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De-Neutrosophication Technique of Pentagonal Neutrosophic Number and Application in Minimal Spanning Tree

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Abstract: In this current era, neutrosophic set theory is a crucial topic to demonstrate the ambiguous information due to existence of three disjunctive components appears in it and it provides a wide range of applications in distinct fields for the researchers. Generally, neutrosophic sets is the extended version of crisp set, fuzzy set and intuitionistic fuzzy sets to focus on the uncertain, hesitant and ambiguous datas of a real life mathematical problem. Demonstration of pentagonal neutrosophic number and its classification in different aspect is focused in this research article. Manifestation of de-neutrosophication technique of linear pentagonal neutrosophic number using removal area method has been developed here which has a remakable impact in crispfication of pentagonal neutrosophic number. Afterthat, utilizing this invented result, a minimal spanning tree problem has been solved in pentagonal neutrosophic environment. Comparision analysis is done with the other established method in this article and this noble design will be benificial for the researchers in neutrosophic domain in future.

1. Introduction

Currently, one of the eminent experimental studies of this era is on the subject of unpredictability and indeterminateness. On this aspect, Conception of Fuzzy set [1] has come up with an efficient way to work on. The theory of uncertainty plays an important role to deal with different issues relating to structure modelling in engineering domain, to do statistical calculation, in the field of social science and in any sort of real life problems relating to decision making and networking. After the invention of fuzzy set theory, researchers from several fields developed triangular [2, 3], trapezoidal [4], pentagonal [5] fuzzy number and its applications in various field of research. Professor Atanassov [6] put forward the concept of intuitionistic fuzzy sets where he considered both the idea of membership and non-membership functions. Later, in 2007 Liu F [7], merged the idea of triangular fuzzy set and intuitionistic set and created triangular intuitionistic fuzzy set. Further, Ye [8] familiarized with a basic concept on trapezoidal intuitionistic fuzzy set which includes both the truthiness and falseness membership function which are trapezoidal number in nature. Disjunctive interesting models in science and technology are developed day by day due to the invention of uncertainty theory.

In year 1995, Smarandache proposed the concept of neutrosophic sets, which was published in 1998 [9], comprised of three distinct logical components: i) truthfulness, ii) skepticism, iii) falsity. Due to the presence of hesitation component this theory gave a high impact in different kind of research domain. Further, Wang et al. [10] proposed single valued neutrosophic sets; Ye [11] formulated the concept of simplified Neutrosophic Sets, and Peng et. al. [12, 13] introduced some ideas on novel operations and aggregation operators. Recently, the concept of several forms of triangular and trapezoidal neutrosophic numbers having membership functions that are dependent or independent was manifested by Chakraborty et.al [14, 15]. In 2015, R. Helen [16] manifested the idea of pentagonal fuzzy number and A.Vigin [17] utilized it in neural network. T.Pathinathan [18] provided with the conception of reverse order triangular, trapezoidal and pentagonal fuzzy number. Several researches on neutrosophic arena were published in different fields like multi criteria decision making [19-26], graph theory [27-31], optimization techniques [32, 33] etc. Recently (2019), Chakraborty A [34] manifested the concept of pentagonal neutrosophic number and its classification component wise and applied it in solving a transportation problem in neutrosophic domain. Demonstration of pentagonal neutrosophic fuzzy number and its de-Neutrosophication value using removal area technique has been developed in this article, moreover it is applied on graph theory problem to evaluate the minimal spanning tree.

In this current epoch, neutrosophic set theory is applied in different sections of graph theory to evaluate the minimum path. Minimal spanning tree is one of the extremely vital concepts in the field of graph theory. Single valued neutrosophic minimal spanning tree and clustering method associated with it was originated by Ye [35]. Mandal & Basu [36] introduced similarity measure in optimum spanning tree problems related with neutrosophic arena. Mullai et.al [37] formulated minimum spanning tree problem in bipolar neutrosophic domain. Further, Broumi et.al [38, 39] manifested the concept of shortest path problem in neutrosophic graphs. Later, Broumi et.al [40] generated the perception of decision-making problem with the help of interval valued neutrosophic number and Kandasamy [41] developed double-valued neutrosophic sets and their application in minimum spanning tree problems. Currently, Broumi et.al [42] formulated neutrosophic shortest path for solving Dijkstra algorithm in graph theory. A few published articles [43-50] are addressed here related with neutrosophic domain which plays an important role in uncertainty research arena. Recently, in 2019 F. Smandache and Abdel Basset developed a concept namely Plithogenic set, which has a great impact in current research arena and its is applied in hospital care system [51], IoT based problem [52], multi criteria decision making problem [53], cancer related problems [54], fractal programming problem [55], hybrid MCDM problem [56,57] and forecasting problems [58] etc.

1.1 Motivation

The invention of uncertainity theory plays a vital role in formulation of real-life scientific mathematical model, structural modelling in engineering domain, multi criteria oriented medical diagonosis problem etc. Recently, a question will arrise if someone choose pentagonal neutrosophic number in any field of research then what will be the crispification value of this said number? How can we convert a pentagonal neutrosophic number equivalent to a crisp number in logical and scientific way? How can we generated some motivating approach in de-neutrosophication technique? Again, The concept of minimal spanning tree is a very well known concept in mathematics field. Now, generally we considered crisp numbers in place of weight in a spanning tree problem. But, suppose the exact value of the weights are unknown to us and decision maker's mind is in dilema in case of putting the exact weights. Thus, it is a conception of neutrosophic number which contains truth, falsity and hesitation components. Here we consider pentagonal neutrosophic numbers to allocate the weights of a spanning tree problem. Now, question will arise at once how can we tackle this problem in neutrosophic environment? From this aspect we shall try to built up this article.

The following table discusses the measurement of uncertaininty, vagueness and hesitation of four disjunctive types of Minimal Spanning Tree including crisp environment, fuzzy environment, intuitionistic fuzzy environment and pentagonal neutrosophic environment.

Edge Parameters in case of Minimal Spanning Tree Problem	**Measurement of Uncertainty**	**Measurement of Hesitation**	**Measurement of Vagueness**
Crisp Number	×	×	×
Crisp Interval Valued Number	×	×	×
Fuzzy Number	Can Determine	×	×
Interval Valued Fuzzy Number	Can Determine	×	×
Intuitionistic Fuzzy Number	Can Determine	×	Can Determine
Interval Valued Intuitionistic Fuzzy Number	Can Determine	×	Can Determine
Pentagonal Neutrosophic Number	Can Determine	Can Determine	Can Determine

From the above table, it is observed that only pentagonal neutrosophic environment can tackle the impreciseness, hesitation and truthiness in a membership function of a uncertain number, which is more reliable, logical and realistic for a decision maker. Thus, we consider our minimal spanning tree model in neutrosophic arena and all the edges of the graph as pentagonal neutrosophic number all the graph.

Advantage and Restrictions of disjunctive categories of set

The below table will shows us the advantage and restrictions of different kind of parameters in our real life mathematical problems.

Disjunctive Categories of Set/Number	Advantages	Restrictions
Crisp Number	Determine the accurate value of a realistic problem perfectly.	Cannot determine the uncertainty information of a realistic problem.
Fuzzy Number	Can describe the uncertainty information of a realistic problem.	Cannot describe the hesitation & falsity information of a realistic problem.
Intuitionistic Fuzzy Number	Can determine the uncertainty &falsity information of a realistic problem.	Cannot determine the hesitation information of a realistic problem.
Pythagorean Fuzzy Number	Can deal with the uncertainty & falsity information of a realistic problem.	Cannot deal with the hesitation information of a realistic problem.
Neutrosophic Fuzzy Number	Can describe the uncertainty, falsity & hesitation information of a realistic problem.	Cannot describe the incomplete weight information of a realistic problem.

1.2 Contribution

In this research article, researchers are primarily focused on pentagonal neutrosophic fuzzy number and its properties. A very engrossing question will arises among the researchers from all around the world that how a neutrosophic number can be transformed into a crisp number? From the last century, researchers are tried to develop lots of new methods associated with the de-Neutrosophication technique for crispification. Here, we generate the idea of crispification of pentagonal neutrosophic fuzzy number is enlarged using removal area skill. Nowadays, researchers are giving their attention to solve the problem of minimal spanning tree in neutrosophic arena. By utilizing the idea of newly generated de-Neutrosophication skill on pentagonal neutrosophic number field, we can able to tackle the problems on minimal spanning tree. Lastly, comparison analysis is done with the established methods to show the importance of this algorithm.

1.3 Novelties

Several research articles had already published in different journals on neutrosophic arena. Researches from different domain applied this concept in distinct areas also. The conception of pentagonal neutrosophic number is totally new in research domain. Thus it can be extended into different fields and can be applied into various research arenas. However a few numbers of articles has been developed in pentagonal neutrosophic environment till now. Thus, our motivation and target is to try to sketch out some unpublished points that are described below.

- Formulation of linear pentagonal neutrosophic number and its classification.
- De-Neutrosophication technique of linear pentagonal neutrosophic number.
- Application in minimal spanning tree problem.

1.4 Structure of the paper

In this reseach article section 1 contains introduction and litrature survey of neutrosophic number, section 2 covers mathematical preliminaries, section 3 admits a de-neutrosophication technique of of linear pentagonal neutrosophic fuzzy number, section 4 covers minimal spanning tree problem in neutrosophic environment, section 5 shows comparison table and lastly section 6 contains the conclusion part of the total reseach work.

2. Mathematical Preliminaries

Definition 2.1: Fuzzy Set: [1] A set $\tilde{C}$, is denoted as $\tilde{C} = \{(x, \mu_{\tilde{C}}(x)): x \in X, \mu_{\tilde{C}}(X) \in [0,1]\}$ and is generally represented by $(x, \mu_{\tilde{C}}(x))$, where $x \in$ the crisp set X and $\mu_{\tilde{C}}(X) \in$ the interval $[0, 1]$, then set $\tilde{C}$ is called an intuitionistic fuzzy set.

Definition 2.2: Intuitionistic Fuzzy Set (IFS): A set [6] $\tilde{P}$, is defined as $\tilde{P} = \{\langle x; [\tau(x), \varphi(x)]\rangle : x \in X\}$, where $\tau(x): X \to [0,1]$ is named as the truth membership function which indicate the degree of assurance, $\varphi(x): X \to [0,1]$ is named the falsity membership and $\tau(x), \varphi(x)$ satisfies the following the relation

$0 \le \tau(x) + \varphi(x) \le 1.$

Definition 2.3: Neutrosophic Set: [9] A set $\widetilde{nA}$ is called a neutrosophic set if $\widetilde{nA} = \{\langle x; [\rho_{\widetilde{nA}}(x), \sigma_{\widetilde{nA}}(x), \omega_{\widetilde{nA}}(x)]\rangle : x \in X\}$, where $\rho_{\widetilde{nA}}(x): X \to [0,1]$ is said to be the truth membership function, $\sigma_{\widetilde{nA}}(x): X \to [0,1]$ is said to be the indeterminacy membership function and $\omega_{\widetilde{nA}}(x): X \to [0,1]$ is said to be the falsity membership function.

$\rho_{\widetilde{nA}}(x), \sigma_{\widetilde{nA}}(x) \& \omega_{\widetilde{nA}}(x)$ exhibits the following relation:

$-0 \le \rho_{\widetilde{nA}}(x) + \sigma_{\widetilde{nA}}(x) + \omega_{\widetilde{nA}}(x) \le 3+$

Definition 2.4: Single-Valued Neutrosophic Set: A Neutrosophic set $\widetilde{nA}$ in the definition 2.1 is said to be a Single-Valued Neutrosophic Set $(\widetilde{SnA})$ if x is a single-valued independent variable. $\widetilde{SnA} = \{\langle x; [\alpha_{\widetilde{SnA}}(x), \beta_{\widetilde{SnA}}(x), \gamma_{\widetilde{SnA}}(x)]\rangle : x \in X\}$, where $\alpha_{\widetilde{SnA}}(x), \beta_{\widetilde{SnA}}(x) \& \gamma_{\widetilde{SnA}}(x)$ denoted the concept of trueness, indeterminacy and falsity memberships function respectively.

If there exist three points $p_0, q_0 \& r_0$, for which $\alpha_{\widetilde{SnA}}(p_0) = 1, \beta_{\widetilde{SnA}}(q_0) = 1 \& \gamma_{\widetilde{SnA}}(r_0) = 1$, then the $\widetilde{SnA}$ is called neut-normal.

$\widetilde{SnS}$ is called neut-convex, which follows the relation:

$\alpha_{\widetilde{SnA}}\langle \delta p_1 + (1-\delta)p_2\rangle \ge min\langle \alpha_{\widetilde{SnA}}(p_1), \alpha_{\widetilde{SnA}}(p_2)\rangle$

$\beta_{\widetilde{SnA}}\langle \delta p_1 + (1-\delta)p_2\rangle \le max\langle \beta_{\widetilde{SnA}}(p_1), \beta_{\widetilde{SnA}}(p_2)\rangle$

$\gamma_{\widetilde{SnA}}\langle \delta p_1 + (1-\delta)p_2\rangle \le max\langle \gamma_{\widetilde{SnA}}(p_1), \gamma_{\widetilde{SnA}}(p_2)\rangle$

where $p_1 \& p_2 \in \mathbb{R}$ and $\delta \in [0,1]$

Definition 2.5: Single-Valued Pentagonal Neutrosophic Number: A Single-Valued Pentagonal Neutrosophic Number $(\tilde{S})$ is defined and described as $\tilde{s} = \langle[(g^1, h^1, i^1, j^1, k^1); \rho], [(g^2, h^2, i^2, j^2, k^2); \sigma], [(g^3, h^3, i^3, j^3, k^3); \omega]\rangle$, where $\rho, \sigma, \omega \in [0,1]$. The truth membership function $(\theta_{\tilde{S}}): \mathbb{R} \rightarrow [0, \rho]$, the indeterminacy membership function $(\phi_{\tilde{S}}): \mathbb{R} \rightarrow [\sigma, 1]$ and the falsity membership function $(\psi_{\tilde{S}}): \mathbb{R} \rightarrow [\omega, 1]$ are given as:

$$\theta_{\tilde{S}}(x) = \begin{cases} \theta_{\widetilde{Sl1}}(x) g^1 \leq x < h^1 \\ \theta_{\widetilde{Sl2}}(x) h^1 \leq x < i^1 \\ \rho \quad x = i^1 \\ \theta_{\widetilde{Sr2}}(x)\, i^1 \leq x < j^1 \\ \theta_{\widetilde{Sr1}}(x) j^1 \leq x < k^1 \\ 0 \quad otherwise \end{cases}, \quad \phi_{\tilde{S}}(x) = \begin{cases} \phi_{\widetilde{Sl1}}(x) g^2 \leq x < h^2 \\ \phi_{\widetilde{Sl2}}(x) h^2 \leq x < i^2 \\ \sigma \quad x = i^2 \\ \phi_{\widetilde{Sr2}}(x)\, i^2 \leq x < j^2 \\ \phi_{\widetilde{Sr1}}(x) j^2 \leq x < k^2 \\ 1 \quad otherwise \end{cases}$$

$$\psi_{\tilde{S}}(x) = \begin{cases} \psi_{\widetilde{Sl1}}(x) g^3 \leq x < h^3 \\ \psi_{\widetilde{Sl2}}(x) h^3 \leq x < i^3 \\ \omega \quad x = i^3 \\ \psi_{\widetilde{Sr2}}(x)\, i^3 \leq x < j^3 \\ \psi_{\widetilde{Sr1}}(x) j^3 \leq x < k^3 \\ 1 \quad otherwise \end{cases}$$

3. De-Neutrosophication of a Linear Neutrosophic Pentagonal Number

On development of the de-Neutrosophication technique, results can be generated into a crisp number according to the results of pentagonal neutrosophic number and its membership functions. Researchers from all around the globe are concerned to know what shall be the crisp value associating the pentagonal neutrosophic number having membership function? By the passing days, they have continuously developed some convenient means to change a fuzzy number to a crisp number and some of these approaches are discussed below:

1. BADD (basic defuzzification distributions)
2. BOA (bisector of area)
3. CDD (constraint decision defuzzification)
4. COA (center of area)
5. COG (center of gravity)
6. ECOA (extended center of area)
7. EQM (extended quality method)
8. FCD (fuzzy clustering defuzzification), etc.

On this pentagonal neutrosophic arena, researches had an ambiguity in finding the suitable method of changing the pentagonal neutrosophic number to a crisp number. There are three distinct membership functions present in pentagonal neutrosophic number. To transform a neutrosophic number to a crisp number, "removal area method" is proposed on this article.

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Suppose, we consider a linear pentagonal neutrosophic number as follows

$$\tilde{A}_{Bineu} = (i_1, i_2, i_3, i_4, i_5; j_1, j_2, j_3, j_4, j_5; k_1, k_2, k_3, k_4, k_5)$$

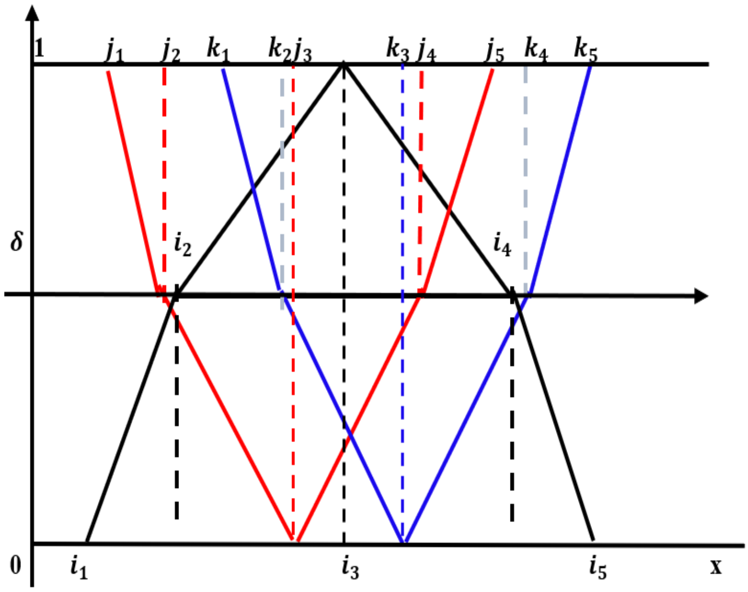

Figure 3.1: Graphical representation of Linear Pentagonal Neutrosophic Number.

Description of above figure: On the above figure we focused on the graphical presentation of linear pentagonal neutrosophic number. The black lined pentagonal represent the truth membership function. Red lined pentagonal represents the falsity membership function and blue lined pentagonal shows indefiniteness membership function of the number. In this, τ follows the relation $0 \leq \tau \leq 1$. The pentagonal number can be altered to triangular neutrosophic number if $\tau = 0 \; or \; 1$.

Let us assume a real number $s \in R$ and a fuzzy number $\breve{P}$ for black line specified pentagons, area of the left side distribution of $\breve{P}$ w.r.t s, is $A_{Neu\,l}(\breve{P}, s)$ that indicates the zone fenced by s and the left side of the fuzzy number $\breve{P}$. Proceeding in this way, the right zone area of $\breve{P}$ w.r.t s is $A_{Neu\,r}(\breve{P}, s)$. Considering a real number $s \in R$ along with the fuzzy number $\breve{Q}$ for the left most top and inverted pentagon, then area of lest side of $\breve{Q}$ wrt s is $A_{Neu\,l}(\breve{Q}, s)$is described as the area bounded by l and the left portion of the fuzzy number $\breve{Q}$. For the second time, the area of right side of Q wr.t l is $A_{Neu\,r}(\breve{Q}, s)$, A fuzzy number $\breve{R}$ for the right most top and inverted pentagon, then left side removal of $\breve{R}$ w.r.t s is $A_{Neu\,l}(\breve{R}, s)$ is described by the area bounded by s and the left side of the fuzzy number $\breve{R}$. similarly, the right portion removal of $\breve{R}$ w.r.t s is $A_{Neu\,r}(\breve{R}, s)$.

Mean is described as $A_{Neu}(\breve{P}, s) = \frac{A_{Neu\,l}(\breve{P},s)+A_{Neu\,r}(\breve{P},s)}{2}$, $A_{Neu}(\breve{Q}, s) = \frac{A_{Neu\,l}(\breve{Q},s)+A_{Neu\,r}(\breve{Q},s)}{2}$,

$A_{Neu}(\breve{R}, s) = \frac{A_{Neu\,l}(\breve{R},s)+A_{Neu\,r}(\breve{R},s)}{2}$

Then, we quantified the de-neutrosophication value of a linear pentagonal neutrosophic number as,

$$A_{Neu}(\widetilde{D_{Pen}}, s) = \frac{A_{Neu}(\breve{P}, s) + A_{Neu}(\breve{Q}, s) + A_{Neu}(\breve{R}, s)}{3}$$

For $s = 0$,

$$A_{Neu}(\check{P},0)=\frac{A_{Neu\,l}(\check{P},0)+A_{Neu\,r}(\check{P},0)}{2},\ A_{Neu}(\check{Q},0)=\frac{A_{Neu\,l}(\check{Q},0)+A_{Neu\,r}(\check{Q},0)}{2},\ A_{Neu}(\check{R},0)=\frac{A_{Neu\,l}(\check{R},0)+A_{Neu\,r}(\check{R},0)}{2}$$

Thus, $A_{Neu}(\widetilde{D_{Pen}},0)=\frac{A_{Neu}(\check{P},0)+A_{Neu}(\check{Q},0)+A_{Neu}(\check{R},0)}{3}$

Here, we take $\check{X}=(i_1,i_2,i_3,i_4,i_5),\check{Y}=(j_1,j_2,j_3,j_4,j_5),\check{Z}=(k_1,k_2,k_3,k_4,k_5)$

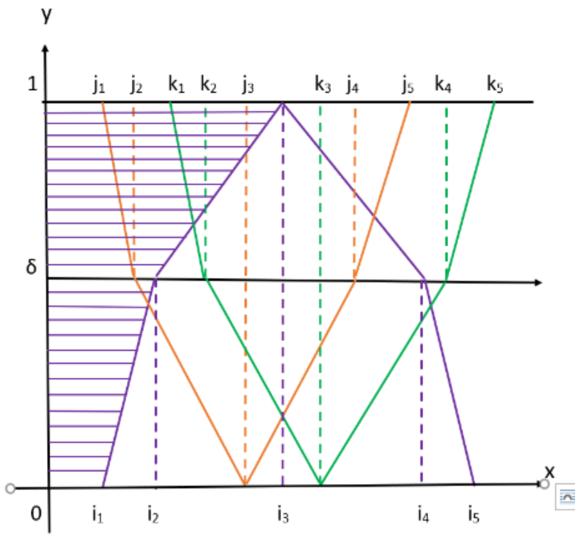

Figure 3.1(a)

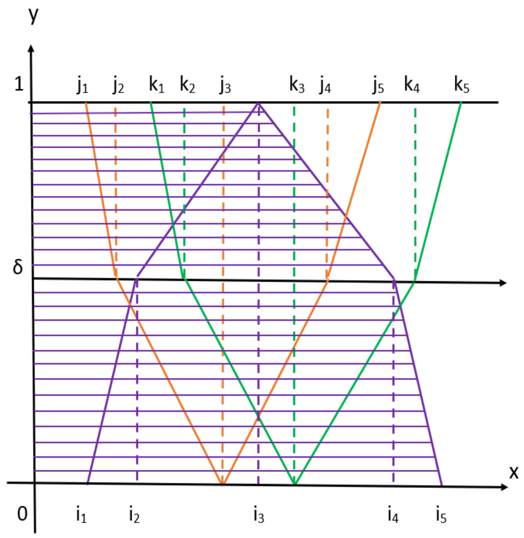

Figure 3.1(b)

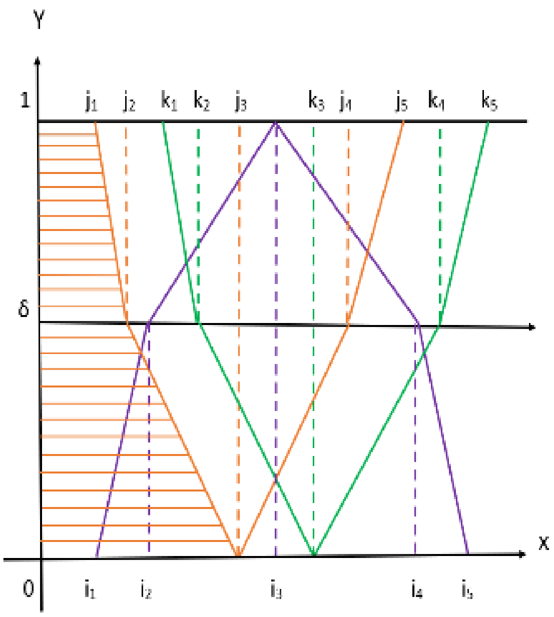

Figure 3.2(a)

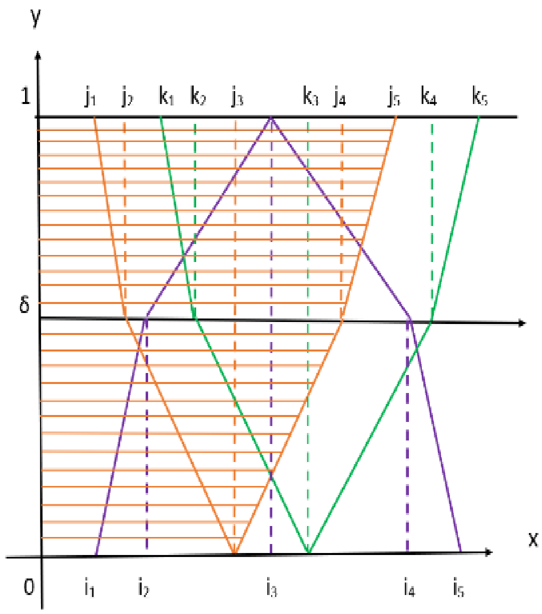

Figure 3.2(b)

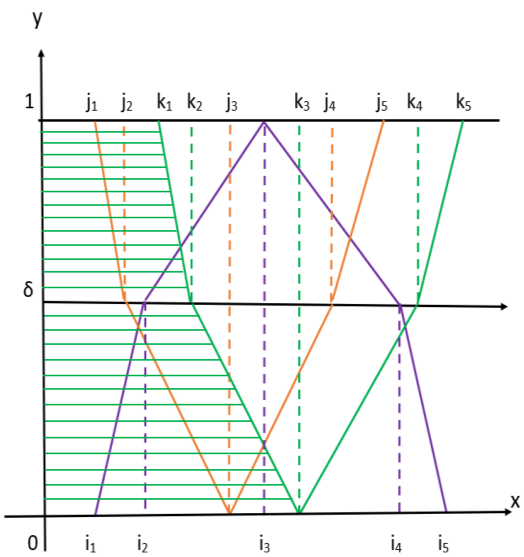

Figure 3.3(a)

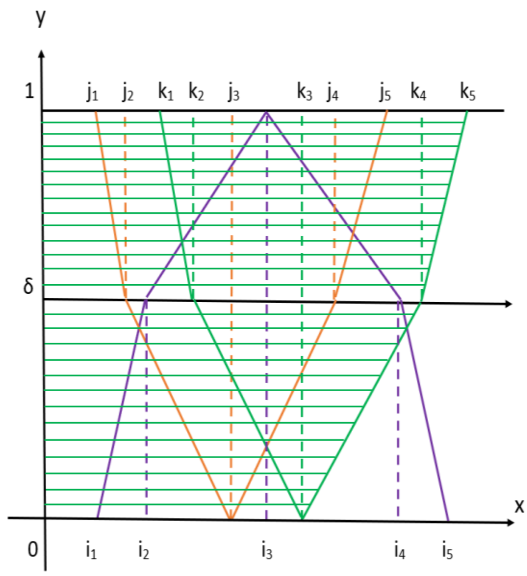

Figure 3.3(b)

Then,

$$A_{Neu\,l}(\check{P},0) = \text{Area of Figure 3.1(a)} = \frac{(i_1+i_2)\delta}{2} + \frac{(i_2+i_3)(1-\delta)}{2}$$

$$A_{Neu\,r}(\check{P},0) = \text{Area of Figure 3.1(b)} = \frac{(i_4+i_5)\delta}{2} + \frac{(i_3+i_4)(1-\delta)}{2}$$

$$A_{Neu\,l}(\check{Q},0) = \text{Area of Figure 3.2(a)} = \frac{(j_1+j_2)(1-\delta)}{2} + \frac{(j_2+j_3)\delta}{2}$$

$$A_{Neu\,r}(\check{Q},0) = \text{Area of Figure 3.2(b)} = \frac{(j_3+j_4)\delta}{2} + \frac{(j_4+j_5)(1-\delta)}{2}$$

$$A_{Neu\,l}(\check{R},0) = \text{Area of Figure 3.3(a)} = \frac{(k_1+k_2)(1-\delta)}{2} + \frac{(k_2+k_3)\delta}{2}$$

$$A_{Neu\,r}(\check{R},0) = \text{Area of Figure 3.3(a)} = \frac{(k_3+k_4)\delta}{2} + \frac{(k_4+k_5)(1-\delta)}{2}$$

$$\text{Hence, } A_{Neu}(\check{P},0) = \frac{\frac{(i_1+i_2)\delta}{2}+\frac{(i_2+i_3)(1-\delta)}{2}+\frac{(i_4+i_5)\delta}{2}+\frac{(i_3+i_4)(1-\delta)}{2}}{2},$$

$$A_{Neu}(\check{Q},0) = \frac{\frac{(j_1+j_2)(1-\delta)}{2}+\frac{(j_2+j_3)\delta}{2}+\frac{(j_3+j_4)\delta}{2}+\frac{(j_4+j_5)(1-\delta)}{2}}{2}, \quad A_{Neu}(\check{R},0) = \frac{\frac{(k_1+k_2)(1-\delta)}{2}+\frac{(k_2+k_3)\delta}{2}+\frac{(k_3+k_4)\delta}{2}+\frac{(k_4+k_3)(1-\delta)}{2}}{2}$$

$$\text{So, } A_{Neu}(\widetilde{D_{Pen}},0) = \frac{(i_1+i_2+i_4+i_5+j_2+2j_3+j_4+k_2+2k_3+k_4)\delta+(i_2+2i_3+i_4+j_1+j_2+j_4+j_5+k_1+k_2+k_4+k_5)(1-\delta)}{12} \quad \text{..........(1)}$$

Table 3.1: Numerical computation of De-Neutrosophication value

Sl. No.	Pentagonal Neutrosophic Number	De-Neutrosophication value
1	(1,2,3,4,5;0.5,1.5,2.5,3.5,4.5;2.2,2.8,3.7,4.5,6)	3.091667
2	(0.5,1.5,2.5,3.5,4.5;0.3,1.3,2.3,3.3,4.3;1.8,2.8,3.8,4.8,5.8)	2.86667
3	(0.7,1.7,2.5,3.5,4.7;0.5,1.5,2.2,3.2,4;1.7,2.7,3.7,4.7,5.7)	2.86250
4	(1.2,2.2,3.2,4.2,5.2;1,2,3,4,5;2.5,3.5,4.5,5,6.5)	3.52500
5	(1,4,7,10,13;0.5,3.5,6.5,9.5,12.5;4.5,7.5,9,12,14.5)	7.66667

4. Minimal Spanning Tree in Pentagonal Neutrosophic Environment

Spanning Tree: Let, G is a graph and T is a subgraph of G. If T is a connected graph having no circuits and covers all vertices of G, then T is called a spanning tree.

Minimal Spanning Tree: A spanning tree which contains the least weight in G is defined as minimal spanning tree. Let us consider a graph in pentagonal neutrosophic domain. Here we developed an algorithm to search out the minimal spanning tree where the weights are pentagonal neutrosophic numbers. Thus this is a problem of neutrosophic graph.

Algorithm:

- Construct an adjacency matrix of the graph.
- Utilize de-Neutrosophication technique and construct crisp matrix.
- Select the least weight and if there is a tie in selection of least weight then take any one edge from the given graph.
- From the edges that are left behind select an edge containing the least edge that doesn't form a loop with the previous established figure.
- Continue this process until all vertices will be covered.
- Stop.

4.1 Illustrative Example:

To acquire a minimal spanning tree of the following graph

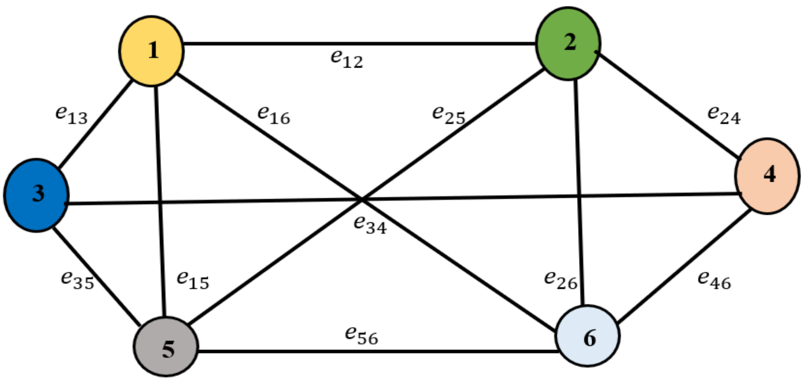

Figure 4.1.1: A Graph with pentagonal neutrosophic number Weight Edges

Table 1: The values of weights related with edges

Edges	Pentagonal Single-Valued Neutrosophic Weights
e_{12}	< 0.5,1,1.5,2,2.5; 0.3,0.7,1.2,1.6,2.2; 0.8,1.3,1.8,2.3,3 >
e_{13}	< 0.7,1.2,1.8,2.4,3; 0.6,1,1.4,2,2.5; 1,1.5,2,2.5,3.5 >
e_{15}	< 0.3,0.8,1.4,2,2.6; 0.2,0.7,1.2,1.8,2.2; 0.5,1,1.5,2.4,3 >
e_{16}	< 1,1.5,2,2.5,3; 0.7,1.2,1.8,2.2,2.6; 1.2,1.6,2.2,2.8,3.5 >
e_{25}	< 0.8,1.4,2,2.6,3.2; 0.6,1.2,1.8,2.2,2.8; 1,1.8,2.4,2.8,3.5 >
e_{26}	< 0.6,1,1.5,2,2.5; 0.4,0.8,1.2,1.8,2.2; 0.8,1.4,2,2.4,3 >
e_{24}	< 0.9,1.4,2,2.5,3; 0.6,1.2,1.6,2.1,2.4; 1.2,1.5,2.3,2.8,3.5 >
e_{34}	< 1.1,1.5,1.9,2.3,2.7; 0.8,1.2,1.7,2.1,2.4; 1.4,1.8,2.2,2.6,3 >
e_{46}	< 0.7,1,1.3,1.6,2; 0.6,0.9,1.2,1.5,1.8; 1,1.4,1.8,2.2,2.5 >
e_{35}	< 0.8,1.2,1.5,1.8,2.4; 0.5,0.9,1.3,1.7,2.1; 1,1.4,1.8,2.2,2.6 >
e_{56}	< 1.2,1.5,1.8,2.4,2.6; 0.9,1.3,1.7,2,2.3; 1.4,1.8,2.2,2.5,2.8 >

Step 1: The associated adjacency matrix of figure 1 is given as follows:

A =

< 0,0,0,0,0; 0,0,0,0,0; 0,0,0,0,0 >	< 0.5,1,1.5,2,2.5; 0.3,0.7,1.2,1.6,2.2; 0.8,1.3,1.8,2.3,3 >	< 0.7,1.2,1.8,2.4,3; 0.6,1,1.4,2,2.5; 1,1.5,2,2.5,3.5 >	< 0,0,0,0,0; 0,0,0,0,0; 0,0,0,0,0 >	< 0.3,0.8,1.4,2,2.6; 0.2,0.7,1.2,1.8,2.2; 0.5,1,1.5,2.4,3 >	< 1,1.5,2,2.5,3; 0.7,1.2,1.8,2.2,2.6; 1.2,1.6,2.2,2.8,3.5 >
< 0.5,1,1.5,2,2.5; 0.3,0.7,1.2,1.6,2.2; 0.8,1.3,1.8,2.3,3 >	< 0,0,0,0,0; 0,0,0,0,0; 0,0,0,0,0 >	–; –; –	< 0.9,1.4,2,2.5,3; 0.6,1.2,1.6,2.1,2.4; 1.2,1.5,2.3,2.8,3.5 >	< 0.8,1.4,2,2.6,3.2; 0.6,1.2,1.8,2.2,2.8; 1,1.8,2.4,2.8,3.5 >	< 0.6,1,1.5,2,2.5; 0.4,0.8,1.2,1.8,2.2; 0.8,1.4,2,2.4,3 >
< 0.7,1.2,1.8,2.4,3; 0.6,1,1.4,2,2.5; 1,1.5,2,2.5,3.5 >	–; –; –	< 0,0,0,0,0; 0,0,0,0,0; 0,0,0,0,0 >	< 1.1,1.5,1.9,2.3,2.7; 0.8,1.2,1.7,2.1,2.4; 1.4,1.8,2.2,2.6,3 >	< 0.8,1.2,1.5,1.8,2.4; 0.5,0.9,1.3,1.7,2.1; 1,1.4,1.8,2.2,2.6 >	–; –; –
< 0,0,0,0,0; 0,0,0,0,0; 0,0,0,0,0 >	< 0.9,1.4,2,2.5,3; 0.6,1.2,1.6,2.1,2.4; 1.2,1.5,2.3,2.8,3.5 >	< 1.1,1.5,1.9,2.3,2.7; 0.8,1.2,1.7,2.1,2.4; 1.4,1.8,2.2,2.6,3 >	< 0,0,0,0,0; 0,0,0,0,0; 0,0,0,0,0 >	–; –; –	< 0.7,1,1.3,1.6,2; 0.6,0.9,1.2,1.5,1.8; 1,1.4,1.8,2.2,2.5 >
< 0.3,0.8,1.4,2,2.6; 0.2,0.7,1.2,1.8,2.2; 0.5,1,1.5,2.4,3 >	< 0.8,1.4,2,2.6,3.2; 0.6,1.2,1.8,2.2,2.8; 1,1.8,2.4,2.8,3.5 >	< 0.8,1.2,1.5,1.8,2.4; 0.5,0.9,1.3,1.7,2.1 1,1.4,1.8,2.2,2.6 >	–; –; –	< 0,0,0,0,0; 0,0,0,0,0; 0,0,0,0,0 >	< 1.2,1.5,1.8,2.4,2.6; 0.9,1.3,1.7,2,2.3; 1.4,1.8,2.2,2.5,2.8 >
< 1,1.5,2,2.5,3; 0.7,1.2,1.8,2.2,2.6; 1.2,1.6,2.2,2.8,3.5 >	< 0.6,1,1.5,2,2.5; 0.4,0.8,1.2,1.8,2.2; 0.8,1.4,2,2.4,3 >	–; –; –	< 0.7,1,1.3,1.6,2; 0.6,0.9,1.2,1.5,1.8; 1,1.4,1.8,2.2,2.5 >	< 1.2,1.5,1.8,2.4,2.6; 0.9,1.3,1.7,2,2.3; 1.4,1.8,2.2,2.5,2.8 >	< 0,0,0,0,0; 0,0,0,0,0; 0,0,0,0,0 >

Step 2: Using the De-neutrosophic value, the associated matrix becomes

$$A = \begin{bmatrix} 0 & 1.504 & 1.788 & - & 1.433 & 1.983 \\ 1.504 & 0 & - & 1.933 & 2.012 & 1.570 \\ 1.788 & - & 0 & 1.917 & 1.542 & - \\ - & 1.933 & 1.917 & 0 & - & 1.433 \\ 1.433 & 2.012 & 1.542 & - & 0 & 1.900 \\ 1.983 & 1.570 & - & 1.433 & 1.900 & 0 \end{bmatrix}$$

Step 3: After examining, the least value is 1.433. So, the edge is selected which is connected with the nodes (1, 5) and thus labelled it. This procedure is repeated till the final spanning tree is found.

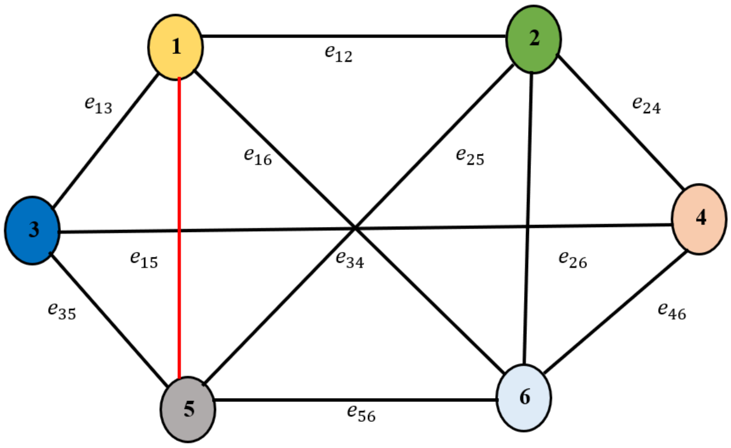

Figure 4.1.2 : Diagrammatic Presentation of Step 3

Step 4: After studying, the least value is 1.433 among the remaining weighted edges. Therefore the edge is selected connecting the nodes $(4,6)$ and label it.

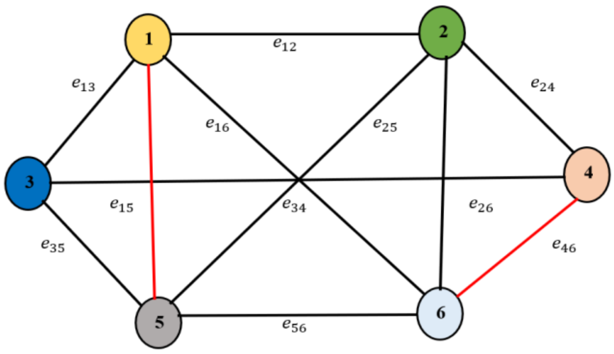

Figure 4.1.3: Diagrammatic Presentation of Step 4

Step 5: After examining, the least value is 1.504 amongst the remaining weighted edges. The edge is selected connecting nodes (1, 2) and thus marked it.

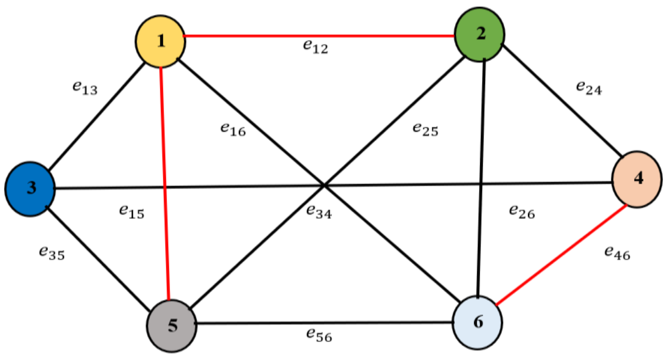

Figure 4.1.4: Diagrammatic Presentation of Step 5

Step 6: Examined that the least value is 1.542 among the remaining weighted edges. Hence, the edge is selected connecting with nodes (3, 5) and labelled it.

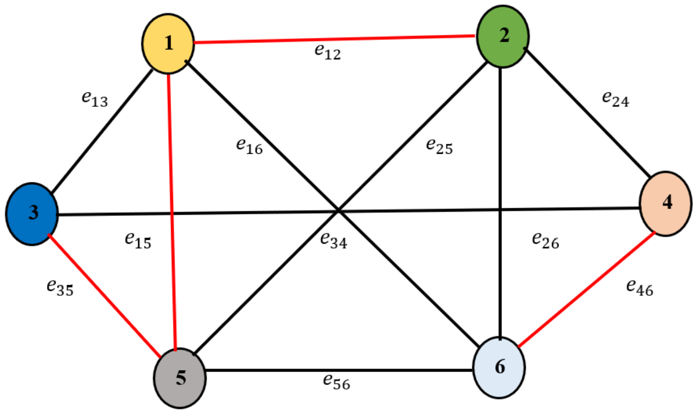

Figure 4.1.5: Diagrammatic Presentation of Step 6

Step 6: Examined that the least value is 1.570 out of the remaining weighted edges. Therefore, the edge is selected connecting the nodes $(2,6)$ and marked it.

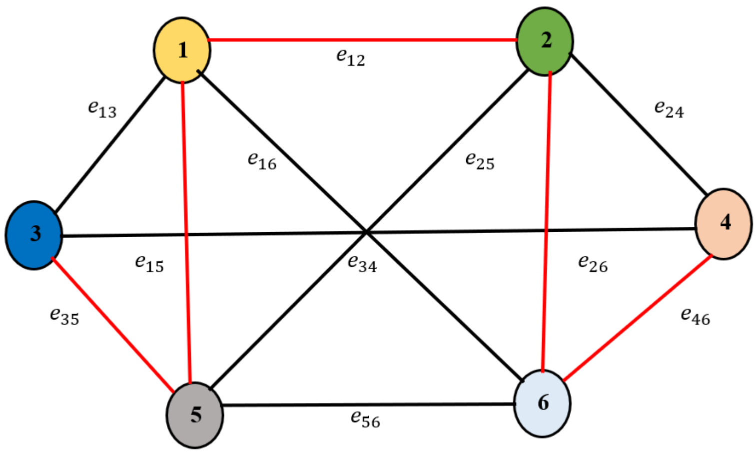

Figure 4.1.6: Diagrammatic Presentation of Step 7

Step 7: After examining all the nodes are joined and if more edges are to be joined it will form a circuit in the figure formed and as stated by the definition of a spanning tree it must not form any circuit but also all the nodes must be connected. Thus, the ultimate minimal spanning tree is followed:

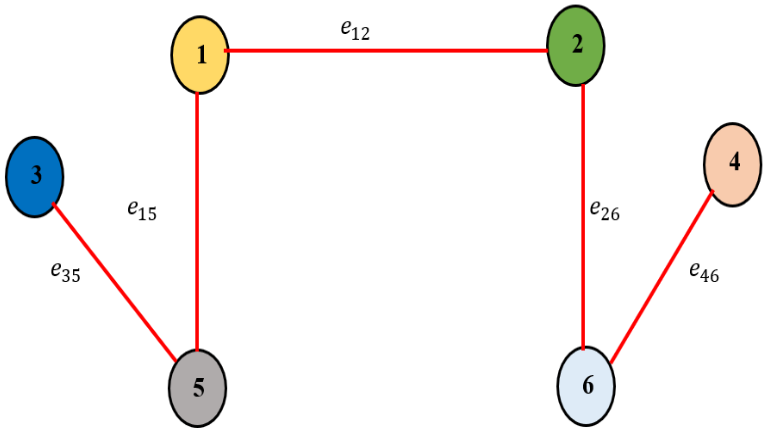

Figure 4.1.7: Diagrammatic Representation of ultimate minimal spanning tree

The least weight of the graph is – $(1.433 + 1.433 + 1.5041.542$+1.570$) = 7.482$ units.

5. Comparison:

Here, we compare our work with Mullai's [37] established algorithm. According to previous concept, the required minimal spanning tree can be obtained from the following steps.

Step 1: Let $S_1 = \{1\}$ *then* $\bar{S_1} = \{2,3,4,5,6\}$

Step 2: Let $S_2 = \{1,5\}$ *then* $\bar{S_2} = \{2,3,4,6\}$

Step 3: Let $S_3 = \{1,5,2\}$ *then* $\bar{S_3} = \{3,4,6\}$

Step 4: Let $S_4 = \{1,5,2,3\}$ *then* $\bar{S_4} = \{4,6\}$

Step 5: Let $S_5 = \{1,5,2,3,6\}$ *then* $\bar{S_5} = \{4\}$

Step 6: Let $S_6 = \{1,5,2,3,6,4\}$ *then* $\bar{S_6} = \{\phi\}$

The Required spanning tree is

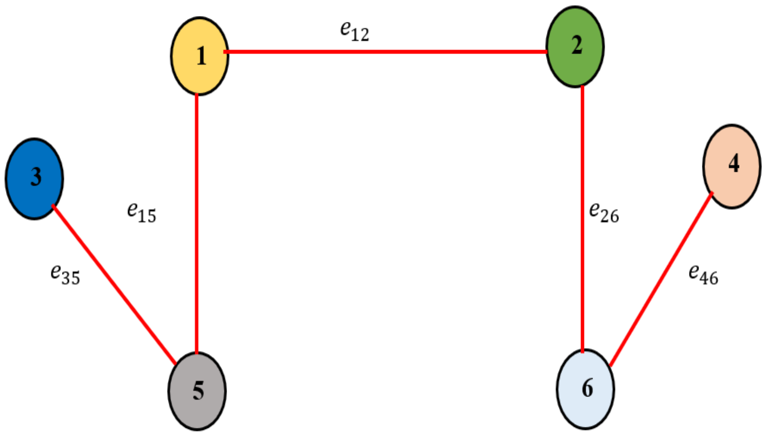

Figure 5.1: Diagrammatic Presentation of minimal spanning tree

Discussion: There is a contrast among the proposed approach and Mullai's technique is that Mullai's formulation is based on edges which are repeatedly evaluated at every steps of the algorithm which leads to the increase of time complexity. However, our technique relating with Matrix can be skillfully handled by utilizing Matlab software. In Mullai's method we need to consider each steps one by one manually but in our proposed method we can solve it using the help of computational software available in mathematics field with the help of computer as it is totally based on matrix concept. Thus we can claim that it will more useful and short time taking approach than any other established algorithm in this research domain.

6. Conclusion

In this research article, the concept of pentagonal neutrosophic number has been developed in a different aspect. Demonstration of De-Neutrosophication method utilizing the removal area technique has been introduced here for conversion of a pentagonal neutrosophic number into a real number. Further, this result is applied in the field of graph theory to evaluate the minimal spanning tree of a general graph. Comparison analysis is done with the established method which gave a crucial impact in this article for the evaluation of minimal spanning tree. Since, no work has been developed in this field so we can claim that this is the best method. Though the stated algorithm able to analyze the solutions of minimal spanning tree problem in pentagonal neutrosophic domain but more reliable, logical and short time taking algorithm maybe established in this field such that it can gives us much more fast, accurate and exact results after the total computation. Thus, these are the limitations of this stated algorithm in neutrosophic scenario.

In future, researchers can developed some interesting algorithms using pentagonal neutrosophic number in various fields like multi criteria decision making problem, image processing problem, pattern recognition problem, cloud computing problem and other mathematical modeling problems. Again, researcher may develop some new structural formulations of pentagonal neutrosophic number in different aspects. Also, researchers can compare this work with the new invented concept in pentagonal neutrosophic environment.

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