Neutrosophic Sets and Systems, Vol. 29, 2019
University of New Mexico

Technique for Reducing Dimensionality of Data in Decision-Making Utilizing Neutrosophic Soft Matrices

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Abstract: The decision-making problems in which there are large numbers of qualitative and quantitative factors involved, the technique of dimensionality reduction plays an important role for simplicity and wider applicability. The impreciseness in the information about these factors are being considered in the neutrosophic perception with the parameters - degree of truth-membership, degree of indeterminacy (neutral) and degree of falsity for a better span of the information. In the present communication, we first propose a technique for finding the threshold value for the information provided in the form of neutrosophic soft matrix. Further, utilizing the proposed definitions of the object-oriented neutrosophic soft matrix and the parameter-oriented neutrosophic soft matrix, we present a new algorithm for the dimensionality reduction process. The proposed algorithm has also been applied in an illustrative example of decision-making problem. Further, a comparative analysis in contrast with the existing methodologies has been successfully presented with comparative remarks and additional advantages.

Keywords: Neutrosophic soft matrix, Dimensionality reduction, multiple criteria decision-making, Object-oriented matrix, Parameter-oriented matrix.

1. Introduction

The methodology of dimensionality reduction is to set out an arrangement of set of high dimensional vectors to a lower dimensionality space while holding systematic measures among them. Due to the inherited disadvantage of dimensionality, there are limitations over using the techniques of machine learning as well as the techniques of data mining for high dimensional data. However, there are two noteworthy dimensionality reduction – procedure where the process of *feature selection* and *feature extraction/feature reduction* is involved. In the procedural steps of feature selection, we select a subset of optimal/most useful features as per the need of the objective function. The prime necessity of the feature selection is to enhance the process of data mining and to increase the speed of learning by reducing the dimensionality and obliterate the noise. Feature extraction or

Feature reduction is the task of mapping the large dimensional data to a smaller dimensional data. The major goals of the dimensionality reduction techniques are to enhance the ability to handle both irrelevant and redundant features, to enhance the cost efficiency in contrast with the existing subset evaluation methods etc. It may be noted that the higher the number of factors, the harder it will be to visualize and work on it.

In case of extreme data modality, dimensionality reduction becomes the center of curiosity to a significant point of study in various fields of application. In the field of soft sets, Chen at al. [1] presented a novel concept of parameterization reduction and compared with the reduction of attributes in rough set theory. There are sequential and simultaneous perspectives to consolidate the selection of samples and for reduction of dimensionality of data whose application structure has been given by Xu et al. [2]. This almost gives the best results while processing of the large-scale training data in comparison to the original data models. In addition to this, they also reached to the conclusion that the selection of samples and the reduction of the data dimensionality are mandatory and helpful for handling the modern large-scale databases. Su et al. [3] introduced a new approach called linear sequence discriminant analysis (LSDA) for reducing the dimensionality of the sequences and devised two new algorithms which differs in the extraction of the statistics. Perfilieva [3] introduced the technique of fuzzy transforms which are in agreement with the technique of dimensionality reduction, based on Laplacian eigenmaps along with an application of fuzzy transform to the mathematical finance.

Konate et al. [5] utilized the principal component analysis (PCA) and linear discriminant analysis (LDA) for the reduction of the dimensionality of the original log set of Chinese Continental Scientific Drilling Main Hole to a convenient size, and then feed these reduced-log set into the three classifiers, i.e., support vector machines, feed forward back propagation and radial basis function neural networks. Further, they also demonstrate and discussed the utilization of the combination of dimensionality reduction methods & classifiers and come up with the result that the reduced log set found from dimensionality reduction separate the metamorphic rocks types better or almost as well as the original log set. Sabitha et al. [6] utilized the three different dimensionality reduction techniques, i.e., principal component analysis, singular value decomposition & learning vector quantization. They applied these three techniques to solar irradiance data set which consists of temperature, solar irradiance, and humidity data and evaluated the efficiency and attain the best technique to be applicable for the data set. Chaterjee et al. [7] proposed a novel hybrid method surround factor relationship and multi-attributive border approximation area comparison (MABAC) methods for selection and evaluation of non-traditional machining processes. The technique condenses the problem of pair wise comparisons for estimating criteria weights in multi-criteria decision-making problem significantly.

Mukhametzyanov and Pamucar [8] presented a model to check the result consistency of MCDM methods and in the process of choosing the best one. Further, issue of sensitivity in the process of decision-making using the different ranking algorithms, e.g., "*SAW, MOORA, VIKOR, COPRAS, CODAS, TOPSIS, D'IDEAL, MABAC, PROMETHEE-I,II, ORESTE-II*" have been analyzed by making necessary perturbations in the entries of the decision matrix within a permissible imprecision value.

In order to deal with the vagueness and impreciseness in various engineering applications, socio-economic problems and other decision-making problems, there are many theories available in literature which have their own limitations due to the involvement of the parameterization tools. Molodtsov [9] proposed a new kind of set, termed as soft set, which has the capability to overcome such limitations and put forward important deliberations based on this. Next, Maji et al. [10-12] extended the notion of soft set to fuzzy soft set & intuitionistic fuzzy soft set and proposed various standard binary operations over it with applications in decision-making. Kahraman et al. [13] studied the fuzzy multi-criteria decision-making literature in detail and presented a literature review on the MCDM techniques. Liu et al. [14] proposed a model for evaluation and selection of a transport service provider based on a single valued neutrosophic number (an extension of interval valued intuitionistic fuzzy number). It was a modified version of the DEMATEL method (Decision-making Trial and Evaluation Laboratory Method) for ranking alternative solutions. Kumar and Bajaj [15] introduced the concept of complex intuitionistic fuzzy soft set and proposed some important distance measures with applications.

Hooda and Hooda [16] used the entropy optimization principles for establishing some criteria for dimension reduction over multivariate data with no external variables. A new criterion for maximum entropy and its relation with other criteria have been established for the selection of principal variables. Maji et al. [17] first introduced the notion of neutrosophic soft set, operations for handling the imprecise & inconsistent information which was further redfined by Deli and Broumi [18] for a better understanding of the belief systems. Further, Peng et al. [19] extended the concept to the Pythagorean fuzzy soft set (PyFSS) with different binary operations and utilized them to solve decision-making problems. Cuong [20] extended the notion of intuitionistic fuzzy soft sets to picture fuzzy soft set. Recently, Guleria and Bajaj [21] successfully proposed the notion of T-spherical fuzzy soft set and studied some new aggregation operators along with some applications in the field of decision-making.

The concept of soft matrices was first introduced by Naim and Serdar [22] for representing the notion of soft set with its successful application in the decision-making problems. This matrix representation of soft set was further extended by Yong et al. [23] and Chetia et al. [24] by incorporating the fuzzy and intuitionistic fuzzy setup to deal the decision-making problems respectively. Also, Deli and Broumi [18] have proposed neutrosophic soft matrices and operators which are more functional to make theoretical studies and application in the neutrosophic soft set theory. Such matrices are helpful in representing a neutrosophic soft set in the memory of computers for a wider applicability. Hooda and Kumari [25] proposed a dimensionality reduction model for finding coherent and logical solution to various real-life problems containing uncertainty, impreciseness and vagueness by utilizing the fuzzy soft set. Recently, Guleria and Bajaj [26] studied the Pythagorean fuzzy soft matrices and its various types along with a new decision-making algorithm to deal the medical diagnosis problem and decision-making problem. In the field of neutrosophic set theory, the new trends have brought important field of research. Abdel-Basset et al. [27] developed a multi-criteria group decision-making method under neutrosophic environment based on analytic network process and VIKOR method to solve a supplier selection problem. Many researchers have worked on neutrosophic set theory and applied these notions in solving various multi-criteria decision-making problems, viz., selection processes [28-30], green supply chain management [31], IoT based problems [32,33].

In the literature available, the problem of dimensionality reduction has not been addressed using the notion of neutrosophic soft matrices yet. In the proposed research work, in order to handle the parameterization tool in a more elaborative way, we have proposed a new methodology to handle the dimensionality reduction of the data in a decision-making problem using the notion of neutrosophic sets in a well structure way and compared it with the existing methodologies & example.

The paper has been organized as follows. The basic notions related to the definitions and operations of neutrosophic soft sets and soft matrices have been presented in Section 2. The definitions of the object-oriented neutrosophic soft matrix, the parameter-oriented neutrosophic soft matrix and its threshold value have been proposed along with an algorithm for the dimensionality reduction in Section 3. In Section 4, an application by taking a decision-making problem into account has been dealt with the help of a numerical example using the proposed methodology. Some comparative remarks depicting the advantages and limitations have also been listed. Finally, the paper is concluded in Section 5 by stating the scope for the future work.

2. Basic Notions & Preliminaries

Some of the basic definitions and fundamental notions related to the neutrosophic soft set and matrix are briefly presented in this section which is easily available in literature. The geometrical extensions and generalizations of fuzzy set are being presented by Figure 1 below:

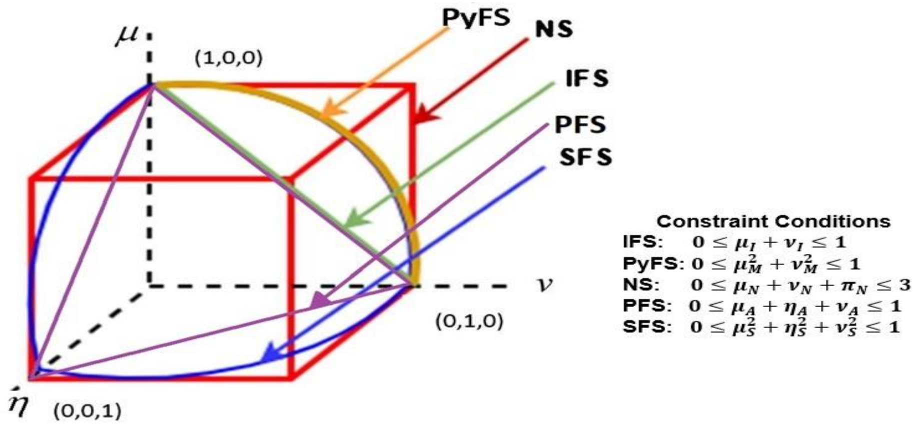

Figure 1: Geometrical Representation of Extensions and Generalizations of Fuzzy Set

In the above Figure 1, the different constraint conditions for the various generalized types of fuzzy sets for intuitionistic fuzzy set (IFS), Pythagorean fuzzy set (PyFS), neutrosophic set (NS), picture fuzzy set (PFS) and spherical fuzzy set (SFS) in terms of membership degree (μ), non-membership degree (ν), indeterminacy or hesitation (η) have been presented. The constraints have been figured out geometrically as per the conditions.

There are different basic notions of matrices, e.g., fuzzy matrices, intuitionistic fuzzy matrices and neutrosophic matrices whose formal definitions are as follows:

Definition 1

Let $U=\{u_1,u_2,\ldots,u_m\}$ be the set of alternatives and $V=\{v_1,v_2,\ldots,v_n\}$ be the set of attributes of every element of U.

- A ***fuzzy matrix*** [34] is defined by $M=\{(u_i,v_j),\mu_M(u_i,v_j)\}$ for all $i=1,2,\ldots,m$ & $j=1,2,\ldots,n$ where, $\mu_M:U\times V\to[0,1]$.
- A ***intuitionistic fuzzy matrix*** [35] is defined by $M=\{(u_i,v_j),\mu_M(u_i,v_j),\nu_M(u_i,v_j)\}$ for all $i=1,2,\ldots,m$ & $j=1,2,\ldots,n$ where, $\mu_M:U\times V\to[0,1]$ and $\nu_M:U\times V\to[0,1]$ satisfying the condition $0\le\mu_M(u_i,v_j)+\nu_M(u_i,v_j)\le 1$.
- A ***neutrosophic fuzzy matrix*** [36] is defined by $M=\{[(a_{ij})]_{m\times n}\,|\,a_{ij}\in K(I)\}$ for all $i=1,2,\ldots,m$ & $j=1,2,\ldots,n$ where, $K(I)$ is the neutrosophic field.

For detailed description, the cited references may be referred.

Definition 2 [37] *A single valued neutrosophic set M in* U *(universal set) is defined by* $M=\{<u,\,T_M(u),\,I_M(u),F_M(u)>|\,u\in U\}$; *with* $T_M:U\to[0,1]$, $I_M:U\to[0,1]$ *and* $F_M:U\to[0,1]$ *being the degree of truth membership, degree of indeterminacy and degree of falsity membership respectively and satisfy the condition*

$$0\le T_M(u)+I_M(u)+F_M(u)\le 3;\ \forall\, u\in U.$$

The sequential development of the notion of soft sets and soft matrices to the concept of Neutrosophic soft sets/matrices can be easily found with necessary illustrative examples in literature [9, 17, 18, 22, 23].

Suppose $U=\{u_1,u_2,u_3,\ldots,u_n\}$ is the universe of discourse and let the collection of parameters $P=\{p_1,p_2,p_3,\ldots,p_n\}$ be under consideration.

- The pair (F,P) is defined to be a ***soft set*** over $U \Leftrightarrow F:P\to\wp(U)$, where $\wp(U)$ is the power set of U.
- Let $FS(U)$ represents the collection of all fuzzy sets of U. A pair $(F,\,P)$ is defined as a fuzzy ***soft set*** over $FS(U)$, where F is a function $F:P\to\wp(FS(U))$.
- The pair $(F,\,P)$ is termed as the ***neutrosophic soft set*** over U if $F:P\to NS(U)$ and can be defined by $(F,\,P)=\{(p,\,F(p)):p\in P,\,F(p)\in NS(U)\}$, where $NS(U)$ is the collection of all neutrosophic sets of U.

- Suppose $(F,\ P)$ be a soft set on U . Then the set $U \times P$ is represented by $R = \{(u,\ p),\ p \in P,\ u \in F(p)\}$ .The characterizing function of R is $\chi_R : U \times P \to [0,\ 1]$ defined as

$$\chi_R(u,\ p) = \begin{cases} 1 & if\ \ (u,\ p) \in R; \\ 0 & if\ \ (u,\ p) \notin R. \end{cases}$$

If $a_{ij} = \chi_R(u_i,\ p_j)$, then a matrix $[a_{ij}] = [\chi_R(u_i,\ p_j)]$ is defined as ***soft matrix*** of the soft set $(F,\ P)$ on U of size $m \times n$.

- If $(F,\ P)$ be a neutrosophic soft set on U , then the set $U \times P$ is represented by

$$R = \{ (u, p), p \in P, u \in F(p) \}.$$

The set R may defined by its characterizing functions- truth function, indeterminacy and falsity function given by $T_R : U \times P \to [0,\ 1]$, $I_R : U \times P \to [0,\ 1]$ and $F_R : U \times P \to [0,\ 1]$ respectively.

If $(T_{ij}, I_{ij}, F_{ij}) = (T_R(u_i, p_j),\ I_R(u_i, p_j),\ F_R(u_i, p_j))$, where $T_R(u_i, p_j)$ represents the belongingness of u_i , $I_R(u_i, p_j)$ represents the indeterminacy of u_i and $F_R(u_i, p_j)$ represents the non-belongingness of u_i in the neutrosophic set $F(p_j)$ respectively, then the **neutrosophic soft matrix** of order $m \times n$ over U, is given by

$$[m_{ij}]_{m \times n} = \left[(T^M_{ij}, I^M_{ij}, F^M_{ij}) \right]_{m \times n} = \begin{bmatrix} (T_{11}, I_{11}, F_{11}) & (T_{12}, I_{12}, F_{12}) & \cdots & (T_{1n}, I_{1n}, F_{1n}) \\ (T_{21}, I_{21}, F_{21}) & (T_{22}, I_{22}, F_{22}) & \cdots & (T_{2n}, I_{2n}, F_{2n}) \\ \vdots & \vdots & \vdots & \vdots \\ (T_{m1}, I_{m1}, F_{m1}) & (T_{m2}, I_{m2}, F_{m2}) & \cdots & (T_{mn}, I_{mn}, F_{mn}) \end{bmatrix}.$$

In order to have a better understanding for constructing a neutrosophic soft matrix, let us consider $U = \{u_1, u_2, u_3\}$ as a universal set and $P = \{p_1, p_2, p_3\}$ as a set of parameters and

$$F(p_1) = \{(u_1, 0.4, 0.5, 0.4),\ (u_2, 0.5, 0.5, 0.3),\ (u_3, 0.9, 0.6, 0.2)\},$$
$$F(p_2) = \{(u_1, 0.2, 0.6, 0.5),\ (u_2, 0.5, 0.6, 0.3),\ (u_3, 0.5, 0.4, 0.2)\},$$
$$F(p_3) = \{(u_1, 0.9, 0.6, 0.2),\ (u_2, 0.5, 0.4, 0.2),\ (u_3, 0.5, 0.4, 0.3)\},$$

then $(F,\ P)$ represents the family of $F(p_1),\ F(p_2),\ F(p_3)$ on U after parameterization. Hence, the neutrosophic soft matrix $[M(F,\ P)]$ may be given by

$$[m_{ij}]_{m \times n} = \left[(T^M_{ij}, I^M_{ij}, F^M_{ij}) \right]_{3 \times 3} = \begin{bmatrix} (0.4,\ 0.5, 0.4) & (0.2, 0.6,\ 0.5) & (0.5,\ 0.3, 0.2) \\ (0.5, 0.5,\ 0.3) & (0.5, 0.6,\ 0.3) & (0.5,\ 0.6, 0.6) \\ (0.9, 0.6,\ 0.2) & (0.5,\ 0.4, 0.2) & (0.5, 0.4,\ 0.3) \end{bmatrix}.$$

Throughout this paper, we take $NSM_{m \times n}$ to represent the collection of all the neutrosophic soft matrices of order $m \times n$.

Operations over Neutrosophic Soft Matrices:

Different types of binary operations for two Neutrosophic soft matrices $M=\left[\left(T_{ij}^{M},\, I_{ij}^{M},F_{ij}^{M}\right)\right]$ and $N=\left[\left(T_{ij}^{N},\, I_{ij}^{N},F_{ij}^{N}\right)\right]\in NSM_{m\times n}$ are as follows [18]:

- $M^{c}=\left[\left(F_{ij}^{M},\,1-I_{ij}^{M},T_{ij}^{M}\right)\right]\quad \forall\, i\ \&\ j.$
- $M\cup N=\left[\max\left(T_{ij}^{M},\,T_{ij}^{N}\right),\,\min\left(I_{ij}^{M},\,I_{ij}^{N}\right),\min\left(F_{ij}^{M},\,F_{ij}^{N}\right)\right]\quad \forall\, i\ \&\ j.$
- $M\cap N=\left[\min\left(T_{ij}^{M},\,T_{ij}^{N}\right),\,\max\left(I_{ij}^{M},\,I_{ij}^{N}\right),\,\max\left(F_{ij}^{M},\,F_{ij}^{N}\right)\right]\quad \forall\, i\ \&\, j.$

3. Algorithm for Dimensionality Reduction

In this section, we first propose two types of matrices - *object-oriented neutrosophic soft matrix* and *parameter-oriented neutrosophic soft matrix*, and then by proposing a new definition for the threshold value we provide a new algorithm for the dimensionality reduction. In general, let $U=\{u_1,u_2,\ldots,u_m\}$ be the universe of discourse and $P=\{p_1,p_2,p_3,\ldots,p_n\}$ be the set of parameters. Consider M to be the neutrosophic soft matrix of the neutrosophic soft set $(F,\,P)$.

Definition 3 *The object-oriented neutrosophic soft matrix with respect to the parameter is defined as:*

$$O_i=\left[\sum_j\frac{T_{ij}}{|P|},\,\sum_j\frac{I_{ij}}{|P|},\sum_j\frac{F_{ij}}{|P|}\right];\ i=1,\ 2,\ldots,m\ \ \&\ \ j=1,\ 2,...,n; \qquad (3.1)$$

where $|\cdot|$ *denotes the cardinality of the set.*

Definition 4 *The parameter-oriented neutrosophic soft matrix with respect to the object is defined as:*

$$P_j=\left[\sum_i\frac{T_{ij}}{|U|},\,\sum_i\frac{I_{ij}}{|U|},\sum_i\frac{F_{ij}}{|U|}\right];\ i=1,\ 2,\ldots,m\ \ \&\ \ j=1,\ 2,...,n; \qquad (3.2)$$

where $|\cdot|$ *denotes the cardinality of the set.*

Definition 5 *If* $M=[(T_{ij}^{M},\,I_{ij}^{M},\,F_{ij}^{M})]\in NSM_{m\times n}$*, then the respective score matrix of neutrosophic soft matrix M is*

$$S(M)=\left[s_{ij}\right]=\left[\left(T_{ij}-I_{ij}F_{ij}\right)\right];\ \forall\ i\ \&\ j. \qquad (3.3)$$

Definition 6 *The threshold value of neutrosophic soft matrix is defined as*

$$S(T)=T_{ij}^{M}-I_{ij}^{M}F_{ij}^{M},\ \textit{where}$$

$$T=\left(T_T,\,I_T,F_T\right)=\left[\sum_{i,j}\frac{T_{ij}}{|U\times P|},\,\sum_{i,j}\frac{I_{ij}}{|U\times P|},\sum_{i,j}\frac{F_{ij}}{|U\times P|}\right];\ i=1,2,\ldots,m\ \ \&\ \ j=1,\,2,\,\ldots,n. \qquad (3.4)$$

where $|\cdot|$ *denotes the cardinality of the set.*

Procedural steps of the proposed algorithm:

The methodology of the proposed algorithm for dimensionality reduction is given by:

- **Step 1.** We first construct the neutrosophic soft matrix as outlined in the beginning of the section.
- **Step 2.** Find the object-oriented matrix for the object O_i and the parameter-oriented matrix for the parameters P_j. Next, compute their score matrix using equation (3.1).
- **Step 3.** Find the threshold element and threshold value of the neutrosophic soft matrix as proposed in equation (3.2).
- **Step 4.** Remove those objects for which $S(O_i) < S(T)$ and those parameters for which $S(P_j) > S(T)$.
- **Step 5.** The new neutrosophic soft matrix is the desired dimensionality reduced matrix.

Based on the neutrosophic soft matrix, the *object-oriented neutrosophic soft matrix*, the *parameter-oriented neutrosophic soft matrix* and the score matrix, the proposed algorithm for dimensionality reduction may be represented with the help of the following flow chart (Figure 2):

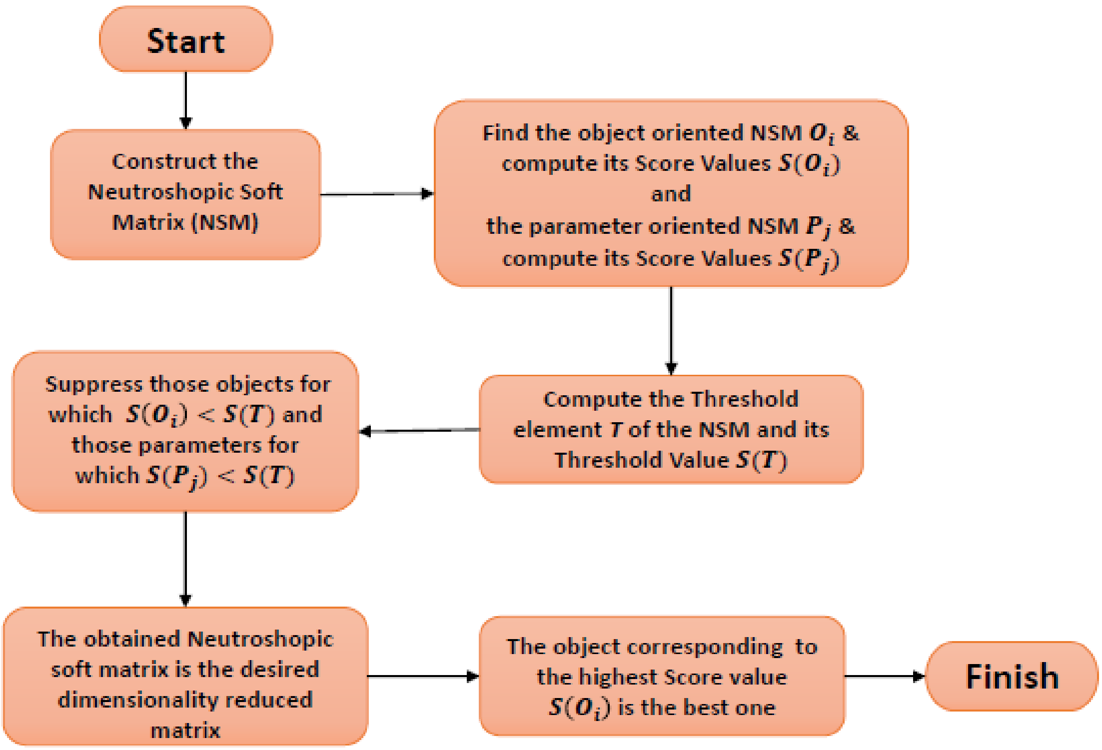

Figure 2: Algorithm for Dimensionality Reduction Using Neutrosophic Soft Matrix

4. Application of Dimensionality Reduction in Decision-Making

We consider an illustrative numerical example in this section for showing the step by step implementation of the proposed algorithm.

Example: Consider there are 5 suppliers (say) $U = \{u_1, u_2, u_3, u_4, u_5\}$ whose proficiencies are being evaluated on the criteria given by $P = \{p_1, p_2, p_3\}$, where" p_1: level of technology innovation",

" p_2 : ability of management", " p_3: level of services". The available data in the form of a neutrosophic soft set is shown below:

$$(F,P)=\{\{F(p_1)=(u_1,0.5,0.6,0.4),(u_2,0.9,0.4,0.1),(u_3,0.6,0.4,0.2),(u_4,0.6,0.4,0.2),(u_5,0.4,0.5,0.3)\}$$
$$\{F(p_2)=(u_1,0.6,0.7,0.2),(u_2,0.5,0.7,0.1),(u_3,0.5,0.4,0.4),(u_4,0.8,0.6,0.2),(u_5,0.6,0.4,0.2)\}$$
$$\{F(p_3)=(u_1,0.5,0.6,0.2),(u_2,0.4,0.8,0.3),(u_3,0.7,0.6,0.2),(u_4,0.6,0.4,0.4),(u_5,0.5,0.5,0.1)\}\}$$

Step 1. First we construct the respective neutrosophic soft matrix.

$$\begin{bmatrix} & p_1 & p_2 & p_3 \\ u_1 & (0.5,0.6,0.4) & (0.6,0.7,0.2) & (0.5,0.6,0.2) \\ u_2 & (0.9,0.4,0.1) & (0.5,0.7,0.1) & (0.4,0.8,0.3) \\ u_3 & (0.6,0.4,0.2) & (0.5,0.4,0.4) & (0.7,0.6,0.2) \\ u_4 & (0.6,0.4,0.2) & (0.8,0.6,0.2) & (0.6,0.6,0.4) \\ u_5 & (0.4,0.5,0.3) & (0.6,0.4,0.2) & (0.5,0.5,0.1) \end{bmatrix}$$

Step 2. Find the *object-oriented neutrosophic soft matrix* O_i for *i* = 1, 2, 3, 4, 5 and the *parameter- oriented neutrosophic soft matrix* P_j for *j* = 1, 2, 3.

$$\begin{bmatrix} & p_1 & p_2 & p_3 & O_i \\ u_1 & (0.5,0.6,0.4) & (0.6,0.7,0.2) & (0.5,0.6,0.2) & (0.533,0.633,0.2667) \\ u_2 & (0.9,0.4,0.1) & (0.5,0.7,0.1) & (0.4,0.8,0.3) & (0.6,0.633,0.1667) \\ u_3 & (0.6,0.4,0.2) & (0.5,0.4,0.4) & (0.7,0.6,0.2) & (0.6,0.4667,0.2667) \\ u_4 & (0.6,0.4,0.2) & (0.8,0.6,0.2) & (0.6,0.6,0.4) & (0.6667,0.4667,0.2667) \\ u_5 & (0.4,0.5,0.3) & (0.6,0.4,0.2) & (0.5,0.5,0.1) & (0.5,0.4667,0.2) \\ P_j & (0.6,0.46,0.24) & (0.6,0.56,0.22) & (0.54,0.58,0.24) & \end{bmatrix}$$

Now, the score matrix of *object-oriented neutrosophic soft matrix* $S(O_i)$ and *parameter- oriented neutrosophic soft matrix* $S(P_j)$ is given as:

$$\begin{bmatrix} & p_1 & p_2 & p_3 & O_i & S(O_i) \\ u_1 & (0.5,0.6,0.4) & (0.6,0.7,0.2) & (0.5,0.6,0.2) & (0.533,0.633,0.2667) & 0.364179 \\ u_2 & (0.9,0.4,0.1) & (0.5,0.7,0.1) & (0.4,0.8,0.3) & (0.6,0.633,0.1667) & 0.494479 \\ u_3 & (0.6,0.4,0.2) & (0.5,0.4,0.4) & (0.7,0.6,0.2) & (0.6,0.4667,0.2667) & 0.475531 \\ u_4 & (0.6,0.4,0.2) & (0.8,0.6,0.2) & (0.6,0.6,0.4) & (0.6667,0.4667,0.2667) & 0.542231 \\ u_5 & (0.4,0.5,0.3) & (0.6,0.4,0.2) & (0.5,0.5,0.1) & (0.5,0.4667,0.2) & 0.40666 \\ P_j & (0.6,0.46,0.24) & (0.6,0.56,0.22) & (0.54,0.58,0.24) & & \\ S(P_j) & 0.4896 & 0.4768 & 0.4008 & & \end{bmatrix}$$

Step 3. Compute the threshold element and threshold value of the neutrosophic soft matrix and its score value:

$$T=\left[(0.58,0.546,0.233)\right] \text{ and } S(T)=0.452782\,.$$

Step 4. Now, we suppress those alternatives for which $S(O_i) < S(T)$ and those parameters for which $S(P_j) > S(T)$. Thus, our new desired matrix M' is given as:

$$M' = \begin{bmatrix} & p_3 \\ u_2 & 0.4945 \\ u_3 & 0.4755 \\ u_4 & 0.5422 \end{bmatrix}$$

Since the score value for supplier u_4 is highest than the other score values, therefore, the supplier u_4 is the best one to choose.

On the other hand, the same problem is studied by Sumathi and Arockiarani [38] and the solution based on their proposed methodology is as follows:

$$A_{AM} = \begin{bmatrix} 0.3664 \\ 0.4944 \\ 0.4755 \\ 0.5422 \\ 0.4067 \end{bmatrix}$$

Therefore, the supplier u_4 is best.

Comparative Remarks:

Based on the above calculations and analysis, the following are the important comparative remarks:

- Sumathi and Arockiarani [38] solved the problem of decision-making without using the concept of dimension reduction and found that the supplier u_4 is highly preferable for any other supplier.
- The proposed methodology has first dimensionally reduced the available data and then worked out that the supplier u_4 is the most suitable one.
- Hence, the proposed method is consistent and better enough for solving decision-making problems.

Advantages of the Proposed Work:

In view of the above detailed analysis, the proposed algorithm for dimensionality reduction by utilizing the concept of neutrosophic soft matrices is found to be worthy enough in contrast with the existing related literatures. The following are the major advantages of the proposed work:

- The proposed methodology has significantly reduced the amount of the data and in addition the decision is found to be equally consistent, reliable and dependable.
- The methodology involves the notions of matrices and hence will prove to be widely applicable in many real-world applications.

- In case of large data set, the proposed methodology may suitably be implemented using the matrices for which we have the built-in-tools.

5. Conclusions and Scope for Future Work

In this paper, the technique for finding threshold value of the neutrosophic soft matrix is successfully provided with the definition of object-oriented and parameter-oriented neutrosophic soft matrix. An algorithm for dimensionality reduction has been properly outlined step by step. A numerical example clearly demonstrates the proposed methodology. In order to exhibit the viability and flexibility of the proposed algorithm, an example related to the decision-making problem has also been presented in detail. The example clearly validates our contribution and demonstrates that the proposed algorithm efficiently applies for the dimension reduction process.

The proposed dimensionality reduction technique may further be applied in the following area:

- **Enhancing the performance of large-scale image retrieval:** In large multimedia databases, it may not be feasible to search through the whole database in order to retrieve the nearest neighbors for a query. For similarity search and indexing, we do need a good data structure. It is quite possible that the existing data structures do not translate well for the high dimensional multimedia descriptors. By utilizing the proposed algorithm for the dimensionality reduction, we can map the nearest neighbors in the high dimensional space to nearest neighbors in the lower dimensional space. Similarly, in the field of content-based image retrieval (CBIR), the utilization of the dimensionality reduction algorithm may be in the images on the basis of textual features and images on the basis of visual features than to apply the traditional methods where all indexes (features) to be used to compare images which will lead to a large size image collection.
- **Face Recognition Algorithm:** In the field of face recognition, a typical face recognition algorithm is 100 x 100 pixels in size i.e., 10000-dimensional vector, not all dimensions are needed. By applying the proposed algorithm for the dimensionality reduction, we can reduce the dimensional vectors. In the intrusion detection/ data mining applications, dimensionality reduction focuses on representing the data with minimum number of dimensions such that its properties are not lost and hence reducing the underlying complexity in the processing of the data. By using the proposed algorithm, we can map a given set of high dimensional data points into a surrogate low dimensional space.

Funding: This research received no external funding.

Conflict of interest: The authors declare no conflict of interest.

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Received: June 21, 2019, Accepted: October 15, 2019