,1]}{x_2}\rangle\right\},\left\{\langle\frac{[0,0],[1,1],[1,1]}{y_1}\rangle\right\}\right),\\ \Psi_{NVB} = \left(\left\{\langle\frac{[1,1],[0,0],[0,0]}{x_1}\rangle,\ \langle\frac{[1,1],[0,0],[0,0]}{x_2}\rangle\right\},\left\{\langle\frac{[1,1],[0,0],[0,0]}{y_1}\rangle\right\}\right)\end{matrix}\right\}$$

is clearly a $NVBDT$ and the corresponding neutrosophic vague topological space is the $NVBDTS$.

Definition 5.12. (Neutrosophic vague binary indiscrete topology and Neutrosophic vague binary discrete topological Space)

A $NVBT$ defined by it's power set is known as neutrosophic vague binary indiscrete topology ($NVBIDT$ in short) and the corresponding neutrosophic vague binary topological space is known as a neutrosophic vague binary indiscrete topological space ($NVBIDTS$ in short).

Definition 5.13. (Neutrosophic vague binary interior and Neutrosophic vague binary closure)

Let $(U_1, U_2, \tau^{NVB}_\Delta)$ be a $NVBTS$ and also

let $M_{NVB} = \left\{\langle\frac{\hat{T}_{M_{NVB}}(x_j),\ \hat{I}_{M_{NVB}}(x_j),\ \hat{F}_{M_{NVB}}(x_j)}{x_j};\ \forall\ x_j \in U_1\rangle\ \langle\frac{\hat{T}_{M_{NVB}}(y_k),\ \hat{I}_{M_{NVB}}(y_k),\ \hat{F}_{M_{NVB}}(y_k)}{y_k};\ \forall\ y_k \in U_2\rangle\right\}$

is a $NVBS$ over a common universe U_1, U_2 defined as in definition 3.1. Then it's neutrosophic vague binary interior (denoted as M^0_{NVB}) and neutrosophic vague binary closure (denoted as $\overline{M}_{NVB}$) are defined as follows:

M^0_{NVB}= $\cup\ \left\{M^i_{NVB}; i \in I \middle| M^i_{NVB}\ is\ a\ NVBOS\ over\ U_1, U_2\ with\ M^i_{NVB} \subseteq M_{NVB}\ ;\ \forall\ i\right\}$

$\overline{M}_{NVB}$= $\cap \left\{M^i_{NVB}; i \in I \middle| M^i_{NVB}\ is\ a\ NVBCS\ over\ U_1, U_2\ with\ M_{NVB} \subseteq M^i_{NVB}\ ;\ \forall\ i\right\}$

Example 5.14.

In example 5.2.

H^0_{NVB}= $\left(\left\{\langle\frac{[0.6,0.7],[0.1,0.8],[0.3,0.4]}{x_1}\rangle,\ \langle\frac{[0.7,0.8],[0.7,0.8],[0.2,0.3]}{x_2}\rangle\right\},\left\{\langle\frac{[0.6,0.8],[0.2,0.5],[0.2,0.4]}{y_1}\rangle\right\}\right) = H_{NVB}$

From example 5.6.

$\overline{M^C_{NVB}}$= $\left(\left\{\langle\frac{[0.6,0.8],[0.2,0.4],[0.2\ 0.4]}{x_1}\rangle,\ \langle\frac{[0.4,0.7],[0.2,0.3],[0.3,0.6]}{x_2}\rangle\right\},\left\{\langle\frac{[0.2,0.4],[0.1.3],[0.6,0.8]}{y_1}\rangle\right\}\right)$= M^C_{NVB}

Proposition 5.15.

(i) M_{NVB} is a $NVBOS \Leftrightarrow M^0_{NVB} = M_{NVB}$

(ii) M_{NVB} is a $NVBCS \Leftrightarrow \overline{M}_{NVB} = M_{NVB}$

Proof

Proof is clear

Proposition 5.16.

(i) $M^1_{NVB} \subseteq M^2_{NVB}$ and $P^1_{NVB} \subseteq P^2_{NVB} \Rightarrow (M^1_{NVB} \cup P^1_{NVB}) \subseteq (M^2_{NVB} \cup P^2_{NVB})$ and $(M^1_{NVB} \cap P^1_{NVB}) \subseteq (M^2_{NVB} \cap P^2_{NVB})$

(ii) $M_{NVB} \subseteq M^1_{NVB}$ and $M_{NVB} \subseteq M^2_{NVB} \Rightarrow M_{NVB} \subseteq (M^1_{NVB} \cap M^2_{NVB})$

$M^1_{NVB} \subseteq M_{NVB}$ and $M^2_{NVB} \subseteq M_{NVB} \Rightarrow (M^1_{NVB} \cup M^2_{NVB}) \subseteq M_{NVB}$

(iii) $\overline{\overline{M}}_{NVB} = M_{NVB}$

(iv) $M_{NVB} \subseteq P_{NVB} \Rightarrow \overline{P}_{NVB} \subseteq \overline{M}_{NVB}$

(v) $\overline{\emptyset}_{NVB} = \psi_{NVB}$

(vi) $\overline{\Psi}_{NVB} = \Phi_{NVB}$

Proof

Proof is clear

6. Continuous mapping for $NVBS'$s

Continuity plays vital role in any topology. In this section image, pre-image and continuity are developed for $NVBS'$s.

Definition 6.1. (Image and Pre-image of neutrosophic vague binary sets)

Let M_{NVB} and P_{NVB} be two non-empty $NVBS'$s defined on two common universes U_1, U_2 and V_1, V_2 respectively. Define a function f : $M_{NVB} \longrightarrow P_{NVB}$, then the following statements hold:

(1)If $D_{NVB} = \left\{ \langle \frac{\hat{T}_{D_{NVB}}(s_i);\, \hat{I}_{D_{NVB}}(s_i);\, \hat{F}_{D_{NVB}}(s_i)}{s_i};\ s_i \in V_1 \rangle ; \langle \frac{\hat{T}_{D_{NVB}}(t_r);\, \hat{I}_{D_{NVB}}(t_r);\, \hat{F}_{D_{NVB}}(t_r)}{t_r};\ t_r \in V_2 \rangle \right\}$ is a $NVBS$ in P_{NVB}, then the preimage of D_{NVB} under f , denoted by $f^{-1}(D_{NVB})$, is a $NVBS$ in M_{NVB} defined by

$f^{-1}(D_{NVB}) =$

$$\left\{ \langle \frac{f^{-1}\left(\hat{T}_{D_{NVB}}(s_i)\right);\ f^{-1}\left(\hat{I}_{D_{NVB}}(s_i)\right);\ f^{-1}\left(\hat{F}_{D_{NVB}}(s_i)\right)}{s_i};\ s_i \in V_1 \rangle ; \langle \frac{f^{-1}\left(\hat{T}_{D_{NVB}}(t_r)\right);\ f^{-1}\left(\hat{I}_{D_{NVB}}(t_r)\right);\ f^{-1}\left(\hat{F}_{D_{NVB}}(t_r)\right)}{t_r};\ t_r \in V_2 \rangle \right\}$$

(2)If $A_{NVB} = \left\{ \langle \frac{\hat{T}_{A_{NVB}}(x_j);\, \hat{I}_{A_{NVB}}(x_j);\, \hat{F}_{A_{NVB}}(x_j)}{x_j};\ x_j \in U_1 \rangle ; \langle \frac{\hat{T}_{A_{NVB}}(y_k);\, \hat{I}_{A_{NVB}}(y_k);\, \hat{F}_{A_{NVB}}(y_k)}{y_k};\ y_k \in U_2 \rangle \right\}$ is a $NVBS$ in M_{NVB}, then the image of A_{NVB} under f , denoted by $f(A_{NVB})$, is a $NVBS$ in P_{NVB} defined by

$f(A_{NVB})$

$$= \left\{ \langle \frac{f_{sup}\left(\hat{T}_{A_{NVB}}(s_i)\right);\ f_{inf}\left(\hat{I}_{A_{NVB}}(s_i)\right);\ f_{inf}\left(\hat{F}_{A_{NVB}}(s_i)\right)}{s_i};\ s_i \in V_1 \rangle ; \langle \frac{f_{sup}\left(\hat{T}_{A_{NVB}}(t_r)\right);\ f_{inf}\left(\hat{I}_{A_{NVB}}(t_r)\right);\ f_{inf}\left(\hat{F}_{A_{NVB}}(t_r)\right)}{t_r};\ t_r \in V_2 \rangle \right\}$$

where

$$\begin{cases} f_{sup}\left(\hat{T}_{A_{NVB}}(s_i)\right) = \begin{cases} sup_{x_j \in f^{-1}(s_i)} \hat{T}_{A_{NVB}}(x_j), & if\ f^{-1}(s_i) \neq \emptyset \\ 0, & otherwise \end{cases} \\ f_{sup}\left(\hat{T}_{A_{NVB}}(t_r)\right) = \begin{cases} sup_{y_k \in f^{-1}(t_r)} \hat{T}_{A_{NVB}}(y_k), & if\ f^{-1}(t_r) \neq \emptyset \\ 0, & otherwise \end{cases} \end{cases}$$

$$\begin{cases} f_{inf}\left(\hat{I}_{A_{NVB}}(s_i)\right) = \begin{cases} inf_{x_j \in f^{-1}(s_i)} \hat{I}_{A_{NVB}}(x_j), & if\ f^{-1}(s_i) \neq \emptyset \\ 0, & otherwise \end{cases} \\ f_{inf}\left(\hat{I}_{A_{NVB}}(t_r)\right) = \begin{cases} inf_{y_k \in f^{-1}(t_r)} \hat{I}_{A_{NVB}}(y_k), & if\ f^{-1}(t_r) \neq \emptyset \\ 0, & otherwise \end{cases} \end{cases}$$

$$\begin{cases} f_{inf}\left(\hat{F}_{A_{NVB}}(s_i)\right) = \begin{cases} inf_{x_j \in f^{-1}(s_i)} \hat{F}_{A_{NVB}}(x_j), & if\ f^{-1}(s_i) \neq \emptyset \\ 0, & otherwise \end{cases} \\ f_{inf}\left(\hat{F}_{A_{NVB}}(t_r)\right) = \begin{cases} inf_{y_k \in f^{-1}(t_r)} \hat{F}_{A_{NVB}}(y_k), & if\ f^{-1}(t_r) \neq \emptyset \\ 0, & otherwise \end{cases} \end{cases}$$

for each $s_i \in V_1$ and for each $t_r \in V_2$

Definition 6.2. (Neutrosophic Vague strongly continous mapping)

Let (X, τ) and (Y, σ) be any two neutrosophic vague topological spaces. A map f : (X, τ) →(Y, σ) is said to be neutrosophic vague strongly continous if inverse image of every neutrosophic vague set in (Y, σ) is neutrosophic vague clopen set [a set which acts simultaneously as neutrosophic vague open set and neutrosophic vague closed set] in (X, τ)

Definition 6.3.

(i) Neutrosophic Vague Binary Continuity:

Let $(U_1, U_2, \tau_\Delta^{NVB})$ and $(V_1, V_2, \sigma_\Delta^{NVB})$ be any two $NVBTS'$s.

A map f : $(U_1, U_2, \tau_\Delta^{NVB}) \longrightarrow (V_1, V_2, \sigma_\Delta^{NVB})$ is said to be neutrosophic vague binary **continuous** (NVB continuous) if forevery $NVBOS$ (or $NVBCS$) M_{NVB} of $(V_1, V_2, \sigma_\Delta^{NVB})$, $f^{-1}(M_{NVB})$ is a $NVBOS$ (or $NVBCS$) in $(U_1, U_2, \tau_\Delta^{NVB})$

(ii)Various kinds of Continuities for $NVBS'$s

Let $(U_1, U_2, \tau_\Delta^{NVB})$ and $(V_1, V_2, \sigma_\Delta^{NVB})$ be any two $NVBTS'$s. A map f : $(U_1, U_2, \tau_\Delta^{NVB}) \longrightarrow (V_1, V_2, \sigma_\Delta^{NVB})$ is said to be

(1) Neutrosophic vague binary **semi-continuous** ($NVBSC$):

if forevery neutrosophic vague binary open set ($NVBOS$ in short) [or neutrosophic vague binary closed set ($NVBCS$ in short)] M_{NVB} of $(V_1, V_2, \sigma_\Delta^{NVB})$, $f^{-1}(M_{NVB})$ is a neutrosophic vague binary semi-open set ($NVBSOS$ in short) [or neutrosophic vague binary semi - closed set ($NVBSCS$ in short)] in $(U_1, U_2, \tau_\Delta^{NVB})$

(2) Neutrosophic vague binary **pre-continuous** ($NVBPC$ continuous):

if forevery $NVBOS$ [or $NVBCS$] M_{NVB} of $(V_1, V_2, \sigma_\Delta^{NVB})$ $f^{-1}(M_{NVB})$ is a neutrosophic vague binary pre-open set ($NVBPOS$ in short) [or neutrosophic vague binary pre-closed set ($NVBPCS$ in short)] in $(U_1, U_2, \tau_\Delta^{NVB})$

(3) Neutrosophic vague binary **strongly-continuous** ($NVBSC$ continuous):

if inverse image of every neutrosophic vague binary set in $(V_1, V_2, \sigma_\Delta^{NVB})$ is neutrosophic vague binary clopen set [a set which acts simultaneously as neutrosophic vague binary open set and neutrosophic vague binary closed set] in $(U_1, U_2, \tau_\Delta^{NVB})$

(4) Neutrosophic vague binary **regular-continuous** ($NVBRC$ continuous):

if forevery $NVBOS$ [or $NVBCS$] M_{NVB} of $(V_1, V_2, \sigma_\Delta^{NVB})$ $f^{-1}(M_{NVB})$ is a neutrosophic vague binary regular- open set ($NVBROS$ in short) [or neutrosophic vague binary regular-closed set ($NVBRCS$ in short)] in $(U_1, U_2, \tau_\Delta^{NVB})$

(5) Neutrosophic vague binary **semi-pre-continuous** ($NVBRC$ continuous):

if forevery $NVBOS$ [or $NVBCS$] M_{NVB} of $(V_1, V_2, \sigma_\Delta^{NVB})$ $f^{-1}(M_{NVB})$ is a neutrosophic vague binary generalized semi- open set ($NVBGSOS$ in short) [or neutrosophic vague binary generalized semi-closed set ($NVBGSCS$ in short)] in $(U_1, U_2, \tau_\Delta^{NVB})$

Example 6.4.

Let $f = (g,h): M_{NVB} \longrightarrow P_{NVB}$ be a function defined as , $f(\Phi_{NVB}^1) = \Phi_{NVB}^2$, $f(M_{NVB}^1) = P_{NVB}^1$, $f(M_{NVB}^2) = P_{NVB}^1$, $f(\Psi_{NVB}^1) = \Psi_{NVB}^2$ where $g: U_1 \longrightarrow V_1$ and $h: U_2 \longrightarrow V_2$ be two functions with $g(x_1) = s_2$, $g(x_2) = s_1$ and $h(y_1) = t_1$, where $U_1 = \{x_1, x_2\}$, $U_2 = \{y_1\}$ and $V_1 = \{s_1, s_2\}$, $U_2 = \{t_1\}$.

Let $\tau_\Delta^{NVB} = \{\Phi_{NVB}^1, M_{NVB}^1, M_{NVB}^2, M_{NVB}^3, M_{NVB}^4, M_{NVB}^5, M_{NVB}^6, M_{NVB}^7, M_{NVB}^8, M_{NVB}^9, M_{NVB}^{10}, M_{NVB}^{11}, \Psi_{NVB}^1\}$ and $\sigma_\Delta^{NVB} = \{\Phi_{NVB}^2, P_{NVB}^1, \Psi_{NVB}^2\}$ be their respective $NVBT'$s.

Here

$$\Phi_{NVB}^1 = \left\{\langle \frac{[0,0],\ [1,1],\ [1,1]}{x_1}; \frac{[0,0],\ [1,1],\ [1,1]}{x_2} \rangle, \ \langle \frac{[0,0],\ [1,1],\ [1,1]}{y_1} \rangle\right\}$$

$$M_{NVB}^1 = \left\{\langle \frac{[0.3,\ 0.4],\ [0.7, 0.8],\ [0.6, 0.7]}{x_1}; \frac{[0.2,\ 0.7],\ [0.1, 0.5],\ [0.3, 0.8]}{x_2} \rangle \langle \frac{[0.4,\ 0.9],\ [0.2, 0.6],\ [0.1, 0.6]}{y_1} \rangle\right\}$$

$$M_{NVB}^2 = \left\{\langle \frac{[0.1,\ 0.6],\ [0.6, 0.9],\ [0.4, 0.9]}{x_1}; \frac{[0.6,\ 0.8],\ [0.3, 0.7],\ [0.2, 0.4]}{x_2} \rangle \langle \frac{[0.2,\ 0.7],\ [0.2, 0.9],\ [0.3, 0.8]}{y_1} \rangle\right\}$$

$$M_{NVB}^3 = \left\{\langle \frac{[0.6,\ 0.8],\ [0.1, 0.5],\ [0.2, 0.4]}{x_1}; \frac{[0.3,\ 0.6],\ [0.6, 0.8],\ [0.4, 0.7]}{x_2} \rangle \langle \frac{[0.2,\ 0.7],\ [0.2, 0.9],\ [0.3, 0.8]}{y_1} \rangle\right\}$$

$$M^4_{NVB} = \left\{\langle \frac{[0.1,\ 0.4],\ [0.7, 0.9],\ [0.6, 0.9]}{x_1}; \frac{[0.2,\ 0.6],\ [0.6, 0.8],\ [0.4, 0.8]}{x_2} \rangle \langle \frac{[0.2,\ 0.7],\ [0.2, 0.9],\ [0.3, 0.8]}{y_1} \rangle\right\}$$

$$M^5_{NVB} = \left\{\langle \frac{[0.6,\ 0.8],\ [0.1, 0.5],\ [0.2, 0.4]}{x_1}; \frac{[0.6,\ 0.8],\ [0.1, 0.5],\ [0.2, 0.4]}{x_2} \rangle \langle \frac{[0.4,\ 0.9],\ [0.2, 0.6],\ [0.1, 0.6]}{y_1} \rangle\right\}$$

$$M^6_{NVB} = \left\{\langle \frac{[0.1,\ 0.4],\ [0.7, 0.9],\ [0.6, 0.9]}{x_1}; \frac{[0.2,\ 0.7],\ [0.3, 0.7],\ [0.3, 0.8]}{x_2} \rangle \langle \frac{[0.2,\ 0.7],\ [0.2, 0.9],\ [0.3, 0.8]}{y_1} \rangle\right\}$$

$$M^7_{NVB} = \left\{\langle \frac{[0.3,\ 0.6],\ [0.6, 0.8],\ [0.4, 0.7]}{x_1}; \frac{[0.6,\ 0.8],\ [0.1, 0.5],\ [0.2, 0.4]}{x_2} \rangle \langle \frac{[0.4,\ 0.9],\ [0.2, 0.6],\ [0.1, 0.6]}{y_1} \rangle\right\}$$

$$M^8_{NVB} = \left\{\langle \frac{[0.1,\ 0.6],\ [0.6, 0.9],\ [0.4, 0.9]}{x_1}; \frac{[0.3,\ 0.6],\ [0.6, 0.8],\ [0.4, 0.7]}{x_2} \rangle \langle \frac{[0.2,\ 0.7],\ [0.2, 0.9],\ [0.3, 0.8]}{y_1} \rangle\right\}$$

$$M^9_{NVB} = \left\{\langle \frac{[0.6,\ 0.8],\ [0.1, 0.5],\ [0.2, 0.4]}{x_1}; \frac{[0.6,\ 0.8],\ [0.3, 0.7],\ [0.2, 0.4]}{x_2} \rangle \langle \frac{[0.2,\ 0.7],\ [0.2, 0.9],\ [0.3, 0.8]}{y_1} \rangle\right\}$$

$$M^{10}_{NVB} = \left\{\langle \frac{[0.3,\ 0.4],\ [0.7, 0.8],\ [0.6, 0.7]}{x_1}; \frac{[0.2,\ 0.6],\ [0.6, 0.8],\ [0.4, 0.8]}{x_2} \rangle \langle \frac{[0.2,\ 0.7],\ [0.2, 0.9],\ [0.3, 0.8]}{y_1} \rangle\right\}$$

$$M^{11}_{NVB} = \left\{\langle \frac{[0.6,\ 0.8],\ [0.1, 0.5],\ [0.2, 0.4]}{x_1}; \frac{[0.3,\ 0.7],\ [0.1, 0.5],\ [0.3, 0.7]}{x_2} \rangle \langle \frac{[0.4,\ 0.9],\ [0.2, 0.6],\ [0.1, 0.6]}{y_1} \rangle\right\}$$

$$\Psi^1_{NVB} = \left\{\langle \frac{[1,\ 1],\ [0, 0],\ [0, 0]}{x_1}; \frac{[1,\ 1],\ [0, 0],\ [0, 0]}{x_2} \rangle,\ \langle \frac{[1,\ 1],\ [0, 0],\ [0, 0]}{y_1} \rangle\right\}$$

and $V_1 = \{s_1,\ s_2\}$, $V_2 = \{t_1\}$ be a common universe with

$$\Phi^2_{NVB} = \left\{\langle \frac{[0,\ 0],\ [1, 1],\ [1, 1]}{s_1}; \frac{[0,\ 0],\ [1, 1],\ [1, 1]}{s_2} \rangle,\ \langle \frac{[0,\ 0],\ [1, 1],\ [1, 1]}{t_1} \rangle\right\}$$

$$P^1_{NVB} = \left\{\langle \frac{[0.2,\ 0.3],\ [0.5, 0.6],\ [0.7, 0.8]}{s_1}; \frac{[0.3,\ 0.7],\ [0.4, 0.5],\ [0.3, 0.7]}{s_2} \rangle \langle \frac{[0.2,\ 0.4],\ [0.5, 0.6],\ [0.6, 0.8]}{t_1} \rangle\right\}$$

$$\Psi^2_{NVB} = \left\{\langle \frac{[1,\ 1],\ [0, 0],\ [0, 0]}{s_1}; \frac{[1,\ 1],\ [0, 0],\ [0, 0]}{s_2} \rangle,\ \langle \frac{[1,\ 1],\ [0, 0],\ [0, 0]}{t_1} \rangle\right\}$$

It is got that, $f^{-1}(\Phi^2_{NVB}) = \Phi^1_{NVB}$, $f^{-1}(\mathrm{P}_{NVB}) = M^3_{NVB}$, $f^{-1}(\Psi^2_{NVB}) = \Psi^1_{NVB}$. Then clearly f is a neutrosophic vague binary contiuous mapping.

7. Distance Measures for $NVBS's$

Let $U_1 = \{x_1,\ x_2, ----,\ x_n\}$; $U_2 = \{y_1,\ y_2, ----,\ y_p\}$ be the common universe. Also let

M_{NVB} and P_{NVB} be two $NVBS's$.

(i) Hamming distance between them is defined as

$$d^H_{NVB}(M_{NVB}, P_{NVB}) =$$

$$\frac{1}{6}\left[\sum_{j=1}^{n}\{[|T^-_{MNB}(x_j) - T^-_{PNB}(x_j)| + |I^-_{MNB}(x_j) - I^-_{PNB}(x_j)| + |F^-_{MNB}(x_j) - F^-_{PNB}(x_j)|] + [|T^+_{MNB}(x_j) - T^+_{PNB}(x_j)| + |I^+_{MNB}(x_j) - I^+_{PNB}(x_j)| + |F^+_{MNB}(x_j) - F^+_{PNB}(x_j)|]\}\right]$$

$$+ \frac{1}{6}\left[\sum_{k=1}^{p}\{[|T^-_{MNB}(y_k) - T^-_{PNB}(y_k)| + |I^-_{MNB}(y_k) - I^-_{PNB}(y_k)| + |F^-_{MNB}(y_k) - F^-_{PNB}(y_k)|] + [|T^+_{MNB}(y_k) - T^+_{PNB}(y_k)| + |I^+_{MNB}(y_k) - I^+_{PNB}(y_k)| + |F^+_{MNB}(y_k) - F^+_{PNB}(y_k)|]\}\right]$$

(ii) Normalized Hamming distance between them is defined as

$d_{NVB}^{nH}(M_{NVB}, P_{NVB})$=

$$\frac{1}{6n}[\sum_{j=1}^{n}\{[|T_{MNB}^{-}(x_j)-T_{PNB}^{-}(x_j)|+|I_{MNB}^{-}(x_j)-I_{PNB}^{-}(x_j)|+|F_{MNB}^{-}(x_j)-F_{PNB}^{-}(x_j)|]+[|T_{MNB}^{+}(x_j)-T_{PNB}^{+}(x_j)|+|I_{MNB}^{+}(x_j)-I_{PNB}^{+}(x_j)|+|F_{MNB}^{+}(x_j)-F_{PNB}^{+}(x_j)|]\}]$$

$$+\frac{1}{6p}[\sum_{k=1}^{p}\{[|T_{MNB}^{-}(y_k)-T_{PNB}^{-}(y_k)|+|I_{MNB}^{-}(y_k)-I_{PNB}^{-}(y_k)|+|F_{MNB}^{-}(y_k)-F_{PNB}^{-}(y_k)|]+[|T_{MNB}^{+}(y_k)-T_{PNB}^{+}(y_k)|+|I_{MNB}^{+}(y_k)-I_{PNB}^{+}(y_k)|+|F_{MNB}^{+}(y_k)-F_{PNB}^{+}(y_k)|]\}]$$

(iii)Euclidean distance between them is defined as

$d_{NVB}^{E}(M_{NVB}, P_{NVB})$ =

$$\sqrt{\begin{array}{c}\left[\frac{1}{6}\left[\sum_{j=1}^{n}\{[|T_{MNB}^{-}(x_j)-T_{PNB}^{-}(x_j)|^2+|I_{MNB}^{-}(x_j)-I_{PNB}^{-}(x_j)|^2+|F_{MNB}^{-}(x_j)-F_{PNB}^{-}(x_j)|^2]+[|T_{MNB}^{+}(x_j)-T_{PNB}^{+}(x_j)|^2+|I_{MNB}^{+}(x_j)-I_{PNB}^{+}(x_j)|^2+|F_{MNB}^{+}(x_j)-F_{PNB}^{+}(x_j)|^2]\}\right]\right] \\ + \\ \left[\frac{1}{6}\left[\sum_{k=1}^{p}\{[|T_{MNB}^{-}(y_k)-T_{PNB}^{-}(y_k)|^2+|I_{MNB}^{-}(y_k)-I_{PNB}^{-}(y_k)|^2+|F_{MNB}^{-}(y_k)-F_{PNB}^{-}(y_k)|^2]+[|T_{MNB}^{+}(y_k)-T_{PNB}^{+}(y_k)|^2+|I_{MNB}^{+}(y_k)-I_{PNB}^{+}(y_k)|^2+|F_{MNB}^{+}(y_k)-F_{PNB}^{+}(y_k)|^2]\}\right]\right]\end{array}}$$

(iv)Normalized Euclidean distance between them is defined as

$d_{NVB}^{nE}(M_{NVB}, P_{NVB})$ =

$$\sqrt{\begin{array}{c}\left[\frac{1}{6n}\left[\sum_{j=1}^{n}\{[|T_{MNB}^{-}(x_j)-T_{PNB}^{-}(x_j)|^2+|I_{MNB}^{-}(x_j)-I_{PNB}^{-}(x_j)|^2+|F_{MNB}^{-}(x_j)-F_{PNB}^{-}(x_j)|^2]+[|T_{MNB}^{+}(x_j)-T_{PNB}^{+}(x_j)|^2+|I_{MNB}^{+}(x_j)-I_{PNB}^{+}(x_j)|^2+|F_{MNB}^{+}(x_j)-F_{PNB}^{+}(x_j)|^2]\}\right]\right] \\ + \\ \left[\frac{1}{6p}\left[\sum_{k=1}^{p}\{[|T_{MNB}^{-}(y_k)-T_{PNB}^{-}(y_k)|^2+|I_{MNB}^{-}(y_k)-I_{PNB}^{-}(y_k)|^2+|F_{MNB}^{-}(y_k)-F_{PNB}^{-}(y_k)|^2]+[|T_{MNB}^{+}(y_k)-T_{PNB}^{+}(y_k)|^2+|I_{MNB}^{+}(y_k)-I_{PNB}^{+}(y_k)|^2+|F_{MNB}^{+}(y_k)-F_{PNB}^{+}(y_k)|^2]\}\right]\right]\end{array}}$$

8. $NVBS's$ in Medical Diagonosis

This section deals with an application of $NVBS's$ in medical diagnosis.

Following table describes datas collected from three patients after conducting liver function test. First set of sample is collected before treatment which describes the first universe. Second set of sample is collected after treatment which describes the second universe. P_{NVB}^1, P_{NVB}^2, P_{NVB}^3 are three NVBS's formed, based on the datas of the three patients under consideration

Before Treatment (BT)	P_1	P_2	P_3
Albumin	[0.042, 0.052]	[0.025, 0.052]	[0.052, 0.064]
Globulin Serum	[0.035, 0.045]	[0.033, 0.035]	[0.011, 0.035]
Bilirubin Total	[0.045, 0.100]	[0.070, 0.100]	[0.093, 0.100]

After Treatment (AT)	P_1	P_2	P_3
Albumin	[0.031, 0.052]	[0.036, 0.052]	[0.052, 0.064]
Globulin Serum	[0.021, 0.035]	[0.035, 0.042]	[0.019, 0.035]
Bilirubin Total	[0.025, 0.100]	[0.017, 0.100]	[0.099,0.100]

Data collected from 3 persons are converted to $NVBS's$ as given below:

$$P_{NVB}^1 = \left\{\begin{array}{l}\langle\frac{[0.042,0.052],\ [0.948,0.958],\ [0.948,0.958]}{P_{BT}^{Albumin}},\frac{[0.035,0.045],\ [0.955,0.965],\ [0.955,0.965]}{P_{BT}^{Globulin\ Serum}},\frac{[0.045,0.100],\ [0.900,0.955],\ [0.900,0.955]}{P_{BT}^{Bilirubin\ Total}}\rangle, \\ \langle\frac{[0.031,0.052],\ [0.948,0.969],\ [0.948,0.969]}{P_{AT}^{Albumin}},\frac{[0.021,0.035],\ [0.965,0.979\ ],\ [0.965,0.979]}{P_{AT}^{Globulin\ Serum}},\frac{[0.025,0.100],\ [0.900,0.975],\ [0.900,0.975]}{P_{AT}^{Bilirubin\ Total}}\rangle\end{array}\right\}$$

$$P_{NVB}^{2} = \left\{ \begin{array}{l} \langle \frac{[0.025, 0.052],\ [0.948, 0.975],\ [0.948, 0.975]}{P_{BT}^{Albumin}}, \frac{[0.033, 0.035],\ [0.965, 0.967],\ [0.965, 0.967]}{P_{BT}^{Globulin\ Serum}}, \frac{[0.070, 0.100],\ [0.900, 0.930],\ [0.900, 0.930]}{P_{BT}^{Bilirubin\ Total}} \rangle, \\ \langle \frac{[0.036, 0.052],\ [0.948, 0.964],\ [0.948, 0.964]}{P_{AT}^{Albumin}}, \frac{[0.035, 0.042],\ [0.958, 0.965],\ [0.958, 0.965]}{P_{AT}^{Globulin\ Serum}}, \frac{[0.017, 0.100],\ [0.900, 0.983],\ [0.900, 0.983]}{P_{AT}^{Bilirubin\ Total}} \rangle \end{array} \right\}$$

$$P_{NVB}^{3} = \left\{ \begin{array}{l} \langle \frac{[0.052, 0.064],\ [0.936, 0.948],\ [0.936, 0.948]}{P_{BT}^{Albumin}}, \frac{[0.011, 0.035],\ [0.965, 0.989],\ [0.965, 0.989]}{P_{BT}^{Globulin\ Serum}}, \frac{[0.011, 0.035],\ [0.965, 0.989],\ [0.965, 0.989]}{P_{BT}^{Bilirubin\ Total}} \rangle, \\ \langle \frac{[0.052, 0.064],\ [0.936, 0.948],\ [0.936, 0.948]}{P_{AT}^{Albumin}}, \frac{[0.019, 0.035],\ [0.965, 0.981],\ [0.965, 0.981]}{P_{AT}^{Globulin\ Serum}}, \frac{[0.099, 0.100],\ [0.900, 0.901],\ [0.900, 0.901]}{P_{AT}^{Bilirubin\ Total}} \rangle \end{array} \right\}$$

D_{NVB}^{LFT} is a NVBS formed, based on the actual range fixed for a liver function test. Ranges for D_{NVB}^{LFT} under a liver function test for albumin, Globulin serum and Bilirubin Total is given as follows:

Before Treatment (BT)	D_{NVB}^{LFT}
Albumin	[0.034, 0.052], [0.948, 0.966], [0.948, 0.966]
Globulin Serum	[0.015, 0.035], [0.965, 0.985], [0.965, 0.985]
Bilirubin Total	[0.000, 0.100], [0.900, 0.100], [0.900, 0.100]

After Treatment (AT)	D_{NVB}^{LFT}
Albumin	[0.034, 0.052], [0.948, 0.966], [0.948, 0.966]
Globulin Serum	[0.015, 0.035], [0.965, 0.985], [0.965, 0.985]
Bilirubin Total	[0.000, 0.100], [0.900, 0.100], [0.900, 0.100]

Above datas are converted to NVBS as below.

$$D_{NVB}^{LFT} = \left\{ \begin{array}{l} \langle \frac{[0.034, 0.052],\ [0.948, 0.966],\ [0.948, 0.966]}{D_{BT}^{Albumin}}, \frac{[0.015, 0.035],\ [0.965, 0.985], [0.965, 0.985]}{D_{BT}^{Globulin-Serum}}, \frac{[0.000, 0.100],\ [0.900, 0.100],\ [0.900, 0.100]}{D_{BT}^{Bilirubin\ Total}} \rangle, \\ \langle \frac{[0.034, 0.052],\ [0.948, 0.966],\ [0.948, 0.966]}{D_{AT}^{Albumin}}, \frac{[0.015, 0.035],\ [0.965, 0.985], [0.965, 0.985]}{D_{AT}^{Globulin-Serum}}, \frac{[0.000, 0.100],\ [0.900, 0.100],\ [0.900, 0.100]}{D_{AT}^{Bilirubin\ Total}} \rangle \end{array} \right\}$$

Neutrosophic vague binary euclidean distance measure can be used to diagonise which patient is more suffering with liver problems even after treatment. Following table gives the neutrosophic vague binary euclidean difference between each of the patients from D_{NVB}^{LFT}

$\boldsymbol{d_{NVBS}^{ED}(P_{NVB}^{1}, D_{NVB}^{LFT})}$	$\boldsymbol{d_{NVBS}^{ED}(P_{NVB}^{2}, D_{NVB}^{LFT})}$	$\boldsymbol{d_{NVBS}^{ED}(P_{NVB}^{3}, D_{NVB}^{LFT})}$
0.014856	0.277330	0.745502

Lowest neutrosophic vague binary euclidean difference is for patient I. So patient I suffers more with liver problems even after treatment

9. Conclusions

Neutrosophic vague binary sets are developed in this paper with some examples and basic concepts. Real life situations demand binary and higher dimensional universes than a unique one. Being the vital concept to homeomorphism - 'which is the underlying principle to any topology' – continuity has an important role in topology. It is also developed for this new concept. Practical applications are tremendous for binary concept in day today life. One real life example in medical diagonosis is discussed above. Several situations demand combined result than 'a unique separate one'- to compare and deal situations in a more fast manner. Neutrosophic vague binary sets is a good tool for comparison in such cases. It could be made use in surveys, case studies and in some other sort of similar situations. Topology are special type of subsets to a universal set- based on which study of all other subsets of the universal set is possible. New study will produce a combined result or net effect than taking a single result. This work could be extended by taking subsets of the common universe.

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