IMT Atlantique
Bretagne-Pays de la Loire
École Mines-Télécom

EPIC

Lab-STICC

Design of Low-Complexity Convolutional codes over $\mathrm{GF}(q)$

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OUTLINE

1. Introduction and motivation
2. Proposed design of NB-CC
3. Results
4. Conclusions

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Introduction and motivation

New standards requirements

- Long block lengths [1] [2]
  - Binary LDPC, polar, turbo codes
- Small block lengths [2]
  - Codes over high order GF

- NB-LDPC are studied in [3] [4]
- No general study on NB-TCs over GF(q) exists in the literature

[1] Adrian Voicila, David Declercq, Francois Verdier, Marc Fossorier, Pascal Urard. Low-Complexity Decoding for Non Binary LDPC Codes in High Order Fields. IEEE Transactions on Communications, Institute of Electrical and Electronics Engineers (IEEE), 2010, 58 (5), pp.1365-1375. <hal-00521074>

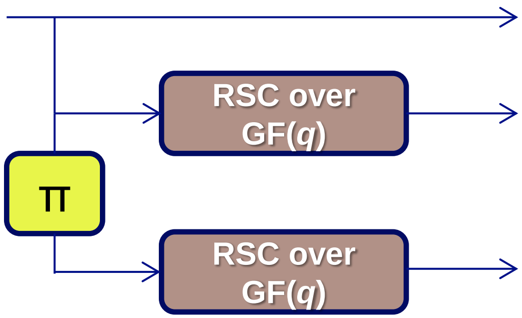
[2] Liva, G., Paolini, E., Matuz, B., Scalise, S., & Chiani, M. (2013). Short Turbo Codes over High Order Fields. IEEE Transactions on Communications, 61(6), 2201–2211. doi:10.1109/TCOMM.2013.041113.120539

[3] M. C. Davey and D. J. MacKay, "Low density parity check codes over GF(q)," IEEE Commun. Lett., vol. 2, no. 6, pp. 165–167, June 1998.

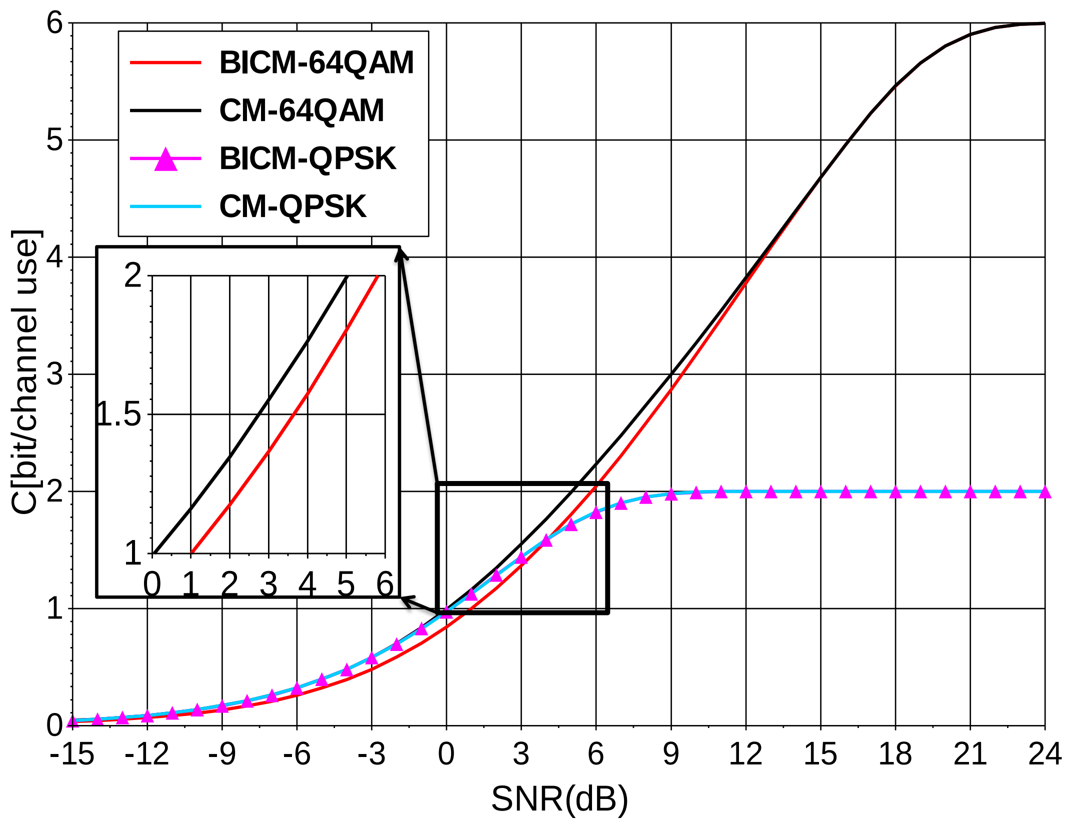
[4] C. Poulliat, M. Fossorier, and D. Declercq, "Design of regular (2,dc)-LDPC codes over GF(q) using their binary images," IEEE Trans. Commun., vol. 56, no. 10, pp. 1626–1635, Oct. 2008.

Introduction and motivation

Turbo codes over GF(q), q>2



BICM-CM



Advantages of NB-TCs

- Lower correlation in the decoding process
- Resisting to burst errors
- Theoretical performance gains [5] [6]
- Offer one of the best performance/complexity compromise [7]

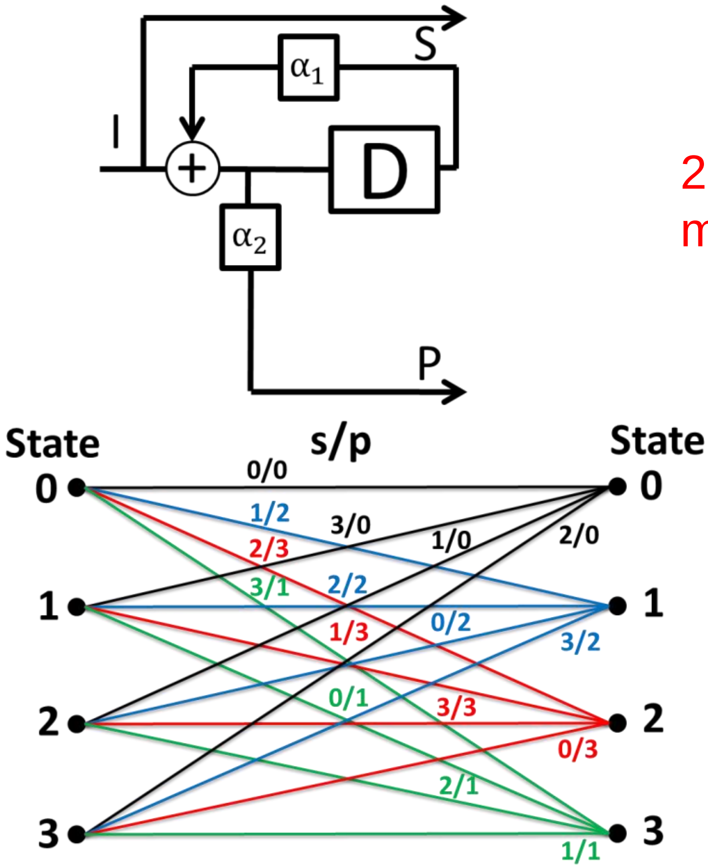
[5] G. Caire, G. Taricco, and E. Biglieri, “Bit-interleaved coded modulation,” IEEE Trans. Inform. Theory, vol. 44, no. 3, pp. 927–946, 1998

[6] A. Alvarado, “On bit-interleaved coded modulation with QAM constellations,” Master’s thesis, Chalmers University of Technology, 2008.

[7] F. Kienle, N. Wehn, and H. Meyr, “On complexity, energy-and implementation-efficiency of channel decoders,” IEEE Trans. On Commun., vol. 59, no. 12, pp. 3301–3310, 2011.

Proposed convolutional code design

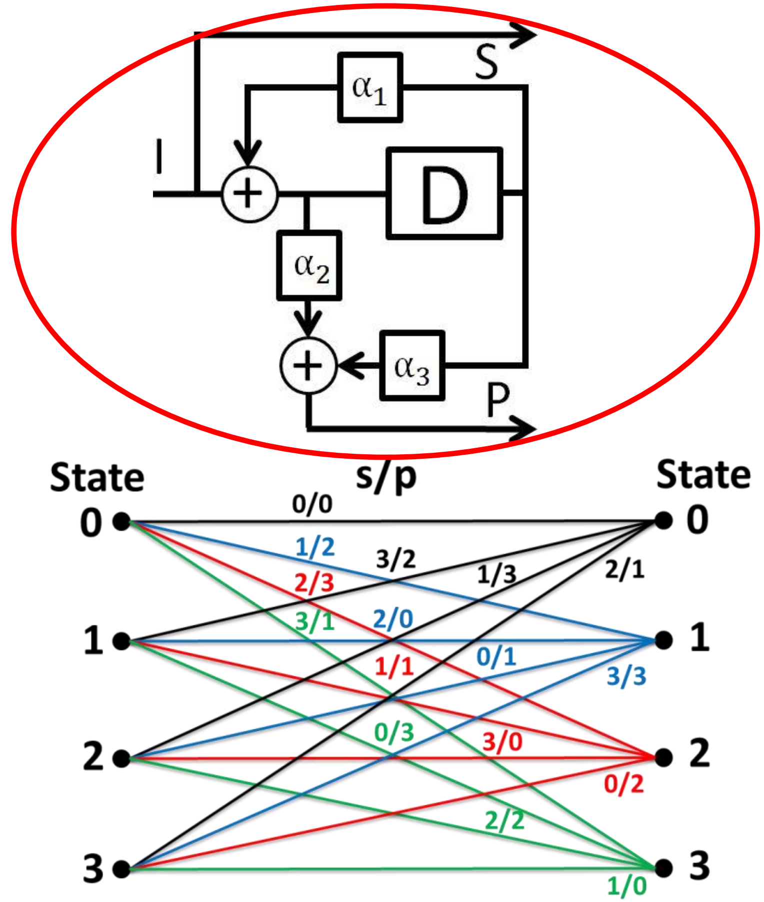
Accumulator encoder over GF(*q*), *q*>2



Same Parity when arriving to a state
→ systematic value responsible for the distance

25% gain in the minimum distance

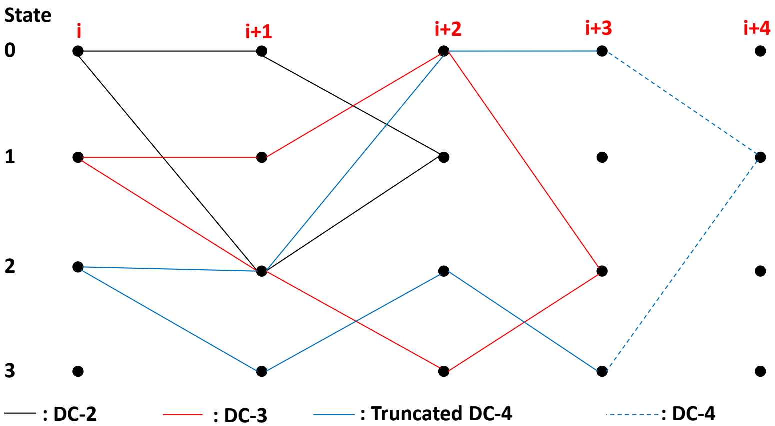
Proposed constituent encoder over GF(*q*), *q*>2



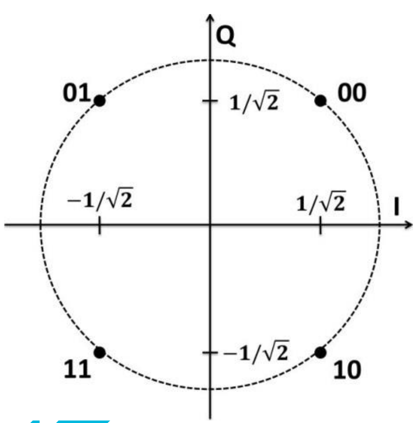
Distinct parity when arriving to a state
→ systematic and parity symbols contribute to the distance

Distance spectrum calculation

Error-prone sequences: Diverging-Converging (DC)



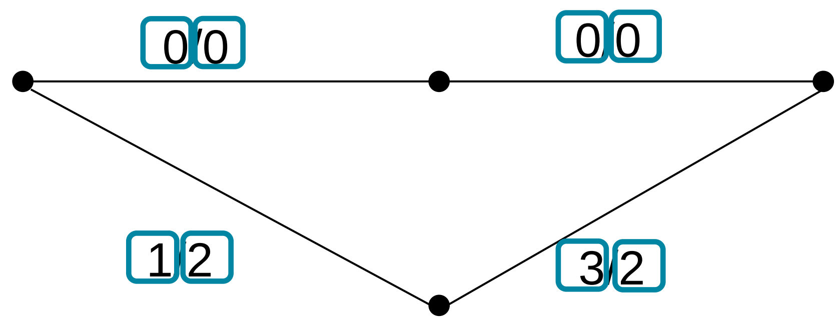
QPSK Constellation



Non-linearity of the non-binary coded modulation

- For high constellation orders:
  - Different protection levels for constellation symbols
  - Non-binary coded modulations are non-linear in signal space
  - All-zero sequence cannot be used as reference

Euclidian Distance Calculation



$$d_{euc}^2 = d_{01}^2 + d_{02}^2 + d_{03}^2 + d_{02}^2$$

Error rates mainly depend on shortest sequences !

Proposed algorithm to find best NB-CC over GF(q)

Define a q-QAM constellation mapping

↓

Define NB-CC by changing $\alpha_i \in \mathrm{GF}(q)$

↓

Calculate the distance spectrum by enumerating all candidate DC sequences

↓

Choose the code with the best distance spectrum

- When the constellation mapping is constant:
  - $(q-1)^3$ possible codes to test
- When the constellation mapping varies:
  - $q! * (q-1)^3$ possible codes to test

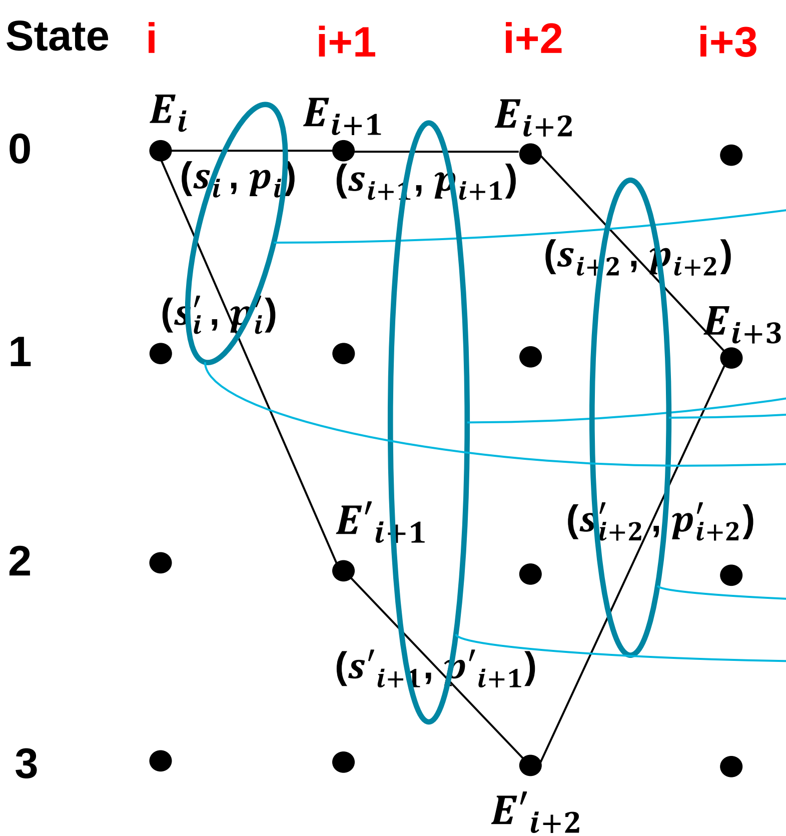
⇒ Prohibitive search complexity.

⇒ Question : Do we really need to consider all these combinations ?

- **Proposal: Constant mapping while limiting the enumeration to DC sequences of length-2 and 3**
  - Acceptable complexity
  - Accurate enumeration of the two lowest distances of the code

Effect of constant constellation mapping

Example of a length-3 DC sequences



$s_{i+1} = s'_{i+1}$ while $p_{i+1} \neq p'_{i+1}$

$p_{i+1} = p'_{i+1}$ while $s_{i+1} \neq s'_{i+1}$

Mapping changes, NB-CC is kept unvaried

$$N_\mu = \binom{q}{2}^2 * \left(\left(\binom{q}{2} + 1\right)^2 - 1\right) * \binom{q}{2}^2$$

Mapping is constant, NB-CC varies

$$N_\alpha = \binom{q}{2}^2 * \binom{q^2}{2} * \binom{q}{2}^2$$

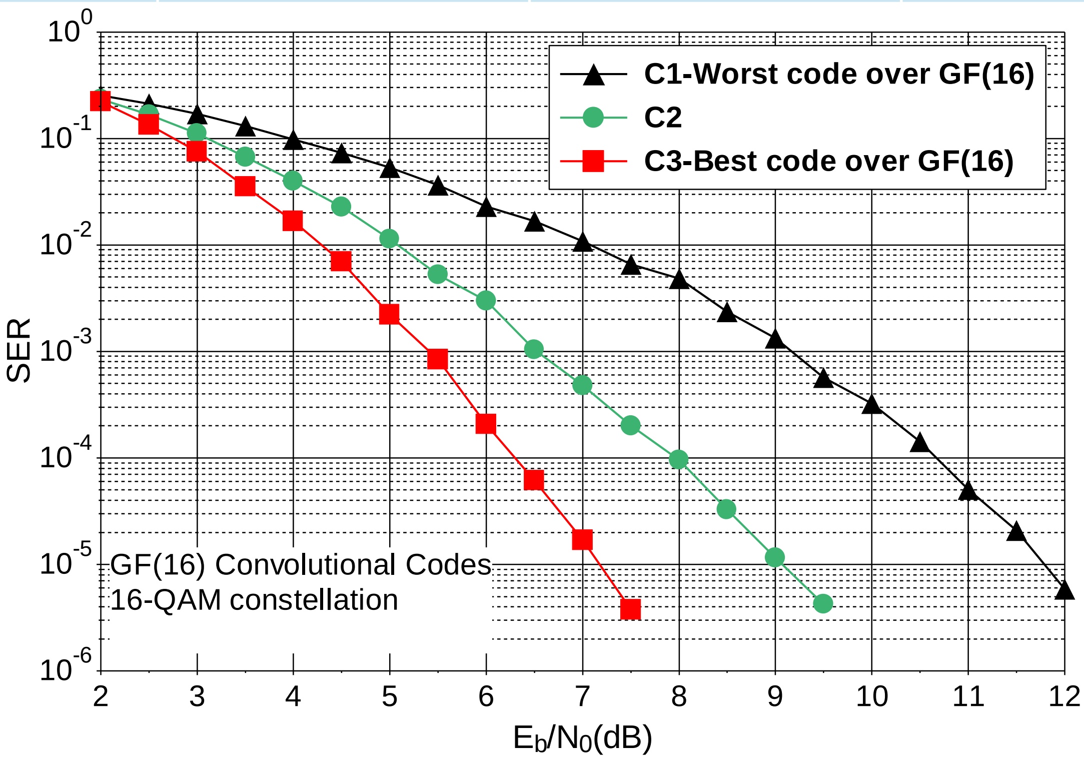
$$\delta_N = \binom{q^2}{2} - \left(\left(\binom{q}{2} + 1\right)^2 + 1\right) = \frac{q(q-1)^2(q+4)}{4} > 0 \ \forall\, q > 0$$

$N_\alpha > N_\mu$ ⇒ A constant mapping is sufficient to find all possible codes

Defined codes over GF(16)

Results over GF(16)

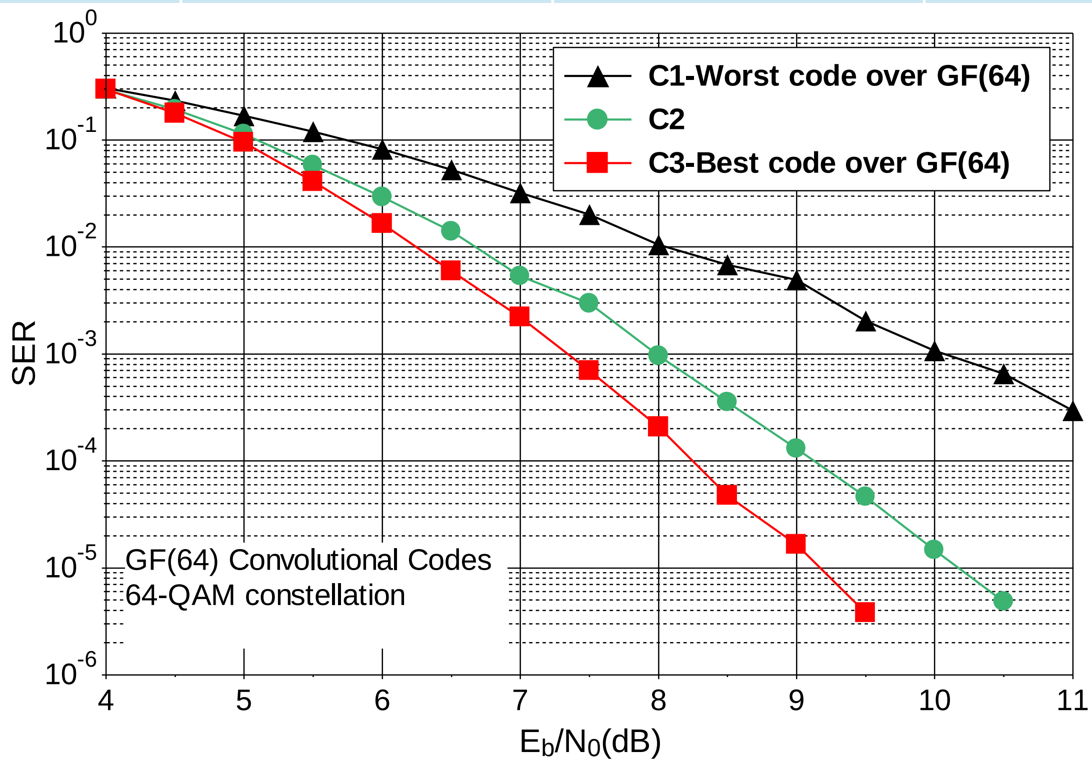
Code	C_1	C_2	C_3
$(\alpha_1, \alpha_2, \alpha_3)$	$(12, 4, 0)$	$(10, 12, 3)$	$(13, 7, 11)$
d_1^2	1,2	2	4
$n(d_1)$	22128	5532	22484
d_2^2	1,6	2,4	4,8
$n(d_2)$	16596	8424	141144



Defined codes over GF(64)

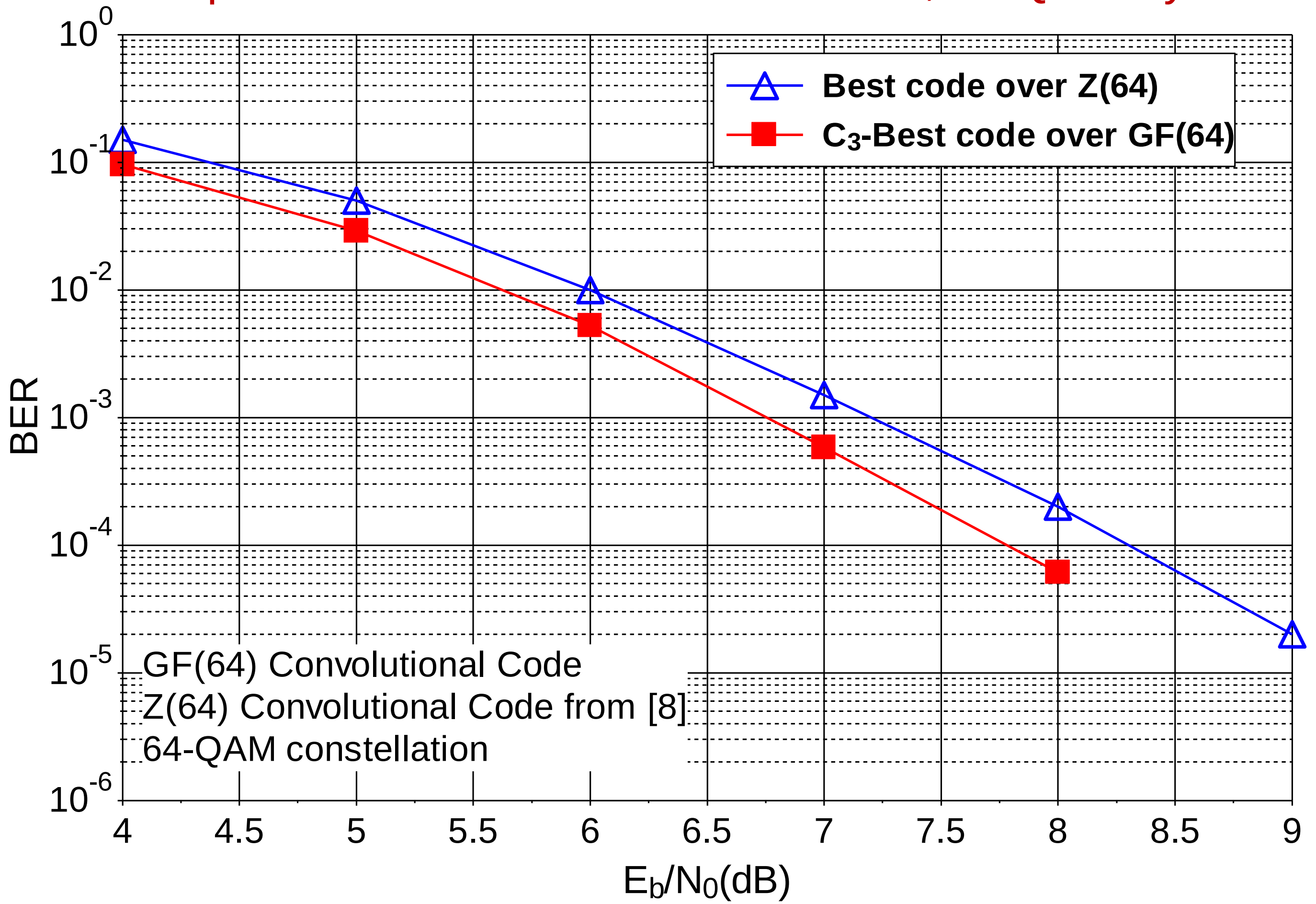
Results over GF(64)

Code	C_1	C_2	C_3
$(\alpha_1, \alpha_2, \alpha_3)$	$(41, 2, 0)$	$(41, 1, 24)$	$(31, 5, 18)$
d_1^2	0,38	1,14	1,52
$n(d_1)$	238422	1542390	652698
d_2^2	0,57	1,23	1,61
$n(d_2)$	230886	4111444	1084014



Comparison with NB-CC over rings

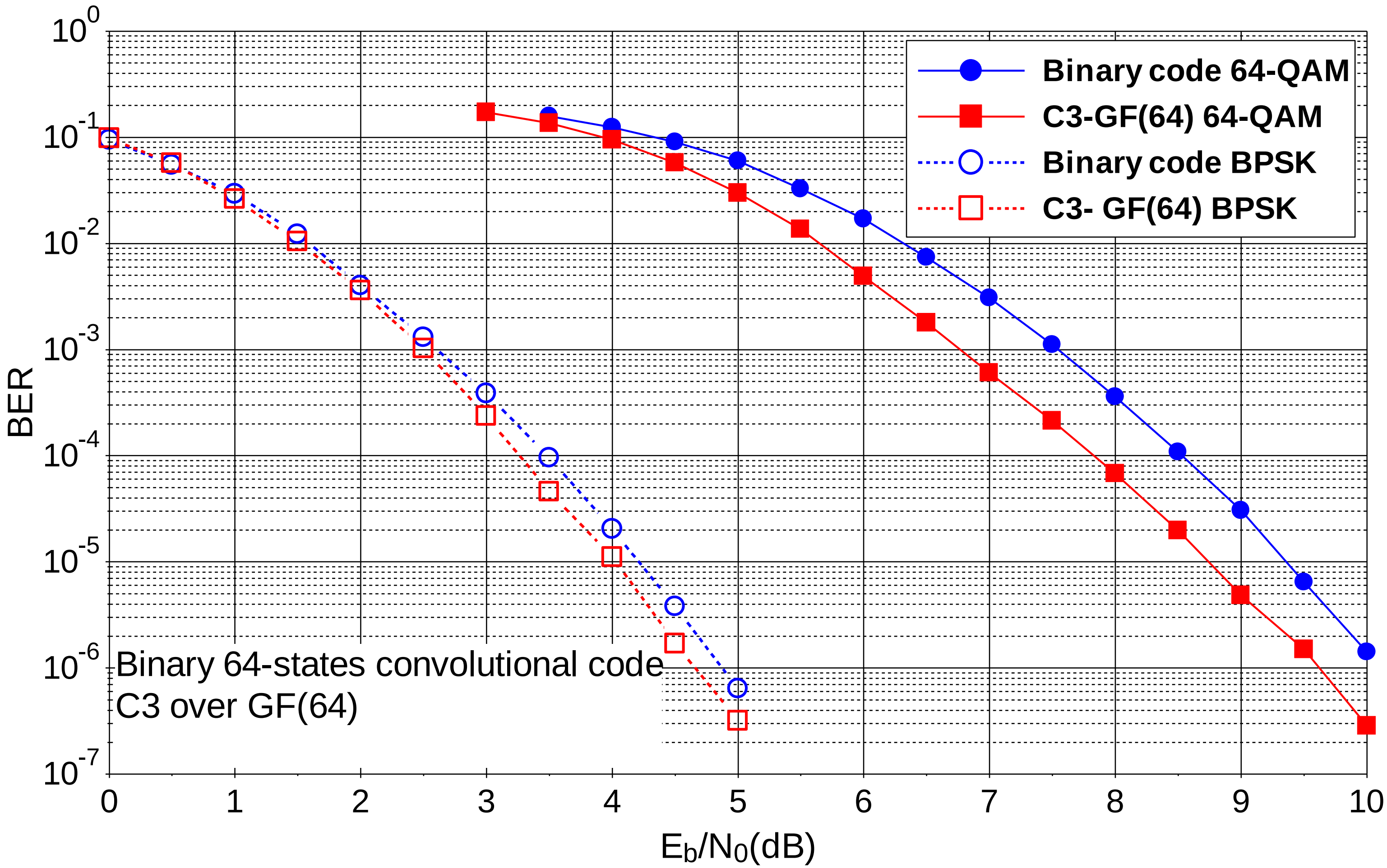
Bit error rate performance evaluation for *Ks*=100, 64-QAM Symbols



[8] T. Konishi, “A coded modulation scheme for 64-QAM with a matched mapping,” in IEEE Int. Symp. Inform. Theory and its Applications (ISITA), Oct. 2014, pp. 191–195.

Comparison with binary CC

Bit error rate performance evaluation for *Ks*=100, 64-QAM Symbols



Conclusion

- Proposal of a new code structure to design NB constituent codes over different GF with up to 25% improvement in distance
- Based on one-memory element to reduce decoding complexity, especially for high order GFs
- Code design based on maximization of the minimum cumulated Euclidean distance spectra along candidate paths
- Proposed structure enables simplified design procedure by limiting the search space:
  - Through the adoption of a constant mapping
  - Through the limitation of candidate sequence lengths to 2 and 3 trellis stages long
- Adopted criterion is validated by performance comparison
- Better results than existing binary and non-binary convolutional codes
- Future works include designing NB-turbo codes and simplifying the decoding algorithm

Thank you
for your attention