DCU
Ollscoil Chathair
Bhaile Átha Cliath
Dublin City University

Proceedings

Seventh Conference on Research in Mathematics Education in Ireland MEI 7

11th and 12th October 2019
Dublin City University

Theme:
Mathematical Literacy, throughout and beyond education

CASTeL
MEI

Editors:
Lorraine Harbison and Aisling Twohill
Institute of Education,
Dublin City University

FOREWORD

It is with great pleasure that we present the following proceedings from the Seventh Conference on Research in Mathematics Education in Ireland (MEI 7), which took place in Dublin City University in October, 2019.

Our conference theme *Mathematical Literacy, throughout and beyond education* aims to foreground learners' engagement with mathematics at all stages of the education system, and their development of mathematical proficiency within and beyond formal classroom settings.

Mathematical literacy encompasses a learner's proficiency to engage fluently with mathematical concepts, and to apply mathematical thinking in non-routine and novel situations. Mathematically literate children and adults recognise the mathematics in a situation, and understand how to mathematise a scenario in order to problem solve. Equally, mathematical literacy includes the capacity to interpret and analyse scenarios presented through mathematics. Mathematically literate individuals are thus less vulnerable to being convinced by inaccurate interpretations of data and mathematics. A key aspect of mathematical literacy is fluency in expressing one's mathematical thinking in a clear and convincing manner.

The notion of what it means to be mathematically literate is to a large extent dependent on the context in which mathematics is used. Some consider performance in examinations to be a determinant of the level of mathematical literacy. However, successfully completing daily activities and routines such as travelling and cooking all rely on fundamental mathematical literacy, for example, knowledge of distance, time, weight and temperature. For educators, mathematical literacy encompasses far more than an ability to do mathematics oneself, but also to be able to ascertain where the learner is at in their mathematical understanding and to scaffold and extend that learning.

The education system in Ireland plays a key role in developing the mathematical literacy of all learners, to support full engagement with 21st century society. Central to such active citizenship is the propensity to apply mathematical concepts beyond the walls of the various classrooms where mathematics is taught; at primary, secondary and third levels. In these proceedings of MEI 7, we present papers that reflect a broad variety of mathematical research that is taking place in Ireland and further afield. Collectively, the authors seek to solidify and progress the research field of mathematics education, throughout and beyond Ireland.

Lorraine Harbison and Aisling Twohill

WHAT IF MATHS IS TAUGHT TO OLDER PUPILS THROUGH MATHEMATICAL STORY PICTURE BOOKS?

Natthapoj Vincent Trakulphadetkrai

Institute of Education, University of Reading

Learning mathematics through reading and creating mathematical story picture books can be a powerful pedagogical strategy for older primary school pupils, but first, what are mathematical story picture books?

KEY CHARACTERISTICS OF MATHEMATICAL STORY PICTURE BOOKS

Just like any story picture books, mathematical story picture books (MSPB) also have a plot, a cast of characters and page illustrations. What makes MSPB unique is that mathematical concepts are either explicitly or implicitly woven into the plot to either demonstrate the concept or show how the concept can be used by characters to solve a problem found in the story.

Take, for example, ‘Fractions in Disguise’ (Einhorn, 2014). This story is about George Cornelius Factor (GCF) who invents a machine called ‘Reducer’ to help him find a very sought-after fraction (5/9) that has been stolen from a fraction auction and has been disguised as another fraction by the villainous Dr. Brok. While at Dr. Brok’s mansion, GCF uses his knowledge of equivalent fractions (in the form of the Reducer machine) to reveal the true form of a range of fractions (e.g. 3/21 is really 1/7; 34/63 is already in its true form; 8/10 is really 4/5, and so on). Finally, GCF comes across 35/63 which is later revealed as the 5/9 fraction he has been looking for.

When we examine the above story structurally, we will see that mathematical knowledge (i.e. knowledge of equivalent fractions) is required to help the character solve the problem. The page illustrations also help readers visually see how 35/63 is in fact the same as 5/9. In brief, MTSB are a specific genre of literature, and they are not (and should never be) mathematics textbooks or worksheets in disguise.

Moreover, as the above story shows, MSPB are more than just counting books. There are several MSPB that focus on mathematics concepts for upper Key Stage 2 pupils (9-11 year olds), such as any titles in the Sir Cumference series. Furthermore, there are also several MSPB with a focus on mathematical concepts for Key Stages 3 and 4 (12-14 year olds), such as ‘What's Your Angle, Pythagoras?’ (Ellis, 2014) for Pythagoras' theorem, and ‘Anno's Magic Seeds’ (Anno, 1999) for exponential growth, among several others.

WHY SHOULD WE TEACH MATHEMATICS TO OLDER PUPILS USING STORY PICTURE BOOKS?

The idea of using MSPB to enrich mathematics learning is not a new idea. In fact, it has been around for almost three decades, particularly in the early years setting. What is less common, particularly in the UK, is using MSPB to enrich mathematics learning beyond the early years level. I have been arguing - and will continue to argue - that the approach could also benefit mathematics learning of older pupils. Specifically, I would argue that the use of MSPB could:

foster pupils' conceptual understanding through multi-representation of mathematical concepts and variation of mathematical situations; develop language skills; and foster engagement with mathematics learning.

Foster conceptual understanding through multi-representation

We can all (hopefully) agree that we do not teach mathematics so that our pupils become a human calculator, that is someone who is good at churning out correct mathematical answers but without conceptually understanding the concept behind it.

As part of one of my research projects, when Jack (pseudonym), a 9-year-old pupil, was asked by me what 20 ÷ 5 equals, he was able to give me the correct answer (4) almost instantly. Then, when he was asked to (contextually) represent 20 ÷ 5 using a word problem, this is what he came up with: "Spanish Yoda had a can of Coke and a bag of bananas and apples and paint. How much did it cost her? Coke: £1.00. Bag of bananas: £2.00. Apples: £8.00. Paint: £9.00. Total £20.00". How Jack's word problem is related to 20 ÷ 5 remains a mystery.

What Jack demonstrates here is a classic example of pupils whose procedural fluency (i.e. the mechanic aspect of mathematical learning) in relation to division is good, but have yet to fully grasp what the concept means conceptually.

As many mathematics education scholars have argued, in order to demonstrate conceptual understanding in mathematics, pupils must be able to represent mathematical concepts in different ways using different representations (e.g. contextualisation, visualisation, etc.). Here, I would argue that key features of MSPB, such as narrative and page illustrations, make learning mathematics conceptually effective as pupils get to learn mathematical concepts through these different representations.

Foster conceptual understanding through variation

Another key strength of teaching mathematics using MSPB is the development of pupils' conceptual understanding in mathematics through what I refer to as the variation of mathematical situations that are often found in well-written MSPB. To explain this concept, take 'Bean Thirteen' (McElligott, 2007) as an example. The story follows two crickets, Ralph and Flora, who have collected twelve beans to bring home for dinner. When Flora decides to pick one more bean (i.e. Bean Thirteen), Ralph is convinced it will bring bad luck. No matter how many friends they invite to try to share the 13 beans equally, it is always impossible.

Situation 1: 13 beans to be shared between 2 crickets (Ralph and Flora) resulting in 1 remaining bean (6 beans each)

Situation 2: 13 beans to be shared between 3 crickets (Ralph, Flora and 1 friend) resulting in 1 remaining bean (4 beans each)

Situation 3: 13 beans to be shared between 4 crickets (Ralph, Flora and 2 friends) resulting in 1 remaining bean (3 beans each)

Situation 4: 13 beans to be shared between 5 crickets (Ralph, Flora and 3 friends) resulting in 3 remaining beans (2 beans each)

Situation 5: 13 beans to be shared between 6 crickets (Ralph, Flora and 4 friends) resulting in 1 remaining bean (2 beans each)

In this example, while the number of crickets varies, the number of beans is invariant. Through this variation of mathematical situations, rich mathematical investigations are made possible. Pupils can be asked, for example, to continue the pattern to prove that 13 cannot be divided evenly by any other numbers except for 13 itself (and hence demonstrating the meaning of prime numbers in the process). I argue that such variation of mathematical situations is crucial to foster pupils' conceptual understanding in mathematics.

Develop language skills

From my earlier research (Trakulphadetkrai, Courtney, Clenton, Treffers-Daller, & Tsakalaki, 2017) and those of others, it has been found that children' mathematical abilities are linked to their language abilities. What is exciting is how recent research (e.g. Hassinger-Das, Jordan, & Dyson, 2015; Purpura, Napoli, Wehrspann, & Gold, 2017) has also found the positive impact of using stories when teaching mathematics concepts to young children on the development of their language abilities particularly their vocabulary knowledge. Why not kill two birds with one stone? Why not teach mathematics using MSPB to develop both pupils' mathematical and language development at the same time?

Engagement through emotional investment

Another key advantage of teaching mathematics using MSPB is that pupils arguably do not see MSPB in the same way that they see, for example, mathematics textbooks or worksheets with word problems after word problems to be solved. They are more likely to view MSPB as something that they can be emotionally invested in, and something that they can enjoy interacting with over and over again either together with the whole class or in their own time at their own pace. Research (e.g. McAndrew, Morris, & Fennell, 2017) has recently found that the use of stories in mathematics teaching can help to foster children's positive attitude towards the subject.

HOW TO USE STORY PICTURE BOOKS IN MATHEMATICS LESSONS?

Reading a mathematical story at the start of a mathematics lesson can help to engage pupils. This also sets the scene and contextualises the mathematics to be taught. Other teachers prefer to wait until the end of the lesson to read the story, to consolidate the learning of mathematics.

Alternatively, teachers might not want to finish reading their chosen story in one go. Quite often, there is a problem for the characters in the story to solve using their mathematical knowledge. Teachers could stop reading the story just before a solution is revealed and use the story's plot to encourage pupils to solve the problem themselves through mathematical investigations.

The beauty of teaching mathematics using MSPB lies in its flexibility: there is not one specific way of integrating MSPB in mathematics teaching. Teachers can be as creative as they like.

WHAT IF PUPILS CREATE THEIR OWN MATHEMATICAL STORY PICTURE BOOKS?

Beyond reading MSPB to pupils, a more innovative mathematics learning strategy that I have been trying to highlight to mathematics teachers (and curriculum developers) in the UK and abroad is the idea of getting pupils to develop their mathematical understanding through creating their own MSPB.

Here, I am not talking about asking pupils to create a full-feature 30-page MSPB. As a mathematics learning activity, pupils can simply be asked to create their own mini MSPB with just 10 pages whereby, for example, the first 2 pages set the scene and the problem to be solved by the characters; the next 6 pages can feature three variations (or attempts) in which the characters try to use their mathematical knowledge to solve the problem; and the story can come to a close on the last two pages.

With this activity, pupils need to think carefully about the storyline, which requires them to consider practical and meaningful applications of the mathematical concept in question. In brief, they need to contextualise abstract mathematical concepts. Additionally, as the focus is on presenting the stories in picture book format, pupils also need to actively think about page illustrations, and how best to communicate abstract mathematical concepts and situations visually to their readers. As previously highlighted, not only could learning mathematics through storytelling benefit pupils mathematically, it could also develop their language and creative writing skills and make possible a great cross-curricular teaching and learning opportunity. Equally important, the approach would allow pupils to see mathematics in a different light – one that is less test-driven, and more fun and imaginative. This is crucial especially if we want to improve pupils' perceptions of the subject.

The preliminary findings of my pilot research with Year 4 pupils on the effectiveness of this mathematics learning activity is promising. Specifically, the results indicate that pupils in the intervention class (i.e. those that were asked to create MSPB on multiplication over the period of a week) had better conceptual understanding of multiplication (as measured through the study's test) than their peers in the comparison class who learned multiplication the normal way (e.g. worksheets and textbooks, etc.).

From a distance, having pupils create their own MSPB might look like a cute, fun activity. However, when one carefully examines this approach, one will see just how pedagogically powerful it can be. I am surprised this approach has not been used more often, because it costs nothing in terms of resources – just a few sheets of A4 paper, a pencil and a splash of imagination!

This mathematics learning activity can also save teachers' time. For example, if the concept in focus is multiplication, teachers could start the day with their mathematics lesson by getting their pupils to consider everyday situations where having knowledge about multiplication can help solve problems, and how the concept can be represented visually. Later in the literacy lesson, they could get their pupils to come up with the plot, characters and setting. They could also get them to work on their draft writing paying attention to things like grammar. After lunch, in an art lesson, the pupils could work on page illustrations, and putting their MSPB together. Before home time, the pupils could read their MSPB with the help of a visualiser to their peers. This one activity can be meaningfully integrated across different curricular subjects throughout the day. What's more – teachers would have just one set of work to mark.

MATHSTHROUGHSTORIES.ORG

Drawing from key findings of my other research project which set out to explore teachers' perceived barriers on the integration of storytelling in mathematics teaching (e.g. Farrugia, & Trakulphadetkrai, In preparation; Markovits, Chen-Haddad, & Trakulphadetkrai, In preparation; Prendergast, Harbison, Miller, & Trakulphadetkrai, 2018; Yang, Su, Chen, & Trakulphadetkrai, In preparation), I designed and created my non-profit research project's website called MathsThroughStories.org.

The website contains the world's largest database of recommendations for MSPB (500+), reviews of MSPB, MSPB-based lesson ideas, exclusive interviews with MSPB authors, and a list of relevant research studies done on the topic, among several other free resources. Since the launch of the website in Spring 2017, the website has now been visited over 300,000 times by more than 60,000 teachers and parents from over 180 countries.

MathsThroughStories.org also organises the Young Mathematical Story Author (YMSA) competition, an annual international competition set up to encourage young mathematics learners (8-13 years old) from around the world to embed their mathematics learning in a meaningful and engaging context through creating their own MSPB. I hope anyone – be it teachers, academics, policy makers or even parents – who is interested in this topic will find this website useful.

FINAL WORDS

Teaching mathematics using MSPB should not only be found in Nursery and Reception classes. This creative, cross-curricular and research-informed mathematics teaching and learning approach should too be utilised by teachers teaching a primary school class (particularly those at the upper Key Stage 2 level). With that in mind, more research on this pedagogical approach for older primary school pupils is desperately needed.

REFERENCES

Anno, M. (1999). *Anno's Magic Seeds*. London, UK: Penguin Putnam Books for Young Readers.

Einhorn, E. (2014). *Fractions in Disguise*. Watertown, MA: Charlesbridge Publishing.

Ellis, J. (2014). *What's Your Angle, Pythagoras?* Watertown, MA: Charlesbridge Publishing.

Farrugia, M. T. & Trakulphadetkrai, N. V. (In preparation). Maltese pre-service and in-service teachers' perceptions on the integration of children's literature in mathematics teaching and learning.

Hassinger-Das, B., Jordan, N. C., & Dyson, N. (2015). Reading stories to learn math: Mathematics vocabulary instruction for children with early numeracy difficulties. *Elementary School Journal*, 116(2), 242–264.

Markovits, Z., Chen-Haddad, N. & Trakulphadetkrai, N. V. (In preparation). Israeli pre-service and in-service teachers' perceptions on the integration of children's literature in mathematics teaching and learning.

McAndrew, E. M., Morris, W. L., & Fennell, F. (2017). Geometry-related children's literature improves the geometry achievement and attitudes of second-grade students. *School Science and Mathematics, 117*(1-2), 34-51.

McElligott, M. (2007). *Bean Thirteen*. New York, NY: Penguin's Putnam Publishing Group.

Prendergast, M., Harbison, L., Miller, S., & Trakulphadetkrai, N.V. (2018). Pre-service and in-service teachers' perceptions on the integration of children's literature in mathematics teaching and learning in Ireland. *Irish Educational Studies, 38*(2),157-175. https://doi.org/10.1080/03323315.2018.1484302

Purpura, D. J., Napoli, A. R., Wehrspann, E. A., & Gold, Z. S. (2017). Causal connections between mathematical language and mathematical knowledge: A dialogic reading Intervention. *Journal of Research on Educational Effectiveness, 10*(1), 116-137.

Trakulphadetkrai, N. V., Courtney, L., Clenton, J., Treffers-Daller, J., & Tsakalaki, A. (2017). The contribution of general language ability, reading comprehension and working memory to mathematics achievement among children with English as additional language (EAL): An exploratory study. *International Journal of Bilingual Education and Bilingualism.* DOI: https://doi.org/10.1080/13670050.2017.1373742

Yang, D.-C., Su, Y.-W., Chen, C.-H., Trakulphadetkrai, N. V. (In preparation). Taiwanese in-service teachers' perceptions on the integration of children's literature in mathematics teaching and learning.

MATHEMATICS EDUCATION IN THE "NEW CLIMATIC REGIME"

Paola Valero

Stockholm University

MEI 7 proposes to explore the topic of mathematical literacy throughout and beyond education. The topic invites us to connect learners' engagement with mathematics in the formal settings of education and the proficiency to work with the tools of mathematics in unexpected ways in a variety of contexts outside of such formal settings. The notion of mathematical literacy proposed in the call of the conference emphasizes citizenship and access, as well as the importance of mathematical literacy for engagement with society today and in the future. Mathematics education and mathematics education research are important not only for promoting mathematical literacy for all, but also for showing new directions for advancing this intention in Ireland. The relationship among researchers in the country and with the international field of mathematics education research is a way of supporting such an aim.

There are many possibilities to address the topic of mathematical literacy following the lines sketched in the conference call. Given my research on the cultural politics of mathematics education (e.g., Valero, 2018), I would like to contribute to the conference by discussing how mathematics education relates to the "New Climatic Regime" (Latour, 2017). Here I sketch aspects of an argument that serves as an invitation to reinvent different notions of mathematics education and of mathematics literacy.

THE "NEW CLIMATIC REGIME"

A mathematics education that engages people —young as well as old— with relevant mathematical literacy talks to the world that people live in. For a long time there have been efforts to create a connection between the happenings in the classroom and the "reality" outside of school. Different forms of pedagogies have tried to "bring reality to the classroom" as a way to generate a link that can promote meaningful learning that can prepare learners to be able to bring what is learned to other contexts of life, study and work (e.g., Gravemeijer, 1994). Even if some success can be documented, still failure to do so is more frequent than desired, or at least so can be argued to be the case if we take results of school achievement that claim to measure students' competence to use mathematics to solve real life problems, such as OECD's PISA studies interested in finding out "what is important for citizens to know and be able to do?" (OECD, 2018, p. 3).

I would like to stop and ask: What is that "real" world that we have now out of school, for which governments and educators alike claim to be preparing new generations to act in? Is it an international competitive market economy? Is it a world of decision-making? Is it that of rapidly changing technology and uncertainty? Is it one of hope, democracy and equal opportunities for all?

In his recent essay "Down to Earth: Politics in the New Climatic Regime", Bruno Latour takes a critical and unexpected look at current events such as the revival of populist nationalisms around the world, the deregulations of globalization, the increase in poverty and inequalities, and the denial of "climate change" understood in a broad sense as the "relations between humans and the material conditions of their existence" (Latour, 2018, p. 1). He argues that looking at these phenomena as distinct, separate trends does not allow us to recognize the conditions and politics of the moment, where the ruling classes seem to act as if there is not a possible common future for them and for all others on Earth. Divisions and tensions are evident all around, and what had been the promises of a better life for all seem to be at stake. Thus, understanding the configuration of what he calls the "New Climatic Regime" —the situation "in which the physical framework that the Moderns had taken for granted, the ground on which their history had always been played out, has become unstable (Latour, 2017, p. 3)— is a way of figuring possibilities for a new political standpoint. To have a political standpoint is a matter that concerns educational thinking and the philosophy of mathematics education, since the question of the direction that such education may take is not an intrinsic matter that is defined a priori by what one may consider to be the mathematical content of education.

The New Climatic Regime is not only the unavoidable recognition that the material conditions of our existence as humans, what we have called "nature", is about to break down in a way that seriously threatens the viability of the way in which we have lived. It is also the fact that the ways in which we have conceived of "nature", "science", "politics" and even the "human" are no longer sustainable. The ideas that since the Enlightenment have dominated the Western rationality to drive humans towards progress and to reach a more desirable future can no longer be maintained in uncritical manners. That the planet almost "turns against us" is no other than the clear sign that the assumed separation of the natural, on the one hand, and the human and culture, on the other hand, is urgently up to revision. The issues of "climate change" are part of political discussions that entangle increasing inequality, poverty and expansion of wealth; migrations, immigrants and the protection of borders, as well as sustainability, CO_2 emissions and aggressive exploitation of natural resources. Latour's idea of a New Climatic Regime is a call to think and act seriously on the basic assumptions that inform the wicked times that we live in.

One could be tempted to pose the question that if this is the situation, then what should mathematics education provide to citizens for coping successfully in a New Climatic Regime? This is the typical question that education in general and mathematics education in particular has always posed. In Modernity, education has served as the governing strategy to solve social problems. This is what historians of education call the "educationalization of social problems" (e.g., Tröhler, 2017). Thus, there is an established idea that education should always respond to the problems of the present to offer solutions to the future by preparing citizens to deal with a future. This has been a strong narrative that can be easily identified in mathematics education. One current example is the identification of programming as a capacity needed for the future workforce, and a current and future lack of qualified persons to do programming according to the expected trends for what will produce value. As a result, many governments suggest to include programming in the school curricula, and this results in initiatives such as one of the most recent changes to the Swedish school curriculum that asks mathematics teachers to incorporate programming as part of mathematics teaching.

But this move would be to continue the same logic that has led us to the current situation. The interesting question, inspired in the Latourian proposal, would be to ask: In which ways is mathematics education now and in the past entangled in the network of people, institutions and materialities that support our current ways of making and thinking about the world? How does mathematics education offer an insight to find an orientation in a world where there seems not to be a common ground? Of course, some critical voices (e.g., Lundin, 2012; Pais, 2012) would call attention on the arrogance of mathematics educators to think that indeed our research and the practices of teaching and learning may offer possibilities of addressing injustice, differentiation, poverty, personal or national success… I would humbly say that as far as mathematics in the school curriculum is increasingly used as an effective biopolitical technology to govern people and populations (e.g., Valero & Knijnik, 2016), the issue of which ideas of us as historical subjects and of the world mathematics teaching and learning is effecting should be a topic of discussion and thinking.

To put in other more familiar terms, mathematics education research has kept a line of reflection on the problem of justification of mathematics education as an area of teaching and learning. More than 20 years ago, Niss (1996) reflected on the multiple reasons that justify the implicit and explicit goals for mathematics education. The cultural reason of learning part of human knowledge creation, the political reason of preparing for citizenship and the economic reasons of qualification for productive functions that Niss identified continue to be present in the way that we conceive of mathematics education and its role in the making of mathematically literate people. Many other proposals of what may count as mathematical literacy or competence have been present in the field. Some emphasize some aspects more than others, but more or less there is an agreement in these basic justifications for why mathematics education. After all, the field of mathematics education research builds on the assumption that Mathematics is still the "queen of the sciences" and that school mathematics is of paramount importance. Our narratives tend to be fixed and stable, and to privilege the intention of being ambassadors for Mathematics.

So far, however, few mathematics educators engage with contemporary philosophical discussions on how science, mathematics and society have changed, and with different possibilities for imagining what may count as mathematical literacy in current times. An example could be the recent book edited by de Freitas, Sinclair and Coles (2017). Expanding the intellectual quest for what is and counts as mathematical in multiple sites of culture, beyond the limits of well-established narratives of Mathematics and of mathematics education, is an important task as we continue to consider the justifications and the overall enterprise of mathematics education. In particular, I am interested in asking not the question of how we should think of mathematics education for the future, but of understanding the configuration of what we consider to be mathematics education, the assumptions that have been familiarized to the point that we consider them to be necessary truths.

MATHEMATICS AND THE MAKING OF THE MODERN CITIZEN

Education through school as an institution has been an effective mechanism of Modern government that relies on knowledge to legitimize power. Within school, mathematics education has historically provided not only knowledge, abilities and possibilities of cognitive development for children; it has most of all provided ways of understanding oneself as a historical subject with and through school mathematics. Studies of learning as change in identity have shown how learners, in the context of the practices of teaching and learning of mathematics, become certain types of persons and this is because knowing is not simply an objectification of knowledge, it is inseparably also a process of subjectification, to use Radford's terms (Radford, 2008).

Studies that have focused on the subjectification that takes place in mathematics education have argued that the routines and procedures in classrooms, while making available a content to acquire, also teach children how to be rational and ordered, how to classify and rank, how to be objective and restrain his/her subject position to produce a detached form of talking about the objects and ideas being manipulated, to desire progress, advancement and growth and even more recently competition and entrepreneurship etc. (e.g., Walkerdine, 1988; Popkewitz, 2004; Andrade-Molina & Valero, 2017; Diaz, 2017; Llewellyn, 2018). If mathematics education indeed builds the discursive and material frame for children to objectify the cultural objects of mathematics and at the same time gain the characteristics of mind, spirit and reason that make the culture of those objects, then mathematics education is an effective technology of governing populations towards being Modern. Of course, this does not mean that each one single individual will indeed be a perfectly designed tin soldier. The point is that we all would have passed through many years of mathematics education that has inserted in our mind, body and forms of thinking important elements of the Modern being. This is not just an oppression. This is also a very productive force to make the type of societies that we have had so far…for good and for bad.

THE LIMITS OF MODERNITY AND OF MATHEMATICS EDUCATION

The use of the expression the "New Climatic Regime" is a way for Latour to playfully indicate that the "New Regime" —the project that the Enlightenment and Modernity brought forward to replace the "Old Regime" of political tyranny based on the authority granted by God— is in urgent need of revision. Knowledge of scientific type, for men to tame nature, was expected to bring a new form of government, a new order, a new hope for the future. The problem is now that we are in the presence of the limits of the project of modernity itself: The forms of production, knowledge, exchange and life are called to question to the extent that no possible project of "modernization" of any nation, developed or developing, can be achieved as planned because there will be no Earth on which to fulfil it.

The predicament for mathematics education emerges when its whole enterprise is geared towards producing modern subjectivities in a time when insisting on continuing to be Modern is a death sentence for the Earth —humans included. This statement can generate questions in very many directions: So, tell us, which contents should then be taught? Which kind of pedagogy should we now bring —to teach the canonical contents of the mathematics curriculum— so that children care for the Earth? What is the mathematics education for a future…if there is a possible future? As I said, I will resist the temptation of following this path; instead, I delve into the configuration of mathematics education as a Modern enterprise with the hope that we can find new possibilities for other articulations of content, curricula and pedagogies. There is a series of ideas that can be called into question. Suffice to say that each one of these deserves a detailed investigation and some of them have indeed already been explored. Here I will only point to some key issues.

Mathematics resides in the mind and makes the good thinker. One of the strongest ideas in current culture is the assumption that mind, thinking and mathematics are connected; and that the materiality of the body is either not important or a simple tool for mathematical thinking. This idea has been challenged (e.g., Lakoff & Núñez, 2000). More recently, de Freitas and Sinclair (2014) have proposed to bring new materialist philosophies to think of the body in/of mathematics. The questioning challenges the duality body/mind that is part of the Western rationalist culture. This duality is also connected to the nature/culture divide that separates the world of the matter from the world of the human. If humans are the superior creation of God and are given the unique capacity to think (and the thinking is in the mind), their mind and ideas produce the possibilities of culture, even the study of nature and its control for the benefit of humanity itself. What is physical or material in nature is to be studied and perceived by the disciplining of the mind. These dualities, so entrenched in the Western, rationalist, Modern epistemology, are at the core of how we conceive of learning and of how we conceive of mathematics as a tool of knowledge that humans have, to distance themselves from the world of nature. In the New Climate Regime Latour proposes that an important realization is that nature and culture are not separated as distinct entities: they are intertwined.

Mathematics is universal and it makes the homeless mind. Another idea about mathematics is that it deals with universal abstractions. When taught in school it should provide people with the capacity to operate on objects and processes on the grounds of true valid rules, in a numerical and symbolic language that transcends the diversity and shortcomings of natural languages and localities. Against a notion of the particular and local, a cosmopolitan form of reason based on mathematics and science fabricates a *homeless mind*, an individuality that sees itself in “relation to transcendental categories that seem to have no particular historical location or author to establish a home” (Popkewitz, 2008, p. 30). Subjects who embody a homeless mind are necessary for moving the desire for a globalized progress. In the New Climatic Regime Latour calls to question the relationship between the global and the local to rethink the role of the territory in a new configuration of threats of environmental, economic and populational type.

Mathematics is a superior creation of dominant cultures and it cannot be acquired by the “Other”. As a privileged form of knowledge of the white, Western culture, mathematics has emerged entangled with political power and the “problem of democracy” which is the establishment of rational means for distributing resources and goods to those who are members of a society (Rose, 1991), be it within the nation or among nations and territories. To produce and know mathematics —and science— has been taken to be a characteristic of the “advanced” cultures and individuals. Thus, the distribution of mathematical competence at individual, community or national levels has been an effective power mechanism to determine who is worthy to have access to different opportunities in society. The persistence of the inequality of access and achievement is connected to the assumption of the epistemic disadvantage of the “Other” (Valero, 2018). In a New Climatic Regime, the increase in inequalities and the explosion of poverty endanger the life opportunities of many people and of the planet itself. How can mathematics education challenge the assumption of the epistemic disadvantage and inequality in inventing new forms of relations with/in mathematics?

NEW POSSIBILITIES?

Latour's hypothesis of the current New Climatic Regime challenges many key ideas and practices that have been taken for granted and that are at the core of assumptions in what constitutes mathematics and school mathematics as cultural forms of thinking, knowing and being. I have pointed here to three such ideas. One could also identify many more, such as the idea that mathematics creates growth and that such growth manifests in increasing economic wealth —an idea that at the moment manifests in the close link between the production of mathematics and science and the strengthening of current financial capitalism (e.g., Valero, 2017). A series of questions for mathematics education can be raised in an attempt to engage with the responsibility of researchers and educators to imagine new possibilities for practice. For example: Can we imagine of ways of reinventing what counts as mathematical beyond the body/mind, nature/culture divide; or to challenge the universalism of abstraction to link to the locality of thinking; or to leave behind assumptions of deficit in different cultures; or to embrace a mathematics of de-growth? When I say here "practice" I am not only thinking about what may actually happen in classrooms, but also and foremost on the intellectual activity of thinking seriously what is and could be the components of a mathematical literacy in a New Climatic Regime. The question is open and far from being exhausted.

REFERENCES

Andrade-Molina, M., & Valero, P. (2017). The effects of school geometry in the shaping of a desired child. In H. Stahler-Pol, N. Bohmann, & A. Pais (Eds.), *The disorder of mathematics education. Challenging the socio-political dimensions of research* (pp. 251-270). New York: Springer.

De Freitas, E., & Sinclair, N. (2014). *Mathematics and the body: Material entanglements in the classroom*. New York: Cambridge University Press.

De Freitas, E., Sinclair, N., & Coles, A. (2017). *What is a mathematical concept?* Cambridge: Cambridge University Press.

Diaz, J. D. (2017*). A cultural history of reforming math for all. The paradox of making in/equality*. New York: Routledge.

Gravemeijer, K. (1994). *Developing realistic mathematics education*. Utrecht: CD-b Press.

Niss, M. (1996). Goals of mathematics teaching. In A. J. Bishop, K. Clements, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.), *International handbook of mathematics education* (Vol. 1, pp. 11-47). Dordrecht: Kluwer.

Lakoff, G., & Núñez, R. (2000). *Where mathematics comes from: How the embodied mind brings mathematics into being*. New York: Basic Books.

Latour, B. (2018). *Down to Earth: Politics in the New Climatic Regime (English edition. ed.)*. Cambridge, UK: Polity Press.

Latour, B. (2017). *Facing Gaia: Eight lectures on the New Climatic Regime*. Cambridge: Polity.

Llewellyn, A. (2018). *Manufacturing the mathematical child: A deconstruction of dominant spaces of production and governance*. New York: Routledge.

Lundin, S. (2012). Hating school, loving mathematics: On the ideological function of critique and reform in mathematics education. *Educational Studies in Mathematics, 80*(2), 73–85.

OECD. (2018). *PISA 2015 results in focus.* Retrieved from https://www.oecd.org/pisa/pisa-2015-results-in-focus.pdf

Pais, A. (2012). A critical approach to equity. In O. Skosvmose & B. Greer (Eds.), *Opening the cage. Critique and politics of mathematics education* (pp. 49–92). Rotterdam: Sense.

Popkewitz, T. S. (2004). The alchemy of the mathematics curriculum: Inscriptions and the fabrication of the child. *American Educational Research Journal, 41*(1), 3-34.

Popkewitz, T. S. (2008). Cosmopolitanism and the age of school reform: science, education, and making society by making the child. New York: Routledge.

Radford, L. (2008). The ethics of being and knowing: Towards a cultural theory of learning. In L. Radford, G. Schubring, & F. Seeger (Eds.), *Semiotics in Mathematics Education: Epistemology, History, Classroom, and Culture* (pp. 215-234). Rotterdam: Sense.

Rose, N. (1991). Governing by numbers: Figuring out democracy. *Accounting, Organizations and Society, 16*(7), 673-692.

Tröhler, D. (2017). Tracking the educationalization of the world: Prospects for an emancipated history of education. *Pedagogika, 67*(3), 211-226.

Valero, P. (2017). Mathematics for all, economic growth, and the making of the citizen-worker. In T. S. Popkewitz, J. Diaz, & C. Kirchgasler (Eds.), *A political sociology of educational knowledge: Studies of exclusions and difference* (pp. 117-132). New York: Routledge.

Valero, P. (2018). Challenging the assumption of the epistemic inequality in the "other". In E. Bergqvist, M. Österholm, C. Granberg, & L. Sumpter (Eds.), *Proceedings of the 42nd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 114-118). Umeå, Sweden: PME.

Valero, P. (2018). Human capitals: School mathematics and the making of the homus oeconomicus. *Journal of Urban Mathematics Education, 11*(1&2), 103-117.

Valero, P., & Knijnik, G. (2016). Mathematics education as a matter of policy. In M. A. Peters (Ed.), *Encyclopedia of Educational Philosophy and Theory* (pp. 1-6). Singapore: Springer Singapore.

Walkerdine, V. (1988). *The mastery of reason: Cognitive development and the production of rationality*. London: Routledge.

THE ONGOING DEVELOPMENT OF THE NEW PRIMARY MATHEMATICS CURRICULUM – FROM RESEARCH TO REALITY

John Behan, Tracy Curran, Jacqueline Fallon and Claire Reidy

National Council for Curriculum and Assessment

In Autumn 2014, the National Council for Curriculum and Assessment (NCCA) published two mathematics research reports (Dunphy et al., 2014; Dooley et al., 2014) at a conference in Dublin Castle entitled Maths is surprisingly important and cognitively fundamental. In addition to these reports, the NCCA also published a commissioned audit of mathematics curriculum policy across 12 jurisdictions (Burke, 2014). Building on this work, Autumn 2016 saw the publication of a background paper and brief to support the development of the new Primary Mathematics Curriculum (PMC). This background paper drew on an extensive suite of evidence, including relevant national and international data and research. In particular, it utilised the NCCA's curriculum reviews (2005, 2008) and evaluations by the Department of Education and Skills (2005, 2010), the two research reports and the international audit of mathematics curricula as outlined above. Findings from focus groups carried out to elicit teachers' and principals' views, beliefs and values regarding mathematics learning and pedagogy, and their ideas regarding the development of a new mathematics curriculum, were also included. The background paper concluded with eight guiding principles for the development of the curriculum. Following its publication, in September 2016, an Early Childhood and Primary Mathematics Development Group (EPMDG) was established, with representatives of stakeholder groups including the DES Inspectorate, management organisations, teacher representatives, SEN and members recruited through a public application process.

Since then, the NCCA executive has worked with the EPMDG, the Board for Early Childhood and Primary, and Council, to fulfil the brief and adhere to the guiding principles set out in the background paper. The initial work of the group endeavoured to formulate a vision for the PMC that would maintain the integrity of mathematics as a discipline in itself, whilst also connecting with the Primary Language Curriculum for junior infants to second class published in 2015. Initial development milestones included the drafting of the curriculum rationale and aims, as well as developing a shared understanding of the different components of the curriculum; their role and purpose. Decisions followed on the organisation of content according to strands and on a model to develop learning outcomes and progression continua. A key focus of the progression continua was to present the key processes alongside the content. Following significant drafting and reviewing work, the draft PMC specification for Junior Infants to second class was published in Autumn 2017. Following approval at the Board for Early Childhood and Primary, and Council, a consultation plan was developed for the specification.

CONSULTATION

A critical component of NCCA's curriculum development processes is consultation with stakeholders. Consultation on the draft Primary Mathematics Curriculum for Junior Infants to Second Class took place between October 2017 and March 2018. The purpose of the consultation was to provide an opportunity for teachers, schools, parents, children and other interested parties to express their views and inform developments of the curriculum going forward.

The consultation was structured around three main strands; an online questionnaire, nationwide seminars and a school network. The questionnaire was open online to teachers, parents and the general public. Three consultative seminars were held in Limerick, Sligo and Dublin, and were attended by teachers, principals, academics and other interested parties. In addition, focused seminars were conducted with the Professional Development Service for Teachers (PDST) and the National Parents Council Primary (NPCP). Finally, the school network consisted of nine schools, identified through a public call for expressions of interest in contributing to the development of the PMC. The network represented both a geographical and contextual spread of school type, including; urban DEIS, rural DEIS, Scoil sa Ghaeltacht, Gaelscoil, special school, school with special classes, small rural and large urban. The network met collectively on three occasions, while NCCA Education Officers visited each school in between each of the gatherings. Data was gathered at the three meetings through field notes and other documentation. Focus groups were conducted during field visits by NCCA staff, in addition to documentation produced by participating teachers. The school network strand also provided an opportunity to explore children's perspectives, based on their mathematical learning experiences from junior infants to 2nd class.

A significant analysis was conducted of all data gathered from each strand of the consultation process. Arising from the consultation findings, key recommendations were derived for the (continued) development of the junior infants to 6th class PMC. These were presented (NCCA, 2018) under seven broad headings: messaging, learning outcomes, progression continua, mathematical proficiency, supporting pedagogy, support material and consultation. The consultation findings contributed to the continued development of the draft specification with a focus on junior infants to sixth class.

CONTINUED DEVELOPMENT

Initially when convened, the focus of the EPMDG was on the development of a PMC to cover junior infants to second class. However, in June 2018, the Department of Education and Skills announced that the PMC will be introduced to schools as a single-stage implementation, from junior infants through to sixth class. The feedback received from stakeholders during the consultation contributed to the evidence base for this decision.

In late 2018, NCCA commissioned a research addendum (Dooley, 2019) to examine considerations for a high-quality mathematics curriculum for middle/upper primary pupils. Development work on the PMC has continued. Drafting of learning outcomes and progression continua for third to sixth class is ongoing. Furthermore, following consultation, mathematical concepts have also been developed for each learning outcome. These are considered essential ideas that underpin each Learning Outcome and may provide useful entry and reference points in relation to planning, teaching and assessment, and may serve to remind teachers of key mathematical knowledge at each stage. Most recently a new chapter 'The Primary Mathematics Curriculum in Practice' was drafted, containing important descriptions of overarching pedagogical practices as highlighted in the research reports. Critical Friends Groups, comprised of academics and teachers in the area of primary mathematics, have been convened to examine initial drafts of this work, with their feedback returned to the EPMDG for consideration. Similar Critical Friends Groups have been used to examine the draft PMC in terms of inclusion, focusing on areas of special education needs and exceptionally able children.

In late 2018, the Minister for Education and Skills announced a revised schedule for the introduction of the new PMC. It is envisaged that the PMC will now be published in Autumn 2021 as a single specification from junior infants through to 6th class.

REFERENCES

Dooley, T. (in press). *Learning and Teaching Primary Mathematics – an addendum to NCCA research reports 17 and 18*. Dublin: National Council of Curriculum and Assessment

Dooley, T., Dunphy, E. & Shiel, G., with D. Butler, D. Corcoran, T. Farrell, S. NicMhuirí, M. O'Connor and J. Travers. (2014). *Mathematics in early childhood and primary education (children aged 3-8 years). Research Report No.18: Teaching and learning*. Available at https://www.ncca.ie/media/2147/ncca_research_report_18.pdf

Dunphy, E., Dooley, T., & Shiel, G with D. Butler, D. Corcoran, M. Ryan, and J. Travers. (2014). *Mathematics in early childhood and primary education (children aged 3-8 years). Research Report No.17: Definitions, theories, stages of development and progression.* Available at https://www.ncca.ie/media/1494/maths_in_ecp_education_theories_progression_researchreport_17.pdf

Burke, D. (2014). *Audit of Mathematics Curriculum Policy across 12 Jurisdictions*. Dublin: National Council for Curriculum and Assessment. Available at https://www.ncca.ie/media/2031/audit-mathematics-curriculum-policy.pdf

Department of Education and Science (2005). *An Evaluation of Primary Curriculum Implementation in Primary Schools: English, Mathematics and Visual Arts.* Dublin: Department of Education and Skills.

Department of Education and Skills (2010). *Incidental Inspection Findings 2010. A Report on the Teaching and Learning of English and Mathematics in Primary Schools*. Dublin: Department of Education and Skills.

National Council for Curriculum and Assessment (NCCA) (2005). *Primary Curriculum Review Phase 1 Final report.* Dublin: National Council for Curriculum and Assessment.

National Council for Curriculum and Assessment (NCCA) (2009). *Primary Curriculum Review Phase 2 Final report.* Dublin: National Council for Curriculum and Assessment.

National Council for Curriculum and Assessment (NCCA) (2018). *Consultation Report on the Primary Mathematics Curriculum for junior infants to second class.* Available at https://www.ncca.ie/media/3605/pmc_consultation_report_july2018.pdf

TRANSFORMING THE PRIMARY MATHEMATICS CURRICULUM: GUIDING PERSPECTIVES

Thérèse Dooley

Dublin City University

INTRODUCTION

In 2014 two research reports were published by the National Council for Curriculum and Assessment (NCCA) to underpin the redeveloped mathematics curriculum for 3–8 year olds (Dooley, Dunphy, Shiel, et al., 2014; Dunphy, Dooley, Shiel, et al., 2014). Recently an addendum to the research reports was published, its aim being to give consideration to aspects of these reports that need particular emphasis or amendment for older primary children (Dooley, 2019). Here the guiding perspectives for the upcoming Primary Mathematics Curriculum (PMC) as proposed in the research reports and the addendum (referred to collectively as 'the research reports' hereafter) are outlined. Some implications of these perspectives for curriculum are discussed. In this discussion I adopt the understanding of mathematics curriculum proposed by Remillard and Heck (2014), that is "*a plan for the experiences* that learners will encounter, as well as the *actual experiences* they do encounter, that are designed to help them reach specified mathematics objectives" (p.707, italics in original).

GUIDING PERSPECTIVES

The research reports endorse an equitable curriculum in which all children have access to mathematics. Equitable access to mathematics is dependent on perspectives on mathematics and how it is learnt. In particular, a view of mathematics as 'absolute and certain' is often regarded as eliminating learners from the discipline whereas a view of mathematics as cultural and context-dependent is more aligned with inclusion. In the research reports, it is proposed that the new mathematics curriculum should be premised on a view of mathematics espoused by Hersh (1997), that is, "mathematics as a human activity, a social phenomenon, part of human culture, historically evolved, and intelligible only in a social context" (p.xi). Such a view has important implications for how mathematics is learnt and taught. Sociocultural theories of learning, in which both the social aspects as well as cultural influences are seen as centrally important to learning, are germane to Hersh's definition above. However, it is acknowledged in the research reports that learning is generally considered to be a complex process not easily explained by a single theory or perspective. Other theories identified as relevant to the teaching, learning and assessment of mathematics are cognitive theories including constructivism in which the construction of knowledge by the individual is emphasised; and social-constructivism which takes account of the central role of social interaction in shaping an individual's learning. As will be seen below, the extent to which these various theories are foregrounded has considerable influence on the planned-for and actual experiences embodied in the PMC.

IMPLICATIONS FOR THE PRIMARY MATHEMATICS CURRICULUM

Construction of the curriculum

An implication of the view of mathematics outlined above is that in the planning and actualization of the PMC, teachers have to take account of learners' interests, backgrounds and ways of knowing. As pupils interact with mathematical tasks, with each other and with the teacher, they contribute to the construction of the curriculum at a local level. As suggested by Remillard and Heck (2014, p.716), "[the curriculum] cannot be fully predesigned and involves design work in the moment." Thus the PMC is dynamic and evolving in nature, and learners have a central role in its construction.

Big Ideas

Big ideas in children's mathematical learning are seen, especially in the US, as crucial to the development of children's mathematical understanding. These ideas are often conflated with curriculum goals. While it is generally accepted that these ideas connect various concepts and procedures within and across domains, there is less agreement on what these big ideas might be. From a cognitivist perspective, the focus of big ideas is on content (e.g., 'adding and subtracting') whereas from a sociocultural perspective, there is a greater emphasis on processes (e.g., 'argumentation'). Arising from the view of mathematics and the learning of mathematics delineated above, both content and processes deserve attention in the specification of goals in the PMC.

Learning Paths

Learning paths are the sequences that apply in a general way to children's development in the different mathematical domains. These are based on developmental progressions which have been constructed for a number of key aspects of mathematics, especially - in the context of early mathematics - by Clements and Sarama (2009). There are different approaches to the explication of learning paths. These include linear/nonlinear presentation, level of detail specified, mapping of paths to age/grade, and role of teaching. Different presentations of learning paths reflect different theoretical perspectives. In the research reports, a presentation of paths that is consistent with a sociocultural view of learning is promoted, that is, paths as provisional, not linked to age and arising from engagement in mathematically rich activity.

Features of Pedagogy

The teaching-learning relationship is at the heart of mathematics education. Pedagogy is regarded as a complex whole where elements related to teaching, learning, and the design of learning environments connect and interact. Features of good pedagogy related to (a) People and Relationships, (b) The Learning Environment and (c) The Learner, are listed in the research reports (although some of these are adapted in the addendum in recognition of the changing nature of 'play' for the older age group). The former two are aligned with sociocultural perspective while the third grouping is more commensurate with a cognitivist approach. This classification exemplifies the significance of co-ordinating learning theories in the planned-for and actual learning experiences in mathematics.

Final Remarks

It is well recognized that teachers play a crucial role in the interpretation and design of curriculum. However, according to Remillard (2012), teachers' interactions with curriculum resources are influenced by their past experience and assumptions about what it is to do and learn mathematics. She goes on to argue that curriculum resources should 'speak to' teachers and that curriculum materials should contribute to their learning. Given the impact of guiding perspectives on mathematics and mathematics learning on the PMC, it is vital that they receive adequate and prominent attention in these curriculum resources.

REFERENCES

Clements, D., & Sarama, J. (2009). *Learning and teaching early math: The learning trajectories approach*. UK: Routledge.

Dooley, T. (in press). *Learning and teaching primary mathematics – an addendum to NCCA research reports 17 and 18*. Dublin: National Council for Curriculum and Assessment.

Dooley, T., Dunphy, E. & Shiel, G., with D. Butler, D. Corcoran, T. Farrell, S.NicMhuirí, M. O'Connor and J. Travers. (2014). *Mathematics in early childhood and primary education (children aged 3-8 years). Research Report No.18: Teaching and learning*. Available at https://www.ncca.ie/media/2147/ncca_research_report_18.pdf

Dunphy, E., Dooley, T., & Shiel, G with D. Butler, D. Corcoran, M. Ryan, and J. Travers. (2014). *Mathematics in early childhood and primary education (children aged 3-8 years). Research Report No.17: Definitions, theories, stages of development and progression.* Available at https://www.ncca.ie/media/1494/maths_in_ecp_education_theories_progression_researchreport_17.pdf

Hersh, R. (1997). *What is mathematics, really?* Oxford, UK: Oxford University Press.

Remillard, J. T. (2012). Modes of engagement: Understanding teachers' transactions with mathematics curriculum resources. In G. Gueudet, B. Pepin, & L. Trouche (Eds.), *From text to 'lived' resources: Mathematics curriculum materials and teacher development* (pp. 105–122). New York: Springer.

Remillard, J. T., & Heck, D. J. (2014). Conceptualizing the curriculum enactment process in mathematics education. *ZDM*, *46*(5), 705-718.

MATHEMATICS REFORM IN POST-PRIMARY SCHOOLS IN IRELAND: OPINIONS OF PRE-SERVICE TEACHERS

Eabhnat Ní Fhloinn[1], Brien C. Nolan[1], Giulio Hoehne Candido[2] and Shannon M. Guerrero[3]

[1]Dublin City University, [2]Technological University Dublin and [3]Northern Arizona University

INTRODUCTION AND BACKGROUND

As part of the OECD Programme for International Student Assessment (PISA) 2012 programme in Ireland, a survey of in-service post-primary mathematics teachers was undertaken, with full results reported by Cosgrove, Perkins, Shiel, Fish & McGuinness (2012). One of the main aims of this questionnaire was to obtain opinions and feedback about the implementation of Project Maths from a nationally representative sample of teachers. In 2015, we gave a subset of this survey to a group of pre-service mathematics teachers immediately before and after a four-month teaching placement. A more complete and detailed description of this project and its outcomes can be found in Ní Fhloinn, Nolan, Hoehne Candido & Guerrero (2018). We also added eight original questions to the survey, and we briefly consider some of the responses in this short paper. A detailed discussion of these is beyond the scope of this short paper, but we have chosen to focus on three questions that yielded some interesting results.

RESULTS AND DISCUSSION

The respondents in this survey were in their third year in university, meaning that they sat their Leaving Certificate in June 2012 at the latest, and so had experienced at most two of the five strands of Project Maths, introduced during their final two years at school, as it was implemented on a phased basis. Therefore, we asked respondents about the difference between their own school experience and that of the Project Maths approach, as well as whether the Project Maths approach agrees well with what content they thought should be taught and how they thought it should be taught. The results are shown in Table 1.

Table 1: Pre-service teachers' responses to survey statements (n=25 for Pre and n=19 for Post)

	Pre/post placement	Strongly Agree	Agree	Disagree	Strongly Disagree
There is a clear distinction between the type of mathematics teaching that I experienced in secondary school and the Project Maths approach	Pre	56%	36%	8%	0%
	Post	26%	47%	16%	11%
The Project Maths approach agrees well with my views on <u>what</u> mathematics content should be taught	Pre	8.7%	82.6%	4.3%	4.3%
	Post	0%	79%	21%	0%
The Project Maths approach agrees well with my views on <u>how</u> mathematics should be taught	Pre	12.5%	83.3%	4.2%	0%
	Post	21%	58%	21%	0%

It can be seen that in relation to the question about there being a clear distinction between their own experience in school and the Project Maths approach, only 8% of pre-placement students disagreed, whereas post-placement, this had increased to 27%. This echoes the findings of Jeffes, Jones, Wilson, Lamont, Straw, Wheater & Dawson (2013), who found that *"there does not appear to have been a substantial shift in what teachers are asking students to do ... traditional approaches to mathematics teaching and learning continue to be widespread"* (p. 4). Similarly, for the question regarding content, 91.3% of pre-placement respondents agreed or strongly agreed with the Project Maths approach, compared to 79% post-placement. For how this content should be taught, 95.8% agreed or strongly agreed pre-placement, which also dropped to 79% afterwards.

Respondents were asked to comment further for the latter two questions, and a thematic analysis of these open-ended responses was conducted. In relation to Project Maths agreeing with their view of *what* content should be taught, the strongest themes emerging pre-placement were "real-life applications", "better understanding", "better for university", and "easier"; whereas post-placement, the first three of these themes still emerged strongly, but "easier" was no longer mentioned, replaced by "topics omitted" and "no real difference". For the question regarding *how* the content was taught, the strongest themes pre-placement were "real-life applications", "better understanding", "too wordy", and "still need to teach skills"; while post-placement, the first two themes were still prominent, but others such as "bad for weaker students", "more problem-solving" and "time pressure" also emerged.

The sample size in question was small and only thirteen students completed both the pre- and post-surveys, meaning that statistical testing would not be reliable; however, the pre-service teachers' responses, combined with those reported in Ní Fhloinn et al (2018), provide an insight into the opinions of teachers who had the experience of being trained during the transitional period in which Project Maths was being introduced on a phased basis into schools.

REFERENCES

Cosgrove, J., Perkins, R., Shiel, G., Fish, R. & McGuinness, L. (2012). *Teaching and Learning in Project Maths: Insights From Teachers who Participated in PISA 2012.* Dublin: Educational Research Centre.

Jeffes, J., Jones, E., Wilson, M., Lamont, E., Straw, S., Wheater, R., & Dawson, A. (2013). *Research into the Impact of Project Maths on Student Achievement, Learning and Motivation: Final Report*. Slough: NFER.

Ní Fhloinn, E., Nolan, B.C., Hoehne Candido, G., & Guerrero, S.M. (2018). Pre-service versus in-service mathematics teachers' opinions of mathematics reform in post-primary schools in Ireland. *Irish Educational Studies, 37*(4), 431-452.

TEACHER SELF-EFFICACY: A BELIEF ABOUT PERSONAL TEACHING CAPABILITIES OR ABOUT CAPABILITIES TO BRING ABOUT DESIRED EDUCATIONAL OUTCOMES?

Gosia Marschall[1,2] and Steven Watson[2]

[1]Stockholm University and [2]University of Cambridge

Although studies on *teacher self-efficacy* over the last 20 years have continued to reveal its value in education research, the accrued theoretical messiness of the concept has a potential to undermine the powerful messages that the research is trying to send. The main theoretical issue regarding the concept relates to its unclear definition, rooted in a historical development of theory from two theoretical strands (*social learning theory* of Rotter (1966) and *social cognitive theory* (SCT) of Bandura (1997)). For example, following Rotter's association of *teacher self-efficacy* with *external control*, some define it as beliefs related to teachers' capabilities to affect the learning outcomes of students (Ozder, 2011); or suggest that *teacher efficacy* comprises of *personal efficacy* (relating to teachers' beliefs about their individual skills), *outcome efficacy* (relating to their actions in bringing about required outcomes) and *teaching efficacy* (relating to *external control*, concerning the ability of teaching in general to overcome external influences) (Soodak & Podell, 1996).

Following Bandura, others depart from the explicit association of *teacher self-efficacy* with *locus of control*, and instead focus on appraisal of teachers' personal capabilities. They define *teacher self-efficacy* as, for example, "the teacher's belief in his or her capability to organise and execute the courses of action required to successfully accomplish a specific teaching task in a particular context" (Tschannen-Moran, Hoy, & Hoy, 1998, p. 22); or "teachers' subjective judgment about their capability to successfully execute a course of action required to fulfil their roles as a teacher" (Cho & Shim, 2013, p. 14); or simply "one's beliefs in their ability to teach mathematics effectively" (Bates, Latham, & Kim, 2011, p. 326).

To complicate the matter further, it is not actually uncommon for researchers to be confounding the two definitions in their own studies. For example, Tschannen-Moran and Hoy define *teacher self-efficacy* as, "a judgment of his or her capabilities to bring about desired outcomes of student engagement and learning, even among those students who may be difficult or unmotivated" (Tschannen-Moran & Hoy, 2001, p. 783), shifting the focus back to the aspect of control and away from teachers' personal attributes.

This deliberation, between beliefs about personal capabilities for executing actions and beliefs about capabilities to bring about desired educational outcomes, left us with a fundamental question about what *teacher self-efficacy* actually is. One might argue that what we are facing is a simple linguistic liberty, which in itself should not be problematic as long as one defines concepts clearly. The issue, however, has serious theoretical and methodological implications. These relate to the confounding treatment of *teacher self-efficacy* and a concept of *control*, subsequently leading to difficulties in interpreting success and failure. In the following section, using Bandura's (1997) theory of self-efficacy, we discuss the fundamental differences between the two concepts, explaining why it is incorrect to treat them synonymously.

TEACHER SELF-EFFICACY BELIEFS AND THE CONCEPT OF CONTROL AS TWO SEPARATE CONCEPTS

Skinner (1996) explains that a sense of control is not synonymous with *teacher self-efficacy* but that it is rather a general concept consisting of two components. *Competence*, in relation to *teacher self-efficacy*, is “conceptualized as a context-specific and malleable belief about what the individual teacher can accomplish given the limitations caused by external factors” (Skaalvik & Skaalvik, 2007, p. 612). *Contingency* is “conceptualized as a general and relatively stable belief about limitations to what can be achieved through education” (Skaalvik & Skaalvik, 2007, p. 612). This reflects two kinds of expectations discussed by Bandura (1997): *efficacy expectation* – an affective “judgement of one’s ability to organise and execute given types of performances”; and *outcome expectancy* - “a judgement of a likely consequence such performances will produce” (p. 21). Here self-efficacy beliefs are clearly defined as "beliefs in one's capabilities to organize and execute the courses of action, required to produce given attainments" (Bandura, 1997, p. 3).

Bandura (1997) explains that although the two are inextricably linked, they cannot be treated as one general efficacy concept. Firstly, although *outcome expectancy* provides an incentive for initiating action, it does not actually directly affect behaviour (Bandura, 1997). Belief that a successfully performed action brings about certain desired outcomes, without a belief a person can execute this action, will not result in action.

Secondly, the relationship between *self-efficacy* and *outcome expectancy* is not of a simple deterministic nature but can exhibit one of three different dependency scenarios: performance determining outcome, performance accounting only for part in the variation of outcomes (joint effect of performance and external factors on outcomes, such as, for example, “students taking responsibility for their own learning” (Wheatley, 2005, p. 754)) and outcome being completely independent of performance (for example in strictly segregated and discriminating circumstances) (Bandura, 1997). Lack of attention to those often leads to an interpretation of personal success in terms of outcomes, as opposed to performance (Morris, Usher, & Chen, 2017). Bandura (1997) explains, however, that the measure of an individual success can be based only on personal attainment. The fact whether this attainment will bring an external outcome has little to do with personal success in reaching a set goal. For example, in an educational context, a teacher might have a high-level of efficacy in teaching specific content or teaching in a certain way, and they might be able to execute this action successfully, yet, the specific approach might not be valued or appreciated by their school, exhibiting low *outcome expectancy*. As an example, one can consider democratic classrooms where there is a clear conflict between well-executed teacher-centred practices and constructivist values, which emphasise the very idea of teachers letting go of classroom control (Wheatley, 2005).

Importantly, attainment and *outcome expectancy* differ conceptually and temporally. *Outcome expectancy* relates to the assessment of physical, social and self-evaluative outcomes and focuses on what will take place following a successful action (Bandura, 1997). In contrast, attainment focuses on a performance that has already taken place and includes self-evaluative and attributive processes. Such an assessment is not based on a simple indicator of success and failure but rather on how these successes and failures are cognised, interpreted, weighted, "organized and reconstructed in memory" (Bandura, 1997, p. 81), based on task difficulty, capabilities and effort expenditure. For example, an individual with high efficacy, when experiencing failure, might not change their self-efficacy beliefs because of attributing the failure to a lack of sufficient effort or an influence of external factors. Charalambous, Philippou and Kyriakides (2008) illustrate this with a quote of a pre-service primary teacher who said: "At first, I assumed sole responsibility [for the failure], but after experimenting with several approaches, I concluded that it was not always my fault. Some pupils are not engaged, simply because they don't care" (p.139).

FINAL REMARKS

There is much current research that uses Bandura's *teacher self-efficacy*, but many use misleading definitions of the concept, confounding the aspects of teacher personal capabilities with that of general *control*. Yet, as discussed above, treating those concepts interchangeably ignores their nuanced differences, standing in the way of interpreting results in valid ways. This liberal treatment of the concept and the definition of *teacher self-efficacy* is in need of addressing, especially in the context of numerous quantitative studies, results of which are often accumulated with a purpose of sending powerful messages.

REFERENCES

Bandura, A. (1997). *Self-efficacy: the exercise of control*. New York: W. H. Freeman and Company.

Bates, A. B., Latham, N., & Kim, J. (2011). Linking preservice teachers' mathematics self-efficacy and mathematics teaching efficacy to their mathematical performance: linking preservice teachers. *School Science and Mathematics, 111*(7), 325–333. https://doi.org/10.1111/j.1949-8594.2011.00095.x

Charalambous, Y. C., Philippou, G. N., & Kyriakides, L. (2008). Tracing the development of preservice teachers' efficacy beliefs in teaching mathematics during fieldwork. *Educational Studies in Mathematics, 67*(2), 125–142. https://doi.org/10.1007/s10649-007-9084-2

Cho, Y.-J., & Shim, S. S. (2013). Predicting teachers' achievement goals for teaching: The role of perceived school goal structure and teachers' sense of efficacy. *Teaching and Teacher Education, 32,* 12–21. https://doi.org/10.1016/j.tate.2012.12.003

Morris, D. B., Usher, E. L., & Chen, J. A. (2017). Reconceptualizing the sources of teaching self-efficacy: a critical review of emerging literature. *Educational Psychology Review, 29*(4), 795–833. https://doi.org/10.1007/s10648-016-9378-y

Ozder, H. (2011). Self-efficacy beliefs of novice teachers and their performance in the classroom. *Australian Journal of Teacher Education, 36*(5), 1–15. https://doi.org/10.14221/ajte.2011v36n5.1

Rotter, J. B. (1966). Generalized expectancies for internal versus external control of reinforcement. *Psychological Monographs, 80*(1).

Skaalvik, E. M., & Skaalvik, S. (2007). Dimensions of teacher self-efficacy and relations with strain factors, perceived collective teacher efficacy, and teacher burnout. *Journal of Educational Psychology, 99*(3), 611–625. https://doi.org/10.1037/0022-0663.99.3.611

Skinner, E. A. (1996). Personality processes and individual differences. *Journal of Personality and Social Psychology, 71*(3), 549–570.

Soodak, L., & Podell, D. (1996). Teacher efficacy: toward the understanding of a multi-faceted construct. *Teaching and Teacher Education, 12*(4), 401–411.

Tschannen-Moran, M., & Hoy, A. W. (2001). Teacher efficacy: capturing an elusive construct. *Teaching and Teacher Education, 17*(7), 783–805. https://doi.org/10.1016/S0742-051X(01)00036-1

Tschannen-Moran, M., Hoy, A. W., & Hoy, W. K. (1998). Teacher Efficacy: Its Meaning and Measure. *Review of Educational Research, 68*(2), 202. https://doi.org/10.2307/1170754

Wheatley, K. F. (2005). The case for reconceptualizing teacher efficacy research. *Teaching and Teacher Education, 21*(7), 747–766. https://doi.org/10.1016/j.tate.2005.05.009

SELF-EFFICACY: CONCEPTUAL FOUNDATIONS AND CRUCIAL FINDINGS

Mark Morgan

Dublin City University

The concept of self-efficacy refers to how well one can perform a course of action that is required in a specific situation. Thus, it is an expectation that will determine whether an individual will be able to exhibit coping behaviour and how long effort will be sustained even in the face of obstacles.

Key Features:

* Not a general trait but a differentiated set of self-beliefs that function differently in specific domains

* Not a fixed capacity but is dependent on the particular area of endeavour

* Self-efficacy is also dependent on the beliefs on what we can do, depending on specific circumstances

* Different from self-esteem, which is a general trait. Yet self-efficacy has a profound effect on perception of the self

* Has major effects on attributions of success and failure

* Also has strong effects on persistence and sustaining motivation

CONCEPTUAL FOUNDATIONS

Albert Bandura (1997) suggests that self-efficacy has a major role in how we approach tasks. He relates the concept to his earlier views on social cognitive theory and the significance of observational and other forms of learning. In his view there are four major sources of self-efficacy.

The first and major source of self-efficacy is through mastery experience. Nothing is more powerful than having a direct experience of mastery to increase self-efficacy. Having a success, for example in mastering a task, builds self-belief in that task whereas a failure will have the opposite effect on efficacy belief.

The second source of self-efficacy comes from our observation of people around us, especially people we consider as role models. Seeing people succeed by their effort raises our beliefs that we too have the capacity to master the activities needed for success in that area.

Influential people in our lives such as parents and teachers can strengthen our beliefs that we have what it takes to succeed. Being persuaded that we possess the ability to master certain activities means that we are more likely to put in the effort and sustain it when problems arise.

A person's physical and psychological state will influence how they judge self-efficacy. Depression, for example, can dampen our confidence. Stress reactions or tension are interpreted as signs of vulnerability to poor performance whereas positive emotions can boost our confidence in our skills.

SELF-EFFICACY VS. GENERAL MEASURES

Traditional measure of attributes served several purposes. There was a tendency to omit information about the specifics of the situation. Measures are usually couched in a general form. Often self-reports on achievement omit specific subject areas, the specific conditions and circumstances that can affect likelihood of success including motivational factors.

For example, the Locus of Control Scale is widely used to gauge the extent to which people see events are within or outside their control. The problem is that there is a variety of factors which impinge on this perception and these need to be taken into account in devising scales, and this is the essence of what self-efficacy measures try to do. Efficacy beliefs vary on several dimensions and these have important implications. A major factor to be taken into account concerns demands that represent various degrees of challenge to achieve a successful performance. If there are no obstacles, the activity is easy and most people have a high perceived self-efficacy for that task, while challenges and obstacles result in relatively lower scores. For these reasons, in devising tests, there is a need to analyse what it takes to succeed in a given domain.

Efficacy beliefs also vary in terms of generality. We can be efficacious across a range of activities or only in specific domains. Efficacy beliefs also vary in strength. In the case of weak efficacy beliefs, these can be wiped out easily by a disconfirming experience while with firm beliefs these will persist even in the face of obstacles.

SELF-EFFICACY IN KEY AREAS

Here we consider the importance of self-efficacy in four areas that have received special interest in recent research. The first of these is concerned with the effect of self-efficacy on the performance of students in higher education while the second focuses on teacher self-efficacy and its impact on classroom interaction and students' achievement as well as teachers' own well-being. We will also look at research on the importance of self-efficacy for brain health of older adults as well as its importance in addressing educational disadvantage.

An especially valuable review of the impact of academic self-efficacy on learning performance (ASE) has been carried out by Honicke & Broadbent (2016) based on 59 studies examining this issue between 2003 and 2015. There were two major questions: (i) what is the strength of the relationship between academic self-efficacy and academic performance? and (ii) what mediating and moderating factors have been investigated to explain the relationship between academic self-efficacy and academic performance of university students and what do they report?

Overall meta-analytic findings suggest that a moderate positive relationship exists between academic self-efficacy and academic performance, but there is significant heterogeneity across studies, which is accounted for partly by inter-study differences in operationalization of self-efficacy and academic performance. Additionally, it appears that the mechanism in which ASE relates to and influences academic performance is mediated through such variables as effort regulation and academic procrastination.

These results suggest that a student's ability to regulate the amount of effort dedicated to learning tasks, in the face of boredom or other distractions, partially facilitates and explains the relationship between self-efficacy and performance. It appears the higher a student's level of academic self-efficacy, the more likely effort will be expended on a learning task, which is likely to result in greater levels of academic performance. This is a logical conclusion and is supported by previous research findings.

A meta-analysis by Zee & Koomen (2016) provides a synthesis of 165 studies that examined the importance of teacher self-efficacy (TSE) for a range of outcomes with a particular focus on quality of classroom interactions, students' academic outcomes and the well-being of teachers and students. Results showed positive relationships between TSE and students' academic achievement as well as their quality of learning. TSE was also found to have an influence on the interaction of teachers and students as well as various factors that contribute to teachers' psychological well-being including job satisfaction and commitment to teaching.

Conversely, there were negative effects on various measures of teacher burnout. Furthermore, a number of students were identified that demonstrated a range of indirect effects of TSE. For example, TSE impacted on academic adjustment and psychological well-being through classroom organisation. These findings underline the range of complexity of the impact of TSE on children and teachers.

Of the recent areas that have been examined, the findings regarding the relationship between self-efficacy and brain health are of particular note. A major concern for our aging population is around the factors that influence cognitive decline. Several factors have been identified that prevent such deterioration including involvement in social and intellectually challenging activities. There is now evidence that one’s personal perceived ability to perform a specific task impacts on performance. A study by Horst & Nagamatsu (2018) sought to explore a relationship between measures of Memory self-efficacy and standardised cognitive tests. They explored this relationship while taking into account other relevant factors including age and level of physical activity. The results of this study suggest that Memory self-efficacy is indeed related to actual performance on tasks requiring remembering. The relationship was strong and accounted for 65% of the variance even after considering other factors.

An important question is around the possible role of self-efficacy in addressing educational disadvantage. This is of special importance given the potential benefits of new approaches to enhancing the performance of children from disadvantaged backgrounds. One line of research by Zahodne et al., (2015) is especially promising. In a national study including adults from 30 to 85 years, it emerged that people with low education, but high self-efficacy performed as well as people with high education. Their study provides evidence that self-efficacy beliefs can buffer against the effects of low levels of education and the associated negative impact on school performance. Thus, there is real promise on how the damaging effect of educational disadvantage can be addressed.

REFERENCES

Bandura, A. (1997). *Self-efficacy: The exercise of control.* New York: Freeman.

Honicke, T., & Broadbent, J. (2016). The influence of academic self-efficacy on academic performance: A systematic review. *Educational Research Review*, *17,* 63-84.

Horst, R., & Nagamatsu, L. (2018). Mind over matter: Understanding the relationship between memory self-efficacy, cognition and brain health in older adults with mild cognitive impairment. *FASEB Journal Supplement 782*.

Zahodne, L., et al. (2015). Self-efficacy buffers the relationship between educational disadvantage and executive functioning. *Journal of the International Neuropsychological Society, 21,* 297-304.

Zee, M., & Koomen, H. (2016). Teacher self-efficacy and its effects on classroom processes, student academic adjustment and teacher well-being: A synthesis of 40 years of research. *Review of Educational Research, 86*, 981-1015.

INVESTIGATING MATHEMATICS TEACHER EFFICACY BELIEFS IN PRIMARY INITIAL TEACHER EDUCATION

Siún Nic Mhuirí, Aisling Twohill and Lorraine Harbison

CASTeL, Dublin City University

INTRODUCTION

This paper will discuss the mathematics teacher efficacy beliefs (MTEB) of primary initial teacher education (ITE) students. We are interested in studying how ITE students' MTEBs are influenced (or not) by mathematics education modules undertaken as part of an undergraduate Bachelor of Education (BEd) programme. We will detail how approximations of practice (Grossman, Compton, Igra, Ronfeldt, Shahan, & Williamson, 2009) have been incorporated into mathematics education modules to create opportunities for the development of MTEBs and will report on focus group interviews which explored MTEBs of ITE students.

Self-efficacy concerns how we perceive our ability to accomplish certain levels of performance (Bandura, 1997). The related concept of teacher efficacy has been defined as "a teacher's sense of ability to organize and execute teaching that promotes learning" (Charalambous, Philippou, & Kyriakides, 2008, p. 126). The theoretical underpinnings of the concept are complex and there is ongoing debate about how it should be measured. Despite this, interest in teacher efficacy persists because outcomes such as teachers' persistence and instructional behaviour as well as student outcomes such as motivation and achievement have been shown to be related to efficacy beliefs (Morris, Usher, & Chen, 2017).

Teacher efficacy is considered to be both context and subject-matter specific (Bandura, 1997). This makes it highly relevant in primary education where (in Ireland) mathematics is taught for the most part by 'generalist' teachers, i.e., teachers who are qualified to teach the range of primary subjects with no specialized qualification in mathematics or mathematics teaching. Bandura suggests that efficacy beliefs are most malleable in their early stages of development however there is little consensus on how efficacy beliefs develop during ITE (Charalambous et al., 2008). The research on MTEBs within primary ITE tends to focus on school placement experience as a key site for the development of MTEBs. Our work adds a new dimension by considering the interplay between taught modules, school placement and MTEBs.

DEVELOPING MTEB IN INITIAL TEACHER EDUCATION

Tschannen-Moran et al. draw on the work of Skinner (1996) to suggest the following:

> Self-efficacy theory is one of the few conceptualizations of human control that describe a distinction between competence, or agent-means relationships (I can execute the actions), and contingency, or means-ends relationships (the actions will attain certain outcomes). (1998, p. 210)

Teacher education has a role in developing both 'agent-means relationships' and 'means-ends relationships'. Skinner (1996, p. 555) maintains that "control beliefs can be arrayed along a continuum from the extremely situation-specific to the extremely general or global". While self-efficacy is generally focused at a specific behavioural level, teacher education must also aim to develop more global beliefs about means-ends relationships in mathematics education. In fact, we suggest that much of our work in the BEd taught modules involves interrogating the 'ends' or goals of mathematics education so that ITE students recognise the different domains of educational purpose (Biesta & Stengel, 2016) and understand how mathematics teaching contributes to the development of dispositions as well as content knowledge.

The focus module of this research study is the first mathematics education module on the BEd. It is designed to engage ITE students in problem-solving to build mathematical knowledge for teaching and to support them in interrogating their preconceptions of mathematics and teaching. All seminars also feature sample classroom activities and opportunities are created to experience multiple, progressive methods of supporting mathematics learning. Furthermore, opportunities are created to experience the four sources of efficacy described by Bandura (*mastery experience, vicarious experience, social persuasion,* and *physiological and affective states)* through the collaborative planning and teaching of a mathematics lesson to peers. This task can be understood as an approximation of practice (Grossman et al., 2009). In relation to *mastery experiences*, the planning element was purposely designed to simulate planning of a lesson for a primary class on school placement. It is an opportunity for authentic (ITE student) experience of planning. For the teaching component, ITE students effectively engage in a live role-play, a less authentic approximation of practice. *Vicarious experiences,* where other individuals are observed carrying out focus activities, are understood to have a more powerful impact on MTEBs if the observer identifies with the individual modelling the activity. Observing peers conduct these planning and teaching tasks is envisaged to hold potential for the development of MTEBs. Bandura described *social persuasion* as "social evaluations of capability" (1997, p. 102). Such feedback was another planned feature of the in-class peer-teaching activity. *Physiological and affective states* concerns how emotional reactions to events can influence efficacy beliefs. The ITE students generally appeared to find the peer-teaching activity challenging but enjoyable. For this reason, we contend that it presents an opportunity for the development of MTEBs.

Morris et al. (2017, p. 819) propose a model showing how efficacy information is integrated and evaluated. The model details the relationship between a) sources, b) integrative and evaluative factors, and c) self-efficacy. In addition to the four sources of efficacy proposed by Bandura (outlined above), they include 'other sources of teacher knowledge' as a possible source of efficacy. This arises from a comprehensive literature review which shows that "knowing the material, and knowing how to teach it well, can improve teachers' sense of efficacy" (Morris et al., 2017, p. 817). Their model, like the earlier work of Tschannen-Moran et al. (1998), emphasizes the role of personal cognition in the formation of self-efficacy. Individuals are envisaged to combine information to make "general appraisals (e.g., of past success, of knowledge, of comparisons with others) that may, in turn, inform self-efficacy" (Morris et al., 2017, p. 820). They also highlight the moderating factors that inform these appraisals as information from various sources is combined and evaluated. An important factor in how a person might weigh information is in how closely (or not) he/she perceives it to relate to the teaching task. As outlined above, the focus module includes mathematical problem solving and pedagogical content consisting of progressive approaches to the teaching of specific mathematics content. We contend that this content supports ITE students' "knowledge of the material" and begins to develop their understanding of "how to teach it well" (Morris et al., 2017, p. 817). The students are directed to draw from the module content to support their in-class enactment of teaching. A core objective of the module structure is thus to support both understanding and enactment of pedagogies that emphasise conceptual understanding, first within the in-class approximation of teaching and later when teaching in schools.

INVESTIGATING MTEB IN INITIAL TEACHER EDUCATION

Three focus group interviews were carried out with 16 ITE students who had completed the first year of the BEd (including the module described above and one teaching placement in a school). Based on the nature of their statements, the ITE students were assigned a place on a four-point MTEB continuum: low efficacy, mixed/low efficacy, mixed/high efficacy, high efficacy. On interrogation of the data, we noticed that some ITE students appeared to see many connections between the taught mathematics education module and their experiences in classrooms while others did not. We created another four-point continuum where the nature of students' statements was used to describe how flexibly they report applying module content to classroom practice. This continuum ranges from 'limited flexibility', where ITE students' statements show a strong focus on specific aspects of mathematics or teaching (e.g., expecting to receive exemplar lesson plans rather than generating their own) to 'Strong Flexibility', where ITE students' statements suggest that they are comfortable generalising theory about mathematics and/or teaching from module content to classroom practice

Looking across these two ways of categorising the ITE students, we found that highly efficacious ITE students appeared to show strong flexibility and professed competence in applying the module content to classroom practice. Less efficacious ITE students appeared to see fewer connections between the mathematics education module and their classroom practice. They appeared to be focused on the finer grained details of planning and teaching (e.g., the nature of planning templates or lesson content for particular class levels) and did not appear to be in a position to take the broader messages about mathematics pedagogy and apply them to the particular contexts of their school placement. Tschannen-Moran et al. (1998) contend that "judgments about the requirements of the teaching task, is an important factor in teacher efficacy" (1998, p. 210). From our data it appears that ITE students with different levels of efficacy may interpret the requirements of teaching tasks quite differently.

Our research design does not support interpretation of whether this connection between efficacy beliefs and flexibility in application of module content, is causal or correlated. We cannot say whether ITE students' strong self-efficacy beliefs support a flexibility of thinking, or vice versa, or indeed whether they are mutually supportive, or whether an additional external understanding or disposition contributes to both. Nevertheless, our findings point to a relationship between the two. Over the coming academic years, we intend to orchestrate opportunities for ITE students to interrogate means-ends relationships (Skinner, 1996) at both broad and fine-grained levels of specificity and to make explicit the links between these two levels. In relation to the development of MTEBs, we theorize that if students consider the broad goals of mathematics education as shaping every teaching task, then particular pedagogies or 'means' aligned with those goals will be adapted or refined according to the specific mathematical content or context of any given situation. Adopting a design research approach (Borko, Liston, & Whitcomb, 2007), we will repeat the data collection and analysis cycles as conducted in the research phase outlined above, and aim to explore whether our modifications to the focus module support the developing efficacy beliefs of our ITE students.

REFERENCES

Bandura, A. (1997). *Self-efficacy: The exercise of control*. New York: Freeman.

Biesta, G. J., & Stengel, B. S. (2016). Thinking philosophically about teaching. *Handbook of research on teaching*, 5, 7-67.

Borko, H., Liston, D., & Whitcomb, J. (2007). Genres of empirical research in teacher education. *Journal of Teacher Education, 58*(1), 3 – 11.

Charalambous, C. Y., Philippou, G. N., & Kyriakides, L. (2008). Tracing the development of preservice teachers' efficacy beliefs in teaching mathematics during fieldwork. *Educational Studies in Mathematics*, *67*(2), 125-142.

Grossman, P., Compton, C., Igra, D., Ronfeldt, M., Shahan, E., & Williamson, P. (2009). Teaching practice: A cross-professional perspective. *Teachers College Record*, *111*(9), 2055-2100.

Morris, D. B., Usher, E. L., & Chen, J. A. (2017). Reconceptualizing the sources of teaching self-efficacy: A critical review of emerging literature. *Educational Psychology Review*, *29*(4), 795-833.

Skinner, E. A. (1996). A guide to constructs of control. *Journal of personality and social psychology*, *71*(3), 549.

Tschannen-Moran, M., Woolfolk Hoy, A., & Hoy, W. K. (1998). Teacher efficacy: Its meaning and measure. *Review of educational research*, *68*(2), 202-248.

THE DEVELOPMENT OF A SET OF LOW-INFERENCE CODES FOR UNCOVERING STUDENTS' UNDERSTANDING OF LINEAR EQUATIONS: FACILITATING COMPARATIVE ANALYSIS

Paul Andrews[1] and Niclas Larson[2]

[1]Stockholm University, Sweden and [2]University of Agder, Norway

In this paper, our goal is to present a methodological contribution to the analysis of students' linear equation solving competence. While other studies have employed a variety of frameworks for analysing students' understanding of this important topic, none have been able to distinguished between solutions based on 'doing the same to both sides', 'swapping the side swapping the sign' or both. Here we describe the development of a set of low-inference codes that facilitate not only this distinction but also the analysis of their interactions. By way of example, data derived from first year primary teacher education students from a large Swedish university are analysed. The results confirm the framework's ease of operation and its propensity for uncovering the complex understandings students have of this important transitional topic. Some implications for cross-cultural studies of students' equations-related knowledge are discussed.

INTRODUCTION

Linear equations, typically construed as one of the most important topics of school mathematics, by providing a meaningful context for students' use of symbols (Brizuela & Schliemann, 2004) offers a strong entry into algebra (Arcavi, 2004). It connects arithmetic to the symbolism of formal mathematics and acts as a *gatekeeper* between school mathematics and higher education and employment (Knuth, Stephens, McNeil, & Alibali, 2006). The literature on equation solving typically distinguishes between two forms of equation. The first, arithmetical equations (Filloy & Rojano, 1989), incorporates the unknown on one side of the equation only. The second, algebraic equations (Andrews & Sayers, 2012), comprises equations with the unknown on both sides. This distinction is didactically significant. On the one hand, an arithmetical equation like $3x + 1 = 13$ can be solved efficiently by means of a straightforward series of operation reversals (Herscovics & Linchevski, 1994). However, if children's first exposure to linear equations is through arithmetical equations then they may face difficulties when faced with algebraic equations like $x + 5 = 4x – 1$ that cannot be solved in such ways. On the other hand, students whose first exposure is to algebraic equations may have an advantage as they will not have learnt a set of procedures that may be superseded.

Historically, algebraic equations have been solved by one of two methods, derived from Viète and Euler respectively (Filloy & Rojano, 1989). The former, warranting a 'swap the side swap the sign' (SSSS) procedure, is based on the transposition of terms from one side of the equation to the other. The latter, justifying a 'do the same to both sides' (DSBS) procedure, draws on operations performed on both sides of the equation simultaneously. Interestingly, intervention studies have typically promoted the balance scale to warrant a DSBS procedure (Caglayan & Olive, 2010; Vlassis, 2002), while analyses of teachers' preferred approaches have found the balance scale being employed with algebraic equations across Europe

(Andrews & Sayers, 2012). In short, the balance scale, despite criticisms (Lima & Tall, 2008), seems positively viewed by both teachers and researchers.

ASSESSING STUDENTS' EQUATIONS-RELATED KNOWLEDGE

Despite this didactical consensus, the limited assessments of students' equation solving have typically found an SSSS procedure (Huntley, Marcus, Kahan & Miller, 2007). Such an approach, reflecting a rote-learnt and arbitrary transposition whereby the unknown finishes on the left-hand side and a value on the right (Filloy & Rojano, 1989), not only perpetuates an operational conception of the equals sign but fails to support students' understanding that such movement does not change the equation's equality. It masks mathematical understanding (Star & Seifert, 2006), and frequently leads to a variety of later difficulties. It is, for many students, a 'magical' (Lima & Tall, 2008) procedure that frequently reduces students "to performing meaningless operations on symbols they do not understand" (Herscovics & Linchevski, 1994, p. 60). Even teacher education students, who may be expected to have a better developed understanding of equations than school students, typically offer solutions indicative of an incomplete understanding of their conceptual basis (Andrews & Xenofontos, 2017; Isik & Kar, 2012). In short, internationally, primary teacher education students have an underdeveloped conceptual understanding of algebra in general and equations in particular, frequently seeing algebra "as a school subject matter dominated by symbols and symbol manipulation" (Stephens, 2008, p. 44).

Procedurally, it is surprising how little attention has been given to the processes that students invoke when solving linear equations. For example, international tests of achievement like PISA and TIMSS typically score correct answer only, as do the national tests in Sweden, the site of the research presented here. From the research perspective, a number of studies have examined aspects of secondary students' equation solving competence in various contexts and by means of various analytical frameworks. However, none of these would have distinguished between a solution based on DSBS and one on SSSS (see, for example, Foster, 2018; Star & Seifert, 2006; Vaiyavutjamai & Clements, 2006). In other words, even when students' solutions to linear equations are under scrutiny, little attention has been paid to the underlining conceptualisation of their thinking.

In this paper, we discuss the development and application of a set of low-inference codes for analysing students' solutions of linear equations. Low-inference code are especially suitable for cross-cultural research, not least because they focus on easily-recognised characteristics of the examined phenomenon, and have been particularly useful in cross-cultural analyses of mathematics classroom activity, both from the perspective of learning outcomes (Andrews, 2009a) and the didactical strategies teachers employ (Andrews, 2009b).

THE CURRENT STUDY

The research presented here draws on an earlier study in which Greek and Greek Cypriot primary teacher education students were invited to explain in writing to a friend, who had been absent when such things had been taught in school, a solution to the equation below, which had been presented with no annotations to indicate the hidden solver's thinking.

$$x + 5 = 4x - 1$$

$$5 = 3x - 1$$
$$6 = 3x$$
$$2 = x$$

The scripts from the two cohorts were subjected to a constant comparison analysis and a set of seven codes identified. While space prevents them being discussed in detail, although their labels can be seen in table 1, we believe that five are self-evident and only two need elaboration. In this respect, an explicit objective referred to the need to find the value of the unknown, while an implicit objective referred to the movement of terms in order to get unknowns on one side of the equation and numbers on the other. Importantly, the seven codes proved adequate for identifying similarities and differences in the two cohorts' scripts, highlighting not only these teacher education students' culturally situated perspectives on linear equations (Andrews & Xenofontos, 2017).

Table 1: The original and two iterations of revised codes

Codes from Greek and Cypriot students' texts	First revised codes for Swedish students	Second revision for Swedish students
The student writes something concerning…		
An aware of unknowns	Unchanged from previous	Unchanged from previous
Defining an unknown	Unchanged from previous	Unchanged from previous
An explicit objective	Unchanged from previous	Unchanged from previous
An implicit objective	Unchanged from previous	
		Particular objective
A rote procedure	Changing side changing sign	
		SSSS general
		SSSS additive
		SSSS division
	Doing the same to both sides	
		DSBS general
		DSBS addition
		DSBS division
	Relational understanding	Unchanged from previous
Inverse operations	Unchanged from previous	
An unspecified process	Uncertain approach	Unchanged from previous
		DSBS induced SSSS
	Checks solution	Unchanged from previous

Developing the codes

Returning to the current study, shortly after beginning their programme and before any exposure to mathematics, one complete cohort of undergraduate Swedish primary teacher education students from a large university were invited to complete the same task. The cohort comprised six classes, each of which was visited by the first author, shortly before the end of a mathematics education lecture, and invited to participate. Although few did so, unwilling students left the room for an early coffee break. Those who remained were given a sheet of paper on which was presented the task and instructions. Additional oral instructions clarified the task and participants wrote their response to their absent school-friend on this paper. After this, scripts were read and reread looking for evidence of the codes applied to the original study of Greek and Cypriot students.

First revised coding schedule

As we browsed these new scripts, the inadequacy of the original coding schedule became apparent. The reasons for this were several. Firstly, many Swedish students wrote of doing the same to both sides, a process not mentioned by any of the Greek or Cypriot students. Secondly, other students wrote of swapping the side and swapping the sign, which aligned with the earlier code concerning the use of a rote rule. Thirdly, a number of Swedish students wrote of the equality of the two sides, which was not something mentioned by either of the Greek-speaking groups. Our first attempt at resolving these tensions were to augment rather than redefine the categories. For example, the first set of revised codes can be seen in table 1. Here, five codes remained unchanged, two were minorly modified and three new ones added. These, as can be seen, reflected the hitherto unseen ‘do the same to both sides’, a solution check and, as a consequence of narratives concerning the equality of both sides, a code concerning a relational understanding of the equals sign, which research had indicated was essential for effective equation solving (Knuth et al., 2006).

Second revised coding schedule

However, as we worked with the Swedish scripts, as well as a second set of data obtained from Norwegian teacher education students, a new set of problems emerged. Firstly, while a code framed by words like explicit may be simple to operationalise, not least because the word explicit implies something visible, implicit codes require the inference of something that is not necessarily visible. Thus, it became apparent as we examined the two sets that the need for interpretation made an implicit objective difficult to discern. Thus, while the original explicit objective, with its general goal, was retained, the implicit objective was replaced with a particular objective, whereby the student had written something concerning getting the x alone and, typically, numbers on the other side. Secondly, a broad code pertaining to SSSS failed to distinguish between students who invoked a general principle and those who applied it to a particular example. In similar vein, DSBS without some form of qualification was too broad. Thus, these two codes became three each, according to whether a student wrote a general statement, a statement concerning a particular additive operation or a particular multiplicative operation. Thirdly, the earlier code concerning inverse operations, was removed as it was thought to require levels of inference unlikely to be found in the data. In a related vein, the earlier code concerning an uncertain approach was frequently observed in relation to the division of 6 by 3, the coefficient of x in the final row of the given solution. On these occasions, whereby it occurred after earlier invocations of an additive DSBS, we concluded that although it was not made explicit it was a derivative of DSBS. This second set of revised codes are summarised in table 1.

Table 2: Working definitions, frequencies and percentages of each code

Code name	The student writes something about…	Absent	Present	%
Mentions unknown	the unknown or variable	128	28	18
Conceptual objective	finding the 'value of x' (addresses the *purpose* of equation solving)	96	60	38
Procedural objective	getting x alone or x on one side (addresses the *process* of equation solving)	68	88	56
SSSS General	the general SSSS movement of objects	147	9	6
SSSS Particular addition	the additive SSSS movement of objects	103	53	34
DSBS General	solving equations by doing the same to both sides in general terms	129	27	17
DSBS General additive	adding to both sides with no reference to the particular objects of the equation	148	8	5
DSBS Particular additive	adding to both sides with reference to the particular objects of the equation	79	77	49
DSBS General division	dividing both sides by the number in front of x	147	9	6
DSBS Particular division	dividing both sides by 3	105	51	33
Unspecified division	dividing by 3; divide 6 by 3 etc., where it's not clear that both sides are divided	104	53	34
Equality of both sides	both sides of the equals sign being equal	118	38	24
Checks solution	checking the solution	136	20	13

Third revised coding schedule

Each time we revised the codes, new tensions emerged as we attempted to apply them to our scripts. Moreover, the more time we went on their development the more obvious it became that unless they were low inference, relying on clear and unambiguous definitions of specific mathematical actions, the more our difficulties would continue. For example, having coded both our Swedish and Norwegian data again, amounting to around 300 students, some ambiguities remained: can we be confident that a student, who had earlier invoked an additive DSBS and was now dividing 6 by 3 had generalised DSBS? Were we clear in our own minds that three forms of objective were not only instantly recognisable but comprehensive? Would we be able to recognise consistently a definition of an unknown? These, and other questions led us to a third revision of the codes, which can be seen in table 2. Here, each code is defined by an action, leaving no need for inference. For example, there were now two forms of objective defined by distinctive forms of action. Also, the code concerning a relational understanding of the equals sign, which we now believed to require too high levels of inference, was replaced by a code concerning a statement of equality between the two sides. In addition, where once we tried to infer either an inverse operation or a DSBS induced SSSS, a code involving an unspecified division was introduced. Finally, SSSS division, which had been included for the sake of completeness, was deleted. Our discussions led us to conclude that while SSSS addition is an intuitively natural action to express in words, SSSS division is not so intuitively expressed and, as later confirmed by the data, unlikely to occur.

Applying the third set of codes

Of the Swedish students involved in the study, six either left their sheets blank or apologised for their lack of equations-related knowledge. Otherwise, 156 students offered mathematically interpretable responses, the analyses of which can be seen in table 3. If, within a student's account, the same code was repeated then only one incidence was recorded. Interestingly, analyses of variance indicated that age had no influence on students' accounts, while Mann-Whitney U-tests showed no influence of gender.

Interactions of low inference codes

Table 4: Cross-tabulation of conceptual objectives against procedural objectives

		Procedural objective: Absent	Procedural objective: Present	
Conceptual objective	Absent	48	48	96
	Present	20	40	60
		68	88	156

With respect to showing how these codes facilitate a deeper understanding of students' equations-related knowledge, a series of cross-tabulations were run. Unfortunately, lack of space prevents many being included but the following two examples offer indications as to how the interactions of the codes play out in meaningful ways. Firstly, table 4 shows the interactions of the two objectives. Here we can see, confirming the figures of table 3, that 60 students (38%) wrote something interpretable as a conceptual objective focused on identifying the value of x, while 88 (56%) indicated a procedural objective, typically about getting unknowns on one side or alone. When the two codes were compared, the scripts of 40 students (26%) yielded both conceptual and procedural objectives, indicating, overall, that 108 individual students (69%) wrote something interpretable as a goal for the equation solving process. Secondly, table 5 shows the interaction of the two particular additive strategies based on DSBS and SSSS respectively, with the former coded for 77 scripts (49%) and the latter for 53 (34%). The cross-tabulation presented in table 5 shows only 12 students (8%) writing of both strategies, with 65 (42% of all students) writing uniquely of a DSBS approach and 41 (26% of all students) writing uniquely of an SSSS strategy. Thus, overall, 118 students (76%) wrote something recognisable as a 'conventional' additive approach. Interestingly, students rarely invoked both conceptualisations, typically offering either a DSBS or an SSSS perspective.

Table 5: Cross-tabulation of DSBS particular addition against SSSS particular addition strategies

		DSBS particular addition		
		Absent	Present	
SSSS particular addition	Absent	38	65	103
	Present	41	12	53
		79	77	156

DISCUSSION

In this somewhat atypical paper, we have narrated the process by which a set of low-inference codes were developed for analysing students' (in this case beginning primary teacher education students) understanding of linear equations. However, our view is that the framework would be applicable to any students and not just adults. Moreover, it does not reflect a hierarchy (Vaiyavutjamai & Clements, 2006) but a set of curriculum-independent and cross-culturally meaningful categories of understanding. From the perspective of the analysis of Swedish teacher education students' equations-related understanding, several of the categories may be redundant due to low frequencies. However, that does not mean to say they will not be important in later evaluations of the Norwegian data mentioned earlier, Spanish data that have recently been collected or reanalyses of the Greek and Cypriot data.

Importantly, particularly for those students who invoked an additive DSBS, equation solving does not involve a 'magical' procedure (Lima & Tall, 2008) involving "meaningless operations on symbols they do not understand" (Herscovics & Linchevski, 1994, p. 60). It is also interesting to note that, compared with their American peers, the Swedish students of this study understood algebra as much more than "a school subject matter dominated by symbols and symbol manipulation" (Stephens, 2008, p. 44). Finally, the two interactions presented above, one based on the two forms of objective and the other the two additive approaches based on DSBS and SSSS respectively, offered an indication of the complexity of students' understanding, both with respect to their objectives for equation solving and their preferred strategies.

REFERENCES

Andrews, P. (2009a). Comparative studies of mathematics teachers' observable learning objectives: Validating low inference codes. *Educational Studies in Mathematics, 71*(2), 97-122.

Andrews, P. (2009b). Mathematics teachers' didactic strategies: Examining the comparative potential of low inference generic descriptors. *Comparative Education Review, 53*(4), 559-581.

Andrews, P. & Sayers, J. (2012). Teaching linear equations: Case studies from Finland, Flanders and Hungary. *The Journal of Mathematical Behavior, 31*(4), 476-488.

Andrews, P. & Xenofontos, C. (2017). Beginning teachers' perspectives on linear equations: A pilot quantitative comparison of Greek and Cypriot students. In T. Dooley & G. Gueudet (Eds.), *Proceedings of the Tenth Congress of the European Society for Research in Mathematics Education* (pp. 1594-1601). Dublin: Institute of Education, Dublin City University.

Arcavi, A. (2004). Solving linear equations: Why, how and when? *For the Learning of Mathematics, 24*(3), 25-28.

Brizuela, B. & Schliemann, A. (2004). Ten-year-old students solving linear equations. *For the Learning of Mathematics*, *24*(2), 33-40.

Caglayan, G. & Olive, J. (2010). Eighth grade students' representations of linear equations based on a cups and tiles model. *Educational Studies in Mathematics, 74*(2), 143-162.

Filloy, E. & Rojano, T. (1989). Solving equations: The transition from arithmetic to algebra. *For the Learning of Mathematics, 9*(2), 19-25.

Herscovics, N. & Linchevski, L. (1994). A cognitive gap between arithmetic and algebra. *Educational Studies in Mathematics, 27*(1), 59-78.

Huntley, M., Marcus, R., Kahan, J. & Miller, J. (2007). Investigating high-school students' reasoning strategies when they solve linear equations. *The Journal of Mathematical Behavior, 26*(2), 115-139.

Isik, C. & Kar, T. (2012). The analysis of the problems posed by the pre-service teachers about equations. *Australian Journal of Teacher Education*, *37*(9), 93–113.

Knuth, E., Stephens, A., McNeil, N. & Alibali, M. (2006). Does understanding the equal sign matter? Evidence from solving equations. *Journal for Research in Mathematics Education*, *37*(4), 297–312.

Lima, R. & Tall, D. (2008). Procedural embodiment and magic in linear equations. *Educational Studies in Mathematics, 67*(3), 3-18.

Star, J. & Seifert, C. (2006). The development of flexibility in equation solving. *Contemporary Educational Psychology, 31*(3), 280-300.

Stephens, A. (2008). What 'counts' as algebra in the eyes of preservice elementary teachers? *The Journal of Mathematical Behavior, 27*(1), 33–47.

Vaiyavutjamai, P. & Clements, M. (2006). Effects of classroom instruction on student performance on, and understanding of, linear equations and linear inequalities. *Mathematical Thinking and Learning, 8*(2), 113–147.

Vlassis, J. (2002). The balance model: Hindrance or support for the solving of linear equations with one unknown. *Educational Studies in Mathematics, 49*(3), 341-359.

Warren, E. & Cooper, T. (2005). Young children's ability to use the balance strategy to solve for unknowns. *Mathematics Education Research Journal, 17*(1), 58-72.

DILEMMAS EXPERIENCED IN LECTURING UNDERGRADUATE CALCULUS

Sinéad Breen[1] and Ann O'Shea[2]

[1]CASTEL, Dublin City University, Ireland; [2]Maynooth University, Ireland

We consider a set of accounts written by two university lecturers describing incidents that took place during their first-year Calculus modules. Analysis of these accounts revealed that the lecturers had to make some difficult decisions while teaching. These situations sometimes involved choices between two or more alternatives each of which had disadvantages. We labelled these choices 'dilemmas'. Here we present and discuss the three most common types of dilemma evident from our data: namely, balancing good practice in teaching with students' feeling of discomfort; balancing the needs of students with different backgrounds; balancing time constraints and active participation by students.

INTRODUCTION

This study arises from a project in which five university mathematics lecturers attempted to follow the Discipline of Noticing (Mason, 2002b) in order to develop their own teaching. They wrote brief-but-vivid accounts of incidents in their classrooms and shared them with each other. Although lectures might be perceived as scripted and non-dynamic, previous analysis of our set of accounts (O'Shea, Breen & Meehan, 2017) revealed that lecturers experience a range of in-the-moment decision points in class. Further analysis has recently caused us to label some of these decision points as dilemmas because of the difficult nature of the choices involved. In this paper we will use as data the accounts written by two of these lecturers (referred to as Lecturer Y and Lecturer Z) while they were teaching modules on Differential Calculus to large first-year groups.

LITERATURE REVIEW

Teaching dilemmas have been a subject of research for some time. Lampert (1985) defined a dilemma as 'a problem forcing a choice between two equally undesirable alternatives…even though choosing would bring problematic consequences' and 'an argument between opposing tendencies within oneself in which neither side can come out the winner' (p. 182). Other authors have used similar definitions, for example Scager, Akkeman, Pilot & Wubbels (2017) considered dilemmas faced by teachers in university settings and considered dilemmas to be 'conflicts in which there are multiple, equally viable alternatives, each of which has advantages and disadvantages'(p. 319). They contend that a dilemma by its nature poses problems for an instructor not just because of the possible negative outcomes arising from any action but also because of the difficulty in trying to take the consequences of the possible actions into account. Schoenfeld (2008) asserted that it is natural that teachers face dilemmas because of the need to 'resolve the inevitable tensions that result from trying to achieve many things and honor many constraints at once' (p. 81). Tripp (1993) noted that dealing with teaching dilemmas and in-the-moment decision-making in general is stressful for teachers and that teachers need to use their professional judgement to make choices (p. 49).

Even though making these choices can be stressful, Lampert (1985) put forward the view that dealing with and reflecting on dilemmas can be useful to teachers and a means to professional growth. In her paper, she outlined two teaching situations which necessitated difficult choices and showed how teachers can use their knowledge of themselves and their goals to overcome problems. She posed a number of questions concerning further study in this area, including the question of how often dilemmas arise in classrooms and how teachers manage them.

At elementary school level, Ball (1993) examined the challenge of creating classroom practises to engage students in authentic mathematical tasks (e.g. formulating and solving problems, experimenting, conjecturing). She believes that trying to teach in such an 'intellectually honest' manner gives rise to dilemmas because of the competing aims of such an approach and the uncertainties inherent in addressing them. She described three types of dilemma: representing the content; respecting the children as mathematical thinkers; and creating and using community. Teachers typically face the dilemmas of their work alone and so Ball advocates forums for professional exchange in which teachers explore one another's practice as a resource for improving teaching and learning.

Very little research has been undertaken into dilemmas arising in mathematics classrooms at university. However, in his *Guide for University and College Lecturers*, Mason (2002a) described some tensions in teaching mathematics and, while he cautioned that there are no universal solutions to such tensions, he suggested they be thought of as sources of energy rather than problems as the latter induce anguish and frustration. In order to discuss the tensions that he believed to be recurrent, he clustered them under three main headings: student and tutor agenda and expectation; doing and construing, knowing and understanding; being subtle and being explicit.

At all levels of education, students need to be challenged to stimulate their learning. However, as Scager et al. (2017) point out, challenging students can conflict with other teacher responsibilities, creating dilemmas for teachers. At university, classes are often large and comprise students of widely differing abilities, and choosing to serve one group of students can have adverse consequences for the learning of others. Scager et al. (2017) conducted a study involving twelve university lecturers from different disciplines reflecting on how challenges for their students were managed. Seven recurrent dilemmas were idenitified, the two most frequent of these being maximising challenge versus maintaining psychological safety of students and maximising challenge versus keeping all students aboard.

Speer and Wagner (2009) considered the tension that arises in the context of whole-class discussions in mathematics lectures between encouraging student ideas and participation and using students' suggestions in a mathematically productive manner. They made use of the terms *social scaffolding* (ways of supporting discourse and participation) and *analytic scaffolding* (ways of supporting mathematical progress) which were previously defined by Williams and Baxter (1996). Their analysis showed that providing both types of scaffolding at once is a very difficult task and requires instructors to draw on their pedagogical content knowledge (to recognise the pedagogical potential of a contribution to the discussion) and their specialised content knowledge (to interpret and evaluate the contributions). They call for further work to study the experience of mathematics lecturers and in particular to analyse situations in which pedagogical and specialised content knowledge could be developed.

We take Lampert's (1985) and Mason's (2002a) view that reflecting on problematic teaching situations or dilemmas can provide opportunities for teachers to develop their knowledge in this way. As a first step, it is important to have information about the types of tensions that lecturers face in the course of teaching a module. We will endeavour to provide some information on the question: What dilemmas do lecturers encounter when teaching large groups of undergraduate calculus students?

METHODOLOGY

In the Discipline of Noticing project mentioned earlier, the group of lecturers attempted to follow the practices outlined by Mason (2002b). In order to ensure that an incident noticed while teaching is available for further reflection, Mason recommends it should be recorded in a 'brief-but-vivid' account. Such an account is

> one which readers readily find relates to their experience. Brevity is obtained by omitting details which divert attention away from the main issue. … Vividness is obtained by sticking as much as possible to descriptions of behaviour which others, had they been present, would have readily agreed to having seen, heard or felt. (p. 57).

The aim is to give an 'account-of' an incident, describing it as objectively as possible, rather than 'accounting-for' (offering interpretation, explanation, value-judgement or criticism). When the purpose of an account is to describe an emotion, then it should do so as physiologically and impartially as possible.

We will consider here only the accounts written by two of the lecturers in one academic year which relate to large group teaching. Both lecturers wrote accounts relating to Differential Calculus modules for first-year students; these modules were aimed at non-specialist students. Lecturer Y's module ran for the entire academic year and about 50 students were enrolled, while Lecturer Z's class consisted of about 150 students and the module ran for one semester only.

The two lecturers wrote 58 accounts in total relating to the modules in question here, and each account usually consisted of between 100 and 150 words. The data analysis process started with us reading and rereading both sets of accounts highlighting what we saw as dilemmas. We adapted the definition of the term *dilemma* given by Lampert (1985); for us a dilemma is a situation in which a difficult choice has to be made by the instructor between two or more undesirable alternatives. Following a general inductive approach (Thomas, 2006), we studied accounts which showed the lecturers' indecision or dissatisfaction stemming from having to make a choice where all courses of action had disadvantages. We met to discuss the accounts selected in this manner and agreed on the final identification of dilemmas. We then compared the issues described, and created categories of dilemmas. In this way the categories emerged from our data, and only later did we consult the research literature for comparison.

We found three main categories of dilemmas in our analysis of the set of accounts. Examples of these categories were evident in both lecturers' accounts and therefore we considered that these problematic situations or choices were general and indicative of the difficulties that other Calculus lecturers might face. Other types of dilemmas occurred less frequently in the accounts and so we will not consider them here.

EXAMPLES OF DILEMMAS

The first category of dilemmas which emerged contains accounts where there is a clash between belief about good practice in teaching and concerns about students feeling uncomfortable (either socially or academically). This category has two important subcategories: creating cognitive conflict or encouraging participation versus not wanting to embarrass students or undermine their confidence (Category 1a); and fostering agency versus providing scaffolding (Category 1b). The second category concerns dilemmas arising from the wish to balance the needs of students with strong mathematical backgrounds versus those with weaker mathematical backgrounds (Category 2). The accounts in this category concern decisions about spending class time on revision or on basic mathematical skills versus keeping all students in the lecture hall engaged. The third category is about trying to balance time constraints with the wish to encourage student participation and/or develop understanding (Category 3). We will explain these categories further below and give examples of accounts which illustrate the choices facing lecturers in these situations.

Category 1: Balancing good practice in teaching with students' feeling of discomfort

The accounts which showed evidence of a dilemma arising from a conflict between the lecturers' views about good teaching practice and their wish not to embarrass or undermine students' confidence (*Category 1a*) occurred in the context of asking questions of the whole class. Both lecturers wrote accounts about incidents where they deliberately introduced cognitive conflict for students with the aim of getting students to recognise the conflict and to develop deeper understanding of a particular concept. However, both lecturers worried about the implications of such a strategy for student confidence and willingness to participate. Both lecturers spoke about asking whole-class questions to identify cognitive conflicts and worried about embarrassing students in public if they were seen to give the 'wrong' answer. On the other hand, they felt that by not causing the conflict in class some students might not be aware of a conflict in their views and/or be able to resolve it by themselves. For example, Lecturer Z wrote:

> Account A: I asked a question that I knew would probably generate a wrong answer. I did this so as to point out the pitfall and misunderstanding. When the wrong answer was given I said 'I'm glad you said that' and explained some more. However, I felt bad that I had more or less deliberately caused someone to give a wrong answer. (Lecturer Z)

Other accounts that dealt with similar situations included lecturers wanting to ask questions to generate a debate but not liking to call on individual students in case they would be embarrassed and lecturers worrying about the effect of being wrong on student confidence.

The second type of dilemma (*Category 1b*) in the first category concerned the tension between the desire to foster agency and independence for students and wanting to provide adequate scaffolding for their learning. This type of situation usually arose when lecturers asked students to work on tasks in lectures or tutorials. Sometimes students looked to the lecturer for validation instead of relying on their own understanding. The lecturer was then faced with a choice of whether to provide the validation (and make the students feel comfortable) or to refuse to do this. A similar situation can occur when a lecturer asks a whole-class question and no one offers a suggestion – the lecturer is faced with a choice between leaving the question unanswered (and possibly causing discomfort) or answering it herself and reducing the opportunity for learning. In the account below, the lecturer noticed that students had problems with an unfamiliar task and provided scaffolding to help them with it. However, she felt that in doing this, she might have compromised the effectiveness of the task:

> Account B: Due to the students' difficulties in yesterday's tutorials in relation to drawing graphs meeting a number of criteria, I changed my plan for today's lecture. The students were given the graphs of functions with various points of discontinuity (but no formulaic description of the functions) and asked to determine whether particular statements were true or false. Asking individuals for their answers indicated that they could correctly determine the truth or otherwise of the statements. In undertaking this exercise, I was a little uneasy that I was perpetuating a type of helplessness by making an unfamiliar problem assigned as homework more manageable for them. (Lecturer Y)

Category 2: Balancing the needs of students with different backgrounds

The second category of dilemmas that emerged from our analysis arose from teaching students with a range of mathematical backgrounds in large class settings. The lecturers faced difficult choices when trying to balance the needs of different groups of students. As we see in the account below, the lecturers sometimes felt the need to review material for students who were less prepared but found it difficult to do this without losing the interest of others. However, they worried that if they moved on then they risked losing many of the students.

> Account C: I started the class by drawing the graphs of sin, cos and tan on the board. We had covered trig functions and domains and ranges last week. My intention was to talk a little about periodicity and then move on to inverse functions. However, I asked the class to tell me the domain and range of the three trig functions and the answers were not great. […] I spent half the class trying to address these issues. I felt that some students were definitely getting bored. (Lecturer Z)

In other accounts related to this category the lecturers spoke of noticing that some students in the class were bored while others were struggling with the material. Most first year university students have already studied quite a lot of calculus at school although some have not developed a deep understanding of the subject. Lecturers face a choice between reviewing topics that some students find difficult and moving on to new material without spending time working on the foundations. One possible solution is to approach these topics from a fresh perspective, however this also is a difficult task and Lecturer Y remarked that she experienced 'a tension between maintaining [students'] interest and motivation and undermining their prior knowledge'. It is common for first-year undergraduate mathematics courses in Ireland to include students with very different levels of mathematical preparation. For example, students who had studied mathematics at Ordinary Level in the State Examinations (Leaving

Certificate) often find themselves in the same module as those who have studied at Higher Level. In contrast, this would be rare in secondary school classrooms.

Category 3: Balancing time constraints and active participation by students

The final category of dilemmas that we identified in our data concerned the tension between wanting to spend time helping students to develop understanding and needing to be cognisant of time constraints. The lecturers spoke about wanting to encourage interaction in their classes but realised that this takes quite a lot of class-time. They had to balance this aim against the need to cover the syllabus and finish the course within a tight timeframe. In the account below, the lecturer speaks about the time implications of inquiry-based learning:

> Account D: I tried to use a 'guided-discovery' approach to facilitate students' realization that the graph of a function and its inverse are mirror images of each other in the line y=x. However, each step of this took a lot longer than I envisaged. Moreover, I wasn't convinced at the end that the students would retain this particular piece of information longer or understand it better for having discovered it themselves as a class community. (Lecturer Y)

Other accounts in this category concerned conflicts in lecturers' priorities, that is, between employing progressive teaching practices and adhering to the syllabus. There was a recognition that some teaching practices take more time than 'lecturing' and that time-pressure places a heavy burden on lecturers.

DISCUSSION

Although it was not our intention when writing accounts of our experiences of teaching first-year Calculus to document the dilemmas inherent in this practice, dilemmas were recorded in 23 of the 58 accounts in question (i.e. 40%). This serves to support Schoenfeld's (2008) view that dilemmas are 'natural' and illustrates their prevalence in day-to-day teaching practice. Moreover, it underlines the importance of examining them with a view to improving practice.

Scager et al. (2017) explored the dilemmas that arise in presenting challenges to students, and subsequently managing these challenges, across a number of disciplines at university level. We found echoes of the two most frequent dilemmas documented by Scager and her colleagues in our study, despite the fact that mathematics was not included in the disciplines they examined. Account A above (introducing a cognitive conflict for students) is an example of a 'maximising challenge versus maintaining psychological safety of students' dilemma. Scager et al. explain how a student's psychological safety could be threatened by being asked difficult questions or by having critical feedback on their work openly communicated to them. Students need to be challenged to move outside of their comfort zones but yet a teacher wants to avoid a student feeling embarrassed or inadequate in order to preserve the student's freedom to contribute to class discussions and activities. On the other hand, Account C clearly describes a situation in which the teacher experiences a dilemma in relation to 'maximising challenge versus keeping all students aboard'. In a class in which there are students with a wide range of different mathematical backgrounds, a teacher tries to avoid setting the challenge too high in order to preserve the self-confidence of some students while simultaneously avoiding demotivating other students with an insufficient level of challenge.

Aside from providing appropriate challenge for students, other dilemmas arise when trying to teach in an authentic or intellectually honest way, respecting the integrity of the discipline of mathematics. The account given above of trying to use a guided discovery approach (Account D) resonates with Ball's (1993) discussion of the dilemmas integral in creating and using a community in which ideas can be developed and critiqued. Ball questions whether the efforts involved in establishing such a community always involve the best use of the (often limited) time. She suggests that constructing a classroom pedagogy to model authentic mathematical practice precisely would be not only inappropriate but also irresponsible because mathematicians have the luxury to focus on a small number of problems while a teacher is usually bound to cover an entire curriculum and develop the associated skills. A teacher must also facilitate the learning of all the learners in her care, in the same room, at the same time. The dilemmas evoked by these constraints are also felt at university level.

Although both lecturers in our study recognised the value in generating debate and cognitive conflict they were both uneasy with the possible consequences for student affect. The setting of a first-year university lecture exacerbates this problem, since the class-sizes are normally large and the lecturers usually do not have an opportunity to get to know the students well. Since students are often reluctant to answer (or even ask) questions in this setting, even when explicitly encouraged to do so (Yoon, Kensington-Miller, Sneddon, & Bartholomew, 2011), lecturers may be excessively cautious not to damage student engagement. The large class size may also result in students having an increased sense of anonymity and may discourage participation from that perspective. Thus, the context of a first-year university lecture may mean the dilemmas encountered in categories 1a, 1b and 3 occur more frequently or are felt more acutely than at primary or secondary school level. More reflection on how to provide social scaffolding in this situation is needed. The situation may be further aggravated by the subject matter, as students often appear to struggle with the mathematical vocabulary needed to participate in a whole class discussion in a mathematics lecture.

Despite Speer and Wagner's (2009) contention that teachers are often more successful at using social scaffolding than analytic scaffolding, the issue of a lecturer having problems providing analytic scaffolding did not appear in the subset of accounts analysed here. Speer and Wagner explain how providing both types of scaffolding at once requires instructors to draw on both their pedagogical content knowledge and their specialised content knowledge. It may be that the specialised content knowledge required to interpret and evaluate a student's contribution is unproblematic in the context of first-year Calculus for non-specialist students.

The dilemmas described here arose from a broader project involving five mathematics lecturers engaging with the Discipline of Noticing (Breen, McCluskey, Meehan, O'Donovan & O'Shea, 2014). Accounts written by each of the lecturers were shared with all and meetings were held periodically to discuss any matters relating to the project. Often these meetings became a forum for the discussion of a dilemma related in one of the accounts and allowed us, a group of teachers, to explore one another's practice and how a lecturer might act when faced with such a dilemma. We found this collaborative aspect of the project to be very beneficial in terms of professional development. As Ball (1993) pointed out, teachers regularly face the dilemmas of their work alone and so forums for professional exchange in which these dilemmas are discussed can provide a necessary opportunity for improving teaching and learning. Scager et al. (2017) also believe that expertise can be developed through collaborative reflection. They assert that reflecting on dilemmas in particular, because the nature of dilemmas allows for 'the evocation of reflection and argumentation, encouraging teachers to talk about choices and considerations' (p. 333), can lead to professional growth.

REFERENCES

Ball, D. L. (1993). With an eye on the mathematical horizon: Dilemmas of teaching elementary school mathematics. *The Elementary School Journal*, *93*(4), 373-397.

Breen, S., McCluskey, A., Meehan, M., O'Donovan, J., & O'Shea, A. (2014). A year of engaging with the discipline of noticing: Five mathematics lecturers' reflections. *Teaching in Higher Education*, *19*(3), 289-300.

Lampert, M. (1985). How do teachers manage to teach? Perspectives on problems in practice, *Harvard Educational Review*, *55*(2), 178-194.

Mason, J. H. (2002a). *Mathematics teaching practice: Guide for university and college lecturers*. Elsevier.

Mason, J. (2002b). Researching your own practice: The discipline of noticing. London: Routledge Farmer.

O'Shea, A., Breen, S., & Meehan, M. (2018). Decision points in mathematics lectures. *Innovations in Education and Teaching International*, *55*(6), 650-659.

Scager, K., Akkerman, S. F., Pilot, A., & Wubbels, T. (2017). Teacher dilemmas in challenging students in higher education. *Teaching in Higher Education*, *22*(3), 318-335.

Schoenfeld, A. H. (2008). On modeling teachers' in-the-moment decision-making. *A study of teaching: multiple lenses, multiple views*, 45-96.

Speer, N. M., & Wagner, J. F. (2009). Knowledge needed by a teacher to provide analytic scaffolding during undergraduate mathematics classroom discussions. *Journal for Research in Mathematics Education*, 530-562.

Thomas, D. R. (2006). A general inductive approach for analyzing qualitative evaluation data. *American Journal of Evaluation*, *27*(2), 237-246.

Tripp, D. (1993). *Critical incidents in teaching (classic edition): Developing professional judgement*. London: Routledge.

Williams, S. R., & Baxter, J. A. (1996). Dilemmas of discourse-oriented teaching in one middle school mathematics classroom. *The Elementary School Journal*, *97*(1), 21-38.

Yoon, C., Kensington-Miller, B., Sneddon, J., & Bartholomew, H. (2011). It's not the done thing: Social norms governing students' passive behaviour in undergraduate mathematics lectures. *International Journal of Mathematical Education in Science and Technology*, *42*(8), 1107-1122.

CONSULTING CHILDREN: MATHS AND ME

Marie Brennan

Department of Education and Skills

This paper examines what teaching and learning mathematics 'looked like' and how it was experienced by children across eight Irish primary classrooms at second (7 and 8 years old) and fifth (10 and 11 years old) class level. Children are at the heart of the learning process and can provide important views of the curriculum and of the teaching and learning. Exploring children's experiences of teaching and learning provide insights into the difficulties and challenges children experience in their learning. A total of twenty four mathematics lessons were observed, while eighteen mathematics lessons were video recorded. Focus group interview with children, child questionnaires and drawings were also employed in the data collection. For the majority of children in this study mathematics is little more than calculation and number work. Despite the Primary Mathematics curriculum advocating mathematical processes such as problem solving, developing logic and reasoning, ad communicating mathematical ideas, in this study few children talked about mathematics in this way. Children's beliefs about mathematics are determined by the mathematics they encounter, the tasks in which they encounter it and their disposition towards mathematics as a result of previous encounters.

INTRODUCTION

Pupil voice has come to be seen as crucially important to understanding the complexities of learning in school. Children's understandings of classroom processes, and their own role in learning, have gained attention in research studies both nationally and internationally (Alexander, 2010; Barber & Houssart, 2013; Devine, 2011; Gipps & Tunstall, 1998). This research seeks to give 'voice' to children's views and perspectives on mathematics learning. In order to best understand how perspectives are formed, it is necessary to go inside classrooms and situate teaching and learning in social context and to examine the variables (hidden and overt) that exert an influence.

According to social practice theory, students come to the classroom as part of many diverse communities in which they have formed their identities and they have to reshape their identities as they participate in the community of the classroom. It is in this reshaping of identity that learning resides (Lee, 2006). "Identities are constructed within a context of activity, pupils build an identity, that is a way that they explain themselves, within each community in which they participate" (Holland, Skinner, Lachicotte, & Cain, 1998 p. 270). Similarly, "Mathematics in practice becomes an issue of identity as well as cognitive process" (Barta & Bremner, 2009 p.91). Enabling students to build an identity as someone who is able to do mathematics is an important aim for a mathematics classroom. Teachers are the most important resource for developing student's mathematical identities (Cobb & Hodge, 2002; Hayes, Mills, Christie, & Lingard, 2006). They influence the ways in which student's think of themselves as learners (Walshaw, 2004). While learning mathematical skills and knowledge, students are also developing beliefs and attitudes about the subject, and themselves as mathematical learners and practitioners (Grootenboer, 2013). Teachers of mathematics are in a powerful position because they can significantly impact on the mathematical identity and the futures of learners through the nature of the relational pedagogy they practice in their classrooms. This is evident throughout the everyday, routine mathematics classes that teachers and students experience. Feedback is a crucial feature of the teaching-learning process. Bloom (1976) identifies feedback, correctives and reinforcements (such as praise, blame, encouragement and other rewards and punishments that are used to sustain learning) as important elements of the instructional process. Feedback is considered to be one of the structuring conditions for learning, and is included alongside such variables as task presentation, sequencing, level and pacing of content and teacher expectations (Gipps & Tunstall, 1998). A major determinant of self-esteem is feedback from others, therefore children's self-evaluations are very often a reflection of significant others' evaluations, such as parents, teachers and peers. As far as academic self-esteem is concerned, teachers' evaluations are the most important, particularly in the early years of schooling. Children develop their 'self-image' in school through observing and feeling not only how the teacher interacts with them, but also how the teacher interacts with the rest of the class (Crocker & Cheeseman, 1988). The development of a positive self-concept in children is dependent upon perceiving themselves as successful, this in turn may depend on the way the child interprets the teachers' reaction to his/her performances.

According to Walls (2009) the right/wrong nature of mathematics as presented by teachers, textbooks, families and peers through social interactions, significantly contribute to student's mathematical identities and construct of themselves as a learner of mathematics. Rowland (1995) argues that a child's level of mathematical competence cannot and should not be judged by the child's offering of a 'correct answer'. Rowland suggests that when a child volunteers an answer that is not the 'expected' teacher answer, it is important to investigate and explicate the child's thinking and reasoning behind it. With reference to the linguist Lakoff, Rowland (1995) demonstrates how in oral explanations, students use "hedges" (such as about, maybe, probably, around) as "a 'shield' against being wrong" (p. 350). The rewards and privileges that come with being correct are great. Rowland (1995) observes that there is a regrettable absence of regard for the role 'uncertainty' plays in the mathematics classroom. Teachers, and in turn students, fail to recognise that being in a state of 'uncertainty' is a necessary precondition to learning and that in "the making and learning of mathematics, uncertainty is to be expected, acknowledged and explicit" (Rowland, 1995 p.328). Recent research carried out by Boaler (2013) into ability and mindset in the mathematics classroom reveals that the types of tasks chosen by teachers communicate powerful messages regarding what mathematics and knowledge is important. Tasks convey what doing mathematics is all about. By engaging in tasks, students develop ideas about the nature of mathematics and mathematics learning (Anthony & Walshaw, 2009; Hodge, Zhao, Visnovska, & Cobb, 2007). If children are assigned short, closed mathematics questions that have right or wrong answers and children are regularly getting them incorrect, it is very difficult to sustain the opinion that high achievement is possible with effort. In contrast, when tasks are open, with opportunities for learning, children can see the possibility of greater achievement and respond to these opportunities to improve (Boaler, 2013).

METHODOLOGY

The study employs a mixed methods design approach (Teddlie & Tashakkori, 2009) across eight primary school classrooms at second and fifth class level. A total of twenty four mathematics lessons were observed, while eighteen mathematics lessons were video recorded. Focus group interviews with children, child questionnaires and drawings were also employed in the data collection.

FINDINGS

Observations of mathematics lessons at second class level showed teaching of both the Measurement and Shape and Space strands of the curriculum. However, the children themselves when interviewed did not make reference to this as mathematics until prompted about the lesson and only then it was about "clocks". Another group of children categorised their mathematics learning into what they termed to be "real maths" and what was not, with importance attached to copy work.

Kim: Real maths is doing sums in our copies

Researcher: And doing 2D shapes like you were today, is that real maths?

Joey: No, no…

Researcher: Well what would you call that?

Kim: I would just call it shapes.

This narrow view of mathematics was supported when children in all the research schools were invited to draw a picture of what 'doing maths' looks like. Many of the drawings consisted of a "table, a chair and me writing down a sum." accompanied with "my maths sheet". The physicality of doing mathematics is described both in the children's own words and in their drawings:

> I am sitting on a chair and my bag is on the back of me chair and I'm doing my maths.

For older children mathematics involved carrying out exercises in computation work, "taking sums off the board", "writing them into my copy", "with my head down and like doing like rough work to try to sort it out." As children got older 'doing maths' became quickly established as a form of individual written task referred to as 'work'. In follow up interviews some teachers appeared to disassociate their teaching practices with children's views of mathematics:

> A lot of kids just find maths you know it's something that you're doing out of a copy or you are just looking at the board.

Unless they have a whole page of numbers to add or take away or something they don't see it as maths, even last week we were doing chance with dice and they said 'we didn't do maths today', I said 'yeah we did'…if it's not written down in adding and things they don't see it as maths. Even when we were doing lines and angles because they weren't specifically doing stuff in a copy, they weren't doing maths, they didn't do maths. Yeah they don't associate it being fun at all so they just think maths is boring and you just have to accept it that's they're understanding of it.

Teacher accounts reveal a lack of awareness of the messages communicated to children within their community of practice. Messages about what constitutes as important mathematics are daily reinforced across classrooms both overtly and covertly through teacher actions and words. One group of second class children welcomed the freedom of using mental strategies over pencil and paper:

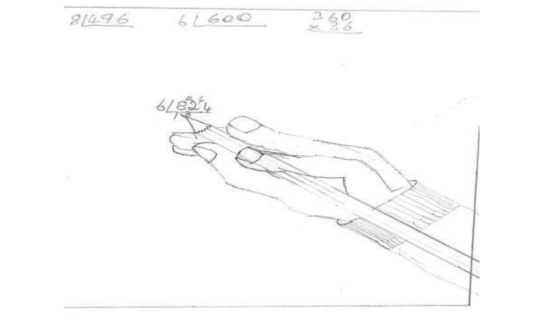
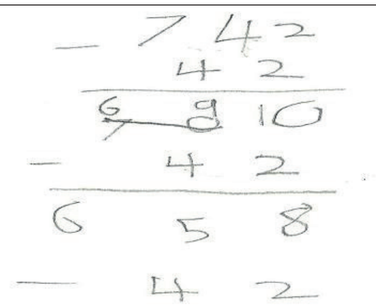
Cody: It is much more fun when you are doing them in your head.

Researcher: Why is it more fun when you do it in your head?

Kim: Because it's a hassle to write it all down.

Cody: Because writing takes up your page.

Children's descriptions and drawings were strongly supported by classroom observations of the salient role writing plays in the mathematics classroom as exampled by Figure 1 below.

Figure 1: Writing in Mathematics Copy	**Figure 2: A Child's Demonstration of Repeated Subtraction**
	

Confusion in the Mathematics Classroom

In this study fifth class children admitted to difficulties in understanding and expressed strong feelings of confusion in their accounts and drawings "I would draw meself confused doing the sums because it does be too hard". For the children in one fifth class feeling confused was a common experience shared by the group:

> Teachers ask more questions than they say answers.
>
> He doesn't really help us…he just tells us what to …like ok 'carry this on, add this number here…and we're just thinking…What?!
>
> Sometimes you don't get something and then you don't get what they are trying to explain and you don't get the other way they are trying to explain at all…
>
> You can pay attention and just not 'get' what is going on.

Feelings of being confused were not confined to fifth class alone as children in one 2nd Class shared their experience:

> This is what happens, sometimes she mixes up the questions and there was a minus sum and then she goes onto an adding sum and then she goes back to the minus sum and then she wants the answer…

The procedures and processes involved in computation caused great difficulty for many children. Some children made reference to tricks used by teachers to help in the explanation and understanding of challenging concepts such as long multiplication and the 'magic zero' "I get mixed up on long multiplication because when there are two digits, I keep just doing it like one digit and I always keep forgetting me magic zero" ;"And then the other night….it was divide 742 divided by 42 and you had to take it away and like you had to take away 17 times, say 742 take away 42." (Keith). Supplied with some paper and a pencil, Keith proceeded to write down the sum as shown in Figure 2 above.

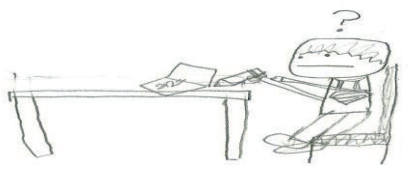
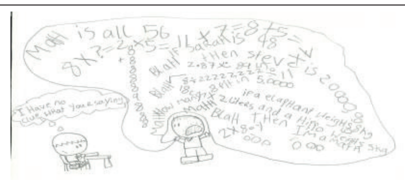
The practice of executing mathematical procedures without understanding was an experience communicated by many children. According to one fifth class child not enough time was allocated by teachers to the teaching and explaining of mathematical content that children were required to know:

> we did an assessment test and there was one thing, use your long division and I skipped it completely because I didn't know what to do…I had just forgotten how to do long division.
>
> I hate it. I think when they are teaching it, they do it for a week and then they just leave it, but I think…because now I forget long division....they should do it all the time.

The emphasis on procedure left many children grappling to understand the mathematical concepts underlying these procedures:

> the yoke where….you had to divide by the bottom, multiply by the top. ..and some of them are a bit annoying because you have to switch them back…
>
> decimal points….I can't get my head around sometimes…I understand how many places you have to go back but then when it gets into bigger numbers…I just get confused.
>
> Time…I don't get time…I just can't get my head around it… it's kind of confusing…because usually when you are carrying over…lets say, 10 minus 11, you would have to carry over something, but when it is with time, you have to bring over like a 6 and everything and leave behind this….it is confusing."

Children were left feeling disempowered and disconnected from the important mathematical processes necessary in order to succeed. Images of being 'confused' when doing mathematics appeared on a number of drawings by fifth class children.

Figure 3: Drawings of Confusion in the Mathematics Classroom	**Figure 4: Being Confused**
	

Other children mentioned processes such as 'thinking' when asked what their drawings would look like "I probably do one with my book and the table and have my pencil banging against my head thinking". "this is what I'd draw, I would have myself like that and I would be just stuck there in time, I would be like 'what the hell, what is this?'

Emotions and Attitudes towards Mathematics

Children expressed different thoughts and feelings about mathematics. Children's attitudes towards mathematics were communicated through their interviews and their drawings.

Researcher:	If I was to mention the word maths to you, what would your image of maths be?
Chelsy:	School…awful…I hate it…
Declan:	Writing.
Diana:	Maths to me would be I don't like it…
Samantha:	Writing or like a book.
Researcher:	A book like your maths book?
Samantha:	Yeah Mathemagic
Researcher:	What about you, what would come to mind?
Eddie:	It's boring.
Chelsy:	I don't really like maths but if someone said to me I would probably think 'my future'. It is kind of everything. I don't want to end up like a hobo on the street because I didn't learn my maths.
Frank:	I like doing the hard sums
Researcher:	And what are hard sums?
Frank:	Hard sums are in the hundreds and the two hundreds and the five hundreds.
Tom:	"I like the challenge of trying to get the answer, say it could be one you never learned before.".

Positive dispositions towards mathematics were represented in children's drawings with 'love hearts' and smiling, happy faces. Negative dispositions towards the subject were accompanied with words in the form of thought bubbles expressing feelings of confusion or boredom.

In a questionnaire of 164 respondents, 91% of children believed that mathematics was different to other subjects. For the children in this study mathematics was different by virtue of being 'easier' or 'harder'. For some fifth class children 'it's easy than most subjects' and for others 'you have to pay attention all the time and it's. The questionnaire found 51% of second class children always liked the work they did in mathematics class with 55% of children feeling they could do a good job on their mathematics tasks. For fifth class children only 20% admitted to always liking what they did in mathematics with 44% of children feeling they could do a good job on their mathematics tasks. Children expressed their desire for 'less writing', preference to 'work in pairs and in groups', together with the call for mathematics to be more hands on 'if we got to do experiments with water to show how litres work'. Among second class children opinion varied from 'maths is cool', 'I don't like it as much as Art, P.E, yard and Science' to 'I learn the most from maths'. 48% of second class children and 70% of fifth class children believed that to be great at mathematics one must work very hard.

The Relevance of Mathematics

“It’s the most important subject but they make it so boring.” Children across all of the eight research schools recognised the importance and relevance of mathematics to their lives both now and in the future. According to one group of fifth class children mathematics was ‘boring’ but “if they could make it interesting, kind of put it in more a story way like, this is millimetre dude or something like that” their experience of this subject would be better. However, despite some children’s rather negative experience of mathematics, its importance as a subject was recognised. Mathematics was considered important when it came to the practical, day to day aspects of life such as shopping “if you go to a shop and you don’t know maths the shopkeeper could make you pay extra” and television “if you want to watch the telly, you won’t know what number the kid’s station is on if you don’t do maths.” . For others mathematics is something that is useful when you are older and “doing something important…say you are a bank accountant or something. Mathematics was recognised by other children as playing a key role in determining their life chances as they get older “it will help you have a better future” and “so I can get in to a good college”.

CONCLUSION

For the majority of children in this study mathematics is little more than calculation and number work. Despite the Revised Primary Mathematics Curriculum 1999 advocating mathematical processes such as problem solving, developing logic and reasoning, and communicating mathematical ideas, in this study few children talked about mathematics in this way. Only two children in fifth class referred to general cognitive processes such as learning and thinking. Children’s beliefs about mathematics are determined by the mathematics they encounter, the tasks in which they encounter it and their disposition towards mathematics as a result of previous encounters. A distinctive feature of their drawings revealed that individual written work was repeatedly experienced by the children during mathematics, and what they most identified as “doing maths”. Observations of mathematics lessons, teachers’ and children’s descriptions of a typical lesson, and examination of children’s mathematics copies for evidence of frequency of written tasks, supported these claims.

REFERENCES

Alexander, R. (Ed.). (2010). *Children, their world, their education: Final report and recommendations of the Cambridge primary review*. London and New York: Routledge Taylor and Francis Group.

Barber, P., & Houssart, J. (2013). Eliciting pupil perspectives in primary mathematics: Possibilities and issues. *International Journal of Primary, Elementary and Early Years Education,* 1-15.

Barta, J., & Bremner, M. (2009). Seeing with many eyes: Connections between anthropology and mathematics. In B. Greer, S. Mukhopadhyay, A. Powell & S. Nelson-Barber (Eds.), *Culturally responsive mathematics education* (pp. 85-109). New York: Routlege.

Boaler, J. (2013). Ability and mathematics: the mindset revolution that is reshaping education. *FORUM, 55*(1), 143-152.

Cobb, P., & Hodge, L. L. (2002). A relational perspective on issues of cultural diversity and equity as they play out in the mathematics classroom. *Mathematical Thinking and Learning, 4*, 249-284.

Crocker, A.C., & Cheeseman, R.G. (1988). The ability of young children to rank themselves for academic ability. *Educational Studies, 14*(1), 105-110.

Devine, D. (2011). *Immigration and schooling in the Republic of Ireland: Making a difference?* Manchester and New York: Manchester University Press.

Gipps, C., & Tunstall, P. (1998). Effort, ability and the teacher: Young children's explanations for success and failure. *Oxford Review of Education, 24*(2), 149-165.

Grootenboer, P. (2013). The praxis of mathematics teaching: developing mathematical identities. Pedagogy, *Culture and Society, 21*(2), 321-342.

Hayes, D., Mills, M., Christie, P., & Lingard, B. (2006). *Teachers and schooling making a difference. Productive pedagogies, assessment and performance*. NSW: Allen & Unwin.

Hodge, L., Zhao, Q., Visnovska, J., & Cobb, P. (2007). What does it mean for an instructional task to be effective? In J. Watson & K. Beswick (Eds.), *Mathematics: Essential research, essential practice (Vol. 1, pp. 329-401)*. Hobart: MERGA.

Holland, D., Skinner, D., Lachicotte, W., & Cain, C. (1998*). Identity and agency in cultural worlds.* Cambridge, MA: Harvard University Press.

Lee, C. (2006). *Language for learning mathematics: Assesment for learning in practice*. Open University Press.

Rowland, T. (1995). Between the lines: The language of mathematics. In J. Anghileri (Ed.), *Children's mathematical thinking in the primary years: Perspectives on children's learning.* London: Continuum.

Walshaw, M. (2004). A powerful theory of active engagement. *For the Learning of Mathematics, 24*(3), 4-10.

Walls, F. (2009). *Mathematical subjects: Children talk about their mathematics lives*: Springer.

SCRATCH AND COMPUTATIONAL THINKING: A COMPUTER PROGRAMMING INITIATIVE IN A GIRLS PRIMARY SCHOOL

Claire Carroll and Aisling Leavy

Mary Immaculate College, Ireland

In recent years there has been an unprecedented push to improve the quality of education, and revitalise interest, in STEM. In this context, Computational Thinking has emerged as an essential twenty-first century competence, as fundamental as reading, writing and arithmetic (Wing, 2006). Mathematics has been identified as a field which can foster the development of computational thinking and the NCCA have committed to embedding computational thinking in the new primary curriculum. Educators and researchers have adopted two main approaches to teaching computational thinking: plugged activities (programming) and unplugged activities (without technology). The aim of this research is to assess what benefits, particularly in relation to computational thinking, can be gained from the use of a visual programming language, Scratch, in a girls primary school. Brennan and Resnick (2012) developed a computational thinking framework that examines three key dimensions of computational thinking: computational concepts, computational practices, and computational perspectives. Using this framework, this study examined the development of students' computational thinking skills during a ten week programming initiative. Data were collected from Project Portfolios Analysis, Design Scenarios and Participant Observation. This paper describes the findings of this research study in relation to one of the key dimensions of computational thinking: computational concepts, developed by the participants as a result of engaging in the programming initiative.

INTRODUCTION

In the face of unpredictable and unprecedented change, European and International policies have acknowledged the importance of developing education systems responsive to the demands of a knowledge-based society. For example, the eEurope 2002 Action Plan observed that 21st century schools require curricula to develop different knowledge, skills and dispositions than those required in the 20th century. Computational thinking is a problem-solving process, which originated in the field of computer science but is increasingly being recognised as an essential competency for all fields. In 2006, Jeannette Wing wrote an influential article on computational thinking, giving a 21st century perspective to the concept. She advocated for adding this new competency to every child's analytical ability, describing it as a vital ingredient of science, technology, engineering and mathematics (STEM) learning. Cuny, Snyder & Wing (2010) describe computational thinking as:

> *The thought processes involved in formulating problems and their solutions so that the solutions are represented in a form that can be effectively carried out by an information-processing agent (p. 1).*

Wing's arguments captured the attention of a broad academic community and since then computational thinking has become a catchphrase, as government education advisors and curriculum developers explore possible directions for 21st century appropriate curricula. In the

Action Plan for Education 2017, the Irish government announced their intention to reform the primary mathematics curriculum, to include computational thinking, creative thinking skills and programming. However, despite considerable international interest in integrating computational thinking to school curricula, its successful integration still faces several challenges, as identified in a report by the ET 2020 Working Group on Digital Skills and Competences (EC, 2016, p. 1):

Is CT a skill that benefits all living in an increasingly digital world?

What features characterise CT instruction?

How does the concept of CT and CT instruction relate to programming/coding, computer science and digital literacy?

How should CT be assessed?

How can teachers be prepared to successfully integrate CT into their teaching practice?

COMPUTATIONAL THINKING IN THE CURRICULUM

Introducing computational thinking to the primary school curriculum, as with the introduction of any new content, brings with it both challenges and possibilities. Teachers are required to equip themselves with both subject knowledge and pedagogical knowledge. With many countries introducing computer science as a core curriculum subject there has been a plethora of research studies, initiatives and policy reports on the topic. Inevitably, the content of curricula in each country will be different, however many of them are incorporating computational thinking. A review of computational thinking initiatives and studies reveals that educators and researchers have adopted two main approaches to teaching computational thinking in schools: plugged activities (computer programming) and unplugged activities (those that do not require the use of technology).

Unplugged Activities

Although many unplugged initiatives predate the rise to prominence of computational thinking, it has become apparent that the unplugged approach emphasises computational thinking (Bell et al., 2012). In the last five years, a small number of studies have reported on the effect of unplugged activities on the development of computational thinking in US classrooms. Many of these have looked at the development of computational thinking of students in middle and high school. There is a dearth of research on the use of unplugged activities to develop the computational thinking of primary school students. Two recent studies that have explored this approach are Faber, Wierdsma, Doornbos, van der Ven & de Vette (2017) and Brackmann, Román-González, Robles, Moreno-León, Casali & Barone (2017). Faber et al. (2017) conducted an exploratory study in twenty six schools in the Netherlands. They designed a series of six ninety-minute unplugged lessons, which were subsequently taught to students in their final year of primary school. They reported that the lessons elicited a positive reaction from both the teachers and students and suggested that the unplugged approach offered a viable alternative to programming. Brackmann et al. (2017) report on a quasi-experiment conducted in two primary schools in Spain. In each of the schools the students were divided into an experimental group who participated in the unplugged lessons and a control group who did not. They assessed the computational thinking skills of both groups of students with a pre-test and a post-test. They found that the computational thinking skills of the students who had participated in the unplugged lessons improved significantly, whereas the skills of those in the control group did not.

Plugged Activities

When computers first became available in schools, there was great enthusiasm to teach children to program. The computer was viewed as an instructional tool, which could be used for the development of higher order thinking skills. Seymour Papert was one of the early advocates of teaching children to program. His research on computers and education was based on the belief that *"children can learn to use computers in a masterful way, and that learning to use computers can change the way they learn everything else"* (Papert 1980, p.8). However, despite Papert's claims, empirical studies failed to find conclusive evidence of this (Kurland, Pea, Clement & Mawby, 1986). Perhaps because of this, the use of computers in schools shifted away from the computer science approach towards a more computer literacy based approach. However, in recent years there has been some resurgence in the use of programming to develop higher order thinking skills in schools. This resurgence has been facilitated by the availability of numerous 'low floor' (easy to learn) programming languages have been developed specifically for novice programmers e.g. Scratch, Alice, Kodu, Toontalk and Stagecast Creator.

In a recent paper funded by the NCCA, Millwood, Bresnihan, Walsh and Hooper (2018) suggested a definition of Computational Thinking for use in the Irish education system:

> "competence in problem solving & design to create useful solutions, informed by the possibilities that Computing offers" (p.8).

This definition suggests that students would exhibit competence in computational thinking by creating with technology. Hence, in this study the plugged approach was chosen as the preferred approach to teaching computational thinking.

Scratch Programming

The development of visual programming languages has provided more accessible ways for younger children to learn programming concepts (Koh, Basawapatna, Bennett, & Repenning, 2010). These languages often require programmers to drag-and-drop icons rather than type code. One such example is the Scratch programming language. Programs are written in Scratch by fitting 'blocks' together like pieces of a jigsaw puzzle (Wilson & Moffat, 2012). The blocks will only fit together in a certain way, so it eliminates the possibility of syntax errors. Even though Scratch is considered a 'low floor' programming language, it is also considered to be a 'high ceiling' (facilitates the creation of complex programs). It is also freely available, provides instant visual feedback and has an offline version, which is essential in areas with poor broadband connection. Therefore Scratch was chosen as the programming language for this study.

Assessing Computational Thinking

The rise in prominence of Scratch, as a key instrument in the development of computational thinking in educational settings, has led to the need for an assessment strategy to address the evaluation of computational thinking of Scratch projects. Brennan and Resnick (2012) developed a computational thinking framework that examines three key dimensions of computational thinking: computational concepts, computational practices, and computational perspectives. Table 1 modified from Lye and Koh (2014) provides a brief summary of the key ideas of each dimension of computational thinking. For the purpose of this paper we will focus on the computational concepts developed by the participants as a result of engaging in the programming initiative.

Table 1: Key Dimensions of Computational Thinking (Lye & Koh, 2014 p. 53).

Dimension	Description	Examples
Computational concepts	Concepts that programmer(s) use	Variables Loops
Computational practices	Problem-solving practices that occurs in the process of programming	Being Incremental and Iterative, Testing and Debugging, Reusing and Remixing Abstracting and Modularizing
Computational perspectives	Students' understanding of themselves, their relationships to others, and the technological world around them.	Expressing and questioning about the technology world

New tools have been developed to assist educators in their assessment of computational thinking in such projects. One such tool is Dr Scratch, a free Web application powered by Hairball, which allows educators to analyse scratch projects by detecting bugs, verifying for the presence of programming constructs and assigning a computational thinking score.

OVERVIEW OF THE PROGRAMMING INITIATIVE

The method of enquiry employed in this study was Case Study Research. Participants were selected using convenience sampling. There were 90 participants from three classes in an urban, girl's, primary school. There were thirty-two third class students, twenty-eight fifth class students and thirty sixth class students. The majority of students had no prior programming experience. They worked in pairs or threes sharing one computer. The initiative took place over a ten week period, with one hour sessions each week, using the visual programming language Scratch. As outlined in Table 2, the first five sessions focussed on helping the children learn the basic functions while creating their first animations.

Table 2: Outline of the activities week by week

Week	Learning Objectives	Activity
1	Control, Movement and Coordinates.	Create an animation incorporating movement and images
2	Sequencing, Time, Iteration and Using Sounds.	Alternate sprite costumes, incorporating time and motion. Import, create and record sounds to use in their project
3	Motion, Direction, Rotation, Sensing and Broadcasting.	Create a joke or other short animation involving at least two characters.
4	Using Conditional Statements	Create a race animation or a pong game.
5	Variables, Time and Sensing.	Create a game which uses variables to record lives and score.

6-10	All previously learned skills.	Plan, create and edit your own Scratch project.

In the following weeks there was no explicit programming instruction instead the sessions were run based on a 'learning on demand' model (Kafai & Ching, 2001) as pupils created their own animations. Various data collection methods were adopted in order to obtain the most comprehensive picture possible of the phenomena being studied. These included Project Portfolio Analysis, Artefact-Based Interviews, Design Scenarios (Brennan & Resnick, 2012) and Participant Observation. Project Portfolio were analysed using Dr Scratch to assign a computational thinking score determined by the competence demonstrated by the students in the seven computational concepts: abstraction and decomposition, logic, data representation, parallelism, synchronisation, flow control and user interactivity (Moreno-León & Robles, 2015). These concepts are explained further in Table 3.

Table 3: Definitions of Computational Thinking concepts (http://www.drscratch.org/learn/Abstraction/)

Concept	Definition
Abstraction and problem decomposition	Breaking a problem into smaller parts that are easier to understand, program and debug.
Logic	Instructions related to logical thinking that create a more dynamic program, so they behave differently depending on the situation.
Data Representation	The set of information about the characters in e.g, the position of each character, the direction it is pointing, size, etc. In addition data such as the level, elapsed time, the rating, the lives, the rewards collected.
Parallelism	Allows several things to occur simultaneously.
Synchronisation	Allow characters to organize things happen in the order we want
Flow Control	Control the behaviour of characters, e.g. certain blocks that are repeated a number of times or until a situation arises.
User Interactivity	The person who running the program can perform actions that provoke new situations in the project.

Each concept is given a score between 0-3 depending on the competence demonstrated in the project. The evaluation of these competences is based on the rules in Table 4.

Table 4: Competence Level for computational thinking concepts (Moreno-León & Robles, 2015 p. 6).

Computational Thinking Concept	**Competence Level**			
	Null (0)	**Basic (1 point)**	**Developing (2 points)**	**Proficiency (3 points)**
Abstraction and problem decomposition	-	More than one script and more than one sprite	Definition of blocks	Use of clones
Logic Thinking	-	If	If else	Logic operations
Data Representation	-	Modifiers of sprites properties	Operations on variables	Operations on lists
Parallelism	-	Two scripts on green flag	Two scripts on key pressed, two scripts on sprite clicked on the same sprite	Two scripts on when I receive message, create clone, two scripts when %s is > %s, two scripts on when backdrop change to
Synchronisation	-	Wait	Broadcast, when I receive message, stop all, stop program, stop programs sprite	Wait until, when backdrop change to, broadcast and wait
Flow Control	-	Sequence of blocks	Repeat, forever	Repeat until
User Interactivity	-	Green flag	Key pressed, sprite clicked, ask and wait,	When %s is >%s, video,

			mouse blocks	audio

The overall computational thinking score is calculated by adding up the partial scores of each computational thinking concept. So, the computational score ranges from 0 to 21 points. Projects with up to 7 points are considered to prove a basic level of computational thinking, while projects between 8 and 14 points are evaluated as developing, and projects with more than 15 points are marked as master.

RESULTS AND DISCUSSIONS

Twenty five of participants' final projects were analysed. There were three projects which the Dr Scratch application was not able to analyse. The projects ranged from fruit drop and hide and seek games to animated stories of classics such as Alice in Wonderland, and original animations. All of the projects analysed were evaluated as developing. The average computational thinking score was 10.2 and the median score was 10. The highest scores were achieved in the *synchronisation* and *parallelism* concepts. 84% of the projects scored three out of three in both these concepts. In programming, a thread is like a mini-program within a program that can run at the same time as other programs. A program with multiple threads can do multiple things at once (Parallelism). When programming either games or stories it is useful to separate threads for conceptually distinct tasks. For example, in the ping pong game that Megan and Sylvia designed they spent some time getting the ball to start moving and the clock to start counting down from two minutes at the start of their game. To make their stories as real as possible several groups who were animating stories wanted their programs to do two things at once. For example, in the Alice in Wonderland story animated by Louise and Bernadette, the pupils wanted their character to hide and scream at the same time to create the perception that Alice was falling down the rabbit hole. Including both parallelism and synchronisation in their programs required the pupils to use algorithmic thinking to design a series of instructions to complete a particular task.

Lower scores were achieved in *flow control* in the projects. Although all projects scored at least one point (which required the creation of a sequence of blocks), only 40% of the projects scored more than one out of three. As computers are only able to execute instructions one at a time, there are times when the order of those instructions is important. Sequencing is the specific order in which instructions are performed in an algorithm. Scoring more than one on the flow control concept required the use of iteration. Iteration is the repetition of a sequence of commands (known as a loop). A loop allows for multiple executions of a command without having to create separate code for each execution. Iteration can be either count-controlled or condition-controlled. Count-controlled loops repeat the same steps a specific number of times, regardless of the outcome. The control blocks 'repeat' and 'forever' are examples of count-controlled. A condition-controlled loop will keep repeating the steps over and over, until it gets a specific result. The control block 'repeat until' is an example of a condition-controlled loop. None of the projects contained condition-controlled loops, which is a more advanced programming construct. The projects that contained the count-controlled loops mostly used it for movement in games or stories. The fruit drop game that Sophie and Bethany designed and the pong game that Megan and Sylvia designed both required the sprite (fruit or ball) to move continuously during the game so the forever block was an important construct in their games.

Data Representation (see table 5) was underutilised in student games and animations. Data such as levels, elapsed time, ratings, lives and rewards collected are all examples of more complex data representation. All of the final projects scored one for this concept. In most cases this meant that they tended to assign information about the sprite at the beginning of the game or animation but not make changes to it during the course of the project.

User interactivity was low in all the projects. Again no project scored more than one out of three. The ‘ask and wait’ block allows us to add user interaction to our programs. The question appears in a voice balloon on the screen. Then the program waits as the user types in a response, or until the Enter key is pressed or the check mark is clicked. The program stores the response (user) input in a temporary variable called ‘Answer’. We can then use our answer in the conditional blocks to determine how the program responds. In most cases this was a personal choice, as pupils did not necessarily require user input to determine how the program ran. However, this was a concept which the pupils found difficult when covered in the early sessions.

The lowest score was achieved in the *logic concept*. Only two of the projects achieved any score in the logic concept, and in both cases this was a score of one out of three. This concept is assessed by checking for the presence of certain constructs which cause the program to behave differently depending on certain conditions. These constructs include if, if else and the logical operations; and, or and not. In stories these constructs are not important as stories usually have a linear structure. However, in games these are essential to perform different actions depending on the condition. For example, if the time equals 0, say game over or if touching fruit change score by 1.

LIMITATIONS AND FURTHER RESEARCH

At this stage in the research the researchers are engaged in preliminary data analysis. This paper outlines initial findings in respect of one of the three computational thinking dimensions, computational concepts. Findings in relation to the other key dimensions of computational thinking will help to give a clearer picture of computational thinking development. There are several limitations to using Dr Scratch to analyse computational thinking and it is hoped that findings from the other data sources will help to alleviate these limitations. These limitations include:

- The use of a particular block or groups of blocks is not enough to confirm fluency on a certain CT concept.
- The examination of a single project might not be as accurate or complete as the analysis of the collection of projects of the user.
- Some key CT competences cannot be measured by analysing the code of a project, such as the debugging or remixing skills.

REFERENCES

Bell, T., Rosamond, F., & Casey, N. (2012). Computer science unplugged and related projects in math and computer science popularization. In H. L. Bodlaender, R. Downey, F. V. Fomin, & D. Marx (Eds.*), The multivariate algorithmic revolution and beyond* (pp. 398-456). Berlin, Heidelberg: Springer.

Brackmann, C. P., Román-González, M., Robles, G., Moreno-León, J., Casali, A., & Barone, D. (2017). Development of computational thinking skills through unplugged activities in primary school. In Proceedings of *The 12th Workshop on Primary and Secondary Computing Education*, (pp. 65-72). New York: ACM.

Brennan, K., & Resnick, M. (2012). New frameworks for studying and assessing the development of computational thinking. Retrieved from https://web.media.mit.edu/~kbrennan/files/Brennan_Resnick_AERA2012_CT.pdf

Cuny, J., Snyder, L., & Wing, J.M. (2010). Demystifying computational thinking for non-computer scientists. Retrieved from http://www.cs.cmu.edu/~CompThink/resources/TheLinkWing.pdf

Faber, H. H., Wierdsma, M. D., Doornbos, R. P., van der Ven, J. S., & de Vette, K. (2017). Teaching computational thinking to primary school students via unplugged programming lessons. *Journal of the European Teacher Education Network*, *12*, 13-24.

Kafai, Y. B., & Ching, C. C. (2001). Affordances of collaborative software design planning for elementary students' science talk. *The Journal of the Learning Sciences*, *10*(3), 323-363.

Koh, K. H., Basawapatna, A., Bennett, V., & Repenning, A. (2010). Towards the automatic recognition of computational thinking for adaptive visual language learning. *IEEE Symposium on Visual Languages and Human-Centric Computing*, (pp. 59-66). USA: IEEE.

Kurland, D. M., Pea, R., Clement, C., & Mawby, R. (1986). A study of the development of programming ability and thinking skills in high school students. *Journal of Educational Computing Research*, *2*(4), 429-458.

Lye, S. Y. & Koh, J. H. L. (2014). Review on teaching and learning of computational thinking through programming: What is next for K-12? *Computers in Human Behavior*, *41*, 51-61.

Millwood, R., Bresnihan, N., Walsh, D. & Hooper, J. (2018). Review of literature on computational thinking. Retrieved from https://www.ncca.ie/media/3557/primary-coding_review-of-literature-on-computational-thinking.pdf

Moreno-León, J., & Robles, G. (2015). Analyze your Scratch projects with Dr. Scratch and assess your computational thinking skills. Retrieved from http://jemole.me/replication/2015scratch/InferCT.pdf

Papert, S. (1980). *Mindstorms: Children, computers, and powerful ideas.* New York: Basic Books.

Wilson, A. & Moffat, D. C. (2012). Evaluating Scratch to introduce younger children to programming. Retrieved from http://scratched.gse.harvard.edu/sites/default/files/wilson-moffat-ppig2010-final.pdf

Wing, J. (2006) Computational thinking. *Communications of the ACM*, *49*(3), 33-35.

SUPPORTING MATHEMATICAL LITERACY IN POST-PRIMARY SCHOOLING: ISSUES TO CONSIDER WHEN USING A CO-TEACHING APPROACH

Audrey Carty and Ann Marie Farrell

School of Inclusive and Special Education, Institute of Education, Dublin City University

This paper reports on the evaluation of co-teaching as a model for supporting mathematical learning from the student perspective. Research instruments included student surveys and focus group interview, along with semi-structured observations of lessons. The findings indicated consensus from students, both with and without SEN, that co-teaching was a favourable way of delivering mathematics lessons. Benefits included increased opportunities to get a teacher's attention; being more comfortable asking questions; greater range of learning experiences; and, the availability of assistance in a discreet way. These benefits afforded by the use of co-teaching provide learning contexts for developing mathematical literacy skills.

INTRODUCTION

This paper reports on the evaluation of co-teaching as an approach to support all students in the mathematics classroom, including those with special educational needs (SEN), and extrapolates implications for supporting the development of mathematical literacy skills. The study evaluated the co-teaching initiative of the special education teacher (SET) and the mathematics teacher from the perspectives of students and teachers involved; this paper reports on the student perspective only and centres around the following research question: What are the viewpoints, both positive and negative, of post-primary school students in relation to their co-taught mathematics lessons?

LITERATURE REVIEW

Mathematical literacy is defined as "formulating, employing and interpreting mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena" (Organisation for Economic Co-operation and Development [OECD], 2017). This requires students to grasp the mathematical concept and to be able to express their understanding clearly. Further, mathematical literacy is associated with social practice and culture (Jablonka, 2003) and, while this is often considered on the macro-level of society generally, it might also be considered at the micro-level of the classroom wherein culture exists and is influenced by teaching strategies and approaches used therein which, in turn, influence the social interactions of both teachers and students. Co-teaching offers a model of supporting student learning by increasing the level of interaction between teacher and student as well as between students themselves and therefore, offers a context for enhancing the development of mathematical literacy skills in an inclusive manner whereby teachers can anticipate and respond to individual differences in the context of everyone (Florian, 2008).

There are six models of co-teaching widely reported in the literature (Friend, Cook, Hurley-Chamberlain & Shamberger, 2010; Moorehead & Grillo, 2013), namely *one teach, one assist;*

station teaching; parallel teaching; alternative teaching; teaming; and *one teach, one observe* (see Figure 1). Research confirms *one teach, one assist* as the most dominant model in practice (Friend et al., 2010).

Figure 1: Diagrammatic Representation of Co-Teaching

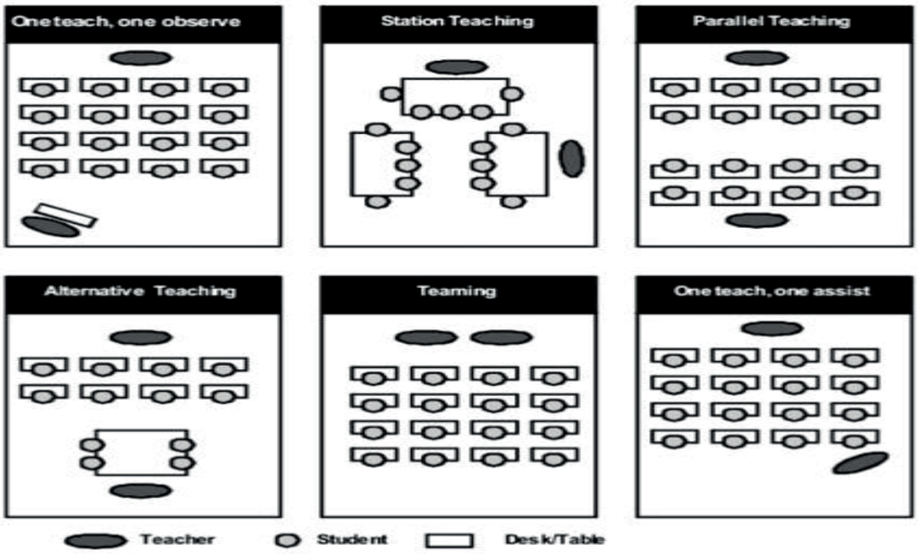


Image from Friend and Bursuck (2009), as cited in Friend et al., 2010, p.12.

It is accepted that the research base surrounding co-teaching in mathematics is extremely limited (Magiera & Zigmond, 2005), with a review of special education and mathematics literature concluding that only one fifth of empirical studies concerned students at second level, while none focused on co-teaching (Van Garderen, Scheuermann, Jackson & Hampton, 2008). Despite this dearth of primary research, some insight into how co-teaching can theoretically be utilised successfully in mathematics classrooms is available. For instance, Moorehead and Grillo (2013) outlined how stations can be arranged for re-teaching, independent practise and problem-solving activities, thus giving opportunities for development of mathematical literacy competencies such as reasoning, argumentation and mathematical communication (Rizki & Priatna, 2018). This use of stations may also facilitate both teachers having a strong voice in the classroom, all the while promoting superior literacy development for students.

Successful strategies for the teaching of mathematics to students with SEN include explicit instruction (Doabler & Fien, 2013) and scaffolding (Bakker, Smit, & Wegerif, 2015). In addition, comprehension can be increased if students are comfortable with mathematical vocabulary, which can be taught by stimulating prior knowledge, repetition and differentiated instruction (Riccomini, Smith, Hughes, & Fries, 2015). Co-teaching may be an effective way of incorporating these strategies into lessons to support the mathematical literacy of all students. The presence of two teachers also maximises the opportunity to assess for learning to ascertain where a student is at in their mathematical understanding informing the scaffolding and extension of that learning. As such, the combination of subject teacher and special education teachers' expertise may develop their vision of practice, knowledge of students and content, as well as their repertoire of tools and practices, thus effectively supporting students during mathematical literacy during lessons (Ghousseini & Herbst, 2016). Of course, teachers need to question whether the second adult present is actually adding value to the lesson. This is particularly relevant at post-primary level, when subject matter is advanced, and special education teachers may not be mathematics specialists. Research suggests that special education teachers may need to become content specialists to ensure their preparedness to work in mathematics classrooms at this level (Murawski & Bernhardt, 2015). As the importance of parity between co-teachers is a recurring theme throughout the literature (Cook & Friend, 1995; Friend, 2008), this is an important factor to be borne in mind when allocating pairs to co-teach mathematics at post-primary level.

METHODOLOGY

The main participants in this study were a class group of 26 students of mathematics, aged 12 or 13 years, in the first year of a mainstream, urban, post-primary school, along with their two co-teachers, both mathematics specialists and one of whom was in the role of SET. Two students had special educational needs. This study comprised three phases, during which data were collected from students, teachers and an independent observer, over the duration of one academic year. This paper focuses on data collected from the student cohort; evaluative data collected from the teacher perspective is reported elsewhere (Carty & Farrell, 2018).

An illuminative evaluation approach (Parlett & Hamilton, 1972) was used to frame the first phase. Using a questionnaire, data were obtained regarding the students' perspectives on the existing co-teaching practice, where co-teaching had been in place for one term using primarily a '*one-lead, one-assist'* model. This illuminated students' perceptions of the positive aspects, as well as the elements of the class they found challenging. During an intervention period of seven weeks (32 class periods), five of the six models of co-teaching were used in the class (*one-teach, one-observe* was not used). Each model was used multiple times so students became familiar with the associated classroom routines. In ten of the lessons, two or three of the models were utilised. The post-intervention framework was evaluative in nature. Students were surveyed again with a focus on eliciting the impact of each co-teaching model from their perspective. In addition, four students took part in a focus group interview.

A grounded theory and content analysis were undertaken. The focus group interview was transcribed and coded. Similarly, responses to student questionnaires were coded ensuring missing data was considered. For each question, variables were defined and labelled. The use of SPSS facilitated interrogation of this data to include frequencies, measures of central tendency and investigation of statistical significance. The data collection instruments were employed to generate a range of data across all student participants, with the focus group interview giving a voice to the less literate students in the class.

RESULTS AND DISCUSSION

After one term of co-teaching, the pre-intervention time period, students were invited to fill in a questionnaire about their experiences in their mathematics class. There was a response rate of 100% (N=26). Students were asked to rate various facets of their experience on a Likert scale, as well as being presented with three open-ended questions, allowing them to elaborate on the aspects of their lessons they enjoyed or those they found challenging.

Students' overall perceptions of existing co-teaching practice (pre-intervention)

It is notable that 24 students either agreed or strongly agreed with the statement that they liked their co-taught class. In addition, 25 students agreed or strongly agreed they could quickly get a teacher's attention when they needed to, while 21 indicated being comfortable asking questions. On the other hand, 8 students agreed they felt distracted at times, with another 8 students undecided on this question. Analysis of the three open ended questions showed the main benefit of co-teaching from the students' perspectives was the availability of help at all times (n=18). The option of asking for help discreetly (n=7) and the availability of a second teacher if there was a difficulty understanding the first one (n=7) were also noted by students. When asked for the main drawbacks, the most popular responses included difficulty with different styles of teaching (n=12), never having a free period in the subject (n=12) and no drawbacks (n=6). Students were also asked to identify if there was anything that would help in their learning of mathematics. More use of *station teaching* (n=8) and the incorporation of more fun activities (n=8) were the most popular responses, along with less homework (n=4) and more use of technology (n=4). Overall, students were very positive about the class, with the majority (n=21) indicating they would choose to be in a co-taught class again. Following analysis of the data from the pre-intervention stage, and in consultation with the class teacher, areas for improvement and refinement during the intervention phase were identified. These centred around development of students' mathematical literacy by utilising a much wider range of co-teaching models in class, as well as increasing the differentiation for all students through the use of technology and active learning methodologies.

Students' perceptions of the usefulness of each co-teaching model (post-intervention)

In analysing the extent to which students were becoming more mathematically literate, the students were happy to raise their hand and ask for assistance from both teachers equally, even during lessons where '*one teach, one assist'* was the model in use. Focus group interviewing reiterated that three out of four students said it did not matter to them which teacher led the class and which assisted the students, illustrating the parity of the teachers in the eyes of the students. The main advantage of co-teaching, and in particular '*one teach, one assist'*, from the students' perspectives was still the availability of help without having to disturb the class, reported in half (n=13) of students' questionnaires.

Station teaching was implemented in the classroom for four full and six partial lessons during the intervention phase. Stations were used for re-teaching, independent practise and problem-solving activities (Moorehead & Grillo, 2013). Analysis of student questionnaires revealed 23 positive statements relating to this model including freedom to choose which station to work at, effective for revision of topics and a feeling of independence during these classes. For instance, one student indicated that *"you could focus on things you were unsure of, and not do things you already knew"*, echoing literature reporting the advantages associated with students taking responsibility for their decisions (Murdock, Finneran & Theve, 2015), and the importance of post-primary students not being subjected to a repeat of sixth class mathematics (O'Meara, Prendergast, Cantley, Harbison & O'Hara, 2019). Only nine negative statements were made by students about station teaching, of which the most common related to not getting to all stations and getting distracted at a station.

Fourteen students made positive statements about *Teaming*, while eight made negative assertions. Points in favour of the model developing mathematical literacy in students included it being an effective way of facilitating students to use appropriate tools strategically and look for, and express patterns in repeated reasoning (Hillman, 2014). Some students reported that concepts were easier to understand and made more interesting if both teachers explained them. Two students outlined that being shown multiple methods of solving a problem was confusing. Both teachers busy at the board, with none available for individual assistance, was also perceived by a minority of students as another drawback to this model. The major advantage of *teaming* is the importance of being able to approach a problem in a variety of ways for examination purposes (Jang, 2006). However, none of the students in this study identified this as a helpful aspect of the *teaming* model of co-teaching. Perhaps their young age, combined with inexperience of formal examinations meant the students in this study could, as yet, not appreciate the benefits of multiple approaches to problem solving.

Alternative Teaching represented the biggest talking point. Responses to the student questionnaire reveal 14 positive statements and 13 negative relating to this model. On closer inspection, it was noted that the responses were linked to whether a student was in the smaller or larger group. Students in the smaller group report it being an excellent way of gaining help following an absence, targeting the people who need the support and availability of assistance without disrupting the rest of the class. Students who found the model unhelpful report never being in the small group, difficulty getting a teacher's attention in the larger group and feeling distracted due to curiosity about what the smaller group were doing. Despite the teachers' concerns surrounding stigmatisation of the students by including them in the smaller group, both students with SEN chose it as their favourite model of co-teaching. The intense and individual instruction available in the small group appealed to the students. It is important to note that the teachers rotated the roles of working with small and large groups, both adding to their parity in the classroom and lessening the stigma associated with receiving additional support.

All participants in the study agreed that *Parallel Teaching* only worked if the two groups did not share the same physical space. During the intervention period, this model was trialled on five occasions, two of which were in the same physical space. For three of the classes, one teacher moved to another location, bringing half of the students. The main benefits of this model for students were that the smaller class size meant it was easier to concentrate and get a teacher's attention. The main drawback identified by students in relation to this model was the noise levels when both groups shared the same physical space, a finding reflected in the limited literature on this model (Cook & Friend, 1995).

Having examined the literature on co-teaching, it is evident there is a requirement for research to focus on specific models (Gurgur & Uzzuner, 2011), which this study has endeavoured to do. From analysis of the students' questionnaires following the intervention in total 125 positive statements were made regarding co-teaching, compared with 46 negative statements, of which, some would be viewed as positives, mirroring findings in the literature (Dieker, 2001; Wilson & Michaels, 2006). For instance, constant monitoring, pressure to stay on task and never getting free periods, though perceived negatively by students, could be positive in terms of developing mathematical literacy.

Several students praised the increased use of digital resources in the classroom. This use of technology, a key competency for mathematical literacy (Rizki & Priatna, 2018) resulted due to collaboration between teachers and was a direct consequence of co-teaching.

CONCLUSIONS

This study, although small-scale, indicates that co-teaching enhances student engagement and participation. It increases opportunities for student-teacher interactions and broadens the range of teaching strategies that can be implemented in a lesson. Many of the benefits of having two teachers as perceived by students provide them with increased exposure to experiences and skill development that enhance their mathematical literacy. For instance, opportunities to self-select the difficulty level of the mathematical task that they work on during station teaching reveal the willingness of students to push and challenge themselves, rather than choosing tasks that they could manage with ease, all the while progressing their mathematical skills. The use of stations also provided students with feelings of independence, which made their classes more enjoyable. As few people generally are willing to view the world through a mathematical lens, and many experience maths anxiety which reduces their willingness to engage in mathematics, let alone enjoy it (Turner, 2016), these findings indicate students show a positive disposition towards mathematics following the support and scaffolding co-teaching affords them.

Both students with SEN had high participation levels during lessons. One of these, who referred to the high levels of monitoring during mathematics as a drawback, conceded it was better for her overall. The other student, encouraged by the competitive aspect, thoroughly enjoyed working at her own level on digital challenges. As both students did not exhibit high levels of self-efficacy pre-intervention, these represented very positive outcomes in terms of developing their mathematical literacy. Considering the social aspect of mathematical literacy (Jablonka, 2003), the fact that students with SEN feel comfortable in the mainstream class is important in providing a context for the enhancement of their mathematical literacy skills in an inclusive setting.

Co-teaching is not firmly established in post-primary schools leading to a dearth of literature, particularly in the Irish context and especially in mathematics classrooms. Utilisation of all models provides teachers with tools to reach all learners and enhances the student experience, which leads to improved outcomes in terms of developing key mathematical competencies and literacy skills. There are implications also for teacher educators who are in the position of modelling the very teaching strategies they wish to develop in their student teachers (Hallett, 2010). Using the practice of co-teaching in their own instruction allows student teachers to experience the approach themselves (Farrell & Logan, 2018; Logan & Farrell, 2018) which may influence their pedagogical decisions and practices when qualified in line with policy expectations (e.g. Department of Education and Skills [DES], 2017).

REFERENCES

Bakker, A., Smit, J., & Wegerif, R. (2015). Scaffolding and dialogic teaching in mathematics education: Introduction and review. *ZDM*, *47*(7), 1047-1065.

Carty, A., & Farrell, A.M. (2018). Co-teaching in a mainstream post-primary mathematics classroom: an evaluation of models of co-teaching from the perspective of the teachers. *Support for Learning, 33*(2), pp. 101-121.

Cook, L., & Friend, M. (1995). Co-teaching: Guidelines for creating effective practices. *Focus on Exceptional Children, 28*(3), 1-16.

Department of Education and Skills. (2017*). Circular 0014/2017: Special Education Teaching Allocation.* Dublin: Author.

Dieker, L. (2001). What are the characteristics of 'effective' middle and high school co-taught teams for students with disabilities? *Preventing School Failure, 46*(1), 14-24.

Doabler, C.T., & Fien, H. (2013). Explicit mathematics instruction: What teachers can do for teaching students with mathematics difficulties. *Intervention in School and Clinic*, *48*(5), 276-285.

Farrell, A.M., & Logan, A. (2018). Increasing engagement and participation in a large, third-level class setting using co-teaching. In J. Domenesch, P. Merello, E. de la Poza & D. Blazquez (Eds.), *HEAd 2019 Conference: 4th International Conference on Higher Education Advances,* (pp. 1393-1402). Valencia: Universitat Politècnica de València DOI: http://dx.doi.org/10.4995/HEAd18.2018.8209

Florian, L. (2008). Special or inclusive education: Future trends. *British Journal of Special Education, 35*(4), pp. 202-208.

Friend, M. (2008). Co-teaching: A simple solution that isn't so simple after all. *Journal of Curriculum and Instruction, 2*(2), 9-19.

Friend, M., Cook, L., Hurley-Chamberlain, D., & Shamberger, C. (2010). Co-teaching: An illustration of the complexity of collaboration in special education. *Journal of Educational and Psychological Consultation, 20*, 9-27.

Ghousseini, H., & Herbst, P. (2016). Pedagogies of practice and opportunities to learn about classroom mathematics discussions. *Journal of Mathematics Teacher Education*, *19*(1), 79-103.

Gurgur, H., & Uzuner, Y. (2011). Examining the implementation of two co-teaching models: Team teaching and station teaching. *International Journal of Inclusive Education, 15*(6), 589-610.

Hallett, F. (2010). Do we practice what we preach? An examination of the pedagogical beliefs of teacher educators. Teaching in Higher Education, 15(4). 435- 448.

Hillman, A. M. (2014). A literature review on disciplinary literacy: How do secondary teachers apprentice students into mathematical literacy? *Journal of Adolescent & Adult Literacy, 57*(5), 397-406. doi:10.1002/jaal.256

Jablonka, E. (2003). Mathematical literacy. In A.J. Bishop, M.A. Clements, C. Keitel, J. Kilpatrick & F.K.S. Leung (Eds.). *Second International Handbook of Mathematics Education* (pp. 75-102). Dordrecht: Kluwer Academic Publishers.

Jang, S.J. (2006). Research on the effects of team teaching upon two secondary school teachers. *Educational Research, 48*(2), 177-194.

Logan, A., & Farrell, A.M. (2018). Co-teaching in a large class in higher education: Working collaboratively in support of engaging pedagogy. *ICEP 2018: 11th International Conference on Engaging Pedagogy: Voice of the Educator.* Retrieved from http://icep.ie/paper-template/?pid=231

Magiera K., & Zigmond, N. (2005). Co-teaching in middle school classrooms under routine conditions: Does the instructional experience differ for students with disabilities in co-taught and solo-taught classrooms. *Learning Disabilities Research and Practice, 20*(2), 79-85.

Moorehead, T., & Grillo, K. (2013). Celebrating the reality of inclusive stem education. *Teaching Exceptional Children, 45*(4), 50-57.

Murawski, W., & Bernhardt, P. (2015). An administrator's guide to co-teaching. *Educational Leadership, 73*(4), 30-34.

Murdock, L., Finneran, D., & Theve, K. (2015). Co-teaching to reach every learner. *Educational Leadership, 73*(4), 42-47.

Organisation for Economic Co-operation and Development (OECD). (2017). *PISA development brief 9: How does PISA for development measure mathematical literacy?* Retrieved 27 May 2019 from https://www.oecd.org/pisa/aboutpisa/9-How-PISA-D-measures-math-literacy.pdf

O'Meara, N., Prendergast, M., Cantley, I., Harbison, L., & O'Hara, C. (2019). Teachers' self-perceptions of mathematical knowledge for teaching at the transition between primary and post-primary school. *International Journal of Mathematical Education in Science and Technology,* 1-23. doi:10.1080/0020739X.2019.1589004

Parlett, M., & Hamilton, D. (1972). *Evaluation as illumination: A new approach to the study of innovatory programs.* Paper presented at the Centre for Research in the Educational Sciences, University of Edinburgh, Scotland.

Riccomini, P.J., Smith, G.W., Hughes, E.M., & Fries, K.M. (2015). The Language of mathematics: The importance of teaching and learning mathematical vocabulary. *Reading & Writing Quarterly*, *31*(3), 235-252.

Rizki, L. M., & Priatna, N. (2019). Mathematical literacy as the 21st century skill. *Journal of Physics: Conference Series, 1157*, 42088. doi:10.1088/1742-6596/1157/4/042088

Turner, R. (2016). Lessons from PISA 2012 about mathematical literacy: An illustrated essay. *Pna, 10*(2), 77-94.

Van Garderen, D., Scheuermann, A., Jackson, C., & Hampton, D. (2008). Supporting the collaboration of special educators and general educators to teach students who struggle with mathematics: An overview of the research. *Psychology in the Schools, 46*(1), 56-78.

Wilson, G.L, & Michaels, C.A. (2006). General and special education students' perceptions of co-teaching: Implications for secondary-level literacy instruction. *Reading & Writing Quarterly: Overcoming Learning Difficulties, 22*(3), 205-225.

BEING ABLE TO DO MATHS BUT YET FEELING KIND OF FREE: USING THE FLAGWAY GAME TO LEARN MATHEMATICS

Majella Dempsey and Ann O'Shea

Education Department and Department of Mathematics and Statistics, Maynooth University.

Over a two-year period in 2016 and 2017 a team led by Bob Moses worked with teachers in Ireland on a project called The Algebra Project. This paper reports on the implementation of the Flagway Game in two primary schools in Ireland as part of this initiative. Data from teacher interviews and student focus groups are analysed using the theoretical framework of Engeström's activity theory (1987). The findings show that both the physical and mental tools developed by Moses and colleagues do function to develop mathematical thinking and improve enjoyment in learning mathematics. Challenges exist in the form of rules that mitigate against devoting the time needed for this kind of engagement and physical infrastructure to support social learning through physical activity.

INTRODUCTION

This paper reports on an evaluation of the implementation of a teaching and learning curriculum initiative called The Algebra Project in Ireland[1] at primary school level. The Algebra Project (AP) was founded in the USA by Bob Moses in the 1980's and has developed its own methods and curricular materials to improve students' algebraic thinking, giving much attention to the professional development of teachers, community and youth workers (Moses and Cobb, 2001). The initiative reported on here, took place in 2016 and continued into 2017. The aim of the project was to introduce the curriculum and pedagogy of the US AP to Irish teachers and students in order to develop mathematical teaching and learning. It was hoped that exposure to the evidence-based proven methods of the AP curriculum would have a positive impact on mathematics education in Ireland and on students' attitude towards mathematics and their belief that they can do mathematics. This paper reports on the implementation of an AP module called the Flagway Game in two primary schools as part of the project. Data from student focus groups and teacher interviews are presented.

LITERATURE REVIEW

The AP set out to help students in disadvantaged communities in the USA to develop their mathematical understanding so that they could progress to university level education (Moses and Cobb, 2001). The methods employed by the AP are based on experiential learning models such as those of Dewey (1938), Piaget (1952) and Kolb (2015). Moses was also heavily influenced by the work of Quine (1992) who wrote about the development of knowledge and understanding from physical experiences and in particular how theoretical language emerges from ordinary language used to describe experiences. The move from arithmetic to algebraic thinking involves students moving from using numbers as physical quantities to working with abstract variables, while simultaneously generalising operations such as addition and subtraction. A fundamental tenet of the AP is that this move from the physical experience to abstraction can be facilitated by using a five-step programme which begins with a physical event or experience and takes students through to a formal symbolic representation of this event. AP's curriculum combines inquiry and experiential learning, which involves mathematics emerging from human experience. Mathematics is also made accessible by using real-life situations that embody rich mathematical concepts. Through the process of mathematizing these situations or events, students are encouraged to actively engage in mathematical discourse by using their everyday language for talking about mathematical concepts. This discourse leads to a focus on important mathematical features about the event and to the process of symbolization (Moses and Cobb, 2001, p122). By actively engaging in the mathematics discovery process, students encounter complex mathematical ideas that they learn to work through. This controlled movement from the concrete to the abstract allows students to build their own meanings for algebraic objects as well as helping them see that algebra is not just a collection of mysterious symbols and operations. The five steps are: 1. Physical event; 2. Picture or model of this event; 3. Intuitive (idiomatic) language description of this event; 4. A description of this event in regimented English; 5. Symbolic representation of the event (Moses, Kamii, McAllister Swap, Howard, 1989, p.433).

Dubinsky and Wilson (2013) investigated the effectiveness of the methods of the AP in a study involving low-achieving high-school students, they found that the students made significant gains over the course of the module and developed understanding comparable to that of a university student. The students took part in a seven-week programme which aimed to develop understanding of the concept of function using a module designed by the AP team on the 'Road-Colouring problem' (Budzban & Feinsilver, 2011). The authors concluded that the AP's five-step method of experiential learning allowed the students to engage meaningfully with non-trivial mathematics. Other studies have found that exposing teachers and young people to mathematically rich tasks in a fun and engaging way has the potential to empower them to see the value of mathematics in their lives and also to develop mathematical fluency (Dunphy, Dooley and Shiel, 2014).

THE FLAGWAY GAME

The module used in primary schools as part of this work was the Flagway Game[2]. This game involves skill and speed – both mathematically and physically. It is based on the Mobius function; this function assigns to each positive whole number one of three possible outputs. In the Flagway game these outputs are the colours red, blue and yellow, and so each natural number is either a red, blue or yellow number. To begin with the players are only told the colours of a few numbers (say the numbers 1-6) and are asked to try to guess the colours of other numbers. The students make conjectures based on the information about the colours of numbers they know and these conjectures are tested when new information about the colours of other numbers is given to them. In this way, a spirit of experimentation is encouraged as well as a reliance on reasoning to check conjectures. In practice, figuring out the rules governing how colours are assigned can take some time but simple versions of the game can be played straight away.

A more complicated version of the game has the students taking three cards from the table and running to a circle in the middle of the play area, this circle has a network of coloured paths leading from it (See Figure 1 below). Each path from the centre is made up of three portions, each coloured red, blue or yellow. Players need to arrange their cards in a sequence and then follow the path dictated by their cards (for example if they have a sequence of 5-6-4 they need to take the red path leading from the centre (for 5), then the blue path (for 6) from that node, and lastly the yellow path (for 4)).

The colours of the numbers are assigned using the Mobius function, and thus depend on the prime factorisation of the number. The three colours correspond to the three possible categories of prime factorisations: the case where the number is divisible by the square of a prime; the case where the number has an odd number of prime factors (with none repeated); the case where the number has an even number of prime factors (with none repeated). Thus, in order to play the Flagway game well (in particular, in order to be quick), it is important to be able to factor numbers and to be able to decide quickly to which category they belong. Pupils are encouraged to represent the factorisations using letters and notice for example that 12 and 18 are both of the form a^2b (and so are both the same colour). Thus, the use of variables is introduced in a natural setting where pupils can appreciate the need for them.

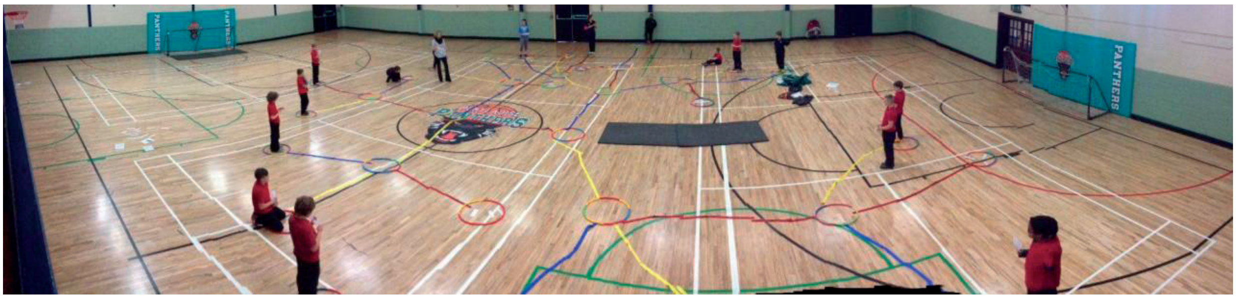


Figure 1: Flagway game in progress

METHODOLOGY

Teachers from nine primary schools around the Kildare region took part in a day-long workshop in February 2016, following an information session with Principals. At this workshop the teachers were introduced to the Flagway game and to the methodology of the Algebra Project. The teachers then introduced these methods into their classrooms over the next few months, under the guidance of the project team. This was followed that summer by a series of intensive workshops facilitated by the US Algebra Project team. Thirteen teachers from six of the original nine primary schools took part in these. At these workshops, the teachers were not told the rules of assigning colours to numbers but had to work together to discover these rules in the same way that pupils would be expected to. The mornings were spent on working through the variants of the Flagway game and discussing how they could be used in the classroom, while the afternoons were spent working with children in a primary school setting along with the AP team.

Four primary teachers from two case study schools who took part in the Flagway module were interviewed at T1 and again a year later at T2 after the participants had used the AP methodologies in their own teaching with 5th and 6th class students. The interviews were analysed to identify any changes to the teaching practices and beliefs of participants as a result of this project. Feedback on the impact of the project from pupils in the case study schools was gathered using focus groups[3]. The researchers also visited schools to observe the methodologies in action [the Flagway game being played].

THEORETICAL FRAMEWORK

The framework for analysis used in this research is Engeström's model of an activity system where contextual artefacts are fundamental in converting external stimuli into internal mental functioning (1987, 2001). There are multiple activity systems at play within any one classroom. This paper takes the student as the unit of analysis and the outcome as developing algebraic thinking and the connections are represented as lines in figure 2 below. In activity theory there are many connections and disconnections that can be observed in any activity system. DeVane and Squire advise that the minimal meaningful unit of analysis is an individual 'engaged in an activity with tools and resources in some social context' (2012, p. 254). In figure 2 we map the AP onto Engeström's (1987) model of an activity system.

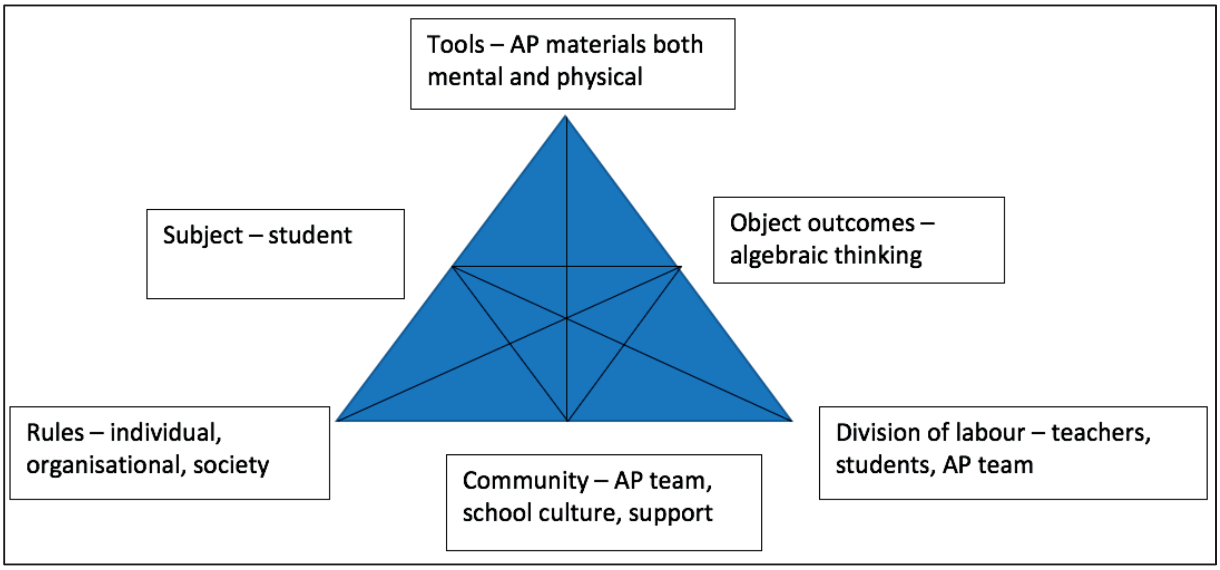


Figure 2: The Algebra Project using Engeström's model of an activity system

RESULTS

We report here on the analysis of the interview and focus group data collected from teachers and pupils who have been involved in the Flagway module. The overall impression from both groups was very positive. For example, one pupil said: *It was not like forced Maths; it was just being able to do Maths but yet feeling kind of free* (T2_CS1_SFG). The themes that emerged concerned the model of professional development used, the implementation of AP methods in schools (benefits and challenges), skills development, opportunities for across school collaboration and fitting Flagway into the Irish curriculum. In this paper we will concentrate on two of these themes.

Implementing the Algebra Project in Schools: Doing Maths and using tools

The teachers spoke about how they implemented the AP methodologies in their schools and what this entails: *It needs an awful lot of drive and it needs an awful lot of commitment to it and it needs an awful lot of organisation but I think it definitely can work* (T2_CS2_T1). Teachers took time to prepare their classes for the mathematics involved. They talked about how they worked ahead of playing the game and the need to work on number sense with students. They all introduced the Flagway game in the way that it was introduced to them in order not to diminish the learning involved. Stein at al. (2000) advises that teachers can limit the level of cognitive demand of tasks in their attempt to make expectations clear to learners. This can result in the learner following a prescribed set of steps rather than engaging in meaning making. Teachers needed to set up the task in such a way so that the learners got time and space to engage with the problem and construct meaning for themselves. This teacher describes how they introduced the game to their classes: *What the children came up with first of all was they found a way to make the numbers fit which was a pattern which was work that they had come up with which they could explain which was brilliant; we initially did it with the numbers 2 to 10 so then I would test them by saying "OK, where would 11 go, where would 12 fit in your pattern?" and see if they could explain to me and see if it could work and then from that step then I told them that they needed to find what Mobius function was; so then I would give them another number and see if they could work out the pattern, see if they could work out the system between them ….so they wanted to do it because it was a challenge and because I would not give in and I would not tell them* (T2_CS2_T2).

The pupils responded to the challenge. They liked the way that the problem was introduced to them and the freedom that this gave them to explore: *The way our teacher explained it to us was really good, the way he is like a mathematician, he questions us about it, he would not just tell us* (T2_CS2_SFG). This exploration gave opportunities for the pupils to engage in sophisticated mathematical thinking: *even though some of them did not get to it they still learned an awful lot and it was a different way of thinking about Maths where they had to try and come up with their own theorems, it was not just me regurgitating information and them learning it* (T2_CS2_T2).The teachers also spoke about how the AP encouraged independent thinking and problem-solving in their pupils and how it can change pupil's perception of mathematics as being about getting the right answer and nothing else.

The time needed for the learners to work through the process of problem solving and the need for the teacher not to offer clarifying help and to have the confidence to allow the process work was evident in the commentary on implementing the game in class in both case study schools. The Flagway game and the card games and tools created affordances for the activity of the class by structuring the kinds of mathematical knowledge that learners got to use and build. The way the game was played was determined by the individual teachers in interaction with the learners in their classrooms. Their pupils recognised the benefits they were reaping: *Yes, it was a really good experience because it was all to do with prime numbers. It will help us definitely in the future for secondary school* (T2_CS1_SFG). Another said: *We did not really think of it as Maths, we kind of thought of it as a game but in our heads, we were kind of doing the strategies and we were learning but we did not really realise it but we did* (T2_CS1_SFG). The ability to do mental arithmetic was highlighted by teachers and pupils.

Another benefit of the AP methodologies was evident in the fact that both teachers and students found the AP to be inclusive and that the task allowed for all to get involved in the learning. The Flagway game seems to offer opportunities to engage pupils who might otherwise encounter difficulties: *I have a child in my class who would be very very weak and by the end she is smiling and she is running and she is looking involved and it is lovely to see* *everyone getting involved* (T2_CS1_T2). At T1 when teachers were asked about a typical maths lesson, they referred to the workbooks and how the learning was very teacher-centred. A typical response was: *It starts out teacher led … and then they would be working a lot with their maths workbooks so it is, they do work individually* (T1_Interview_T2). The descriptions of how the AP methodologies were implemented in the case study schools paint a very different picture; the classes seem to involve inquiry-based learning.

The teachers spoke about two main challenges with the implementation of the AP in schools. One was the physical space needed for the setting up of the game, and the second was the time needed to play the game. CS2 had tried to adapt the game and to do it inside on the whiteboard. CS2_T1 lamented the lack of a recreation space in his school, however, he had improvised and the day the researcher visited the students were very engaged in 'doing mathematics'. *Just trying to do it as much as you can in the small space, clearing the tables and make them walk the game rather than run it, which kind of defeats the active side of the game, and as well as setting it up outside takes time* (T2_CS2_T1). CS1 had a recreation space which meant that they got to play the AP Flagway game more often. They also got to host other schools coming to play the game, however the teachers explained that the Irish weather hampers the availability of space for the game. The pupils expressed very similar opinions; it is difficult to play the game outside unless the weather is good and you need a big hall to play it properly inside. *I prefer to do it outside rather than in the hall because sometimes inside it was really squished because it is smaller. I like doing it outside because we have lots more room to run around* (T2_CS1_SFG).

The time needed to give students the opportunity to problem solve and 'do maths' was a common theme in both teacher and student interviews. One teacher talked about how they had got the setting up time for the game down to a minimum but they also lamented that to really use the game to its potential takes time. *I think there is an awful lot of Maths in it, there is an awful lot of benefit to it but you have to invest in it, we are doing a lot of work, investing time in it, we are able to do it here because we have it in our policy that we do Maths games once a week* (T2_CS2_T1). The division of labour and community are at odds if adequate time for mathematics teaching and learning is not included in the rules of the schools. CS1_T1 talked at T1 about the perceived challenges she anticipated for implementing the AP in her school. At T1 she talked about physical challenges such as the size of the hall, the time table and so on. At T2 she talked about how to integrate the AP more with the curriculum. We would contend that this represents a move in this school to embedding the AP as a part of the curriculum. *The real challenge I think with the Flagway is trying to adapt it now to be most effective within the Irish curriculum; I think that's the biggest challenge*. It is moving from AP being seen as a game to being seen as doing maths.

Development of skills

The development of skills such as working with others, peer teaching, empathy and communication was very obvious from all the student focus groups when they talked about playing the game. They talked about how they taught pupils from the visiting schools, delegated work within teams and encouraged all to get involved. In some of the responses the learning of the mathematics was almost seen as insignificant in comparison to the development of skills. *I liked it also because it was team work and if you made a mistake nobody was going to give out to you or anything because you just like, it was all to do with your team, one girl might be taking up sets of cards that have not been taken and you might have one girl trying to run around. I liked it because it was pretty fair* (T2_CS1_SFG). Peer tutoring came up in all teacher interviews; the AP was seen as a vehicle for peer tutoring and teachers spoke about how this enhanced the development of skills such as working with others and communication. *If you got the strong kid in with a couple of weak kids some of the games are set up so that it has to be a team answer so the whole team has to stand up and shout the answer, so they've got, it's in their interest to make sure that the weak guy knows what's happening* (T1_CS2_Interview_T1). The ability to build self-efficacy in maths while also doing physical activity was significant for some of the learners in both case study schools. For some, combining the two activities made learning mathematics more enjoyable: *I thought it was good because it gave us an opportunity to get active and to enjoy the Maths, because some people don't enjoy Maths but, in a way, we were kind of doing PE with it and we were still learning* (T2_CS1_SFG). While for others it was the other way around: *I am not that fast as everyone else, I am horrible at running so that is why Flagway, I like the Flagway, it gave me a reason to be good at PE. You need to be fast to win* (T2_CS2_SFG).

DISCUSSION

We have seen the themes that emerged from the interview and focus group data. Overall, pupils and teachers were very positive about the AP and the Flagway game. Both the teachers and the pupils spoke eloquently about the various benefits that they observed. The AP five-step process, with its emphasis on moving from a physical experience to an abstract concept, seems to lead to deeper and more persistent understanding. This echoes the findings of Dubinsky and Wilson (2013). Furthermore, we have seen evidence that the methodologies used can lead to the development of key skills such as group work and communication.

For the pupils in this research the tools, the physical events of working with the colours, numbers and modelling, pushed their problem identification and problem solving skills and encouraged learning. The move from the intuitive language to structured language was evident from what we observed and from teacher and pupil interview data. An outcome not envisaged at the outset was that of the development of the skills of peer tutoring, team work and communication. The tools (the AP pedagogical process) in interaction with the division of labour between the teacher, pupil and curriculum impacted on pupils' development of skills and in doing so enhanced their attitude to maths, as the title says they experienced *being able to do maths and feeling kind of free*. The Flagway game seems to fit very well into the Irish primary school curriculum. However, teachers expressed worry about covering content areas and about preparation for various tests at the end of primary education. This may be indicative of their conceptions of mathematics as a bundle of isolated facts that need to be 'covered', and may indicate a need for a reconceptualization of mathematical teaching and learning.

It is interesting to look at the teacher as a unit of analysis in tandem with the student in two parallel activity systems. What is clear is that the model of professional development used by the AP team where the participants were the pupils in the morning and then implemented their learning in a school setting with the AP team in the afternoon had a big impact on moving the teachers' thinking and understanding of how the tools support the development of algebraic thinking. This merits further analysis. The dissonance in the activity system was the lack of physical space and time. While this may appear to be easy to solve, it involves a complex and potentially difficult interaction between the rules at organisational level and community to support the investment in this kind of experiential learning and meaning-making.

BIBLIOGRAPHY

Budzban, G., & Feinsilver, P. (2011). The generalized road colouring problem and periodic digraphs. *Applicable Algebra in Engineering, Communication and Computing*, 22, 21–35.

DeVane, B., & Squire, K. (2012). Activity Theory in the Learning Technologies. In D. Jonassen & S. Land (Eds.), *Theoretical Foundations of Learning Environments* 2nd Edition (pp242-267). New York: Routledge.

Dewey, J. (1938). *Experience and learning*. New York: Touchstone.

Dubinsky, E. & Wilson, R.T. (2013). High school students' understanding of the function concept. *Journal of Mathematical Behavior*, 32, 83– 101

Dunphy, E., Dooley, T., & Shiel, G. (2014). *Mathematics in Early Childhood and Primary Education (3–8 years).* Dublin: NCCA

Engeström, Y. (1987). *Learning by expanding: An activity-theoretical approach to developmental research*. Helsinki: Orienta-Konsultit.

Engeström, Y. (2001). Journal of Education and Work Expansive Learning at Work: toward an activity theoretical reconceptualization. *Journal of Education and Work*, 14(1), 133–156.

Kolb, D. (2015). *Experiential Learning: Experience as the source of learning and development*, 2nd edition. Pearson Education, New Jersey.

Moses, R., Kamii, M., McAllister Swap, S., Howard, J. (1989). The Algebra Project: Organising in the spirit of Ella. *Harvard Educational Review*. 59(4) 423-444.

Piaget, J. (1952). *The origins of intelligence in children*. New York, NY: International Universities Press.

Quine, W. V. O. (1992). *Pursuit of truth.* Boston: Harvard University Press.

Stein, M. K., Smith, M. S., Henningsen, M., & Silver, E. A. (2000). *Implementing standards-based mathematics instruction*: *A casebook for professional development*. New York: Teachers College Press.

---

[1] The Algebra Project in Ireland (API) study was a year-long initiative funded by SFI through the SFI Discover Call 2015 scheme and sponsored by the NCCA and the Kildare Education Centre. Additional support for the project came from the PDST and KWETB.

[2] Bob Moses developed the Flagway Game in 1995 and patented it in 1996 (Moses, U.S. Pat #5520542 & 5704790).

[3] Data are coded as follows: T1 and T2 refer to pre implementation and post implementation data. CS1 refers to case study school one. SFG is student focus group. For example, T1_CS1_T1 refers to pre implementation interview in case study school one with teacher one.

INVESTIGATING THE EFFECTS OF SHARED PICTURE BOOK READING ON PARENTAL INVOLVEMENT IN MATHEMATICS

Edward Fitzgerald and Aisling Twohill

Dublin City University

The intervention described in this paper facilitated mathematical discussions between parents and children within the context of picture books. Parental involvement has been shown to have the potential to impact significantly on a child's attainment in school (Epstein, 1995; Anthony & Walshaw, 2007). The intervention took place in a rural school in County Kildare and lasted for three weeks. The research focus encompassing the intervention was the parents' involvement with their children's mathematical learning. Data collection included parent interviews and a reflective journal maintained by the teacher-researcher. Findings indicated that the majority of parents felt they were more involved in their child's learning of mathematics through the intervention. Furthermore, the participants noticed a number of benefits when using the picture books, including a greater understanding in children's mathematics, the children having a greater motivation to do mathematics, and an increase in mathematical discussions.

PARENTAL INVOLVEMENT IN MATHEMATICS AND THE ROLE OF PICTURE BOOKS

Parental involvement can have a significant impact on children's mathematical development (Henderson & Berla, 2004: Anthony & Walshaw, 2007). Recent reforms have emphasised the need to work with parents to enable them to support their children's mathematical development. For this to happen, there is a need to educate parents on current practices in mathematics education (Civil, 2006) and to address parents' own concerns and attitudes towards mathematics (Muir, 2012). In recent times, a number of initiatives between the school and the home environment have been successful in enabling parents to have a greater role in their child's mathematics learning (Merttens, 2005; Civil, 2006; Muir, 2012; Bleach, 2010).

Parents are the biggest factors in determining children's success in mathematics, with children's attitudes to learning mathematics shaped by the home environment (Merttens, 2005). Parental involvement in mathematics begins at home in the early years. Furthermore, differences in mathematical abilities of young children can be linked to the social activities they engage in at home (Benigno & Ellis, 2004). LeFevre, Skwarchuk, Smith-Chant, Fast, Kamawar and Bisanz (2009) describe the indirect and direct mathematical activities that parents and children engage in at home. Direct activities are focused on counting and numbers for the purpose of developing quantitative numerical skills. On the other hand, indirect activities include games and everyday tasks where the learning of mathematics is incidental (ibid). Boaler (2009) advocates the use of indirect mathematical activities at home and advises that we cannot overlook the "role of simple interactions in the home, and the role of puzzles, games and patterns, in the mathematical development and aspirations of young children" (p. 108). Children's understanding of mathematical concepts is shaped by the experiences they encounter at home. Parental support at home can help children develop mathematical skills and in turn their confidence in performing in mathematics can develop (Pomerantz, Moorman

& Litwack, 2007). Moreover, children's overall mathematical skills may be improved when families engage in discussion while participating in these mathematical activities (Sheldon & Epstein, 2005).

Furthermore, mathematical discussions which take place during *direct* and *indirect* mathematical activities can have an even greater impact on a child's development (Sheldon & Epstein, 2005). Parents can play a role in this by stimulating mathematical conversations at home or in the environment during everyday activities such as going to the shops, waiting at a bus stop or when cooking a meal. Traditional games such as playing cards, board games and puzzles can also be used to good effect. Skwarchuk (2008) argues that parents will often engage in discussions which focus on numbers but are unsure on how to incorporate other aspects of mathematics into the discussion.

Using Picture Books to Teach Mathematics

In recent years, the use of picture books to support mathematical development has become more common as teachers seek an integrated approach to teaching mathematics (Haury, 2001). The use of picture books for mathematical purposes has been espoused by recent literature which has advocated its use (Casey, Kersh & Young, 2004; Hong, 1996; van den Heuvel-Panhuizen & Elia, 2012; Anderson, Anderson & Shapiro, 2005). The recently established Maths Through Stories initiative reflects the growing trend of using picture books in mathematics lesson and gives teachers recommendations on suitable picture books to use. In picture books the illustrations, along with the text, play a vital role in telling the story and conveying the meaning (Elia, van den Heuvel-Panhuizen & Georgiou, 2010). Furthermore, a mathematical picture book can be defined as, "picture books with mathematical content present in both the text and images" (Marston, 2010, p. 383). Three types of mathematical picture books exist. These include: picture books where the mathematics is explicitly referenced; the mathematics is embedded within the story; or where the mathematics is perceived to be occurring (ibid).

Theoretical Perspectives underpinning the Use of Picture Books for Mathematical Purposes

The use of picture books for mathematical purposes is espoused by three long established theories of learning: the sociocultural theory of learning; the constructive approach to learning; and the importance of contextualised learning (van den Heuvel-Panhuizen & van den Boogaard, 2008). In the constructivist approach picture books offer children the opportunity to actively construct new mathematical knowledge (Phillips, 1995). The storyline or pictures often present a problem in which the children use their prior knowledge to try and come up with a solution. When doing this they develop new ideas, structures, and schemas and achieve a higher level of understanding (van den Heuvel-Panhuizen & van den Boogaard, 2008, p. 343). Similarly, the use of picture books for mathematical purposes is rooted in the sociocultural theory of learning. Shared reading experiences with a teacher or a parent, and the discussions which ensue, allow children to actively construct new knowledge in a social environment (Cobb, 1994). Picture books also allow children to encounter problems in a meaningful context. Children are able to understand and solve problems which are in context, more readily than more formal questions (Donaldson, 1978).

The role of the teacher is vital during picture book reading in the classroom. Van den Heuvel-Panhuizen and Elia (2012) contend that the dialogic book reading allows for greater mathematical understanding as the child and the teacher co construct the knowledge. Furthermore, the teacher also needs to ensure that the mathematical concept of the story is identified as many children may struggle to realise this (Pramling & Pramling-Samuelson, 2008). Casey et al., (2004) report that picture books should be read to children in a manner similar to that of storytelling which allows the reader to connect more with their audience through eye contact, facial gestures and body language.

Several factors support the use of children's literature in mathematics. Research on picture books indicates that they can lead to an improvement in mathematical attainment (van den Heuvel-Panhuizen & Elia, 2012). Hong (1996) notes, that in one particular study, the children who experienced story book reading and complimentary activities outperformed the children in the control group in classification, number combination and shape tasks. An experiment conducted by Jennings, Jennings, Richey and Dixon-Kraus (1992) showed that children's mathematical test scores improved considerably when picture books were used as part of the curriculum. It was also noted that the instances where children would use mathematical terms increased during free play time.

Shared book reading can contribute to a child's language development (Anderson et al., 2005; Jennings et al., 1992) as the illustrations in picture books can stimulate discussion between the reader and the child (Anderson et al., 2005). As a result, these discussions enable children to learn new vocabulary and concept development (ibid). During picture book reading, teachers and pupils have the opportunity to discuss the mathematical problems highlighted in the book and come up with possible solutions. Such use of picture books can also facilitate the use of mathematical discussions at home. Van den Heuvel-Panhuizen and van den Boogaard (2008) note that children used mathematical language spontaneously during shared picture book reading with their parents. In their study, Anderson et al., (2005) videotaped thirty-nine parents and their children as they engaged in picture book reading at home. The results showed a wide diversity in the mathematical concepts that were discussed.

Picture books can also be used to develop mathematical concepts (Whitin & Whitin, 2000). For example, previous research has shown that picture books have the capacity to improve mathematical knowledge in the areas of measurement (van den Heuvel-Panhuizen & Iliada, 2011). Hong (1996) also reported that kindergarten children showed significant improvements in classification, number combinations and shape recognition when exposed to picture book reading. Similarly, Casey et al. (2004) reported that students developed their spatial and analytical awareness when exposed to storytelling sagas in the mathematics lessons. The use of storytelling sagas allows mathematical concepts to be taught in a systematic way over a number of lessons (ibid, 2004). Bjorklund and Pramling-Samuelson (2012) argue that in order to maximise a child's mathematical understanding teachers should approach picture book reading with a particular mathematical concept in mind, thus noting the importance of teacher preparation and planning.

The use of picture books also allows mathematics to be placed in context, in real life situations, which makes learning more meaningful for the children (van den Heuvel-

Panhuizen & Elia, 2012). Moreover, children can only construct new meaning when it makes sense to them. Picture books can enable children to "encounter problematic situations, may stimulate them to ask their own questions, search for answers, consider different points of view, exchange views with others and incorporate their own findings with existing knowledge." (van den Heuvel-Panhuizen et al., 2016, p. 324). Exposure to problems which are centred round everyday life makes the learning more meaningful for the children (Donaldson, 1978).

Picture books can have a positive impact on the way children view mathematics. They have the potential to motivate children and to foster an appreciation of mathematics (Von Drasek, 2006; Jennings et al., 1992). Picture books also have the power to engage and focus the attention of children (Van den Heuvel-Panhuizen, van den Boogaard & Doig, 2009). Hong (1996) explored the impact that children's literature and follow up activities had during free play time. She noted that the children who experienced story book reading were more likely to engage in mathematical activities during free play time than the children who were in the control group. Furthermore, a number of picture books used in mathematics can also be used to teach students how to solve personal problems, cope with conflict and to take responsibility for their actions (Hong, 1996).

METHODOLOGY

The research was carried out in a rural primary school in County Kildare. The school is a Catholic School and has a mixed cohort of approximately 230 students. The research was undertaken with first class in which there were 28 students, 13 boys and 15 girls. The school has a very active parent's association which is affiliated to the National Parents Council-Primary.

The research question pertaining to this study is whether parental involvement in mathematics can be enhanced through shared picture book reading? From this research question a number of subsequent sub questions emerge. These are: to what extent are parents involved in their child's mathematics education? what is the current understanding of parents in relation to their role in their child's mathematics education? how can parents be enabled to become more involved in their children's mathematics' development?, and can picture books encourage parents to become more aware of the possibilities of engaging in mathematics with their child?

Prior to the intervention, all parents completed a questionnaire, in order to gauge their existing level of parental involvement in mathematics. Following this the parents were categorised into three groups: those who identified themselves as having a high, medium, or low level of involvement. One parent from each group was chosen at random and invited to participate in this study. All three parents were interviewed in order to gain a greater understanding of their perceptions of parental involvement and to discuss their level of involvement. A classroom observation then followed where the parents were invited into the classroom to observe a mathematics lesson which used a picture book. Following the classroom observation, each child took a picture book home once a week over three consecutive weeks. The child and their parent engaged in mathematical discussions based on the content of the picture book. Each parent filled in a Parent Report Sheet at the end of each session to reflect on the process. At the end of the intervention the parents completed a final evaluation sheet which detailed their entire experience of taking part in the research. A second interview took place with the parents to gain a deeper understanding of their experience in using picture books at home.

FINDINGS

The findings from this study have shown that some parents' current attitudes towards mathematics education had been influenced by their past experiences of mathematics in school. When reflecting on her own experiences of mathematics in primary school Ann recalls, "I suppose I just found it difficult and challenging and I suppose some elements I liked and other parts I didn't like". Both Ann and Sarah stated they had a negative experience of mathematics while in primary school and both also believed that, aside from helping with homework and answering some incidental questions, they were not involved in their child's mathematics education. This correlates with the research which states that parents' who have negative experience of mathematics in school may be less likely to become involved in their children's mathematics (Boaler, 2015). The findings also suggest that Sarah may lack the knowledge on how to become more involved in their child's mathematics education, "I actually don't really know to be honest with ya cos I need to start learning maths myself to be honest". Skwarchuk (2008) highlighted how parents may be aware of how to engage in literacy activities at home with their children, but often struggle when it comes to mathematics.

The findings also highlight that parental involvement in mathematics needs to be extended beyond the scope of helping with homework and include engagement in mathematical activities which contribute more to children's mathematical understanding (Sheldon & Epstein, 2005). On analysing the Parent Report Sheets, it appears that a number of parents view parental involvement as simply helping with homework. For this change to happen parents need to be educated on how they can become more involved in their child's mathematics education at home.

This study has also shown the need to empower parents to become more involved in their child's mathematics education. This initiative allowed parents to become more involved in their children's mathematics and also established links between the home and the school. During the intervention the parents enjoyed having a greater role to play in their child's mathematics education. Ciara describes the benefit it had on her family, "definitely as a family we all became more aware of how you can learn maths through reading and through books". Similarly, Sarah enjoyed spending time with her son during the intervention, "it kinda makes you spend a bit more time with him as well". This initiative correlates with similar projects where partnerships between the school and the home were established to enhance parental involvement in mathematics (Civil, 2006). The use of picture books for mathematical purposes was a new experience for the researcher and parents and overall was very positive.

A number of parents also reported increased motivation on the part of the children after the three-week intervention and that their child looked forward to taking home a picture book each week. Not only had parental involvement increased but a number of parents noted that their confidence in engaging with mathematics improved as a result of the three week intervention. It was apparent that by week three the parents had become confident enough to come up with their own questions to ask their child when reading the picture book.

CONCLUSION

The potential of using picture books for mathematical purposes, as evidenced in this study, is also supported by current research which advocates their use (Hong 1996; Anderson et al., 2005). In this study, parents noted that the use of picture books increased children's motivation to do mathematics (Hong,1996), allowed them to engage in mathematical problems which were in context (van den Heuvel-Panhuizen & Elia, 2012) and lead to rich mathematical discussions (Anderson & Anderson, 1995). Furthermore, the participants also felt more confident in engaging in mathematics with their child. The need to work with parents and include them in their child's mathematical learning is a key goal of mathematics reform (Government of Ireland, 1999). Partnerships and initiatives between the school and the home can lead to enhanced parental involvement. While recent policy documents have emphasised the need to work in partnership with parents, limited guidance has been given to schools and teachers on how they can develop partnerships with parents (Gilleece, Shiel, Clerkin & Millar, 2012). Perhaps, future policy documents could include explicit examples on how to effectively involve parents in mathematics. In many cases, it is the leadership shown by teachers and principals that provide the catalyst for new initiatives which promote parental involvement (INTO, 1997). On a wider scale there is the potential for more schools and teachers to show greater initiative in establishing partnerships with parents. Perhaps, teachers need to engage in high quality professional development so that they can be exposed to current research and gain in depth knowledge on the subject. Furthermore, they must also be informed on how best they can include parents.

There is a host of activities in which parents can engage in with schools which can bring about a greater understanding of children's learning. However, research has shown that there is a lack of knowledge amongst parents about how to become more involved in mathematics at home (Anthony & Walshaw, 2009). Parents' past experiences of mathematics plays a significant role in how shaping their attitude towards mathematics today. Therefore, there is a need to educate parents on the value of mathematics in our everyday lives. Once parents come to realise the value of mathematics they are more likely to become involved in their child's mathematics at home, which in turn will benefit the child.

The findings show that many parents in this research view parental involvement as helping with homework. Consequently, it is suggested that picture books could be used as an alternative to traditional mathematics homework. Sheldon and Epstein (2005) contend that homework is a wonderful opportunity for parental participation in their child's learning. Similarly, picture books are an easy and enjoyable way for parents to play an active role in their child's homework. However, for this to happen schools will need to invest resources in supplying a wide variety of picture books which can be used from Junior Infants right through to Sixth Class. While this may be an expensive outlay the benefits of picture books far outweigh the cost element. Furthermore, there is a need for strong leadership from teachers and principals who are willing to establish a culture in their school whereby parents are included in their child's mathematics education at home through the use of picture books.

REFERENCES

Anderson, A., & Anderson, J. (1995). Learning mathematics through children's literature: A case study. *Canadian Journal of Research in Early Childhood Education*, 4, 1–9.

Anderson, A., Anderson, J., & Shapiro, J. (2005). Supporting multiple literacies: Parents' and children's mathematical talk within storybook reading. *Mathematics Education Research Journal*, 16 (3), 5–26.

Anthony, G., & Walshaw, M. (2007). *Effective pedagogy in mathematics.* Wellington, NZ: Ministry of Education.

Benigno, J. P., & Ellis, S. (2004). Two is greater than three: Effects of older siblings on parental support of preschoolers' counting in middle-income families. *Early Childhood Research Quarterly*, 19(1), 4–20.

Björklund, C. & Pramling Samuelsson, I. (2012). Challenges of teaching mathematics within the frame of a story-A case study. *Early Child Development and Care*, 183(9), 1339–1354.

Bleach, Margaret, J. (2010). *Parental involvement in primary education in Ireland*. Dublin: Liffey Press.

Boaler, J. (2009). *What's math got to do with it? How parents and teachers can help children learn to love their least favourite subject*. New York: Penguin.

Boaler, J. (2015). *Mathematical mind-sets: Unleashing students' potential through creative math, inspiring messages and innovative teaching*. San Francisco, CA: Jossey-Bass.

Casey, B., Kersh. J. & Young. J. (2004). Storytelling sagas: An effective medium for teaching early childhood mathematics. *Early Childhood Research Quarterly*, 19, 167–172.

Civil, M. (2006). Working towards equity in mathematics education: A focus on learners, teachers, and parents. In S. Alatorre, J.L. Cortina, M. Sáiz, & A. Méndez (Eds.), *Proceedings of the twenty eighth annual meeting of the North American chapter of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 30–50). Mérida, Mexico: Universidad Pedagógica Nacional.

Cobb, P. (1994). Where is the mind? Constructivist and sociocultural perspectives on mathematical development. *Educational Researcher*, 23, 13–20.

Donaldson, M. (1978). *Children's minds*. New York: W.W. Norton.

Elia, I., van den Heuvel-Panhuizen, M. & Georgiou, A. (2010). The role of pictures in picture books on children's cognitive engagement with mathematics. *European Early Childhood Education Research Journal*, 18(3), 125–147.

Epstein, J. L. (1995). School/family/community partnerships: Caring for the children we share. *Phi Delta Kappan*, 76, 701–712.

Gilleece, L., Shiel, G., Clerkin, A., & Millar, D. (2012). *The 2010 national assessments of English reading and mathematics in Irish-medium schools*. Dublin, Ireland: Educational Research Centre.

Government of Ireland (1999). *Primary school curriculum: Mathematics*. Dublin, Ireland: The Stationery Office.

Haury, D. (2001). *Literature-based mathematics in elementary school*. Columbus, OH: ERIC Clearinghouse for Science, Mathematics, and Environmental Education.

Henderson, A. T. and Berla, N. (2004). *A new generation of evidence: The family is critical to student achievement*. Washington DC: National Committee for Citizens in Education.

Hong, H. (1996). Effects of mathematics learning through children's literature on math achievement and dispositional outcomes. *Early Childhood Research Quarterly,* 11, 477-494.

INTO, (1997). *Parental involvement: Possibilities for partnerships*, Dublin: Author.

Jennings, C., J. Jennings, J. Richey, and L. Dixon-Kraus. (1992). Increasing interest and achievement in mathematics through children's literature. *Early Childhood Research Quarterly*, 7, 263–276.

LeFevre, J., Skwarchuk, S. L., Smith-Chant, B. L., Fast, L., Kamawar, D., & Bisanz, J. (2009). Home numeracy experiences and children's math performance in the early school years. *Canadian Journal of Behavioural Science*, 41, 55–66.

Marston, J. (2010). Developing a framework for the selection of picture books to promote early mathematical development. In C. Hurst, B. Kissane, & L. Sparrow (Eds). In *Shaping the future of education.* Proceedings of the annual conference of the Mathematics Education Research Group of Australasia. Fremantle WA: MERGA.

Merttens, R. (2005). Family numeracy. In I. Thompson (Ed.), *Issues in teaching numeracy in primary schools* (pp. 78-90). Berkshire, UK: Open University Press.

Muir, T. (2012). Numeracy at home: Involving parents in mathematics education. *International Journal for Mathematics Teaching and Learning*, 1–13. Retrieved from http://www.cimt.plymouth.ac.uk/journal/muir.pdf

Phillips, D. C. (1995). The good, the bad, and the ugly: The many faces of constructivism. *Educational Researcher*, 24(7), 5–12.

Pomerantz, E., Moorman, E., & Litwack, S. (2007). The how, whom, and why of parents' involvement in children's academic lives: More is not always better. *Review of Educational Research*, 77(3), 373- 410.

Pramling, N., & Pramling-Samuelsson, I. (2008). Identifying and solving problems: Making sense of basic mathematics through storytelling in the preschool class. *International Journal of Early Childhood*, 40(1), 65–79.

Sheldon, S. B., & Epstein, J. L. (2005). Involvement counts: Family and community partnerships and mathematics achievement. *The Journal of Educational Research*, 98(4), 196–206.

Skwarchuk, S-L. (2008). Look who's counting! The 123s of children's mathematical development during the early school years. *Encyclopaedia of Language and Literacy Development* (pp. 1–9). Retrieved from http://www.literacyencyclopedia.ca/pdfs/topic.php?topId=243

van den Heuvel-Panhuizen, M., & van den Boogaard, S. (2008). Picture books as an impetus for kindergartners' mathematical thinking. *Mathematical Thinking and Learning*, 10(4), 341- 373.

van den Heuvel-Panhuizen, M., & Elia, H. (2012). Developing a framework for the evaluation of picture books that support kindergartners' learning of mathematics. *Research in Mathematics Education*, 14(1), 17–47.

van den Heuvel-Panhuizen, M. & Elia, I. (2011). Kindergartners' performance in length measurement and the effect of picture book reading. *Mathematics Education* 43: 621

van den Heuvel-Panhuizen, M., van den Boogaard, S. & Doig, B. (2009). Picture books stimulate the learning of mathematics. *Australasian Journal of Early Childhood*, 34 (2), 30-39.

Von Drasek, L. (2006). Teaching with children's books: The "wow" factor. *Teaching Pre K-8*, 36(4), 62-63.

Whitin, D. J., & Whitin, P. (2000). *New visions for linking literature and mathematics*. Reston, VA: National Council of Teachers of Mathematics

DEVELOPING A PROBLEM-SOLVING MODULE IN MATHEMATICS FOR HIGHLY-ABLE POST-PRIMARY SCHOOL STUDENTS

Aidan Fitzsimons and Eabhnat Ní Fhloinn

School of Mathematical Sciences, Dublin City University

Even with the introduction of a new mathematics syllabus at post-primary-level in Ireland, there has been ongoing doubt raised as to the effectiveness of our mathematics education for highly-able students. Prior research has indicated the use of problem-solving and open-ended questioning as important tools in students' mathematical development. This paper discusses the design of a module for post-primary students using these methods.

INTRODUCTION

Research has emphasised the importance of problem-solving in the study of mathematics, as well as its ability to develop students' higher-level thinking skills (Lewis & Smith, 1993; Shen, 2012). The introduction of Project Maths, the latest approach to teaching and learning mathematics at post-primary level in Ireland, has meant a renewed focus on problem-solving in the classroom (NCCA, 2013). In relation to highly-able students, teachers face the difficult task of differentiating their lessons to try to cater for such students, with teachers citing reasons such as overloaded curricula, large class sizes, and an emphasis on catering for weaker students as some of the reasons why it was not always possible to support highly-able students (Riedl Cross, Cross, O'Reilly, & Mammadov, 2014). Problem-solving, together with open-ended questioning, allows highly-able students to explore topics further, investigating various approaches and depth within a problem, creating a rich learning experience for these students (Hmelo-Silver, 2004; Kwon, Park, & Park, 2006). Otherwise, out-of-school programmes such as the Centre for Talented Youth, Ireland (CTYI) (https://www.dcu.ie/ctyi/index.shtml), and mathematical enrichment programmes (e.g. http://www.irmo.ie/) provide opportunities to further their knowledge and skills in mathematics.

In this paper, we describe a problem-solving mathematics module developed for highly-able post-primary students. This module, run in conjunction with CTYI, aims to improve highly-able students' problem-solving abilities, while tracking their mindset and resilience throughout a fourteen-week programme, through standard tests and reflective diaries kept by the students throughout the programme. The results of these investigations are beyond the scope of this paper, and therefore we shall focus here instead upon the design and development of the module itself and the research underpinning its foundations.

BACKGROUND

Gifted Education refers to the strategies, techniques or programmes employed by schools, parents and educators to provide highly-able students with differentiated learning that meets their needs (Gallagher, 2003, pp. 17–21). The vast array of terms used to describe students within Gifted Education provides an insight into the often-fragmented nature of research in the area. "Gifted", "talented", "gifted and talented", "highly-able" and "exceptionally-able" are just some of those used, with little consensus as to which one is best suited, or to whom they refer. Traditional definitions of giftedness were based on IQ or standardised testing measures (Fasko, 2001; Sternberg, 1999), and debate remains as to whether this is a constant throughout a person's life. In recent decades, research has explored giftedness as a more complex issue, involving, for example, a wide range of intelligences (Gardner, 1999; Gardner & Hatch, 1989), or a person's ability to interact with their environment (Sternberg, 1984). Numerous studies have also highlighted the necessity to include potential within the discussion of giftedness (Subotnik, Olszewski-Kubilius, & Worrell, 2011).

The National Council for Curriculum and Assessment (NCCA, 2007, p. 7) in Ireland use the term "exceptionally able" for students "who require opportunities for enrichment and extension that go beyond those provided for the general cohort of students". Whilst this report was prepared as draft guidelines it remains the most recent definition to date regarding Gifted Education in Ireland. It estimates that 10-15% of students may be exceptionally able, but the exact requirements of qualifying students may vary greatly in different school contexts. With the introduction of Project Maths, some parties expressed fears that, although this change may benefit students as a whole, it may not prove sufficiently challenging for more able students (NCCA, 2012), with an apparent lack of attention paid to the needs of the highly-able student (DES, 2010). Although Ireland performed slightly above average in the PISA mathematics rankings since 2006 it finished below the OECD average in terms of students rated as top performers (those scoring at the highest two levels of PISA) (O'Reilly, 2014, p. 8). While the latest rankings have improved in terms of Ireland's highly-able students, with 15.5% performing at the top level (slightly above the OECD average of 15.3%), there is still work to do to further develop the abilities of these students (OECD, 2016, p. 5).

Although students thought to be highly-able may often be grouped to allow for easier differentiation within lessons, the differences in ability between these students may be complex (Gallagher, 2003). At the moment, students with learning difficulties in Ireland may be able to avail of additional teaching within a resource classroom setting, but highly-able students have no such additional support network (NCSE, 2013). Acceleration or enrichment classes, such as AP Potential in the United States, offer students the opportunity to further develop their subject knowledge in school (Schiever & Maker, 2003). Traditionally, approaches in Ireland have favoured differentiation in the classroom as opposed to acceleration of students (Riedl Cross et al., 2014).The Consortium of Institutions for Development and Research in Education in Europe (CIDREE, 2010) evaluated the preparedness of the Irish, Swiss and Dutch education systems to address the needs of highly-able students, and highlighted the effectiveness of enrichment activities and pull-out programmes in both the Dutch and Swiss systems. The report suggested that both the Dutch and Swiss systems offered more opportunities for students who show characteristics of high-ability.

Research into best practice in the teaching of highly-able students has noted that characteristics often associated with highly-able students seem to indicate that a problem-solving approach to learning is a highly effective tool in developing understanding. Sriraman (2003, p. 163) echoed findings from the 1980s (Burton, 1984), showing that highly-able students flourish when their attention is captured through the use of open-ended questioning.

Problem-based-learning (PBL) has been shown to be particularly effective with highly-able students as they are often highly motivated, with the confidence to attempt more difficult or challenging tasks (Hmelo-Silver, 2004). Stepien, Gallagher & Workman (1993) found that highly-able students who were taught through PBL outscored their traditionally taught counterparts in multiple-choice testing, while highly-able students in an action research study showed greater levels of skill retention under PBL than a traditional setting (Dods, 1997).

Problem-solving in mathematics

In 1945, Polya published his book 'How to Solve It' , becoming a prominent figure in the research of problem-solving in mathematics education (Polya, 1957). Polya developed a four-step approach to problem-solving: understand the problem; devise a plan - which involves applying some problem solving strategies; carry out the plan; and look back. Polya is credited as a key influence in the influential problem-solving book "Thinking Mathematically" (Mason, Burton, & Stacey, 2010), and the system suggested therein by Mason et al follows a very similar dynamic. Once again, an emphasis is placed upon reflection after solving, or attempting to solve, a problem. The four-step process created and publicised by Polya remains strongly relevant to mathematics education today.

MODULE DESIGN

The module nurtures students' problem-solving abilities through open-ended problems in a collaborative learning environment (Gokhale, 1995), with students working in small groups throughout. These groups are randomly assigned at the start of the module and remain the same for the duration, with three/four students per group. The teacher, acting as a facilitator to learning, encourages dialogue within groups and utilises probing questions to promote understanding as students learn to communicate their reasoning (Hmelo-Silver & Barrows, 2006).

The module generally operates once a week, over a 3-hour period (a 2-hour class and 1-hour tutorial), where the final hour tutorial revolves around one designated problem that students must reflect on in the form of a diary entry. This practice is modelled on the final problem-solving step outlined by Polya (1957), and corroborated by Mason et al. (2010). Based on a combination of these models and observation of the processes taken by students in class, the model shown in Figure 1 was created to represent the path taken in problem-solving groups:

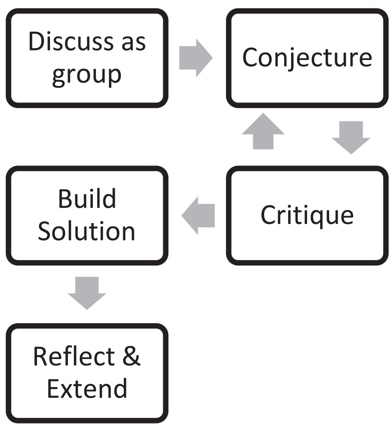


Figure 1 Problem-Solving Process

Where students have disproven a prior conjecture through critique, they return to the conjecture stage, and this movement between steps 2 and 3 may occur multiple times within a problem.

Problem-solving strategies provide students with a framework to approach mathematical problems (Katz, Segal, & Stupel, 2016). This project utilises several common strategies, and implements them as weekly themes, allowing students to discover them while solving problems. The themes for the first seven weeks are: visuals; patterns; specialising and generalising; conjectures; assumptions and questioning; structure; and working backwards. Problems chosen within each week emphasise the theme, although many problems utilise multiple strategies in their solution. Students are encouraged to find multiple ways to solve the problem by the facilitator, and thus can discover new strategies through inquiry. At the end of the tutorial the facilitator draws attention to the weekly theme so students are aware of future use of the strategy. The next four weeks focus on problems aligned to the four contextual strands of the Junior Cycle Mathematics: Number, Algebra and Functions, Geometry and Trigonometry, and Probability and Statistics (DES, 2017, p. 9). Problems during the remaining weeks are more general, following no contextual or strategical theme.

We will now look in greater detail at specific examples that the students meet in the module, and discuss typical reactions to these.

Example 1

In the first tutorial, under the theme "visuals", students meet the question:

> "How many squares are on a regular chessboard?"
>
> (Mason et al., 2010, p. 17)

Students often begin this problem by drawing out a chessboard to help, as shown in Figure 2:

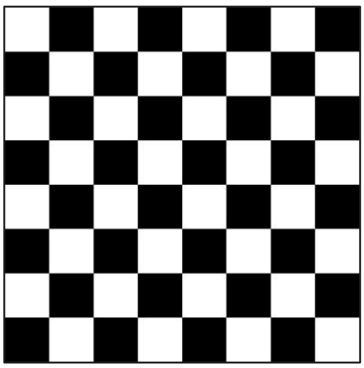


Figure 2 Regular Chessboard

Whilst this makes it possible to solve the problem, first exploring smaller versions of the pattern board will aid them in coming to this solution, as shown in Figure 3:

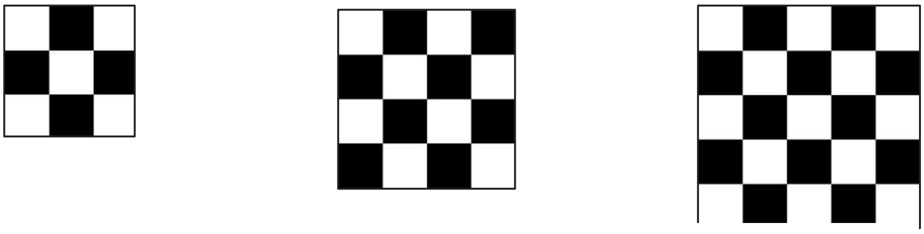


Figure 3 Specialised Chessboard Pattern

Developing a visual aid for the problem helps students with their problem-solving; however, within the above visuals we have also specialised the problem. Rather than continue to work with the next stage of the diagram, most students will begin to notice a pattern emerging, and thus find the answer and perhaps move on to generalise the problem. This emphasises the overlap of different strategies within each problem, and the facilitator ensures students recall this overlap as the module progresses.

The initial problem may be solved without too much difficulty, and so the facilitator is also responsible for pressing the students to explore the problem further. Students may determine an algebraic generalisation, but also display mathematical creativity in developing an extension to the problem. The creation of an extension allows students to take a problem which may not overly challenge them, and alter it to meet their own needs. Two examples of extensions to the problem have been to use a 3D model, or to seek the number of rectangles in a regular chessboard.

Example 2

The tutorial problem during week 5, under the theme “assumptions and questioning”:

> 25 coins are in a 5 by 5 array. A fly lands on a coin and wants to hop to every coin exactly once, at each stage moving only to an adjacent coin in the same row or column. Is it possible?
>
> (Mason et al., 2010, p. 161)

Once more, a visual aid will provide an introductory strategy to this problem. This may take the form of a diagram, such as that in Figure 4, or by students using physical objects to represent the fly and coins.

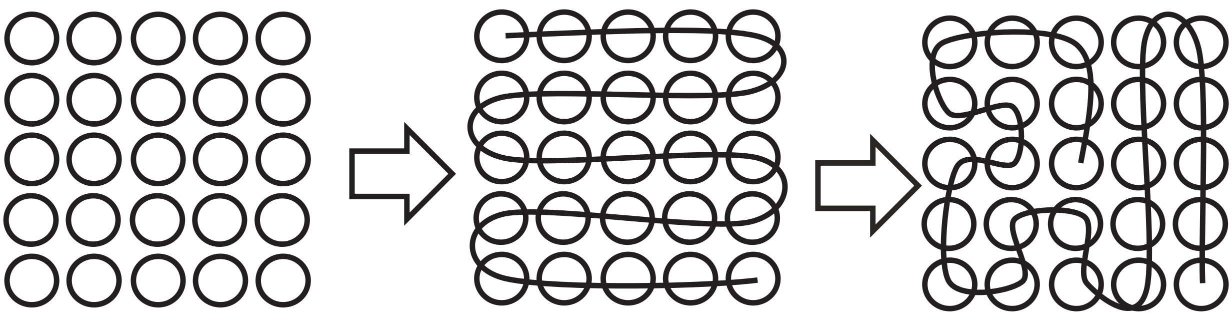


Figure 4 Fly Hopping Routes

Students usually begin with an opening array of coins on which they can attempt to solve the fly's path, and then proceed to choose various 'starting coins' to show that it works. The common assertion at this point is that it works and they declare the problem solved. While they have utilised a visual aid to reach this conclusion, students often 'assume true' for all coins. This assumption leads to an incorrect solution, and thus students are encouraged to ask more questions of the problem, such as "does it matter what coin the fly lands on", or "what happens if I change the number of coins?", exploring the problem in greater depth.

CONCLUSION

A problem-solving approach to instruction has been well-researched in recent decades in the education of highly-able students (Schiever & Maker, 2003; Strobel & Van Barneveld, 2009). When students are encouraged to ask further questions about a problem, they begin to 'think mathematically'(Schoenfeld, 1992), and develop the intrinsic motivation to understand the inner-workings of a problem. The module discussed in this paper utilises the problem-solving approach in an environment designed to nurture students' problem-solving abilities and higher-order thinking skills. Schoenfeld (1992) outlined the role the learning environment plays in developing this mode of thought within students. Collaborative learning through group-work gives students the opportunity to communicate their mathematical reasoning, and discuss alternative views on a problem. The role of the facilitator is to aid the student in these processes by encouraging them, asking probing questions, and promoting the desire to understand "why" a problem may be solved. In the early weeks of this module, the facilitator takes a more central role in questioning, to probe students' understanding of a problem. As students grow accustomed to this approach, they begin to ask the question of themselves and one another, thus lessening the role of the facilitator from week to week, and enabling them to develop as independent problem-solvers. Problem-solving is at the forefront of the design of Project Maths, and thus this module fits well into the Irish context, and also offers highly-able students the opportunity for challenge.

REFERENCES

Burton, L. (1984). Mathematical Thinking: The Struggle for Meaning. *Journal for Research in Mathematics Education*, *15*(1), 35–49. Retrieved from https://www.jstor.org/stable/748986?seq=1#metadata_info_tab_contents

Consortium of Institutions for Development and Research in Education in Europe (CIDREE). (2010). *Curriculum Provision for Exceptionally Able Students*. Retrieved from https://www.sess.ie/sites/default/files/Projects/Equality_of_Challenge/CIDREE.pdf

Department of Education and Skills (DES). (2010). *Report of the Project Maths Implementation Support Group*. Retrieved from https://www.education.ie/en/Publications/Policy-Reports/Report-of-the-Project-Maths-Implementation-Group.pdf

Department of Education and Skills (DES). (2017). *Junior Cycle Mathematics*. Retrieved from https://www.curriculumonline.ie/getmedia/6a7f1ff5-9b9e-4d71-8e1f-6d4f932191db/JC_Mathematics_Specification.pdf

Dods, R. F. (1997). An action research study of the effectiveness of problem-based learning in promoting the acquisition and retention of knowledge. *Journal for the Education of the*

Gifted, *20*(4), 423–437.

Fasko Jr., D. (2001). An Analysis of Multiple Intelligences Theory and Its Use with the Gifted and Talented. *Roeper Review*, *23*(3), 126–130. https://doi.org/10.1080/02783190109554083

Gallagher, J. J. (2003). Issues and Challenges in the Education of Gifted Students. In N. Colangelo & G. A. Davis (Eds.), *Handbook of Gifted Education* (3rd ed., pp. 11–23). Boston: Allyn and Bacon.

Gardner, H. (1999). *Intelligence Reframed: Multiple Intelligences for the 21st Century*. New York: Basic Books.

Gardner, H., & Hatch, T. (1989). Multiple Intelligences Go to School: Educational Implications of the Theory Multiple Intelligences. *Educational Researcher*, *18*(8), 4–10. Retrieved from http://www.jstor.org/stable/1176460

Gokhale, A. A. (1995). Collaborative Learning Enhances Critical Thinking. *Journal of Technology Education*, *7*(1), 22–30. Retrieved from http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.77.1338&rep=rep1&type=pdf#page=23

Hmelo-Silver, C. E. (2004). Problem-Based Learning: What and How Do Students Learn ? *Educational Psychology Review*, *16*(3), 235–267.

Hmelo-Silver, C. E., & Barrows, H. S. (2006). Goals and Strategies of a Problem-based Learning Facilitator. *Interdisciplinary Journal of Problem-Based Learning*, *1*(1), 21–39.

Katz, S., Segal, R., & Stupel, M. (2016). Using the Working Backwards Strategy of Problem-Solving in Teaching Mathematics to Foster Mathematics Self-Efficacy. *Pure and Applied Mathematics: Advances and Applications*, *15*(2), 107–144.

Kwon, O. N., Park, J. S., & Park, J. H. (2006). Cultivating Divergent Thinking in Mathematics through an Open-Ended Approach. *Asia Pacific Educational Review*, *7*(1), 51–61. Retrieved from https://link.springer.com/content/pdf/10.1007/BF03036784.pdf

Lewis, A., & Smith, D. (1993). Teaching for Higher Order Thinking (Summer, 1993). *Theory into Practice*, *32*(3), 131–137.

Mason, J., Burton, L., & Stacey, K. (2010). *Thinking Mathematically* (Second). Essex: Pearson Education Limited.

National Council for Curriculum and Assessment (NCCA). (2007). *Exceptionally Able Students: Draft Guidelines for Teachers*. Dublin.

National Council for Curriculum and Assessment (NCCA). (2012). *PROJECT MATHS: Responding to current debate*. Retrieved from https://www.ncca.ie/media/2275/project-maths-responding-to-current-debate.pdf

National Council for Curriculum and Assessment (NCCA). (2013). Post-Primary Overview: Project Maths. Retrieved from https://www.ncca.ie/media/3153/project-maths-research_en.pdf

National Council for Special Education (NCSE). (2013). *Supporting Students with Special Educational Needs in Schools*. Trim. Retrieved from http://ncse.ie/wp-content/uploads/2014/09/Supporting_14_05_13_web.pdf

O'Reilly, C. (2014). *Understanding Gifted Children*. Dublin.

Organisation for Economic Co-operation and Development (OECD). (2016). *Pisa 2015:*

Results in Focus. Paris.

Polya, G. (1957). *How To Solve It: A New Aspect of Mathematical Method* (2nd ed.). New York: Doubleday Anchor Books. Retrieved from https://is.muni.cz/el/1441/podzim2013/MA2MP_SMR2/um/polya--how_to_solve_it.pdf

Riedl Cross, J., Cross, T. L., O'Reilly, C., & Mammadov, S. (2014). *Gifted Education in Ireland: Educators' Beliefs and Practices*. Dublin.

Schiever, S. W., & Maker, C. J. (2003). New Directios in Enrichment and Acceleration. In V. Lanigan (Ed.), *Handbook of Gifted Education* (3rd ed., pp. 163–173). Boston: Allyn and Bacon.

Schoenfeld, A. H. (1992). Learning to Think Mathematically: Problem Solving, Metacognition, and Sense-Making in Mathematics. In D. Grouws (Ed.), *Handbook for Research on Mathematics Teaching and Learning* (pp. 334–370). New York: MacMillan.

Shen, Y. (2012). *Scaffolding Higher-Order Thinking during Ill-Structured Problem-Solving: A Conceptual Framework*. University of Georgia. Retrieved from https://getd.libs.uga.edu/pdfs/shen_yan_201205_phd.pdf

Sriraman, B. (2003). Mathematical Giftedness, Problem Solving, and the Ability to Formulate Generalizations: The Problem-Solving Experiences of Four Gifted Students. *Journal of Secondary Gifted Education*, *XIV*(3), 151–165.

Stepien, W. J., Gallagher, S. A., & Workman, D. (1993). Problem-Based Learning for Traditional and Interdisciplinary Classrooms. *Journal for the Education of the Giftedal*, *16*(4), 338–357. https://doi.org/10.1177/016235329301600402

Sternberg, R. J. (1984). Toward a triarchic theory of human intelligence. *The Behavioral and Brain Sciences*, *7*, 269–315. Retrieved from http://arthurjensen.net/wp-content/uploads/2014/06/1984-sternberg.pdf

Sternberg, R. J. (1999). The Theory of Successful Intelligence. *Review of General Psychology*, *3*(4), 292–316.

Strobel, J., & Van Barneveld, A. (2009). When is PBL More Effective? A Meta-synthesis of Meta-analyses Comparing PBL to Conventional Classrooms. *Interdisciplinary Journal of Problem-Based Learning*, *3*(1), 44–58. https://doi.org/10.7771/1541-5015.1046

Subotnik, R. F., Olszewski-Kubilius, P., & Worrell, F. C. (2011). Rethinking Giftedness and Gifted Education: A Proposed Direction Forward Based on Psychological Science. *Psychological Science in the Public Interest*, *12*(1), 3–54. https://doi.org/10.1177/1529100611418056

COOPERATIVE LEARNING IN INCLUSIVE SETTINGS IN PRIMARY MATHEMATICS – CONNECTING TASK DESIGN AND STUDENTS' SOLUTION STRATEGIES

Nina Flottmann

Bielefeld University, Institute for Didactics of Mathematics, Germany

As a primary teacher I find myself in an area of conflict between the heterogeneity of the primary students – irrespective of their specific intellectual, physical and social-emotional capacities, the design of tasks and learning environments and the aim to allow for cooperative learning, which plays an important role for motivation, development of social skills and social integration in school. Hence, the design-based research project that forms the basis of this paper seeks to investigate cooperative learning processes in inclusive classroom settings. Two crucial aspects of the study are the task design and the monitoring and documentation of children's solution strategies in cooperative settings.

INTRODUCTION

Following Feuser's (1998) paradigm with respect to 'joint learning on a shared topic' for inclusive education, teachers in Germany and other countries all over the world are presented with the challenge to develop learning environments and tasks that address students with a wide range of abilities and promote (mathematics) learning for all with and from each other. However, the implementation of this paradigm into practice, i.e. to include and to accept all the specific intellectual, physical, social as well as emotional capacities of students and their individual as well as collective mathematics learning processes, appears to be a special problem in mathematics classrooms, as research of teacher knowledge and beliefs by Korff (2015) suggests.

In order to overcome this theory-practice dilemma, teachers need to develop and install learning trajectories based on task design as well as children's solution strategies which are proven to foster mathematics learning in order to understand and connect individual learning with patterns of cooperation. Furthermore, teachers need to be able to

- assess where students stand in their mathematical development and understanding,
- know their students' special needs and
- scaffold and support their individual and collective learning.

To be able to do all of this in their classrooms, teachers need a good understanding of what constitutes a suitable task for individual and collective mathematics learning in inclusive settings as well as information about what solution strategies students might develop and execute and how an appropriate scaffolding can help them to overcome hurdles and difficulties they encounter along the way.

In this context, the design-based research project that forms the basis of this paper seeks to investigate cooperative learning processes in small group work. A crucial aspect of the study is the task design. Fermi problems have been found suitable for cooperative learning in mixed-ability groups (e.g. Peter-Koop, 2004). Hence, the following section will provide an overview of the literature on cooperative problem solving and modelling and identify requirements with respect to the study's task design. Furthermore, the methodological approach that guided the design of the study – design-based research – and its adaptation to the specific research interest will be explained. Finally, first results from a pilot study with third- and fourth-graders will be presented.

THEORETICAL FRAMEWORK

Fermi Problems in Cooperative Group Work

The curriculum for mathematics teaching and learning in German primary schools has got different topics; one of them is “measurement” (Ministerium für Schule und Weiterentwicklung des Landes NRW, 2008). The pupils develop sustainable fundamentals and knowledge skills to handle with different measurement (e.g. length, weight, time, money). One main goal is the acquisition of competences in modelling for skills that are needed in everyday life (Peter-Koop, 2003, p. 113). Modelling means complex and realistic problem solving with developing a mathematical model (Maaß, 2011). In access to international competitive studies different mathematical tasks are designed, which are challenging, meaningful, allow different strategies and solutions, proceeding competences (like modeling) and includes all facets of pupils´ abilities (Walther, Granzer, & Köller, 2008).

The idea of Fermi problems goes back to the Physician and Noble prize winner Enrico Fermi (1901-1954), who developed tasks for his students, which implicate several approaches with different answers and solutions (e.g. “How many piano turner live in Chicago?”). These problems have a high degree of complexity and can only be solved by giving a reasonable estimate. Peter-Koop (2004) investigated primary children’s mathematical modelling of Fermi problems. Furthermore, Fermi problems such as *How many cars will be caught in a 3 km traffic jam on the motorway?* share the characteristic that the initial response of the problem solver is that the problem could not possibly be solved without recourse to further reference material. However, while individuals frequently reject these problems as too difficult, Clarke and McDonough (1989) pointed out that “pupils, working in cooperative groups, come to see that the knowledge and processes to solve the problem already reside within the group” (p. 22).

The analysis of the classroom-based data (Peter-Koop, 2004) indicated that Fermi problems can be solved in sensible and appropriate ways by third and fourth graders in mixed ability groups and even in groups with typically rather low achieving children. While in traditional problem solving at primary school level only one modelling cycle is needed, Fermi problems can serve as “model-eliciting tasks” (Lesh & Doerr, 2000, p. 380), because the required modelling process extends beyond the application of a standard algorithm and necessitates multiple modelling cycles with multiple ways of thinking about givens, goals and solution paths (Bell, 1993). Lesh and Doerr (2000) point out that model development is learning. Hence, the outcome of the modelling activity can be a conceptual tool that exceeds the solution of a specific problem. The results of the study by Peter-Koop agrees with the findings from the analyses of secondary students’ modelling processes by Lesh and Doerr (2000) who highlighted “the need for teachers to examine students’ developing models in order to assess student knowledge and understanding and to foster continued model development in ways that evolve as the student models evolve” (p. 375).

The curriculum for mathematics teaching and learning in German primary schools requires that teachers initiate and foster cooperative learning among their students (Ministerium für Schule und Weiterentwicklung des Landes NRW, 2008, p.14). The social integration and the cooperative learning of being involved with all individuals’ abilities is one of demands. Pupils should be lead to argue, accept other perspectives, deal with other vies and express their own thoughts and strategies (Ministerium für Schule und Weiterentwicklung des Landes NRW, 2008, p.10 ff). Teachers can best accommodate this requirement by developing cooperative settings in which communication and cooperation are also the motor for individual learning. Johnson and Johnson (1999) have established core elements for cooperative teamwork, i.e. positive interdependence, individuality, accountability, social skills, face-to-face interaction and group processing. Based on their initial work following publications introduce a wide scope of teaching strategies that aim to initiate and foster cooperative learning (e.g. Green & Green, 2005; Johnson, Johnson, & Holubec, 2005). However, these strategies rather address teacher-initiated group work.

In contrast to these approaches, in her doctoral dissertation Röhr (1995) advocates and investigates student cooperative group work that results from the mathematical tasks themselves and is not initiated 'on top' of a task by the teacher or the textbook. She argues that cooperative skills can best develop when the need to cooperate with others is not initiated or requested by the teacher but rather comes from within the problem solving task that requires the students to jointly look into the ask, to discuss possible strategies, to argue for specific approaches and to jointly develop a solution as their cooperation has a shared goal (ibid, p. 75). Cooperative group should be explored by a significant Fermi problem, which leads by discussion and exploration to a solution.

However, typically not all of the students choose to participate in these joint activities. Engaging these children in cooperative group discussions presents a special challenge for teachers. It is therefore a particular research interest of this project to analyze how primary students cooperatively solve Fermi problems that relate to their real-world experiences as well as to their mathematical competencies with respect to their mathematical learning processes and the arising cooperative patterns of interaction.

The methodological approach chosen with respect to this specific research interest is design-based research as it allows to include and connect the design of the specific tasks to be used in the classroom-based study with empirical insights into the occurring learning processes (e.g. Prediger, 2018).

Design-based Research

The fundamental intention of design-based research in education as described by Bakker (2018) is also known under various other related terms characterizing similar approaches: *educational design research* (Plomp & Nieveen, 2013; van den Akker, Gravemeijer, McKenney, & Nieveen, 2006), *design experiments* (Brown, 1992; Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003), *design science (*Collins*,* 1990; Wittmann, 1995) or *didactical design research* (Prediger, 2018) to name the most prominent terms. All these approaches have a common goal, that is to investigate a problem arising from and identified through classroom practice. In a first step the identified problem is related to theory. Following is the development, implementation and analysis of an intervention resulting in the characterization and analysis of the design and develop it for re-design. Plomp and Nieveen (2013) describe this approach as *"the systematic analysis, design and evaluation of educational interventions with the dual aim of generating research-based solutions for complex problems in educational practice, and advancing our knowledge about the characteristics of these interventions and the processes of designing and developing them"* (p. 18).

Prediger (2018, p. 33) highlights two main aims of design research: (1) to develop learning arrangements for supporting classroom practice, and (2) to gain insights in mathematical arrangements for didactical theory formation. In order to reach these aims and to classify for design-based research, according to Bakker (2018, p. 18) five significant characteristics have to be fulfilled: (1) Development of theories about learning and how to support learning, (2) Interventionist nature of the research, (3) Prospective and reflective components, (4) Invention and revision in order to form an iterative process, and (5) Transferability. Plomp & Nieveen (2013, p. 17) provide the following illustration of the research process and its design cycles:

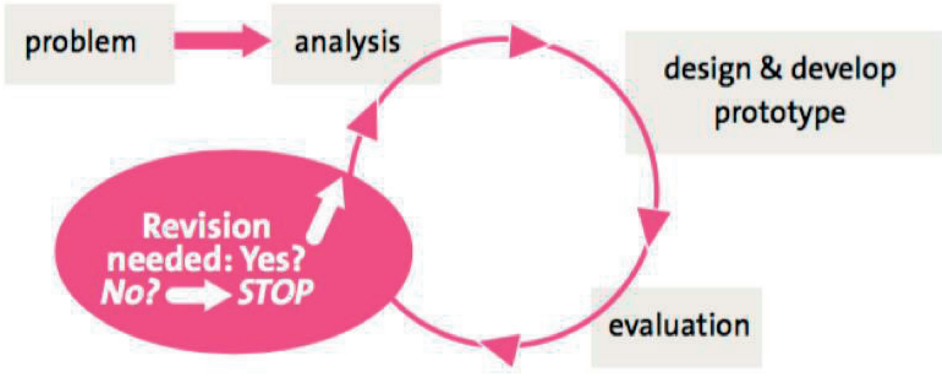


Figure 1: Iterations of systematic design cycles (Plomp & Nieveen, 2013, p.17)

METHODOLOGY

Investigating Cooperative Group Work through Design-based Research

Transferring the theory of design-based research to the empirical study described in this paper the main *problem* can be summarized as the tension between the demand for providing inclusive mathematics education and the need to support and foster all students irrespective of their individual capacities and special needs. The special focus is solving Fermi problems in cooperative group work. The *design and development of a prototype* is the investigation of a mathematical learning environment (Fermi problems) to be *evaluated* and optimized for students and teachers. The following figure based on the original illustration by Plomp and Nieveen (2013) (see Fig. 1) depicts this cyclic process and identifies the main characteristics of this project in the process of design-based research.

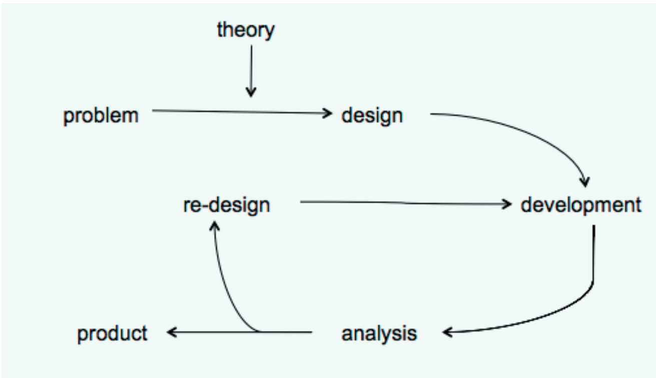


Figure 2: Adapted cyclical process

With respect to cooperative learning another research setting aims at the optimization of the learning environment. The challenge is to move beyond cooperative learning as a kind of 'social decoration' to cooperative group work that is predominantly inherent in the mathematical task. In this view the key question is how to support and scaffold students in their cooperative learning without restricting their mathematical thinking and understanding. The assignments must be designed that cooperative learning is useful and necessary.

Research Questions

The following research questions guide this design-based research project:

- How can mathematical learning environments be developed that promote and encourage students with a wide range of abilities?
- Which strategies do students demonstrate and use to solve a Fermi problem? What kind of support is helpful and/or needed?
- Which processes are mappable in cooperative group work? What constitutes mathematical learning environments that are suitable for inclusive settings?

Another aspect that erases from the analysis is the crucial role the teacher plays, which must be focused as well (Henningsen & Stein, 1997; Leiss, 2007).

Data Collection and Analysis

For the data collection of the pilot study small groups of up to five children from inclusive Grade 3 and Grade 4 mathematics classrooms have been videotaped while solving a Fermi problem. The selection of the groups was determined by parental consent.
During the first round the groups were determined by the home group teacher who favored ability grouping. In the second round the mathematics teacher was responsible for the grouping and chose mixed-ability groups. Data from the third round was collected at the university math lab which is visited by classes from the local primary schools after prior registration. Since neither the children nor their potential special needs were known to the lab staff, here the groups were chosen arbitrarily. This heterogeneity of the groups is important for researching if cooperative learning that is task based is possible, useful and meaningful.
The video recordings of the groups were transcribed following explicit and previously determined transcription rules. In addition, the course of the group work was recorded in the form of episode plans (i.e. a protocol of the chronological sequence of the main steps followed by the group). Children's solution strategies were analyzed based on a multi-cyclic model of the modelling process (as described by Peter-Koop (2004)). Further analyses will include the use of MaxQDA on the written transcripts in order to investigate and describe the cooperative processes displayed during the group work.

FIRST RESULTS OF THE PILOT STUDY

In order to relate problems from the students' real-world experiences (e.g. see Maaß, 2004, p.14ff) and their classroom experience, three settings have been chosen as starting points for further mathematical investigations:
Setting 1: Towards the end of the school year a Grade 4 class was planning a church service focussing the symbol "door" and had worked on the topic interdisciplinary from a variety of perspectives in several school subjects. The students started to think about, how often they themselves went through different doors within the school building and the question arose: "How many times in the past four school years have the students of our class been going through our classroom door?"
Setting 2: All the Grade 3 students of a local primary school had to attend a bicycle training and pass a test to be allowed to come to school by bike. As in the years before, on the days of the training, there were bicycles parked all over the school yard. This lead to the following question: "If all the bicycles of the third graders are lined up – one behind the other – how long would that 'chain of bicycles' be?
Setting 3: The Fermi problem that the groups who visited the university lab school encountered, is related to a specific feature of the university building and its various elevators – a feature that provides a certain degree of fascination for the visiting students. Hence, they could easily relate to the question: "How many children can simultaneously ride in all the elevators of the main building?"

Initial analyses of the video transcripts and episode plans indicate that the approaches taken by the different groups vary substantially. Each group chose a different starting point for the modelling activities and subsequently followed different strategies. Some of the groups quickly developed an expedient approach, while it took other groups much longer to agree on a solution strategy.
As an example is shown an episode plan in figure 3 and one part of the modelling cycle of a third grade group dealing with the task of setting 2 - "bicycle chain" in figure 4
.

time	phase / abstract
start till #00:02:02-2#	introduction / task:
#00:02:02-3# till #00:06:27-0#	argumentation and processing I: finding a representative; agreement: 1,60 m as a representative for 1 bicycle
#00:06:27-1# till #00:12:08-2#	argumentation and processing II: group calculates length of bicycles in a bicycle chain for one class
#00:12:08-4# till #00:12:24-6#	introduction / reminding the task
#00:12:24-7# till #00:14:57-6#	processing III: group calculates lenght of bicycles for the next third grade
#00:14:57-7# till #00:15:32-7#	agreement: How many classes belong to the third graders?
#00:15:32-8# till #00:16:17-6#	processing IV: group calculates lenght of bicycles for the next third grade
#00:16:17-6# till #00:17:06-9#	processing V: group calculates lenght of bicycles for the next third grade
#00:17:07-0# till #00:18:38-9#	processing VI: group calculates lenght of bicycles for the next third grade
#00:18:39-0# till #00:23:40-1#	processing and calculation: total length of a bicycle chain of all bicycles of the third graders

Figure 3: episode plan / task "bicycle chain" / group D

The detailed transcribed episode will be shown during the conference.
Further analyses of the videos as well as the transcripts show that all groups have worked intensively and productively. They could identify with the respective task and work together cooperatively. Only few children chose to work subsidiary or co-existent (Wocken, 1998). Further stages of the project will focus on these children and the optimization of the task design in this respect.
Another focus of analysis will be the different grouping strategies – ability grouping, mixed-ability grouping and random grouping. With only one exception, all groups worked productively and naturally together.
During the conference presentation more detailed results from the different groups will be introduced and discussed, including excerpts from the group discussions, the episode plans and the respective modelling cycles.

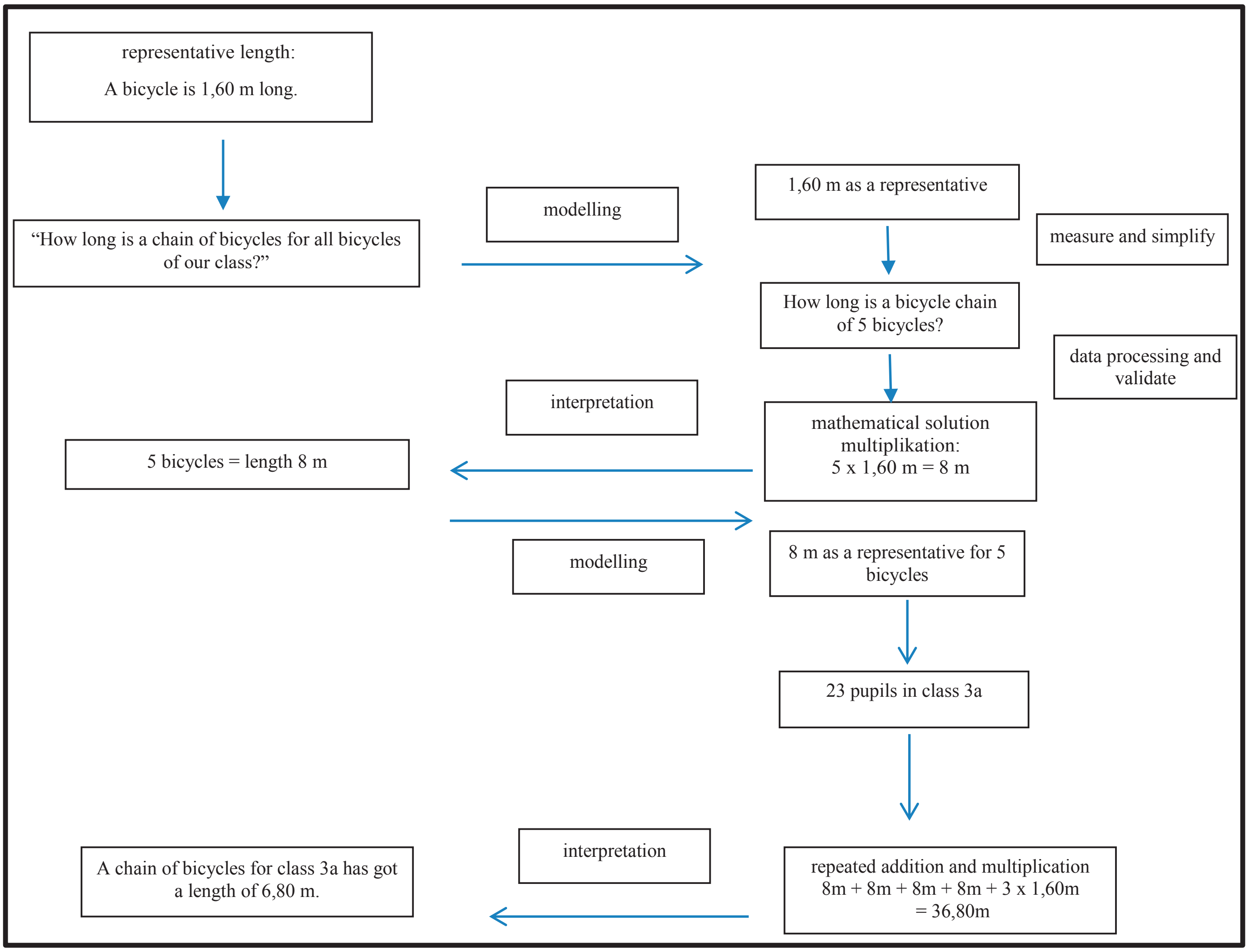


Figure 4: modelling cycle / task "bicycle chain" / group D

REFERENCES

Brown, A. L. (1992). Designs Experiments: Theoretical and methodological challenges in creating complex interventions in classroom settings. *The Journal of the Learning Sciences, 2*(2), 141–172.

Clarke, D. J., & McDonough, A. (1989). The problems of the problem solving classroom. *Australian Mathematics Teacher, 45*(2), 20–24.

Cobb, P., Confrey, J., diSessa, A., Lehrer, L., & Schauble, L. (2003). Design experiments in educational research. *Educational researcher, 32*(1), 9–13.

Collins, A. (1990). *Toward a Design Science of Education: Technical report No.1.* New York: Center for Technology in Education.

Feuser, G. (1998). Gemeinsames Lernen am Gemeinsamen Gegenstand: Didaktisches Fundamentum einer allegmeinen (integrativen) Pädagogik. In A. Hildeschmidt & I. Schnell (Eds.), *Integrationspädagogik. Auf dem Weg zu einer Schule für alle* (pp. 19–35). Weinheim: Juventa.

Green, N., & Green, K. (2005). *Kooperatives Lernen im Klassenraum und Kollegium: Das Trainingsbuch.* Seelze: Kallmeyer.

Henningsen, M., & Stein, M. K. (1997). Mathematical Tasks and Student Cognition: Classroom-Based Factors That Support and Inhibit High-Level Mathematical Thinking and Reasoning. *Journal for Research in Mathematics Education, 28*(5), 524–549.

Johnson, D. W., Johnson, R., & Holubec, E. (2005). *Kooperatives Lernen, kooperative Schule: Tipps - Praxishilfen - Konzepte.* Mülheim: Verlag an der Ruhr.

Johnson, D. W., & Johnson, R. (1999). *Learning together and alone: Cooperative, competive and individualistic learning.* Boston: Allyn & Bacon.

Korff, N. (2015). *Inklusiver Mathematikunterricht in der Grundschule: Erfahrungen, Perspektiven und Herausforderungen* (1. Aufl.). *Basiswissen Grundschule: Vol. 31.* Baltmannsweiler: Schneider Hohengehren.

Leiss, D. (2007). *"Hilf mir, es selbst zu tun": Lehrerinterventionen beim mathematischen Modellieren. Texte zur mathematischen Forschung und Lehre: Vol. 57.* Hildesheim, Berlin: Franzbecker.

Lesh, R., & Doerr, H. (2000). Symbolizing, communicatin and mathematizing: Key components of models and modeling. In P. Cobb, E. Yackel, & K. McClain (Eds.), *Symbolizing and communicating in mathematics classrooms* (pp. 361–383). Mahwah, New York: Erlbaum.

Maaß, K. (2004). *Mathematisches Modellieren im Unterricht: Ergebnisse einer empirischen Studie. Texte zur mathematischen Forschung und Lehre: Vol. 30.* Hildesheim: Franzbecker.

Maaß, K. (2011). *Mathematisches Modellieren in der Grundschule.* Kiel: IPN Leibniz-Institut f. d. Pädagogik d. Naturwissenschaften an d. Universität Kiel.

Ministerium für Schule und Weiterentwicklung des Landes NRW (2008). *Richtlinien und Lehrpläne für die Grundschule in Nordrhein-Westfalen.* Frechen: Ritterbach.

Peter-Koop, A. (2003). "Wie viele Autos stehen in einem 3-km-Stau?" - Modellbildungsprozesse beim Bearbeiten von Fermi-Problemen in Kleingruppen. In S. Ruwisch & A. Peter-Koop (Eds.), *Gute Aufgaben im Mathematikunterricht der Grundschule* (pp. 111–130). Offenburg: Mildenberger.

Peter-Koop, A. (2004). Fermi problems in primary mathematics classroom: Pupils' interactive modelling process. In I. Putt, R. Faragher, & M. McLean (Eds.), *Mathematics education for the third millenium: towards 2010. Proceeding of the 27th annual conference of the Mathematics Education Research Group of Australia, Townsville* (pp. 454–461). Sydney: MERGA.

Plomp, T., & Nieveen, N. (2013). *Educational Deign Research.* Enschede: SLO.

Prediger, S. (2018). Design-Research als fachdidaktisches Forschungsformat. In Fachgruppe für Didaktik der Mathematik der Universität Paderborn (Ed.), *Beiträge zum Mathematikunterricht* (pp. 33–40). Münster: WTM.

Röhr, M. (1995). *Kooperatives Lernen im Mathematikunterricht der Primarstufe.* Wiesbaden: Deutscher Universitätsverlag.

van den Akker, J., Gravemeijer, K., McKenney, S., & Nieveen, N. (2006). *Educational design research.* London: Routledge.

Walther, G., Granzer, D., & Köller, O. (Eds.) (2008). *Lehrer-Bücherei: Grundschule. Bildungsstandards für die Grundschule: Mathematik konkret* (7. Auflage). Berlin: Cornelsen.

Wittmann, E. C. (1995). Mathematics Education as a 'design science'. *Educational Studies in Mathematics, 29*(4), 355–374.

Wocken, H. (1998). Gemeinsame Lernsituationen: Eine Skizze zur Theorie des gemeinsamen Unterrichts. In A. Hildeschmidt & I. Schnell (Eds.), *Integrationspädagogik. Auf dem Weg zu einer Schule für alle* (pp. 37–52). Weinheim: Juventa.

LESSON PLAY: SUPPORTING PRE-SERVICE TEACHERS TO ENVISAGE PUPILS' SENSE-MAKING IN MATHEMATICS LESSONS

Paul Grimes[1], Thérèse Dooley[1] and Sara O' Donnell[2]

[1]CASTeL & School of STEM Education, Innovation & Global Studies, Dublin City University

[2]St Joseph's Primary School, Dún Laoghaire

In this paper the potential of Lesson Play in mathematics teacher education is explored. Through the process of script writing in Lesson Play, teachers imagine their own responses to classroom situations. We describe how script writing has the potential to help pre-service teachers envisage ways in which pupils make sense of mathematics, and become more aware of the teacher moves that allow pupils to articulate and modify ideas in mathematics lessons. We analyse the lesson script of one pre-service teacher with reference to Grice's Conversational Maxims, and discuss ways in which Lesson Play can be developed to further enhance pre-service teachers' ability to facilitate classroom discussions.

INTRODUCTION

Socio-constructivist perspectives on mathematics teaching and learning have gained considerable traction in recent years. From these perspectives, the learning of mathematics is seen as a social process in which the teacher and students co-construct ideas within the domain through talk and argumentation. While the relationship between mathematics and language has various interpretations in the research literature, the position we take is that "doing mathematics essentially entails speaking mathematically" (Morgan, Craig, Schuette & Wagner, 2014, p.846). As elaborated by Rowland (2000), this is strongly linked with a view of mathematics as the product of human activity and interpersonal dialogue, leading to classroom practices where pupils are encouraged to articulate ideas and modify them as necessary in order to make sense of mathematics.

The importance of discussion and communication in mathematics lessons is emphasized in the 1999 Irish Primary School Mathematics Curriculum (Gov. of Ire., 1999). However, there is considerable evidence that teachers continue to control much of the talking that occurs. For example, one of the findings of TIMSS 2015, in which fourth class children's mathematical performance was assessed, was that 73% of pupils in Ireland were asked to listen to their teacher explaining new content in 'every or almost every lesson' (Clerkin, Perkins, & Chubb, 2017). In contrast with this, 34% of pupils work on problems together in the whole class with direct guidance from the teacher in most or all lessons. While the orchestration of mathematical discussion is challenging for teachers, it is particularly so for pre-service teachers (PTs) who are often uncomfortable in a classroom environment where they cannot take complete control of the direction of a lesson (e.g., McGlynn-Stewart, 2010).

In this paper we explore how Lesson Play (LP) offers a means of helping PTs to envisage the ways in which pupils use language to make sense of new mathematical ideas and, moreover, the moves a teacher might make to facilitate the development of this sense-making. LP allows teachers to imagine their own responses to particular classroom situations, and envisage how the conversation between the learner and a teacher might proceed. We use an LP script created by a PT to demonstrate this and consider the implications for further development of this approach.

CONVERSATIONAL MAXIMS

Taking the position on language and mathematics outlined in the introduction, we argue that pupils make sense of mathematics by articulating their ideas and modifying them as necessary. This suggests that mathematics lessons in which sense-making is at the core are characterised by a 'to-ing and fro-ing of ideas' such as applies in a conversation. If this is the case, it could be expected that conversational maxims would apply. The philosopher, Paul Grice, proposed that normal conversation is based on co-operative principles, meaning for which can be found in 'maxims' of conversation that specify what the participants have to do to ensure that their conversation is co-operative and rational (Grice, as cited by Rowland, 2000, p.81-82):

- Quality: Let your contribution be truthful; do not say what you believe to be false.
- Quantity: Let your contribution be as informative as required (for the current purposes) and not be more informative than is required.
- Manner: Let your contribution be clearly expressed, e.g., be brief, orderly, unambiguous.
- Relevance: let your contribution be relevant to the matter in hand.

The maxims are supposed to apply both to the delivery and the interpretation of messages but it is not the case that they are always observed. Grice maintains, however, that participants of a conversation behave as if cooperative principles are being upheld. The following interaction is a case in point:

Teacher: Where is your home exercise?

Student: My aunty called last night

Although it might seem that the student is not addressing the teacher's question, the teacher might infer that she did not do her home exercise because her aunt called on the previous evening. In other words, the student's input is interpreted by the teacher as if there is conformance to the maxims at least at some level.

Rowland (2000) reminds us that 'co-operative' in the Gricean sense is not necessarily associated with pleasantness but has more to do with the 'sense-making' of spoken interactions of the participants of a conversation. He also contends that Grice's Cooperative Principles, can account for many of the vague features of conversation. For example, citing Brockway (1981), Rowland describes the word 'well' as a maxim hedge - it is often used by speakers to notify the hearer that a contribution will in some respect fall short of one or more of Grice's maxims. For example, in calculating the sum of two numbers, say 25 + 27, a pupil might make the following contribution in whole-class conversation:

Áine: Well, I got 52.

Here Áine uses 'well' to indicate that her input might not meet the requirement of the maxim of quality, that is, she is not entirely sure that her contribution is truthful. There are other ways that speakers might convey to their audience the awareness that they are violating the Gricean principles, for example, pausing, giving hints and clues, under- and over-elaborating statements, being ironic and using rhetorical questions (Bills, 2000; Rowland, 2000).

Teacher moves can also be described in terms of the Gricean principles (Forman and Larreamendy-Joerns, 1998). Among teacher moves associated with sense-making mathematics lessons are those of 'press' and 'revoicing' (Brodie, 2011). A press move occurs when a teacher asks a learner to elaborate, clarify, justify or explain an idea while a revoicing move is seen when a teacher repeats or rephrases a student's idea. Forman and Larreamendy-Joerns (1998) contend that there is often a discrepancy between what students take for granted as understood and what teachers are willing to accept as explicit information. While everyday and mathematical conversations both depend on the co-operative principle, the degree of accountability differs in the case of each. The degree of accountability is concerned with the level of explanation that participants are expected to make. Everyday explanations are highly condensed because of familiarity, shared history, trust etc. More extensive explanations are required in the sciences. In the mathematics classroom, requests by the teacher for further explanation serve in general to develop appropriate socio-mathematical norms. These are norms that pertain to normative aspects of students' mathematical activity, for example, what counts as a different solution, a sophisticated solution, an efficient solution, and an acceptable explanation as constituted in classroom interaction (Cobb and Yackel, 1998). Teachers' conversational meta-messages, of which revoicing and requests for explanation are examples, invoke the Gricean maxims by conveying to students the need to provide explanations that are 'explicit, relevant, orderly, precise and informative' (Forman and Larreamendy-Joerns, 1998, p.111) and thus help to build a bridge between every day and mathematical explanations. It would seem then that PTs should be aware of these maxims. LP, described next, is a context where this awareness might be developed.

FICTIONAL DIALOGUES AND LESSON PLAY

The use of fictional dialogues in mathematics education has had many different purposes over the past number of years (see Crespo, Oslan & Parks, 2011). In mathematics teacher education, one approach in which fictional dialogues are utilised is LP. Here, teachers write a script of an imagined dialogue between the teacher and students or between a group of students (Zazkis, Liljedhal & Sinclair, 2009). It was first introduced as an alternative way to allow teachers to anticipate students' ideas, providing "an opportunity to imagine the future, being informed by the past" (p. 46). It usually follows a prompt, e.g., the beginning of a dialogue in which there is a misconception or gap in a learner's understanding. Following this prompt a script is written, usually involving an interaction between teacher and pupil. The script is informed by the writers' (PTs') own learning, teaching and research experience. LP as presented in this paper did not begin with a prompt. The reason for this was that we believed it would allow the PTs to draw on their own experience to produce the script, and not focus only on addressing the issue pertaining to the prompt. The PTs' experience encompassed both a practicum (school placement) and a literature review conducted as part of the LP process.

LESSON PLAY: AN EXAMPLE

The lesson script outlined in this paper was written by a PT, Sara, in the 4th year of a Bachelor of Education programme. At this stage, PTs have undergone a number of weeks of school placement. The PTs in this programme complete a final year undergraduate research project in a subject area of their choice. The grade awarded for this project contributes to the final marks they receive for their degree. Sara was one of a group who chose to conduct research in mathematics education using LP. As part of this, PTs had to reflect on a previously taught lesson, then design a new lesson plan based on this reflection and a literature review. Finally, they engaged in LP. PTs were asked to imagine a scene (or several scenes) that might occur in the lesson, and to write and analyse a script for the interaction between students in the class and the teacher during that scene. We chose Sara's script because it exemplified, more than scripts written by other PTs in the group, some of the conversational maxims described above.

Sara explored the idea of differentiation in multi-grade classroom consisting of 3rd and 4th class children (aged 8-10 years). The focus of the lesson in 3rd class was 'regular tessellations', while 4th class children considered 'semi-regular tessellations' [1]. For her LP, Sara wrote a script for a scene that involved the teacher and six children (three from 3rd class and three from 4th class). Sara analysed this lesson script with reference to her own research question. However, for the purpose of this paper we are focussing not on her analysis but on the script itself, in particular, the ways in which she presented the classroom interactions. We analyse her script from the perspective of conversational maxims and teacher moves, although these were not explicitly taught to PTs as part of the undergraduate research module.

Analysis of the script

In Sara's script (see Appendix) we can see some examples of her use of the 'press' move. For example, in the interchange:

Teacher:	Good. Now that we know that squares tessellate. What do we know about tessellation?
Shane (3rd):	It means that when you make a pattern, the shapes fit together perfectly.
Teacher:	Exactly Shane. But what do we need to be careful about when making patterns that tessellate?
Kevin (3rd):	Shapes don't tessellate if there are any gaps... or overlapping shapes in the pattern.
Teacher:	That's correct.

In everyday conversation about, say, tiling the explanation given by Shane that tessellation means that '…shapes fit together perfectly' would be adequate. The meaning of 'perfectly' could well be inferred by the other party in the conversation to mean 'without gaps'. It seems that the teacher is happy that Shane has an adequate understanding of the concept ('Exactly') but her follow-up question ('What do we need to be careful about?') suggests that she feels some duty to the other pupils in the setting. Here she is pressing them for an explanation that fulfils the Gricean maxim of quantity (i.e., 'Let your contribution be as informative as required for the current purposes'). Kevin does exactly that when he proposes that 'Shapes don't tessellate if there are any gaps... or overlapping shapes in the pattern'. This explanation is sufficient for this group of children since the topic of tessellation is first introduced in 3rd class (Gov. of Ire., 1999).

There are other reasons that the teacher may have been happy with Shane's description of tessellation. His use of the pronoun 'you' tells something of his understanding. Rowland (1999) suggests that pupils seldom use the term 'you' to address a teacher in classroom because of asymmetrical power relationship in adult-child mathematical conversations. However, pupils often use the pronoun 'you' in such conversations. He contends that 'you' in such instances tends to refer to something rather than someone – that is, it can function as a 'generaliser' pointing to what happens 'every time'. It can be inferred from Shane's use of the word 'you' that he had generalised his understanding of tessellation.

The follow-up conversation on tessellation with the 3rd class pupils is characterised by greater certainty on the part of the pupils. There is very little hesitation in their deliberations and in general they use declarative sentences, that is, sentences that assert how things are (Vanderveken, 1990). For example, when asked to identify shapes that form a regular tessellation, Ciara says, 'And triangles! Because equilateral triangles have the same length of side and their angles are the same size too so that means they tessellate'. Although further press on the matter of equal angles might have injected more vagueness into the pupils' input, the next example of a violation of the maxims of conversation occurs when Sara introduces semi-regular tessellation to the older pupils. For example, Anna uses the maxim hedge 'Well' in the following exchange:

Teacher: Well done. Now, 4th class, watch carefully as to how I make this pattern (pause). How is it different to the last pattern?

Anna (4th): Well, you used more than one shape.

It seems that Anna understands that her suggestion might fall short of the maxim of quality and her 'Well' serves to give notice of this. While it is true that more than one shape has been used in the pattern, Anna is probably aware that this response will not satisfy this classrooms norms for a satisfactory explanation – as has already been displayed in the conversation with the third-class children. In fact, Sara demonstrates in her script that the description of a semi-regular tessellation might prove difficult for these children as Lucy's contribution is laced with hesitation:

Lucy: Isn't it that all the corners in the pattern have to be the same? So for that pattern with hexagons and squares, if you picked one corner at the top of the square, each square would have to always have two hexagons touching it... is that right Ms.?

In Sara's LP, the pattern shown to the children consisted of one made by regular octagons and squares (see Figure 1). This is significant since a semi-regular tessellation with hexagons and squares also includes equilateral triangles. Moreover, in the semi-regular tessellation of regular octagons and squares, the 'corner' of each square does have two regular octagons touching it.

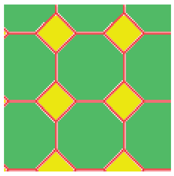


Figure 1: Semi-regular tessellation created using regular octagons and squares

Lucy's explanation is correct in terms of the tessellation presented in the lesson (Fig. 1) and it can be assumed that her use of the word, 'hexagons' is a slip occasioned by Sara in her writing of the lesson script. However, she prefaces her input with a question which we can assume to be rhetorical since, as evidenced in the transcript, she does not appear to pause for a response. This shows that she is aware that her input might not comply with the maxim of quality. Her next sentence is more convincing. Like Shane from 3rd class, her use of the pronoun 'you' indicates her belief that the polygons in question are arranged the same at every vertex, a definition that is key to semi-regular tessellation. Her generalisation of this can also be inferred by her use of the word 'always' later in the sentence. Her question - 'is that right, Ms?' serves a different purpose to that at the beginning of this turn. It reveals her awareness (and Sara's) that the teacher has asked a question to which she knows the answer - a common trait of classroom discussion. Sara's affirmation of Lucy's input and what seems to be her oversight of the slip ('hexagons') is also consistent with classroom practice. As described by O'Connor (2001), at any one moment there are several demands competing for the teacher's attention – the alignment of students with each other, sensitivity to individual students, the maintenance of mutual respect and trust, the development of social norms and socio-mathematical norms, the coordination of a student's own ideas with those of the class and with the accepted mathematical practices of the school and wider community. In a real life sense-making lesson, it is very likely that a teacher would be impressed by the sophistication of Lucy's understanding of semi-regular tessellation and consequently might not notice the slip. While Sara may not have deliberately planned this error in her script, it represents a reflection of actual talk in a sense-making classroom. In the development of LP with PTs, an example such as this could serve as an important reflective piece - reminding PTs that conversation in a sense-making classroom can have many twists and turns.

CONCLUSION

In this paper we explored the potential of LP to support PTs' engagement in sense-making conversations with pupils in mathematics lessons. It is important to note we did not provide PTs with an opening prompt, e.g. a classroom scenario where there was either a misconception or an alternative understanding on the part of a pupil. It would seem that in not providing a prompt, Sara was encouraged to focus the discussion not on ways to correct student misconceptions, but rather on how she could facilitate a sense-making discussion in the classroom. In her script Sara, showed an awareness of (a) the ways in which pupils 'try out' new ideas in sense-making mathematical conversations and (b) the teacher moves that prompt the accountability that is necessary for development of disciplinary understanding. We believe that this indicates her engagement in a fictional dialogue, that is, she entered into the classroom as if it were real. It is reasonable to expect that she will carry some of these teacher moves into her mathematics lessons in the future. It is also reasonable to suggest that LP offers a realistic way in which PTs begin to give careful consideration to how children make sense of new mathematical ideas. While Sara analysed her LP from a different perspective, consideration should be given to introducing PTs to conversational maxims in future courses. This might enable greater focus by PTs on sense-making mathematical discussions but this warrants further investigation.

NOTES

1. For the purpose of these lessons, regular tessellations were defined as tessellations made using a single regular polygon, and semi-regular tessellations were defined as tessellations made using a combination of two or more regular polygons.

REFERENCES

Bills, L. (2000). Politeness in teacher-student dialogue in mathematics: A socio-linguistic analysis. *For the Learning of Mathematics*, 20(2), 40-47.

Brodie, K. (2011). Working with learners' mathematical thinking: Towards a language of description for changing pedagogy. *Teaching and Teacher Education*, 27(1), 174-186.

Clerkin, A., Perkins, R. & Chubb, E. (2017). *Inside the primary classroom: What happens in Fourth Class?* Dublin: Educational Research Centre.

Crespo, S., Oslund, J. A., & Parks, A. N. (2011). Imagining mathematics teaching practice: Prospective teachers generate representations of a class discussion. *ZDM*, 43(1), 119-131.

Cobb, P., and Yackel, E. (1998). A constructivist perspective on the culture of the mathematics classroom. In F. Seeger, J. Voigt and U. Waschescio (Eds.), *The culture of the mathematics classroom* (pp. 158-190). Cambridge: Cambridge University Press.

Forman, E. A., and Larreamendy-Joerns, J. (1998). Making explicit the implicit: Classroom explanations and conversational implicatures. *Mind, Culture and Activity*, 5(2), 105-113.

Gov. of Ire. (1999). *Primary curriculum: Mathematics*. Dublin: The Stationery Office.

McGlynn-Stewart, M. (2010). Listening to students, listening to myself: Addressing pre-service teachers' fears of mathematics and teaching mathematics. *Studying Teacher Education*, 6(2), 175-186.

Morgan, C., Craig, T., Schuette, M., & Wagner, D. (2014). Language and communication in mathematics education: An overview of research in the field. *ZDM*, 46(6), 843-853.

O'Connor, M. C. (2001). "Can any fraction be turned into a decimal?" A case study of a mathematical group discussion. *Educational Studies in Mathematics*, 46, 143-185.

Rowland, T. (2000). *The pragmatics of mathematics education: Vagueness in mathematical discourse*. London: Falmer Press.

Vanderveken, D. (1990). *Meaning and Speech Acts: Principles of Language Use (Vol. 1)*. Cambridge: Cambridge University Press.

Zazkis, R., Liljedahl, P., & Sinclair, N. (2009). Lesson plays: Planning teaching versus teaching planning. *For the learning of mathematics*, 29(1), 40-47.

APPENDIX

T = Teacher, K = Kevin, L = Lucy, S = Shane, C = Ciara, M = Max, A = Anna

T: So, boys and girls, how do we identify squares?

K (3rd): Squares have sides that are the same length.

T: Yes. Can anyone help him out further?

L (4th): Squares also have equal angles that are all 90 degrees.

T: Indeed. So is a square a regular shape or an irregular shape? Yes Lucy?

L (4th): It's a regular shape because all sides are the same length and all angles are the same size.

T: Good. Now we know that squares tessellate. What do we know about tessellation?

S (3rd): It means that when you make a pattern, the shapes fit together perfectly.

T: Exactly Shane. But what do we need to be careful about when making patterns that tessellate?

K (3rd): Shapes don't tessellate if there are any gaps... or overlapping shapes in the pattern.

T: That's correct.

L (4th): There are other shapes that tessellate though, not just squares!

T: And you say so because?

L (4th): The honeycomb cells make up lots of hexagons stuck together and they don't overlap either.

T: Great observation Lucy.

C (3rd): And triangles! Because equilateral triangles have the same length of sides and their angles are the same size too so that means they tessellate.

T: Excellent Ciara. Equilateral triangles are one of the three 2D shapes that make up regular tessellations. Now, I will make a pattern on the board using the tangrams (pause). Does my pattern tessellate?

S (3rd): Yes, because you used squares and squares have the same length of sides and the same angles and they don't overlap.

M (4th): There are no gaps either!

T: Well done. Now, 4th class, watch carefully as to how I make this pattern (pause). How is it different to the last pattern?

A (4th): Well, you used more than one shape.

T: Yes and what name is given to a tessellation pattern with more than one shape?

L (4th): Semi-regular tessellation.

M (4th): But how do you actually know it is semi-regular?

T: How could we help Max?

L (4th): Isn't it that all the corners in the pattern have to be the same? So for that pattern with hexagons and squares, if you picked one corner at the top of the square, each square would have to always have two hexagons touching it... is that right Ms.?

T: Yes Lucy, that is correct. Do you understand now Max?

M (4th): Yes.

STEM FOR FUN

Mairéad Holden

St. Teresa's Primary School, Balbriggan, Co. Dublin

STEM education has become a hot topic, to the point where it has been described as an area of "universal preoccupation" (English, 2016). However, equality of access to STEM Education and STEM careers is not universal. Lack of access to STEM Education and careers is especially challenging for struggling learners in DEIS settings. The STEM for Fun project explored the effectiveness of a small-group DEIS-centric programme designed to develop primary pupils' engagement, critical thinking skills and language using STEM-based activities over a six-week period. This paper reports on one of the aspects of the STEM for FUN project: pupils' engagement. The project was undertaken in a DEIS Band 2 Urban primary school in North County Dublin with three 4th class pupils, all of whom were categorised as "struggling learners" according to their performance in standardised Maths and English tests. Analysis of data suggests that the STEM for Fun intervention had a positive effect on participants' engagement in STEM learning and that the STEM for Fun tasks also supported the development of positive learning dispositions.

INTRODUCTION

STEM for Fun is an intervention which formed part of a small scale participatory action research project, designed to promote engagement, critical thinking skills and STEM language. The intervention was designed in response to the needs of a small group of pupils I had been working with in my own classroom context. These pupils were deemed "struggling learners" having scored below the 10th percentile in their most recent Maths and English standardised tests. I was troubled by my pupils' low confidence, lack of motivation and negative learning dispositions. STEM for Fun aimed to capitalise on the success of the DEIS programme "Maths for Fun". This programme had been effective at addressing engagement and confidence in Maths within my own school and its effectiveness in other DEIS schools had been noted in existing literature (DES, 2011; Weir, Archer, O'Flaherty & Gilleece, 2011). Recent literature pertaining to STEM in Irish education suggests a need for innovative projects which promote "engagement, enjoyment and excellence in STEM learning" (STEM Education Review Group, 2016). Aside from content-related aspects of STEM, recent research also underlines the importance of developing 21st century skills, referred to as "the 5 C's": creativity, communication, collaboration, cooperation and critical thinking (Dede, 2010; Butler, 2014). I wondered could STEM for FUN weave all of these aspects together, in order to support the learner needs I had identified?

LITERATURE REVIEW

In terms of supporting pupils' engagement and confidence, I examined research relating to the development of positive dispositions (as described by Katz, 1993). She defines a disposition as "an enduring habit of mind and action or a tendency to respond to situations in characteristic ways. A productive disposition in STEM is an aspect of STEM Proficiency. It involves pupils seeing STEM as useful and relevant, practical and enjoyable, engaging and motivating; recognising the benefits of perseverance and developing confidence in STEM knowledge and ability. While teachers may aspire to develop productive dispositions in their pupils, FitzPatrick, Twohig and Morgan (2014) also flag that learning dispositions is a priority area as identified by parents of primary school pupils.

While reflections documented in my reflective diary suggested my pupils were struggling with negative dispositions and lack of engagement, I looked to literature to guide me in how I might document the development of pupil dispositions in a more structured manner. While frameworks relating to dispositions exist in early years and post-primary curricula (NCCA, 2009; 2015) a review of existing literature revealed a gap in relation to dispositions from 2nd to 6th class. The STEM for Fun project would involve design of a draft dispositions framework to ameliorate this.

While I had considered the area of dispositions from the perspective of teachers and parents, I had not taken cognisance of my pupils' views. I explored literature for ways to give my pupils the opportunity to have their voices heard as well as reflect on and take ownership of their own learning. Baird, Fensham, Gunstone and White (1993) suggest the usefulness of reflective response slips as a tool to capture pupil voice and support reflection, which can also contribute to the development of metacognition. The main idea behind such pupil reflection is that

> *..the meaning any learner derives from a lesson depends on number of factors: the person's attitudinal state, perception of the nature, purpose and progress of the lesson, existing knowledge, and decisions about what to do as learning proceeds… Attitudes, abilities and knowledge are all involved in the processing of information*
>
> (Baird et al, 1993, p.63).

In addition to documenting dispositions, I wondered what kinds of tasks would be most appropriate and effective in developing and enhancing them. The low-threshold high-ceiling (LTHC) nature of the Maths for Fun activities was a key contributory factor in developing pupil engagement (Boaler, 2016; DES, 2011). LTHC tasks, as described by McClure, Woodham and Borthwick, (2011) are tasks which are accessible enough for all levels of pupils to experience an initial degree of success, but also present additional layers of sophistication for pupils who require extra challenge. Teachers and their pupils reported that the open-ended nature of Maths for Fun tasks also fostered confidence and engagement, as pupils had the opportunity to explore activities from multiple perspectives (DES, 2011).

A further effective feature of Maths for Fun tasks was their hands-on nature. This resonates with the constructionist principles as described by Papert (1987). While "fun" was important to engage pupils and build confidence, "hard fun" using meaningful hands-on tasks where pupils got the opportunity to manipulate different materials served to help pupils' development of Science, Technology, Engineering and Maths concepts.

OVERVIEW OF THE STEM FOR FUN INTERVENTION

The STEM for Fun programme involved a six-week intervention incorporating a variety of hands-on STEM-based activities arranged by theme: Electronics; Robotics; Shapes and Patterns; Engineering and Forces; Coding and Digital Presentation skills. Themes had a cross-curricular focus incorporating elements of Digital Technology, Language, Science, Maths, Visual Arts and SPHE. Each 45 minute lesson concluded with a short reflective discussion using reflective response slips. The final week of the programme involved the pupil participants presenting, demonstrating and explaining an activity of their choice to their classmates, using a Google Slides presentation.

METHODOLOGY

The STEM for Fun project was conducted within the paradigm of participatory action research (Groundwater-Smith, Dockett & Bottrell, 2015; McNiff, 2002), through generating a personal living educational theory. Whitehead (1989, p.41) describes living educational theory as "an explanation produced by an individual for their educational influence in their own learning, in the learning of others and in the learning of the social formation in which they live and work". Whitehead articulates the challenge presented when we find ourselves living in contradiction to our values. I identify care (as described by Noddings, 1992) and social justice (as described by Bernstein, 2000) as values which underpin my work as an educator. The lack of engagement of my pupils due to their low confidence and negative dispositions conflicted with my values. I felt compelled to act to support these pupils, with the aim of developing their agency as STEM learners.

The STEM for Fun intervention occurred in partnership and dialogue (as described by Groundwater-Smith et al, 2015) with my pupils, rather than an externally imposed study, done "to" them. This allowed reflexivity (as described by Brookfield, 2002) where I amended my practice in response to my own, my pupils and my colleague's reflections. Data collection methods were chosen to reflect and respect the voice of the pupils, and their perceived progress in their learning.

Rigour and Potential Bias

To enhance the rigour of the study, I used a combination of data collection tools: Reflective diary, pupil reflective response slips, semi-structured interviews, attitude questionnaires and I also invited a colleague to act as critical friend. Due to the nature and design of the study, it was open to the effects of various biases. These included acquiescence and the Hawthorne effect. My critical friend helped to limit confirmation bias through our engagement in professional dialogue and through her observation of some of the STEM for Fun activities in action.

Dispositions Framework

In order to assess and track dispositions identified during pre- and post- semi-structured interviews, I developed a Dispositions Assessment rubric (See Figure 1) drawing from existing Aistear and Junior Cycle frameworks as well as drawing from the work of Deakin-Crick, Broadfoot & Claxton, (2004).

Frequency➔ **Disposition🡻**	**Rarely**	**Occasionally**	**Regularly**	**Almost Always**	**Always**
Curiosity Questions the world around them. Seeks answers to questions posed by self and/or others.					
Perseverance Makes continued effort to work towards a solution despite challenges.					
Confidence ***(Have a go attitude)*** Approaches tasks through thinking or doing without hesitation					
Self-Awareness of Learning Can self-identify strengths & needs related to their own learning.					
Resourcefulness Can find appropriate solutions to problems.					
Meaning Making Connects new STEM ideas or information and prior knowledge. Uses STEM to make sense of their world.					

Figure 1. Dispositions Rubric (Adapted from Deakin-Crick, Broadfoot & Claxton (2004); NCCA (2009; 2015).

Pupil reflective response slips

After each STEM for Fun session, pupils were asked to complete a short response slip with a number of question stems and incomplete sentences (McKernan, 1996). The questions used in the response slip were adapted for the primary STEM context from those used by Baird et al. (1993) whose work related to Science specifically. The pupil response slips not only acted as qualitative data but also served as a valuable formative and self-assessment tool. They allowed me to track pupil attitudes, pupil learning and pupil perceptions of my teaching. The response slips also provided an important record of pupils' use of STEM language.

DATA ANALYSIS

For the purpose of this paper, data analysis relating to engagement and dispositions will be presented.

Engagement

Data emerged across qualitative sources, showing the development of pupils' engagement in and enthusiasm for STEM for Fun and STEM in general. The STEM for Fun pupil group regularly asked "Can we stay in to work on this at break time?" (Reflective diary, 01/02/17). Pupils in the STEM for Fun group stated that they "liked" or in some cases "loved" activities. They even stated this in instances where they also stated that activities were difficult. In fact, the pupils were so engaged and enthusiastic, not only did they stay in over break time (at their own request) but they also brought the activities home to work on them.

Pupil preferences

The STEM for Fun group also selected "Scribblebots" as their topic to present to their classmates. The creative and interactive nature of the Scribblebot activity correlates with data from the attitude questionnaires. Responses from these questionnaires suggest that almost 90% of pupils in 3rd and 4th class enjoy designing and building activities. This was also true for the STEM for Fun group who expressed similar strong preferences for such active STEM learning activities as outlined earlier. Creativity represents one of the 5 C's of 21st Century Skills (Dede, 2010; Butler, 2013) as mentioned earlier. Data suggesting pupils' preference for hands-on activities also correlate with existing literature (Varley, Murphy and Veale, 2008) suggesting that pupils find activities where they get to use the equipment themselves (rather than a teacher demonstration) more engaging.

Making STEM connections

An aspect of engagement requires pupils to make connections between STEM and other areas, for example, with other subjects or with life outside of the classroom including at home. While strong connections were made in some areas, in the case of Science, connections may need reinforcement. Although in their questionnaires, almost 70% of pupils in 3rd and 4th class stated that they viewed Science as useful, this connection with home is not made in the same strong way as Maths and Technology. Almost 90% of pupils reported using Maths and Technology at home, whereas only 41% of pupils stated that they used Science at home.

STEM at home

Pupils in 3rd and 4th class appeared to make strong connections with Maths, Technology and home, according to responses in Pre and Post Attitudes Questionnaires. Pupils who took part in the STEM for Fun intervention also made further connections by bringing their projects home to work on. Sarah asked "Can I bring this home to show my Mam?" when working on her 3-D shape construction task, where she had made a bird structure using straws (Reflective diary, 7/02/17, week 4). Debbie brought her Scribblebot home one weekend to play with in her Nana's.

> "*Scribblebots won't work on the carpet, I tried it in my Nana's". When I asked her what her Nana thought of her Scribblebot, she replied, "My Nana said I'm gonna be off working in NASA someday".*
>
> (Reflective diary, 15/02/17, week 4).

These statements reveal the pupils' interest in bringing their STEM for Fun work home. They talked about it with friends and family, connecting STEM with their daily lives outside the classroom. For example, when Conor was working on circuits during Week One, he remarked, "This is actually kind of handy coz if a bulb goes in your house you can fix it!" (Reflective Diary, 30/01/17, week 1).

DISPOSITIONS

Analysis of data gathered suggested that some dispositions are more susceptible to change than others. Existing literature suggests that a holistic approach involving both home and whole school are effective in developing positive dispositions to learning (Katz, 1993). While a number of findings emerged relating to the various elements which comprise dispositions, for the purpose of this paper, I report on findings relating to dispositions of confidence and perseverance.

Confidence

During interviews, the STEM for Fun pupils were asked a number of questions which related to their perceptions of their own learning. Prior to the intervention, the responses given by the pupils revealed a general lack of self-confidence and lack of self-efficacy in relation to classroom learning. The pupils seemed acutely aware of the gaps in their knowledge. Analysis of data from my diary showed a social dynamic unfolding between the pupils during the intervention. Some of the pupils developed a notable expertise in some activities to the point that they could act as "expert". I encouraged the more expert pupil to help and support pupils who were finding the task challenging. An example of this was Conor's proficiency at assembling successful circuits. He had created several complex circuits while the other two pupils in the group had not managed to complete the initial basic task. He paused his work on own activities in order to help out the other pupils.

> *Without prompting, Debbie took up the flashcards [which had STEM vocabulary printed on them] and started helping Conor to say the words. Later in the lesson, Conor helped Debbie and Sarah to assemble their circuits.*
>
> (Reflective diary, 30/01/17, week 1).

This extract describes Debbie and Conor acting as "experts". This was a notable development for these particular pupils who had earlier showed low levels of confidence and motivation. I felt acting as "expert" offered an opportunity for a confidence and self-esteem boost for those pupils. It also allowed the pupils to see each other as sources of information, rather than always relying on the teacher. Acting as experts for each other also demonstrated progress in pupil collaboration and communication. These represent two of the "5 C's" of 21st Century Skills (Dede, 2010; Butler 2014) as described earlier.

Perseverance

Prior to the intervention, the pupils showed low self confidence in Maths was also reported by my critical colleague who was their class teacher. I wondered if the pupils' perception of their "failure" in standardised Maths tests had contributed to their low self-confidence. The people best placed to answer this were the pupils themselves, but I felt it would be inappropriate and unhelpful to ask the pupils this directly. However, I was able to observe their anxiety when presented with certain types of Maths questions. These I describe as "high-risk" items for the pupils. They had built up a degree of anxiety based on previous failure. Not only had they failed (gotten the wrong answer or not known how to do the question), but they had done so publicly, in front of their classmates. The anxiety which stemmed from such public failure could not be helpful when trying to develop perseverance. This anxiety also appeared deep-rooted and resistant to change.

> *Today's session made me realise how unsure of herself Debbie is in general. Has she gotten into a habit of not bothering to think for herself? Has a spoon-feeding approach in learning support exacerbated this? Her [Debbie's] behaviour today has made me wonder about how dispositions can become quite deep-rooted, and whether something like STEM for Fun alone can really alter such seemingly deep-rooted behaviours.*
>
> (Reflective diary, 06/02/17, week 2).

How could I support Debbie to ameliorate and develop her "deep-rooted" lack of perseverance in six weeks? The extract below describes an encounter I had with Debbie whose lack of perseverance was frustrating me and her class teacher. I eventually decided to broach this directly with Debbie herself

> *I* ***have*** *seen her [Debbie's] confidence improve during the STEM for Fun sessions. She is no longer "afraid" to handle the materials [wires and batteries] - she was very timid to begin with. I will consult with her class teacher on this and maybe try to push her on to be a bit more autonomous in her sums [high-risk items for Debbie]. I have seen her work independently in other areas- maybe I will highlight this observation to Debbie herself and see if it will make a difference?*
>
> (Reflective diary, 6/02/17, week 2).

The above extract also demonstrates the potential of the STEM for Fun activities to develop perseverance at little or no risk of failure, anxiety or humiliation. The "low-risk" nature of the STEM for Fun activities seemed to be more conducive to developing perseverance in the pupils. While the activities had an end goal, such as getting the marble into the pot in tumble tracks, there were many different paths to arrive at the same goal. It was apparent to me from the analysis of the data that the disposition of perseverance is complex. Six weeks is a short time to expect a significant change, but there was some evidence of pupils transferring perseverance to maths sums which they previously would have given up on.

> *When it came to doing the division sums, she [Debbie] seemed to approach them confidently, looking for minimum assistance from me. Conor, although noticeably tired, worked well and got on with doing his sums. He finished his work [including a difficult item on tally marks which featured a lot of text] without any prompting or pestering from me, a big improvement!*
>
> (Reflective diary, 13/02/17, week 3).

Supporting pupil engagement in reflective discussion

The context of the project supported pupils' engagement in reflective discussion. The pupils had very little prior experience of using reflective discussion. The conversation and discussion around the reflective response slips grew to become a natural habit. The pupils would tidy up their materials and then reconvene to have a "chat" about how the session went. This also demanded a degree of accountability from the pupils, as they knew they would be required to discuss and explain what they had worked on. I felt that my classroom was becoming a place where it was ok to make mistakes, which in turn provided talking points and informed our future action. Sometimes "bits of stuff fall apart" or don't work the way they're meant to. In our STEM for Fun group, it was our job to experiment and discuss the "why". This experimentation was becoming valued and seen by the pupils as providing opportunities for learning. Research from existing literature supports this "mistake-valuing environment" (as described by Boaler, 2016) as an important factor in further developing pupils' engagement as well as their critical thinking skills.

SUMMARY OF MAIN FINDINGS, IMPLICATIONS & RECOMMENDATIONS

Engagement

The pupil participants demonstrated an increased level of engagement in learning, during and after the intervention. This increased engagement was across learning in general, but particularly in relation to STEM. A possible explanation for this improved engagement may have been hands-on, creative and collaborative nature of STEM for Fun activities. Pupils stated a preference for activities of this nature, both in existing research and in data gathered during the course of this study. This study was conducted in a specific context with a small sample group. Further study in a diverse range of contexts with a large sample would be needed, in order to establish if similar improvements in engagement arising from using STEM for Fun- style approaches could be achieved in different settings. The STEM in Irish Education Report (STEM Education Review Group, 2016) underlined the urgent need for innovative projects which promote engagement, enjoyment and excellence in STEM teaching and learning. STEM for Fun offers a possible intervention to develop such engagement. It also helps pupils and their families, particularly those from a disadvantaged context, to see STEM as useful, relevant and accessible to all. It is a recommendation of this study that the STEM for Fun model be trialled in a variety of school contexts (both DEIS and non-DEIS) in order to ascertain whether similar findings emerge in other contexts.

Positive dispositions

Pupil participants demonstrated an increase in positive dispositions to learning during and following the STEM for Fun intervention. The positive dispositions which showed the greatest increase were curiosity, perseverance and confidence. The "have a go" attitude in relation to Maths has been sustained following conclusion of the intervention. A whole-school approach would help these dispositions to become embedded.

A further finding of this study is that negative dispositions can be deep-rooted. A holistic intervention on a whole-school basis over a longer time-frame along with parental support could help in addressing such negative dispositions.

This study revealed a significant gap in curriculum policy relating to learning dispositions for children from 2nd-6th class. While this study focused on 3rd and 4th class only, this study did identify the important role played by these learning dispositions in supporting pupil learning. It is a recommendation of this study that policy be developed to build a developmentally appropriate dispositions framework for pupils from 2nd- 6th class. To this end, the study presented a possible Dispositions assessment rubric. While used for teacher-led assessment in STEM as part of this project, this rubric could also be adapted for use as a pupil self-assessment tool across multiple subjects.

This study demonstrated the value of reframing mistakes as opportunities for learning in order to build positive dispositions. Such mistakes can offer rich opportunities for discussion and learning, acting as a springboard for learning, in the spirit of formative assessment. This study has also identified the potential of low floor high ceiling tasks to help promote confidence and engagement in struggling learners within my specific school context. I recommend that further study could be conducted into the impact of such activities on the STEM learning of average and above average ability pupils.

REFERENCES

Baird, J., Fensham, P., Gunstone, R., White, R. in Whitelegg, E. Tresman, S., Thomas, J. (1993). *The importance of reflection in improving science teaching and learning. Challenges and opportunities for science education.* Open University.

Bernstein, B. B. (2000). *Pedagogy, symbolic control, and identity: Theory, research, critique* (No. 4). Rowman & Littlefield.

Boaler, J. (2016). *Building a maths mindset community. Youcubed*. University of Stanford.

Brookfield, S. D. (2002). Using the lenses of critically reflective teaching in the community college classroom. *New directions for community colleges*, *2002*(118), 31-38.

Butler, D. (2014). *Embedding the development and assessment of key skills into lesson design*. Workshop presented at NCCA Assessment for Key Competencies Conference, November 2014.

Deakin-Crick, R, Broadfoot, P., & Claxton, G. (2004). Developing an effective lifelong learning inventory: The ELLI project. *Assessment in Education: Principles, Policy & Practice*, *11*(3), 247-272.

DES. (2005*). DEIS–Delivering Equality of Opportunity in Schools*. DES.

DES. (2011). *National Strategy: Literacy and Numeracy for Learning and Life 2011-2019*. DES.

Dede, C. (2010). Comparing frameworks for 21st century skills. *21st century skills: Rethinking How Students Learn*, *20*, 51-76.

English, L. D. (2016). STEM education K-12: perspectives on integration. *International Journal of STEM education*, *3*(1), 3.

FitzPatrick, S., Twohig, M. & Morgan, M. (2014): Priorities for primary education? From subjects to life-skills and children's social and emotional development. *Irish Educational Studies. 33*(3), 269-286.

Groundwater-Smith, S., Dockett, S., Bottrell, D. (2015). *Participatory research with children and young people.* Sage: London

Katz, L. G. (1993). Dispositions as educational goals. *ERIC Digest*.

McClure, L., Woodham, L. & Borthwick, A. (2011). Using Low Threshold High Ceiling Tasks. Nrich. University of Cambridge. Retrieved https://nrich.maths.org/7701/index

McKernan, J. (1996). *Curriculum action research: A handbook of methods and resources for the reflective practitioner*. London: Routledge.

McNiff, J. (2002). *Action research for professional development.*

National Council for Curriculum and Assessment (NCCA). (2009) *Aistear: The early childhood curriculum framework*. Dublin: NCCA.

NCCA. (2015). *Key Skills of Junior Cycle.* NCCA.

Noddings, N. (1992). *The challenge to care in schools*. Teachers College Press.

Papert, S. (1987). Microworlds: transforming education. In *Artificial intelligence and education* (Vol. 1, pp. 79-94). Ablex Utrecht.

STEM Review Group. (2016). *STEM Education in the Irish School System*. ESRI.

Varley, J., Murphy, C. & Veale, O. (2008*). Science in primary schools, Phase 2: Final report*. NCCA.

Weir, S., Archer, P., O'Flaherty, A., & Gilleece, L. (2011). *A report on the first phase of the evaluation of DEIS.* Dublin: Educational Research Centre

Whitehead, J. (1989). Creating a living educational theory from questions of the kind, 'How do I improve my practice?' *Cambridge Journal of Education*, *19*(1), 41-52.

DEVELOPING MATHEMATICAL LITERACY: A REPORT OF THE POTENTIAL USE OF LESSON STUDY WITHIN ITE

Mairéad Hourigan and Aisling Leavy

Mary Immaculate College, University of Limerick, Ireland

All students should have access to quality mathematics education. Given the documented impact teachers have on learning outcomes in mathematics, it is essential that initial teacher education develop the relevant knowledge and aptitudes among pre-service teachers to facilitate them to teach mathematics effectively. This paper focuses on the potential role of Lesson Study within initial teacher education in meeting this goal. In particular, we examine the various opportunities and benefits that are available to each of the 'partners' engaging in Lesson Study. Over a decade we have explored the benefits of Lesson Study using a three-tier teaching experiment approach (Lesh & Kelly, 2000). This approach facilitates insights into the effects of participation in Lesson Study from the perspectives of the three partners: teacher educator, the student teacher and the pupils.

LESSON STUDY

Lesson study (LS) originated in Japan and has traditionally been used within schools as a bottom-up school-based form of professional development. Through this process, a cycle of planning, teaching and reflection takes place. Initially, a group of teachers come together to form a lesson study group (LSG). This LSG selects a subject, teaching approach or skill they wish to improve and work together to plan a detailed single lesson called a research lesson. This lesson plan anticipates learners' strategies, misconceptions and responses. One of the LSG members teaches this first lesson (teach 1) and all other members observe the lesson whilst focusing in particular on pupil learning. Through this process of observation, reflection and discussion the LSG revise the initial research lesson and a second LSG member reteaches the revised research lesson to a different class group (teach 2). This cycle of planning, observation, reflection and discussion can be repeated many times over weeks or even months. In some cases, 'knowledgeable others' engage with the LSG to provide support at various stages. LS concludes with the LSG sharing their learning through a written report (Murata, 2011).

The above approach is referred to as 'traditional' or 'formal' LS. In Japan, the majority of teachers engage with formal LS annually. Outside of Japan, the use of LS has spread over time among qualified teachers. Research reports various benefits associated with engaging in formal LS including improved practice with a greater emphasis on student learning, enhanced teacher knowledge, increased teacher commitment and collaboration, and the development of learning resources (Chassels and Melville, 2009; Murata, 2011; Ní Shuilleabhain, 2016). However, implementing LS within schools presents a series of challenges including cost (e.g. substitute teachers), sustainability (engagement for one term/year versus ongoing participation) and issues in supporting effective teaching practices among teachers with limited content knowledge (Murata, 2011).

Lesson Study in Initial Teacher Education

Over time, interest in the potential of LS within initial teacher education (ITE) has increased. However, due to limited time within already crowded ITE courses LS has inevitably taken many forms (Cohan and Honigsfeld, 2007). Adaptations include 'Microteaching LS', where pre-service teachers (PSTs) teach research lessons to their peers (Fernandez, 2005). Other modifications include compressed LS cycles (1 day) (McMahon and Hines, 2008) or Lesson Plan Study where PSTs engage in the collaborative planning phase only (no implementation) (Cavey and Berenson, 2005). Other variants situate the planning phase in the university setting, involve research lessons being taught and videoed during school practicum and subsequently evaluated on return to university (Cohan and Honigsfeld, 2007). Despite these restrictions placed on adapted models of LS, studies report positive outcomes for PSTs including improved content knowledge (Cavey and Berenson, 2005), the development of collaborative and reflective practice as well as a move towards more learner-centred pedagogy (Fernandez, 2005).

A relatively small number of studies use a more traditional LS structure (Leavy and Hourigan, 2016; Marble, 2006; Chassels and Melville, 2009; Sims and Walsh, 2009; Corcoran and Pepperell, 2011; Cajkler et al., 2013; Cajkler and Wood, 2016a; 2016b; 2018). However, the majority of these LSs were implemented as part of the school practicum component of ITE with the co-operation of mentor teachers (Chassels and Melville, 2009; Cajkler and Wood, 2016a; 2016b; 2018). Reported challenges associated with implementing LS within ITE programmes include scheduling problems, limited time for collaboration and debriefing, the inability to access a suitable class for the reteach, difficulties securing suitable qualified mentors (Chassels and Melville, 2009; Cajkler and Wood, 2016b) and insufficient knowledge of classroom students (Chassels and Melville, 2009). Despite these issues, the reported benefits of implementing formal LS in ITE generally reflect those reported for qualified teachers (Chassels and Melville, 2009; Bjuland and Mosvold, 2015; Cajkler and Wood, 2018; Leavy, 2010, 2015; Leavy and Hourigan, 2016, 2018a).

This paper examines the various outcomes of a particular model of formal LS within an ITE programme. This model differs from the majority of others in ITE, as the LS was not based around or within the PSTs' school practicum. Instead, the implementation of the research lessons was facilitated by co-ordinating with local partner schools.

METHODOLOGY

To support the observation and analysis of outcomes arising from engaging in LS in ITE, annually a three-tier perspective (Lesh and Kelly, 2000) was taken. Tier 1 focuses on the impact of LS on primary school children (for example the nature of their developing mathematical knowledge and abilities). Tier 2 focuses on the impact of LS participation on PSTs (for example their developing knowledge, assumptions about the nature of students' mathematical knowledge and abilities). Tier 3 concentrates on researcher (teacher educators) characteristics (for example their evolving teacher educator identity or their developing conceptions about the nature of children's and pre-service teachers' developing knowledge and abilities). The design involves the ongoing collection of data at each stage of the LS cycle from multiple sources.

Participants

Annually (2008-2017) the researchers worked with a small group of undergraduate primary PSTs (maximum of 25) who had opted to participate in an elective mathematics education course during the 3rd year of their 4-year ITE course in Ireland. These PSTs had completed all 5 compulsory mathematics education modules which addressed the content and pedagogy related to the range of curriculum strands. In terms of school placement, prior to the study PSTs had engaged in at least 10 weeks of placement in a range of classes. LS research lessons were taught predominantly within 2 local schools with whom we have an informal partnership. All ethical requirements were completed including institutional ethical approval, school Board of Management permission, and information and consent provided in the form of information letters and consent forms for all the relevant participants.

The nature of formal Lesson Study within this study

Within this study, participating PSTs experienced a formal LS approach within their ITE programme.

Stage 1: At the start of the module (Stage 1 of the LS cycle; Table 1), PSTs were introduced to the process of LS as implemented in Japan. They engaged with various key LS readings such as Stigler and Hiebert (1999) and considered the strengths and weaknesses of this model and its applicability to the Irish context.

Stage 2: Subsequently, the PSTs moved into the planning stage (approximately 6 weeks) of the LS cycle (Stage 2; Table 1). In order to facilitate ongoing co-operation from volunteering schools and teachers, each year a sequence of 5 research lessons (developed by 5 lesson study groups) over 5 consecutive days replaced the class teacher's mathematics lessons for a teaching week. While the researchers (in their role of teacher educators) decided the mathematical area (e.g. strand of Algebra) and/or mathematical concepts (e.g. Equality, Functions, Variables) which would be addressed, the PSTs had a central role in deciding the teaching sequence, methodologies, contexts, materials and in related pedagogical and content decision making. To inform their decisions, PSTs were assigned relevant readings (e.g. academic and practitioner articles) and various international curricula. This process led to discussions regarding the assignment of concepts to specific research lessons and subsequently appropriate contexts that could motivate children to engage. PSTs were assigned to a LSG; with each LSG responsible for planning and teaching a designated lesson. We use the lesson plan format devised by Ertle, Chokshi and Fernandez's (2001) as it required PSTs to explicitly attend to expected student responses and the teacher's response to student activity/response. The researchers (in their role as knowledgeable others) met each LSG weekly both during and outside of lecture time to work on the planning of the research lesson. Subsequently, each LSG implemented their research lesson (in sequence) to a designated class level in school 1 across 5 consecutive days. While one PST taught the first lesson (teach 1), the remaining LSG members assumed the role of observers. Immediately after lesson implementation, the researchers and the LSG met to share and discuss observations and determine the necessary revisions to the research lesson. Teach 2, which took place two weeks later in school 2 to a different group of children at the same grade level, was video-recorded by a professional video crew. Again, the process of observation, discussion and reflection resulted in further refining of the research lesson. Class teachers (in both settings) provided informal feedback based on observations.

Stage 3: The final stage, which lasted two weeks (Stage 3; Table 1), focused on reflection and reporting on the LS process. Each LSG made a presentation describing and critiquing their research lesson. PSTs also completed reflective assignments at the end of semester focusing on areas such as the development of children's understandings of the relevant concepts, perceptions of their own learning and experience of the LS process.

Table 1. Structure and focus of each LS cycle stage and related data collection methods

LS Stages		Primary Activities	Data Collection Methods
Stage 1		• Introduce LS • Explore and discuss key readings relating to LS	• Researcher field notes taken during tutorials and work sessions • Researcher reflective journal entries
Stage 2	Step 1: Collaborative planning of the research lesson	• Identify the relevant mathematics concepts • Develop a trajectory of instruction • Identify the key foci for each LSG • Design the research lessons	• Researcher field notes taken during tutorials and work sessions • Researcher reflective journal entries • Content analysis of lesson
	Step 2: 'First teach': Seeing the research lesson in action	Each of the LSGs: • Teach the lesson • Observe the lesson and make notes	• Observations of first lesson implementation • Researcher reflective journal entries
	Step 3: Reflection, discussion and revision	Each of the LSGs: • Reflect on the taught lesson • Revise the original research lesson	• Researcher field notes taken during feedback session • Participant reflections • Record of changes made to revised lesson and justification of those changes • Researcher reflective journal entries
	Step 4: 'Second teach': Seeing the research lesson in action	Each of the LSGs: • Teach the revised lesson • Observe the lesson and make notes	• Observations of second lesson implementation • Video records of second lesson • Researcher reflective journal entries
	Step 5: Reflection, discussion and revision	Each of the LSGs: • Reflect on the taught lesson • Make final revisions to the research lesson	• Researcher field notes taken during feedback session • Record of changes made to revised lesson and justification of those changes • Researcher reflective journal entries
Stage 3		• LSG presentations • Individual pre-service teacher reflections	• Records and observations of LSG presentations • Participant reflections • Researcher reflective journal entries

Data collection and analysis

A mixed-method approach was taken to data collection. Table 1 summarises the nature, source and range of data collected at each stage of the LS cycle. These included field notes and recorded conversations from all stages (planning meetings, post-teach meetings), lesson plans, samples of children's work, researcher observations, videos of teach 2, PST and researcher reflections and group presentations. While acknowledging the limitations of self-report data, the variety of data sources serve to strengthen the findings. The issues of reliability and validity are particularly important when utilising qualitative methodologies as much of the data is in the form of communicated opinions, attitudes and beliefs which may well contain a certain degree of bias. A number of measures were taken to combat bias and ensure verisimilitude. Firstly, the researchers collected data from a number of perspectives and sources throughout the various LS stages (See Table 1). Transcripts reflected verbatim accounts of both researchers', PSTs' and children's ideas, opinions and understandings (McMillan and Schumacher, 2001). Secondly, a systematic process of data analysis was undertaken where the raw data were initially organised into natural units using representative codes (Creswell, 2009). Successive examinations of the data let to the identification of relationships between codes and subsequently the creation of overarching themes. Thus, across all studies, data were analysed using a grounded theory approach, where the data steered the emerging theory.

SUMMARY OF CONCLUSIONS AND OUTPUTS FROM LS

Over the decade, while the approach to formal Lesson Study has been consistent, our research focus has varied. Our research interests shift in response to local and national areas of priority and interest. Some examples of research foci include our own evolving identity as teacher educators, PSTs' perceptions of what they had learned, PSTs' content knowledge and children's evolving understandings as a result of engaging in research lessons. The three tier model alongside the range of potential foci within each tier illustrates the usefulness, fluidity and productivity of Lesson Study as a process for all participants (teacher educators, PSTs and learners).

Focusing on Tier 1, annually we are interested in examining the growth of children's understandings of various mathematical concepts as a result of engaging in the research lessons arising from the LS. For example, we have examined the nature of young children's understandings of various statistical concepts including their use of inscriptions in statistical investigations (Leavy and Hourigan, 2018b) as well as the selection of attributes in data modelling environments (Leavy and Hourigan, 2018c). At Tier 2, our research has examined the impact of engaging in LS on PSTs mathematical knowledge for teaching. This model of Lesson Study has been found to have a positive impact on developing both mathematics content knowledge as well as pedagogical content knowledge (Leavy, 2010, 2015; Leavy and Hourigan, 2016, 2018a; O' Ceallaigh, Hourigan and Leavy, 2019). Throughout, the researchers are constantly reflecting and learning themselves (Tier 3) about the mathematical and pedagogical considerations of the mathematical concepts under study any one year. When deciding to engage in LS within an Irish immersion setting, while the authors were considered 'old timers' in the world of LS, they were 'newcomers' to the world of immersion teacher education. As part of this process, we examined the nature of our evolving immersion teacher educator identity over the course of this project (Leavy, Hourigan and O'Ceallaigh, 2018).

We acknowledge that, in its present form, LS has limited impact with only a small proportion of PSTs annually having the opportunity to engage with LS. However, we believe that Lesson Study can potentially have a more widespread and positive impact on the PSTs and qualified teachers beyond those directly involved. One way this is possible is through the sharing of research lessons created within the LS process in relevant national and international practitioner journals (see this link as an example of how we endeavour to make our outputs freely accessible to educators https://www.mic.ul.ie/faculty-of-education/department/stem-education?index=5). These dissemination efforts provide both practicing and prospective primary teachers with relatable experiences; providing examples of reform-oriented teaching contexts, learner-centred activities, varied pedagogies. These articles also explore children's thinking and evolving conceptions. Such resources have the potential to provide support for teachers and enhance their mathematics knowledge for teaching.

Another valuable means of increasing the impact of LS is through the digital recording of the research lessons, particularly those revised lessons taught in the second cycle of LS. These video case studies can be used within ITE mathematics education sessions to give the general population of PSTs access to the concepts, contexts, pedagogies and children's thinking for the various areas of the mathematics curriculum. Rather than use dated video footage, or footage of qualified teachers, or classes from another context, our experiences is that PSTs relate better to footage of Irish PSTs teaching in Irish primary schools.

This paper highlights that Lesson Study can potentially contribute to all of the partners' understandings of various pertinent aspects of effective mathematics education.

REFERENCES

Bjuland, R. & Mosvold, R. (2015). Lesson study in teacher education: Learning from a challenging case, *Teaching and Teacher Education, 52*, 83-90.

Cajkler, W., Wood, P., Norton, J., & Pedder, D. (2013). Lesson study: Towards a collaborative approach to learning in Initial Teacher Education. *Cambridge Journal of Education, 43*(3), 537-554.

Cajkler, W. & Wood, P. (2016a). Adapting 'lesson study' to investigate classroom pedagogy in initial teacher education: What pre-service teachers think. *Cambridge Journal of Education, 46*(1), 1-18.

Cajkler, W. & Wood, P. (2016b). Lesson study and pedagogic literacy in initial teacher education: Challenging reductive models. *British Journal of Educational Studies, 64*(4), 503-521.

Cajkler, W. & Wood, P. (2018). Mentors and student-teachers 'lesson studying' in initial teacher education. *International Journal for Lesson and Learning Studies, 5*(2), 84-98.

Cavey, L. O. & Berenson, S. B. (2005). Learning to teach high school mathematics: Patterns of growth in understanding right triangle trigonometry during lesson plan study. *Journal of Mathematical Behavior*, *24*, 171-190.

Chassels, C. & Melville, W. (2009). Collaborative, reflective, and iterative Japanese lesson study in initial teacher education programme: Benefits and challenges. *Canadian Journal of Education*, *32*(4), 734-763.

Cohan, A. & Honigsfeld, A. (2007). Incorporating 'lesson study' in teacher preparation. *The Educational Forum, 71*(1), 81-88.

Corcoran, D. & Pepperell, S. (2011). Learning to teach mathematics using lesson study. In T. Rowland & K. Ruthven (Eds.) *Mathematical knowledge in teaching* (pp. 213-230). Dordrecht: Springer.

Creswell, J.W. (2009). *Research design: Qualitative, quantitative and mixed methods approaches* (3rd ed.). Los Angeles, CA: Sage.

Ertle, B., S. Chokshi, and C. Fernandez (2001). *Lesson planning pool*. Retrieved from https://sarahbsd.files.wordpress.com/2014/09/lesson_planning_tool.pdf

Fernandez, M. L. (2005). Learning through microteaching lesson study in teacher preparation. *Action in Teacher Education*, *26*(4), 37-47.

Hourigan, M. & Leavy, A.M. (2019, on-line first). Learning from teaching: Pre-service elementary teachers' perceived learning from engaging in 'formal' lesson study. *Irish Educational Studies.*

Leavy, A. M. (2010). The challenge of preparing preservice teachers to teach informal inferential reasoning. *Statistics Education Research Journal*, *9*(1), 46-67.

Leavy, A. M. (2015). Looking at practice: Revealing the knowledge demands of teaching data handling in the primary classroom. *Mathematics Education Research Journal*, *27*(3), 283-309.

Leavy, A.M. & Hourigan, M. (2016). Using lesson study to support knowledge development in initial teacher education: Insights from early number classrooms. *Teaching and Teacher Education, 57,* 161-175.

Leavy, A.M., Hourigan, M. & O' Ceallaigh, T.J. (2018, on-line first). Unpacking dimensions of immersion teacher educator identity, *Journal of Immersion and Content-based Language Education* – Special Issue: 'Teacher education and professional development in immersion and content-based language education contexts: Current research on programs, practices, and teacher educators'.

Leavy, A.M. & Hourigan, M. (2018a). Using lesson study to support the teaching of early number concepts: Examining the development of prospective teachers' specialized content knowledge. *Early Childhood Research Quarterly, 46*(1), 47-60.

Leavy, A.M & Hourigan, M. (2018b). Inscriptional capacities of young children engaged in statistical investigations. In **A.** Leavy, M. Meletiou-Mavrotheris, & E. Paparistodemou (Eds.), *Statistics in early childhood and primary education: Supporting early statistical and probabilistic thinking* (pp.89-107). Singapore: Springer.

Leavy, A. & Hourigan, M. (2018c). The role of perceptual similarity, data context and task context when selecting attributes: Examination of considerations made by 5-6 year olds in data modelling environments. *Educational Studies in Mathematics*. *97*(2), 163-183.

Lesh, R., Kelly, A., (2000) Multitiered teaching experiments. In A. Kelly, & R. Lesh (Eds.), *Research design in mathematics and science education* (pp. 197-230). Mahwah, New Jersey: Lawrence Erlbaum Associates.

Marble, S.T. (2006). Learning to teach through lesson study. *Action in Teacher Education, 28*(3), 86-96.

McMahon, M. & Hines, E. (2008). Lesson study with preservice teachers. *Mathematics Teacher, 102*(3), 186-191.

McMillan, J. H. & Schumacher, S. (2001). *Research in education: A conceptual introduction* (5th ed.). New York: Longman.

Murata, A. (2011). Introduction: Conceptual overview of lesson study. In L. Hart, A. Alston, & A. Murata (Eds.), Lesson Study research and practice in mathematics: Learning together (pp. 1-12). New York: Springer.

Ní Shuilleabhain, A. (2016). Developing mathematics teachers' pedagogical content knowledge in lesson study: Case study findings. *International Journal for Lesson and Learning Studies, 5*(3), 212-226.

O' Ceallaigh, T.J., Hourigan, M. & Leavy, A.M. (2018, on-line first). Developing potentiality: Pre-service elementary teachers as learners of language immersion teaching, *Journal of Bilingual Education and Bilingualism.*

Sims, L. & Walsh, D. (2009). Lesson study with preservice teachers: Lessons from lessons, *Teaching and Teacher Education, 25*(5), 724-733.

Stigler, J. W. & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom.* New York: The Free Press.

ESCALATING INTERVENTIONS TO IMPROVE BEHAVIOUR AND PERFORMANCE IN AN UNDERGRADUATE STATISTICS MODULE

Emma Howard[1], Maria Meehan[1] and Andrew Parnell[2]

[1]University College Dublin and [2]Maynooth University

In this study, we present the implementation of an early warning system in a large introductory statistics module which is escalated over four semester offerings. An early warning system identifies students who are at risk of failing or dropping out of a module, and provides them with supporting interventions. While familiar undergraduate mathematics supports include formative assessment and peer-assisted learning, our interventions tried to encourage student engagement through personalised emails which detailed supports and how students were progressing in the module. In later escalations, at-risk students received weekly emails encouraging them to use the Maths Support Centre. We believe our module-based interventions had limited impact upon the at-risk students. In hindsight, we believe these students needed programme-level interventions. Overall, this study provides insights for others into implementing learning analytics interventions.

INTRODUCTION

With the availability of data, learning analytics has become an international trend with positive implementations reported worldwide. Campbell and Oblinger (2007) consider the five steps in the analytics cycle to be: Capture; Report; Predict; Act; and, Refine. There is currently a wealth of papers on prediction models for learning analytics. However, research on the ‘Act’ and ‘Refine’ steps are not as prevalent with a limited amount of evidence-based studies available. Familiar supports in third-level mathematics modules may include formative feedback or peer-support. Recently learning analytics interventions have been implemented in STEM modules (Cai, Lewis, & Higdon, 2015).

Here, in our quasi-experimental study, we evaluate an escalating intervention provided to students in four offerings of a large undergraduate statistics module, *Practical Statistics*. In the first two offerings of the module, a feedback email was sent to all students containing study advice and details allowing them to compare their continuous assessment at that time to that of their peers. On finding no positive benefit of the intervention, in subsequent offerings of the module, the intervention was escalated. In the third offering, students were provided with their predicted final module mark in the feedback email, and at-risk students were targeted with weekly emails from the manager of the university Maths Support Centre (MSC). In addition to these escalations, the fourth offering catered for a face-to-face meeting between each at-risk student and their lecturer, an author of this paper. We measure the effectiveness of these supports by examining changes in behaviour and performance. To do this, we analyse Learning Management System (LMS) data, academic marks, and MSC attendance data from six offerings of the module (N = 876). The six offerings include four offerings where an intervention occurred and two offerings which were used for comparison purposes in the statistical analysis. Whilst some hypothesis tests of our interventions were statistically significant, we believe that these were not of practical significance for the students. However, we argue that our study shows how an intervention may be refined and affirms the need to focus on how to evaluate the impact of interventions.

LITERATURE REVIEW

A popular learning analytics intervention is an early warning system. This consists of identifying students at risk of failing or dropping out and providing them with support (an intervention). The identification of at-risk students is commonly performed through prediction modelling, although some universities have dedicated support staff to identify them (based on students' GPA, whether they are repeating, inconsistent grades et cetera). There have been a number of studies that investigate how to develop accurate prediction models for early warning systems, including in engineering and mathematics modules (Corrigan, Smeaton, Glynn, & Smyth, 2015). For this study, we focus on the intervention and the evaluation of its effectiveness. Dawson, Jovanovic, Gašević and Pardo (2017) assert that a limited number of studies provide examples of applying and measuring the effectiveness of interventions.

We focus on module-level interventions where students are identified as at-risk and are provided with an intervention based on their progress in a single module. Choi, Lam, Li, and Wong (2018) suggest an intervention strategy should encourage a good student-lecturer relationship (starting with a welcome email), and involve ranking of students by their prediction results with a proactive strategy that initially focuses on students at high risk of failing and which shifts to low-risk students in the final stages of the module. Na and Tasir (2017) conducted a systematic review of learning analytics interventions specifically in online learning, and found that the interventions implemented could be classified as: providing additional, or substantially changing, teaching materials; emailing students; providing advice to students; posting of a signal to a student dashboard; arranging a face-to-face meeting; improving module materials; and, texting students. Na and Tasir (2017) note that no study used tutoring as an intervention. However, they hypothesise that this may be owing to challenges faced by the organisation and implementation of tutoring in online learning.

Cai et al. (2015) piloted an intervention scheme in an intermediate algebra module. This included a dashboard which allowed instructors and teaching assistants to view students' class participation details, examination results and an at-risk indicator. Based on their assessment results, students were sent personalised emails which included a list of resources to work on in their tutor centre. Using descriptive statistics and *t*-tests they found a significant difference between those who attended and those who did not attend the tutor centre. Following the use of prediction modelling, Corrigan et al. (2015) sent emails to students which included support details and a module position indicator. Dawson et al. (2017) identified at-risk students in seventeen modules, including STEM modules. In order to increase retention rates, trained personnel telephoned the at-risk students and detailed the available supports. They investigated the impact of this through χ^2 tests, logistic regression and mixed-effects modelling. Initially, the intervention appeared to have caused a significant increase in retention but after further rigorous evaluation this was proved untrue. As this is a relatively new area, the ‘Refine’ step of the learning analytics cycle (Campbell & Oblinger, 2007) is under-researched, and the potential for university mathematics modules has not fully been explored.

MODULE AND METHOD

Practical Statistics is an introductory online statistics module for approximately 150 students, offered each semester in University College Dublin. Over the 12-week teaching semester, there are no face-to-face lectures, however, there are two one-hour face-to-face software laboratory lessons per week. The lecturer has taught *Practical Statistics* for five years with little change to the module content for the duration of this study. The only exception to this occurred in semester 2 of 2015/16 when a colleague taught *Practical Statistics*, although they covered the same content and used the same format as previous iterations. *Practical Statistics'* continuous assessment contributes 40% to a student’s final module mark with the remaining 60% based on an end-of-semester 2-hour written examination. The continuous assessment consists of: accessed videos (2%); weekly lecture questions (6%); weekly Minitab laboratory tutorials (3%); a Minitab examination (10%); weekly R laboratory tutorials (4%); and an R examination (15%). In both semesters of 2017/18, the university closed for a few days owing to severe weather conditions. This resulted in the cancellation of the Minitab examination and a re-weighting of the continuous assessment components. For our statistical analysis, we focus on written examination marks rather than final module mark as these represent 60% of the final module mark for all students for every semester of this study. Unlike semester 1, in semester 2 there is a two-week midterm break for students between weeks 7 and 8 of the teaching semester.

Data Collection

Since 2015/16, we have collected: students’ demographic data, continuous assessment and examination marks; LMS log files; and, students’ MSC attendance. The LMS resources (for example online videos and lectures slides) are divided into fifteen folders based on the week which the material content relates to (week 1 module material, ..., week 12 module material, lecture questions solutions, module information, and past examination questions). We collated the data for each student under a pseudo number. In accordance with our ethics permission from the university ethics committee, students were provided with information sheets outlining the nature of this study and provided with the option of having their data removed from the study. However, no student withdrew.

In semester 1 of 2015/16, we conducted an online survey of *Practical Statistics'* students aimed at understanding students’ resource usage, to which approximately 30% responded (n = 38). We analysed the responses to this survey and incorporated this information into the advice of our intervention email for future years. When asked about resource usage, students reported predominantly using resources provided by the lecturer, with external resources including Khan academy videos and *How to Lie with Statistics* by Huff. When asked about online learning, students’ responses were mixed with some students praising the flexibility of online learning and others finding it difficult to adjust to the freedom accompanying it.

However, students noted that the weekly lecture questions helped them to regulate their learning.

The Interventions

The aim of each of our interventions was to improve students' academic achievement through encouraging engagement with module resources and reflection on prior material. This research is considered quasi-experimental as students receive an intervention based on a set-criteria rather than random assignment (see Table 1). For our initial intervention, in semester 1 of 2016/17, we emailed students of Practical Statistics at the beginning of weeks 6-7. Previous research (Howard, Meehan, & Parnell, 2018) identified weeks 5-6 as an "optimal time" to provide support to students and obtain a sufficient level of prediction accuracy. While the core information provided to all students in the email was the same, the email was phrased slightly differently depending on the tertile of students' continuous assessment to-date. The email included the distribution of continuous assessment to-date (to allow students to compare their own continuous assessment mark to their peers) and following from the survey, study suggestions included: visiting statistics tutors in the MSC for free one-to-one tutoring; external resource recommendations (based on results of the survey distributed in 2015/16); suggestion of studying at the same times every week; and, a suggestion to study with friends in the university active learning environment rooms.

Table 1: Interventions provided to students by year and semester. The control group were identified retrospectively as at risk by using the criteria that their predicted module mark was less than 50.

Year	2015/16		2016/17		2017/18	
Semester	**S1**	**S2**	**S1**	**S2**	**S1**	**S2**
Email with advice and continuous assessment graphs	No	No	Yes	Yes	No	No
Email with advice and predicted mark	No	No	No	No	Yes	Yes
At-risk students receive weekly MSC emails	No	No	No	No	Yes	Yes
At-risk students invited to meet lecturer	No	No	No	No	No	Yes
Number of students in study	139	115	148	158	152	164
Criteria for at-risk students	Control Group	Control Group	Lowest Third	Predicted Mark <60	Predicted Mark <50	Predicted Mark <50
Number of at-risk students	37	19	47	38	30	26

From our initial exploratory analysis of the effectiveness of the email intervention, we found that it had no measurable impact upon students' final examination mark. Therefore, in subsequent semesters, we escalated our interventions and/or changed our criteria for an at-risk student (Table 1). From semester 2 of 2016/17 onwards, at-risk students were identified using a prediction model, Bayesian Additive Regression Trees (Chipman, George, & McCulloch, 2010), and this was based on their LMS interactions, continuous assessment and demographic data. In semester 2 of 2016/17 the intervention remained the same. Starting in semester 1 of

2017/18, for our intervention escalation, rather than providing students with the distribution of the module's continuous assessment, at weeks 6-7 we provided them with their predicted module mark, and a disclaimer that this was not their final module mark. We emphasised that the predicted mark could be changed by studying. The MSC manager agreed to send weekly emails to students from week 7 who were identified as at-risk. These emails informed the students that their lecturer recommended they attend the MSC for additional support. However, this intervention had no effect in semester 1 of 2017/18. For semester 2 of 2017/18, in addition to the MSC emails, 26 at-risk students were emailed by the lecturer of the module, offering them a one-to-one meeting to discuss issues and/or their study plans for the remainder of the semester in relation to *Practical Statistics*. Upon receiving only three replies, the lecturer tried to phone the remaining 23 students. Several students had not provided correct contact details on the university system. Of the 26 students, the lecturer arranged meetings with 12 of them.

Evaluation of the Interventions

To analyse whether the interventions had a behavioural or/and academic impact on students, the final written examination marks, LMS usage and MSC attendance of students are compared over multiple offerings of the module while controlling for at-risk students. LMS and MSC are used as proxies for engagement. We acknowledge that engagement is complex and these proxies do not consider the quality of time spent on learning. In the case of LMS data, smoothed daily LMS activity with 95% confidence intervals are used for comparisons. Examination marks are compared using descriptive statistics (boxplots), analysis of variance (ANOVA) tests and linear regression. In addition, the level of MSC attendance is compared over multiple semesters. Survey responses suggested that some students had difficulty adjusting to and motivating themselves to engage with online modules. Following from the advice given to students to study at the same times each week, we examined whether there was a relationship between the dominant periodicity of students' resource usage and written examination marks using spectral analysis.

RESULTS: MEASURING THE IMPACT OF THE INTERVENTIONS

To allow for comparisons between at-risk students, we retrospectively identified students at-risk for 2015/16 using a prediction mark cut-off of 50% and Bayesian Additive Regression Trees. In Figure 1, we visually compare *Practical Statistics'* smoothed daily LMS activity for semester 1 and written examination marks over three years (similarly this can be done for semester 2). These comparisons are shown for the at-risk and not at-risk cohorts. The initial increase in LMS activity correlates with the first major continuous assessment, the Minitab examination in Week 6. The second increase in week 12 correlates with the second main continuous assessment, the R examination. Consistently the at-risk students had lower LMS activity. However, for semester 1 2017/18, the LMS activity for both student cohorts are nearly perfectly aligned. This cannot be associated with the intervention as this alignment in LMS activity occurs both pre- and post-intervention.

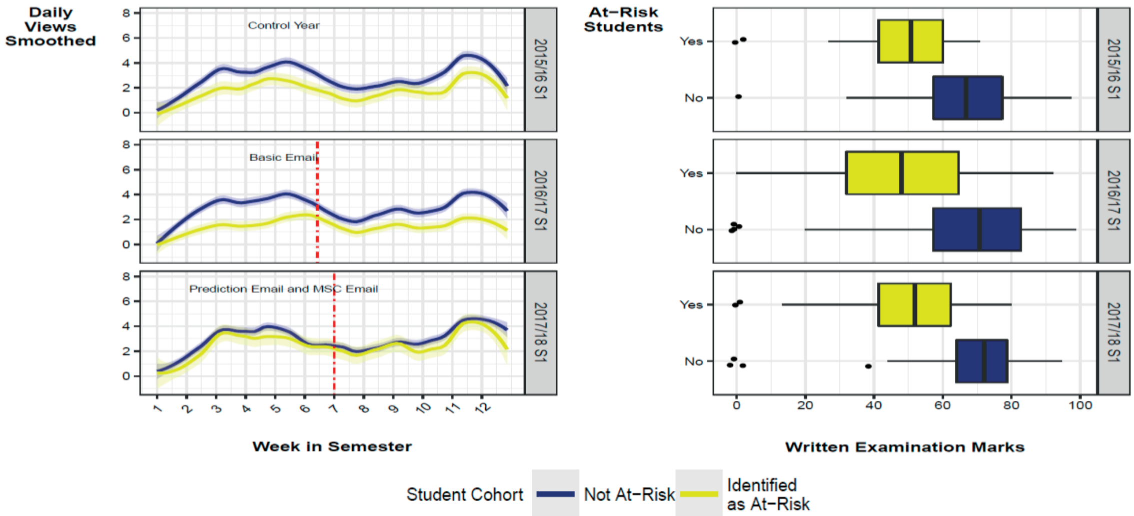


Figure SEQ Figure * ARABIC 1: Daily LMS activity combined with boxplot of written examination marks for semester one factorised by whether students were identified as at-risk or not. The dashed line in a panel represents the date on which the initial intervention of that semester occurred.

Hypothesis testing is a popular method of analysing whether learning interventions have been effective. The ANOVA test is used for multiple mean comparisons. One of the assumptions of an ANOVA test is homogeneity of variances. To confirm whether the data follows this assumption, Bartlett's test was used. Bartlett's test was significant, which indicates that the assumption of equal variances is violated, and should be accounted for when using ANOVA. Using Welch's ANOVA, to take into account the variance, we found no statistical difference in semester 1. For semester 2 a significant result was found with an F-value of 23.984 (p-value < 0.01, $v = 2$). We also used linear regression to investigate whether the year of intervention impacted at-risk students' written examination mark as the intervention is escalated over the course of the study. For at-risk students, the year of intervention is not a significant explanatory variable. In our opinion, these changes in the distribution of marks do not provide sufficient evidence to endorse a positive impact of the learning analytics interventions. Rather the changes could have been caused by natural fluctuations having occurred owing to differences in student cohorts, end-of-semester examination paper et cetera.

The university has a free drop-in MSC for students taking mathematics/statistics modules. The MSC maintains detailed recordings of students visits to the centre. In recent years, the number of visits from *Practical Statistics'* students has remained low (Table 2) despite the strong encouragement given to students to attend the MSC through the lecturer-student meetings.

Table 2: Number of Maths Support Centre visits for *Practical Statistics*.

Year	2015/16		2016/17		2017/18	
Semester	**S1**	**S2**	**S1**	**S2**	**S1**	**S2**
Total number of visits from students	20	2	20	7	16	12
Number of visits from at-risk students	2	0	2	1	0	2

One of the suggestions given to *Practical Statistics'* students, was that "students new to learning online have found that setting specific times every week to complete online material is beneficial". The hypothesis being that there is a relationship between the dominant periodicity of students' resource usage and written examination marks. From analysing time series plots of students' engagement with online resources, there was evidence of students consistently studying pre- and post-receipt of the intervention email; in other words, the email itself did not have an effect on the consistency of students' study patterns. To investigate whether adhering to a regular pattern benefited students, we used spectral analysis on the semester 2 2017/18 cohort. We chose this cohort as they were exposed to the final escalation of the intervention. Spectral analysis is the decomposition of a time series into underlying sine and cosine functions of different frequencies (Hill & Lewicki, 2006). This allows for the isolation of strong or important frequencies. We included a taper effect of 0.2 to reduce the importance of the beginning and end of our LMS activity series. Following from this, we identified each student's dominant periodicity of their study (the number of days for one full cycle of study). Unsurprisingly, the majority of students had a dominant study pattern which is less than seven days. We considered and found no relation between the dominant study cycle of students and their examination marks received. Upon closer examination, only 19% of the dominant periodicities were statistically significant (based on a p-value of 0.05).

DISCUSSION AND CONCLUSION

Learning analytics are not always implemented smoothly or without adaptation; as we have progressed in our longitudinal study, we have changed the at-risk criteria to focus our efforts, escalated the intervention implemented and altered the continuous assessment breakdown owing to university closures. This is a limitation in terms of the statistical analysis. However, it can be a beneficial effect of longitudinal studies since the key focus is to support students. In our study, we compared students' academic results, LMS data and MSC attendance. We also investigated students' dominant study patterns. While some studies use a stringent approach of a control versus an experiment group, ethically this can be harder to achieve in an education setting. We do not believe our interventions had a significant impact upon students' behaviour but rather this study emphasises the debate around the non-engagement of at-risk students. This idea of at-risk students failing to pursue help is not a new topic to education but more research is needed into encouraging engagement in at-risk students. This may involve investigating whether there is a stigma attached to accessing specific student supports, for example the MSC, or, whether students are unable to access supports owing to work or caring duties.

Na and Tasir (2017, p. 65) explain that "different at-risk students have different learning problems and, thus, different interventions are required". In our study, we escalated the intervention in a module, however we did not consider interventions on a global scale - whether the students' problems related to all statistics (or mathematics) modules or were across their programme. Problems could be caused by personal reasons and not necessarily owing to students' non-engagement. From the twelve face-to-face meetings between the lecturer and students, it became apparent that students were at-risk for a range of reasons.

Each student's case was unique, and this emphasises the need for a broader range of interventions as well as the inclusion of pastoral academic support. We concur with Na and Tasir (2017) that different at-risk students require different interventions, however, we propose approaching this by firstly identifying whether the required intervention should be targeted towards a specific module or should support the student across their programme. We hypothesise that at-risk students are more likely to have difficulties at the programme level whereas module-level interventions are more likely to resonate with the "average" student. Alternatively, Choi et al., (2018) conjecture that at-risk students may have metacognitive skills which constrain their ability to benefit from reflective learning analytics interventions. In Atif, Richard and Bilgin's (2015) examination of students' perceptions of learning analytics, factors affecting students' success included personal responsibilities, daily travel, work commitments, financial issues and emotional and physical health issues, with a limited number suggesting module-level factors of under-preparedness and support. Future work will investigate programme-level interventions. This includes pastoral support, for example peer-to-peer mentoring, meetings between students and student advisors, and campaigns to raise awareness of the university supports available.

REFERENCES

Atif, A., Richards, D., & Bilgin, A. (2015). Student preferences and attitudes to the use of early alerts. In T. Bandyopadhyay, D. Beyene, & S. Negash (Eds.), *Americas Conference on Information Systems,* AMCIS 2015 (pp. 1-14). Puerto Rico: AMCIS.

Cai, Q., Lewis, C. L., & Higdon, J. (2015). Developing an early-alert system to promote student visits to tutor centre. *Learning Assistance Review*, *20*(1), 61-72.

Campbell, J. P., & Oblinger, D. G. (2007). *Academic analytics.* Educause. Retrieved from https://www.educause:edu/ir/library/pdf/PUB6101.pdf.

Chipman, H. A., George, E. I., & McCulloch, R. E. (2010). BART: Bayesian additive regression trees. *The Annals of Applied Statistics, 4*(1), 266-298.

Choi, S. P. M., Lam, S. S., Li, K. C., & Wong, B. T. M. (2018). Learning analytics at low cost: At-risk student prediction with clicker data and systematic proactive interventions. *Educational Technology & Society, 21*(2), 273-290.

Corrigan, O., Smeaton, A. F., Glynn, M., & Smyth, S. (2015). Using educational analytics to improve test performance. In G. Conole, T. Klobučar, C. Rensing, J. Konert, & E. Lavoué (Eds.), *Proceedings of Design for Teaching and Learning in a Networked World, 10th European Conference on Technology Enhanced Learning* (pp. 42-55). Toledo: Springer.

Dawson, S., Jovanovic, J., Gašević, D., & Pardo, A. (2017). From prediction to impact: Evaluation of a learning analytics retention program. In *Proceedings of the 7th International Learning Analytics and Knowledge Conference* (pp. 474-478). New York, USA: ACM.

Howard, E., Meehan, M., & Parnell, A. (2018). Contrasting prediction methods for early warning systems at undergraduate level. *The Internet and Higher Education, 37*(1), 66-75.

Na, K. S., & Tasir, Z. (2017). A systematic review of learning analytics intervention contributing to student success in online learning. In *2017 International Conference on Learning and Teaching in Computing and Engineering* (LaTICE) (pp. 62-68). Retrieved from https://ieeexplore.ieee.org/document/8064433.

MATHEMATICAL IDENTITY OF SCIENCE AND ENGINEERING STUDENTS IN AN IRISH UNIVERSITY

Fionnán Howard

Dublin City University

This paper presents the initial findings of my PhD study investigating science and engineering undergraduates' relationship with mathematics and the contexts that inform this relationship. Thirty-two students completed an online questionnaire consisting of three open-ended questions. The data was analysed using thematic analysis with both inductive (data-driven) and deductive (theoretical) coding. A summary of the background, theoretical perspective and conceptual framework will be presented followed by some initial results. Student responses illustrating two emergent themes will be described.

INTRODUCTION

The purpose of this research is to explore science and engineering students' relationship with mathematics and the contexts that inform this relationship as they transition to higher level education. To investigate this, the concept of mathematical identity was used and the study is thus titled Mathematical Identity of Science and Engineering students (MISE). Understanding mathematical identity helps a teacher to teach more effectively. Not least because

> "... the role of the teacher includes fostering change for the better in students' mathematical identity. To effect such change, requires knowledge about students' mathematical identity in the first instance" (Eaton & OReilly, 2009, p. 234).

Identifying issues relating to pedagogy or the learning experience of students can combat feelings of marginalisation and influence students' decision to continue, or not, their mathematical studies (Grootenboer & Zevenbergen, 2008). Thus, teachers influence the relationship that each student has with mathematics, but by understanding this relationship they can improve the students' learning experience and the efficacy of their own teaching.

Students' preconceptions about mathematics influence their learning and are further complicated by the transition to higher level education where they will be required to learn and demonstrate knowledge in new ways. It is well documented that students struggle with the kind of abstraction that is common in higher level mathematics (Breen, O'Shea, & Pfeiffer, 2013, p. 2317). Reflecting on their own mathematical identity can help students engage more effectively as mathematics learners during this transition (Kaasila, 2007). In the same vein, Sfard and Prusak (2005, p. 16) suggest that "identity talk makes us able to cope with new situations in terms of our past experience and gives us tools to plan for the future."

Background to the study

I was motivated to conduct the current research by my teaching experience. When conducting tutorials with science and engineering students in Trinity College Dublin, it appeared that some of the school of mathematics lecturers who communicated very well with mathematics students, struggled to engage effectively with (or were embraced more fully by) science and engineering students. Research has shown that lecturers teach mathematics students differently to ‘service mathematics’ students (Bingolbali, Monaghan, & Roper, 2006) and I claim that differences in mathematical identity could explain why the lecturers find different approaches more effective and can help guide such approaches.

LITERATURE REVIEW

Grootenboer & Zevenbergen (2008, p. 244) proposed that identity incorporates students’ “knowledge, abilities, skills, beliefs, dispositions, attitudes and emotions” thus placing identity as an important precursor to the learning of mathematics. In their seminal paper, Sfard and Prusak (2005, p. 16) wished to operationalise identity by avoiding the notion of “who one is.” They claimed that other authors had relied on this timeless, agentless essence in their definitions, and that this had rendered the concept untenable. Although they treated narrative and identity as equivalent, it has become more common to consider the concepts as related rather than synonymous (Eaton, 2013; Kaasila, 2007) i.e. to see narratives as the action through which identity is revealed.

A series of previous studies in Ireland developed an instrument for exploring mathematical identity of pre-service teachers which has been adapted for this new context. Mathematical Identity of Student Teachers (MIST) used grounded theory to develop a questionnaire consisting of two open-ended questions: a broad opening question and a follow-up question which included some prompts. These researchers wanted to allow the nine participants to make responses that were “indicative of their personal mathematical identity” but not leave them without any direction (Eaton & OReilly, 2009, p. 229). The prevalence of self-reflection as part of mathematical identity became evident (Eaton & OReilly, 2009a) and as a result, the subsequent study ‘Mathematical Identity using Narrative as a Tool’ (MINT) added a third question about self-reflection. Since MINT involved 99 students from four institutions, they migrated the questionnaire to an online tool (Eaton, Horn, Liston, Oldham, & OReilly, 2013).

METHODOLOGY

Theoretical Perspective

I follow a constructionist epistemology meaning “[t]here is no objective truth waiting for us to discover it” (Crotty, 1998, p. 8). The focus of the research is not events themselves but the meaning of experiences from the point of view of the participants (Creswell, 2009, p. 16) since “... people may construct meaning in different ways, even in relation to the same phenomenon” (Crotty, 1998, p. 9). I believe that it is not possible for a researcher to step outside their biases and conduct research impartially as an objective observer and therefore I present my conclusions as justified beliefs rather than absolute truths. The narratives I produce through data analysis are co-constructed by researcher and participant and thus the research process is shaped by both (Cohen, Manion, & Morrison, 2007, p. 37). What I bring to the study from my own background and identity is embraced as experiential knowledge which informs, while not dominating, the research design since “[s]eparating your research from other aspects of your life cuts you off from a major source of insights, hypotheses, and validity checks” (Maxwell, 2013, p. 45).

Conceptual Framework

Mathematical identity is defined as the multi-faceted relationship that an individual has with mathematics, including knowledge, experiences and perceptions of oneself and others (Eaton and OReilly, 2009, p. 228, see also Grootenboer & Zevenbergen, 2008). This study embraces the view of Kaasila (2007) who explains that "one's mathematical identity is manifested when telling stories about one's relationship to mathematics, its learning and teaching." He acknowledges the narrative mode of thought proposed by Bruner (1986, p. 13) which attempts to deal with "the particulars of experience, and to locate the experience in time and place" emphasising the contextual and situated nature of identity as defined in this paper.

A hybrid process of inductive (data-driven) and deductive (theoretical) coding was used to analyse the open-ended responses to the online questionnaire followed by a thematic analysis guided by the theoretical perspective and research questions. This was adapted predominantly from Fereday and Muir-Cochrane (2006) with influence from Braun and Clarke (2006) and Crabtree and Miller (1999). (For a full discussion of the methodology see Howard, OReilly, & Nic Mhuirí, 2019, in print) The aim of the analysis was to produce a group narrative co-constructed by the participants and the researcher using thematic maps within NVivo. This paper reports on the results of applying the aforementioned methodology with this aim.

Methods

Science (SCI) and Engineering (ENG) students represent a significant portion of DCU's undergraduate population and of students taking mathematics modules, but they have not previously been included in research on mathematical identity. We identified 16 cohorts of SCI and ENG students in DCU who study mathematics in their first year. An adapted version of the online questionnaire from MIST/MINT was used (See Figure 1) with each question appearing on a separate page. There were 32 respondents to the main study (22 SCI and 10 ENG students representing 14 of the 16 chosen cohorts), contributing more than 6500 words.

Q1. Think about your total experience of mathematics. Tell me about the dominant features that come to mind.

Q2. Now think carefully about all stages of your mathematical journey from primary school to university mathematics. Consider:

- Your feelings or attitudes to mathematics
- Influential people
- Critical incidents or events
- Specific mathematical content or topics
- How mathematics compares to other subjects
- Why you chose to study a course which includes mathematics at third level

With these and other thoughts in mind, describe some further features of your relationship with mathematics over time.

Figure 1: Online questionnaire where Q1 and Q2 appeared on separate pages.

To analyse this data, a codebook of forty-seven codes was developed based on a literature review and a pilot study conducted in November 2017. These codes provided the deductive (theoretical) dimension of the analysis. During analysis of the dataset for the main study, twenty new codes were developed. These provided the inductive (data-driven) dimension. Firstly, these new inductive codes were given a definition before all codes were reviewed to develop a narrative for each one. I asked myself the basic question: “What are the extracts in this code saying?” I noted observations, interpretations or ideas using memos in NVivo.

I began a thematic map by including the inductive codes so I could build it from the new ideas expressed by the MISE participants. I included the deductive codes in groups of three or four by reviewing the content of each code and forming some connections in the thematic map with other codes e.g. *I need it explained to me* and *I took charge* are connected since students say that when the teacher is bad at explaining they take the initiative to find another source from which to get better explanations. Each connection in the map was assigned a brief phrase to explain the connection while the memos in NVivo catalogue each one in more detail.

Due to the complexity of the data, it was not possible to partition the codes into themes without any connections between the themes themselves. The aim was to cluster well-connected parts of the thematic map to minimise (rather than eliminate) the connections between themes. Some codes were very well connected to the rest of the thematic map (e.g., -‘Teachers’ and ‘Exams and LC subject choice’) due to the number of extracts they contained and the broad range of issues within these extracts. I removed these temporarily and moved the other codes and potential themes to group them into well-connected clusters. I included the remaining codes in the most sensible potential theme and used a miscellaneous theme to hold codes temporarily before they were placed elsewhere. This allowed external and internal heterogeneity to be analysed (Braun & Clarke, 2006, p. 91).

RESULTS

The analysis resulted in the development of five main themes of which, the first two will be the focus of this paper:

1. Ways of learning mathematics
2. Mindsets and getting started
3. What is Mathematics?
4. Mathematics gets harder as you progress: transitions and realisations
5. Mathematics is a means to an end

Theme 1: Ways of Learning Mathematics

A triad of learning, understanding and teaching mathematics emerged at an early stage. My rationale for this triad came from several perspectives:

1. Learning: Some students mention exams and ways of learning mathematics to obtain good results or improve performance:

> ID118: We just learned it for the sake of learning and making the deadline for the Leaving so we could pass.

2. Understanding: Others refer to concepts behind calculations and how these types of building blocks are important for understanding. They want to understand rather than memorise it and they are aware of a difference between these two types of learning:

ID54: Understanding the maths we were studying instead of just learning off an equation.

ID66: I started to actually understand maths, rather than just do it.

3. Teaching: Many students gave critical evaluations of their classroom experience, comparing teaching methods and teachers themselves. They have developed strong opinions on best practice for teaching, drawing from these experiences.

ID66: The importance of learning through concepts rather than through questions should be stressed a lot more.

The teacher is the first port of call when students encounter difficulty:

ID86: I found that teacher very bad in terms of her ability to explain maths. I moved myself to higher mathematics (2nd year) because of an amazing teacher that was teaching it.

ID125: I sat higher level as the teachers in ordinary level classes were not that good.

As can be seen above, some of the MISE participants (8 out of 32) took matters into their own hands (working by themselves, going to grinds teachers) when searching for a source to help them learn or understand mathematics.

ID66: I was only doing bad in it because of how I was taught. Once I began teaching myself and actually understanding the concepts, I started to really enjoy it.

ID86: Maths became very easy only when I had a private tutor.

The MIST study found that student teachers took a "team approach" to mathematics at higher level, much more so than at post-primary level (Eaton & OReilly, 2009, p. 232). In contrast, no MISE participants' reported that they work collaboratively. This suggested that the individual learning styles and objectives of MISE participants would play a more prominent role than first expected, as demonstrated above. This individual approach to learning had two main elements which I called "I work on my own" and "I took charge." Participants describe working by themselves (by choice or necessity) with one participant stating succinctly that:

ID118 It is between you and the numbers. You have to learn and put in the work or you don't succeed.

Many participants demonstrate responsibility for directing their own learning, by finding a new source of learning or setting their own goals rather than relying on advice from others:

ID34: I dropped into ordinary before the leaving to ensure I would get a good enough grade.

ID66: Was told to drop to ordinary. After that, I took matters into my own hands, and came out with a H3 in the leaving.

ID112: When I took it upon myself to improve my maths as well as the help of my great grinds teacher, I found maths more appealing and myself more capable

I was struck by the depth and awareness with which the students discuss types of learning as well as the self-direction that they demonstrate when making decisions about how, where and when to source such learning. I find it particularly interesting that they appear cognisant that there are other ways of learning that don’t suit them. They acknowledge that they want to see the steps, have it explained or understand the concepts, and they seek out sources that provide that type of learning.

Theme 2: Mindsets and getting started

Although not prominent in previous studies among students in Ireland, it has been reported that students sometimes see mathematics as an objective subject, especially compared to other subjects, where pursuit of the right answer is the goal (Eaton & OReilly, 2009, p. 233). This view came through strongly in this study, as did the view that this objectivity does not necessarily make the process easier since:

ID91 I love how in maths there's only one correct answer but lots of different ways to get there.

Participants are clear that you need to think a certain way to work well in mathematics

ID7 Thinking outside the box … seeing the bigger picture.

They particularly focus on getting started as a keystone which includes understanding what is being asked, being able to see non-obvious solutions and trying different approaches:

ID124 I would find difficulty trying to start a problem but once I know how to start I’d be able to finish it … Maths involves more thinking and trying different methods to get to the answer.

ID86 Figuring out what to do and where to start was challenging.

A range of ideas are mentioned by participants that inform this belief. These include problem solving skills, analytical skills, lateral thinking and breaking down a problem. Confidence could improve one’s ability to think as above:

ID55 I generally did well in maths exams compared with other subjects, which made me enjoy the subject more, and gave me confidence and a belief that I could do well in the field.

However, participants identify several barriers to such positive growth mindsets including anxiety, the will to persevere and language barriers:

ID69 Maths is very heavy and stress inducing.

ID118 It’s the work that has to be put in to persevere … I know I can do it, and I want to, it’s just tough

ID15 A lot of maths were taught through Irish which I feel made it slightly harder to grasp certain topics.

Overall The participants are clear that their mindset affects their ability to get started with a question or problem, to persevere with multiple attempts or to interpret and analyse the information given. They think these are key elements to improve upon in order to succeed in mathematics.

DISCUSSION AND FURTHER RESEARCH

In this paper I have presented two initial themes from my research on mathematical identity of undergraduate science and engineering students in DCU. Theme 1 deals with ways of learning mathematics whereas theme 2 deals with how mindset facilitates or hinders effective application of knowledge. Although ways of studying mathematics has featured as a theme in previous mathematical identity research in Ireland (Eaton & OReilly, 2009a), the duality in types of knowledge/learning has not been the focus of such research. Consequently, I was surprised by the maturity and clarity with which participants presented this framework for doing mathematics, especially in only their first year at university.

The small sample size is a limitation of the study. However, the small number of participants allowed an in-depth analysis that would not have been practical with a larger group. In this paper, the validity of the themes is established by multiple references and participants describing the same phenomenon. However, since some students wrote much less than others, themes may not represent the views of all 32 participants, and this remains to be verified. Once the analysis has been completed, the next phase of the study involves a focus group. The purpose of this is twofold: to present the themes and narratives to a selection of participants in order to check the validity of the themes (Creswell, 2009, p. 190-191) and to allow the participants to elaborate on, or add to, their questionnaire submissions.

During analysis I found that some codes were not helpful for developing a group narrative but may be useful for interrogating the data for specifics (e.g. for finding out which students think mathematics at secondary school is about understanding or rote learning.) The third phase of my PhD study will focus more on such codes when I look longitudinally at the participants' individual journeys as they transition to higher level education. I expect the benefit of hindsight through some experience of mathematics at university level to affect the mathematical identities expressed by the participants in this first phase of data collection.

REFERENCES

Bingolbali, E., Monaghan, J., & Roper, T. (2006). Service teaching: Mathematical education of students of client departments. In J. Novotná, H. Moraová, M. Krátká, & N. Stehlíková (Eds.), *Proceedings of the 30th Conference of the International Group for the Psychology of Mathematics Education* (pp. 169−176). Prague, Czech Republic: PME.

Braun, V., & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology*, *3*(2), 77−101.

Breen, S., O'Shea, A., & Pfeiffer, K. (2013). The use of unfamiliar tasks in first year calculus courses to aid the transition from school to university mathematics. In B. Ubuz, C. Haser & M. A. Mariotti (Eds.), *Proceedings of the Eighth Congress of the European Society for Research in Mathematics Education (CERME8)* (pp. 2316−2325). Ankara, Turkey: Middle East Technical University and ERME.

Bruner, J. (1986). *Actual Minds, Possible Worlds.* Cambridge, MA: Harvard University Press.

Cohen, L., Manion, L., & Morrison, K. (2007). *Research methods in education.* Abingdon, UK: Routledge.

Crabtree, B., & Miller, W. (1999). *Doing qualitative research.* Thousand Oaks, CA: SAGE Publications Inc.

Creswell, J. W. (2009). *Research design: Qualitative, quantitative and mixed methods approaches.* Thousand Oaks, CA: SAGE Publications Inc.

Crotty, M. (1998). *The Foundations of Social Research.* Thousand Oaks, CA: SAGE Publications Inc.

Eaton, P., Horn, C., Liston, M., Oldham, E., & OReilly, M. (2013). Developing an instrument to explore mathematical identity: A study of students from several third level institutions in Ireland. In E. Arntzen (Ed.), *Proceedings of the Association for Teacher Education in Europe 38th Annual Conference* (pp. 280−296). Halden, Norway: ATEE.

Eaton, P., & OReilly, M. (2009). Who am I and how did I get here? Exploring the mathematical identity of student teachers. In D. Corcoran, T. Dooley, S. Close & R. Ward (Eds.), *Proceedings of Third National Conference on Research in Mathematics Education (MEI 3)* (pp. 228−236). Dublin, Ireland: St. Patrick's College.

Eaton, P., & OReilly, M. (2009a). Exploring mathematical identity as a tool for self-reflection amongst pre-service primary school teachers: "I think you have to be able to explain something in about 100 different ways." In Paditz, L., & Rogerson, A. (Eds.), *Models in Developing Education: Proceedings of the 10th International Conference on Mathematics Education into the 21st Century* (pp. 153−156). Dresden, Germany: ME21.

Fereday, J., & Muir-Cochrane, E. (2006). Demonstrating rigor using thematic analysis: A hybrid approach of inductive and deductive coding and theme development. *International Journal of Qualitative Methods*, *5*(1), 80−92.

Grootenboer, P. J., & Zevenbergen, R. (2008). Identity as a lens to understand learning mathematics: Developing a model. In M. Goos, R. Brown, & K. Makar (Eds.), *Proceedings of the 31st Annual Conference of the Mathematics Education Research Group of Australasia* (pp. 243−250). Brisbane, Australia: MERGA.

Howard, F., OReilly, M., & Nic Mhuirí, S. (2019, in print). "I don't like Maths as a subject but I like doing it": A methodology for understanding mathematical identity. In *Proceedings of the 11th Congress of the European Society for Research in Mathematics Education (CERME11)*. Utrecht, Netherlands: University of Utrecht and ERME.

Kaasila, R. (2007). Using narrative inquiry for investigating the becoming of a mathematics teacher. *ZDM: The International Journal on Mathematics Education*, *39*(3), 205–213.

Maxwell, J. A. (2013). *Qualitative Research Design: An Interactive Approach.* Thousand Oaks, CA: SAGE Publications Inc.

Sfard, A., & Prusak, A. (2005). Telling identities: In search of an analytic tool for investigating learning as a culturally shaped activity. *Educational Researcher*, *34*(4), 14–22.

Strauss, A. L., & Corbin, J. M. (1998). *Basics of qualitative research: Techniques and procedures for developing grounded theory.* Thousand Oaks, CA: SAGE Publications Inc.

FACILITATING MATHEMATICAL DISCUSSION THROUGH THE USE OF PICTURE BOOKS IN AN IRISH SENIOR INFANT CLASSROOM

Mary Kearney

St. John's N.S, Drogheda.

This study explored how mathematical discussion could be facilitated through the use of picture books in a senior infant classroom. A qualitative inquiry was conducted over a three-and-a-half-week period in a large urban disadvantaged junior primary school in Ireland. Data comprised of observations of the children as they engaged in small groups during mathematics lessons. The data was collated by the methods of observations in situ, video recordings and extended fieldnotes. The findings of this research indicated that children's participation in a mathematical discussion can support the development and use of mathematical language and also support their mathematical literacy. Evidence revealed that mathematical discussion also supports children's mathematical thinking through the engagement in mathematical processes such as problem-solving, reasoning and connecting. The findings of this research have implications for early childhood mathematics policy and curriculum which advocate the need for increased opportunities for children to engage in mathematical discussion. One effective pedagogy to facilitate mathematical discussion is the use of picture books which support young children's overall mathematical proficiency.

INTRODUCTION

The fundamental aim of mathematics education for young children should be the provision of experiences and opportunities that cultivate their understanding of mathematics in such a way that they can use mathematics "in real-life situations as a meaningful tool to describe their quantitative world" (Hong, 1999, p. 162). According to the National Council for Curriculum and Assessment (NCCA) research report No. 17 on *Mathematics in Early Childhood and Primary Education (3-8 years),* (Dunphy, Dooley & Shiel, 2014), mathematical proficiency must be considered central in early childhood mathematics.

Mathematics Education in Ireland

Research indicates that more focus is needed on the development of mathematical language particularly in Irish classrooms (Shiel, Cregan, McGough & Archer, 2012). Following the evaluation of the execution of the *Primary School Curriculum* (GoI, 1999a), the Inspectorate discovered that 25% of teachers did not spend sufficient time on the development of mathematical language during mathematics lessons (DES, 2005a). Additionally, it was highlighted that in cases where teachers spent adequate time teaching mathematical language, the teachers "planned for the teaching of mathematical language, used appropriate terminology, provided opportunities for children to use mathematical language and referred to mathematical words and symbols" (p. 30). *Aistear* (NCCA, 2009) outlines in one of its main themes, *Exploring and Thinking,* that children form understanding of mathematics by interacting with others which enables them to explore, question and refine ideas. A significant factor to consider is that interactions need to be supported by teachers' engagement with children which "builds on children's abilities, interests, experiences, cultures, provides for their needs and facilitates them to initiate activities" (NCCA, 2009, p. 27). This endeavours to contribute to children's overall mathematical learning and development and improve their mathematical literacy.

Literature Review

When children engage in a mathematical discussion or *math talk,* they are afforded the opportunity to talk about their mathematical thinking and communicate using mathematical language (Fuson, Kalchman & Bransford, 2005). They are supported in talking about their mathematical thinking which incorporates their formal and informal representations of mathematical ideas and symbols (Dunphy et al., 2014). Cheeseman (2015) noted that a mathematical discussion provides teachers with opportunities to understand children's thinking but also provides occasions "to challenge children's thinking to make meaning in the moment and to stimulate new learning right 'on the edge' of the child's thinking" (p. 275). Therefore, during a mathematical discussion, mathematical language development and mathematical vocabulary acquisition can be facilitated because children may be required to express their mathematical thinking, which may enable them to apply problem solving and reasoning skills and so demonstrate their level of mathematical understanding (Anderson, Anderson & Shapiro, 2004).

Nic Mhuiri (2011) notes that the discourse of mathematics lessons which appears to engage children incorporates "patterns of dialogue that involve making conjectures, and examining and justifying one's own mathematical thinking and the mathematical thinking of others" (p. 320). Nic Mhuirí observed that children were more interested in activities which involved participation in discussion as opposed to traditional mathematics lessons which involved repetition and lower-order questioning. Although young children often lack the essential mathematical language to communicate their understanding, the use of playful practices including a discussion centred on a picture book can enable them to experience correct use of mathematical language (Hong 1999).

During mathematical discussion children are provided with opportunities to talk about their mathematical thinking and ask questions which deepens their understanding and cultivates mathematical language (van den Heuvel-Panhuizen, Elia & Robitzsch, 2014). Evidently, mathematical language development is a vital component of children's overall mathematical proficiency. At first children talk about their mathematical experiences using everyday informal language but gradually progress to using more mathematical language (Montague-Smith & Price, 2012).

Mathematization is a crucial factor in supporting children's understanding of mathematical processes and content and developing their overall mathematical proficiency (NRC, 2009). Dunphy (2015) argues that in order to sufficiently accommodate for mathematization in early childhood mathematics there is a need for "the intervention of the teacher who not only recognises opportunities to encourage and support mathematization but who proactively seeks to engage children with mathematization processes" (p. 3). One effective means of facilitating mathematization is through mathematical discussion where children are challenged to mathematize. Children engaged in mathematical discussion are “collaborating and building upon each other’s contributions” (Suggate, Davis & Goulding, 2010, p. 25).

Children’s literature, such as picture books, can act as a catalyst to support children’s ability to participate in mathematical discussion where they can interact, mathematize and make learning connections. From a socio-cultural perspective, children’s mathematical language and discussion is enriched when children share their individual connections to the story (Whitin, 1992).

METHOD

Participants

Some groups in society are “known to struggle with general language acquisition, including children living in disadvantaged circumstances, children who speak a language other than the language of instruction at home, and children who have a language impairment” (Dunphy et al., 2014, p. 65). This statement is a true depiction of my research setting which consists of children with a range of abilities that come from diverse backgrounds. The Delivering Equality of Opportunity in Schools (DEIS) action plan states that every child should be afforded the opportunity to access, share in and reap the benefits from education regardless of their circumstances (DES, 2005b). Hence, this study endeavoured to attend to the needs of 5-6 year old children in senior infants, the second year of their formal education, by providing them with a context that they can connect with, enabling them to build on their prior mathematical experiences through a mathematical discussion.

I participated in team-teaching with this class every day in my role as a learning support teacher. The class who were my sample consist of twenty children, twelve boys and eight girls who come from diverse backgrounds. Six children are non-Irish Nationals whose second language is English, two children are from the Travelling Community and one child has special educational needs.

Qualitative Research Methodology

A qualitative research design was deemed most appropriate as this study proposed to gain an understanding of children’s overall mathematical proficiency during mathematical discussion based on picture books. Therefore, the investigative and descriptive characteristics of a qualitative study (Maykut & Morehouse, 1994) suited this particular research and facilitated the inquiry into how children participated in mathematical discussion using three picture books. As well as this, the research was conducted in the “natural setting” (Creswell, 2007, p. 37) of the children’s own classroom which involved gathering multiple sources of data and subsequent inductive data analysis (Creswell, 2007).

In conducting this qualitative inquiry, triangulation of data collection was used to check whether the responses from each method were consistent and to ensure the credibility and validity of the data (Cohen, Manion & Morrison, 2011). Within the interpretivist paradigm the methods of data collection that were used were observations on situ, video recordings and extended fieldnotes based on the video recordings. They were selected in order to facilitate the investigation of children’s understandings through an interactive process. Through continuous analysis of the data the following recurring themes emerged: mathematical language, mathematical processes and mathematical content.

FINDINGS AND DISCUSSION

Mathematical Language

The picture books presented extensive opportunities to model correct use of mathematical language through a meaningful context. The following extract from a discussion on *Oliver's Milkshake* (French, 2000) supports this claim. We discussed an image where Lily and Oliver were peering over the half-door at the goats with Auntie Jen. The children engaged in a discussion about their height and Deon used the comparative language "bigger" (Transcript, Thursday 2nd February 2016) on one occasion during the interaction. The content area of measures was being developed here. There is also evidence of teacher modelling where both comparative and superlative mathematical language was used.

MK:	Have a look... what are Lily and Oliver doing, Jill?
Jill:	Points at Lily and Oliver
MK:	Ok…they are looking at the goats. They have to go up on their tippee-toes I'd say.
Derek:	Or maybe...or maybe Oliver is a bit bigger?
MK:	Maybe Oliver is just a little bit taller. I think you are right. I can see more of Oliver's head than Lily's head so I think Oliver is taller. Do you think Oliver is taller Deon?
Dn:	Yes
MK:	Who is the tallest out of everyone?
Deon:	Points to Auntie Jen
MK:	Auntie...
Deon:	Jen
MK:	Auntie Jen is the tallest. Who is the smallest Jill?
Jill:	Points to Lily
MK:	Lily is the smallest and they are peeping over the door to see the goats.

It is clear from the above extract that the children did not have the appropriate mathematical language to engage in the discussion. The children showed their understanding of measurement in relation to height by pointing to the images and were supported by teacher modelling the correct use of mathematical language.

Mathematical Processes

It was found that the picture books contained ample opportunities to enhance children's engagement in mathematical processes such as reasoning. The following extract from video footage of a discussion on *The Runaway Dinner* (Ahlberg, 2006) illustrates this. They were presented with the problem that there were not enough chairs for all of the chips. Some of the chips were sitting on chairs but others were standing up.

MK:	Were there enough chairs for all the chips?
May:	No
MK:	How many more chairs would we need for those chips Matt?
Matt:	2
MK:	2 more chairs
Evelyn:	No 4!
MK:	4 more chairs?
Evelyn:	For those 2 there (pointed to two more chips in the boat on the other page). They are going to sail back and stop.
MK:	Oh yes! Because there are 2 here and 2 there and that makes 4. Well done.
Evelyn:	And they are sailing back to the path

The use of comparative language can be seen in the above extract as well as evidence of the counting principle of one-to-one correspondence where the children had to determine how many more chairs were needed so that each chip would have one. Evelyn noticed that there were two more chips in the picture and justified why she thought four more chairs were needed. The language of addition was modelled to illustrate that two and two makes four. During this discussion the children co-constructed knowledge when Evelyn extended Matt's response to determine that four more chairs were needed (Suggate et al., 2010).

Mathematical Content

Over the course of the research the children began to make connections between the content that they had discussed in the different picture books and this supported their understanding of algebra. *Oliver's Milkshake* (French, 2000) contained ample opportunities to develop the concept of recognising patterns. When the children looked at the image of Oliver's bedroom, they noticed many different patterns (Figure 1.). Savine (5 years 8 months) noticed a red and white pattern "On his pyjamas", Hayden (6 years 5 months) commented that the pattern on the duvet was "Indigo, white, indigo, white" and Matt (5 years 8 months) observed that the pattern on the cat was "White, orange, white, orange" (Transcript, Friday 5th February 2016).

***Figure 1*. An image of the patterns in Oliver's bedroom from the story *Oliver's Milkshake* (French, 2000)**

The picture books presented abundant opportunities to explore number and supported the children in making estimations. The following extract displays how they engaged in a discussion about the relationship between the number of fruits in the basket and the number of people. They discussed whether there would be enough fruit for the fifteen people in the picture (Figure 2.).

MK: Could you guess how many tangerines are in that basket Polly?

Polly: A million!

MK: You think a million!

Hayden: I think a million hundred

MK: Do you remember we were estimating during the week? …we were making guesses. I'm just looking at that and I think there could be about 150..?

Evan: I think there's 100

MK: …how many do you think Victor?

Victor: I think 145

MK: Well done…Hayden do you think there are enough tangerines in the basket for everyone to get one?

Hayden: yes

MK: …now we have 15 people

Evan: But there's more tangerines

MK: So do you think some people might get 2 tangerines

Hayden: No a lot of them

MK: Some people could even get 3 tangerines

Polly: or 4?

Hayden: or 6!

This extract showed that the children had an awareness of large numbers when they commented on the basket over-flowing with tangerines. During this discussion they engaged in approximation and estimation of number. I suggested that each person could get two or three tangerines each but Evan (6 years 3 months) and Hayden (6 years 5 months) disagreed and used comparative language when justifying how many tangerines each person could get. Polly (5 years 11 months) suggested that each person could get four tangerines each which demonstrated her understanding of the stable-order principle of counting. This example shows the range of mathematical processes and mathematical concepts that can be developed when children engage in a discussion about an image that is meaningful to them.

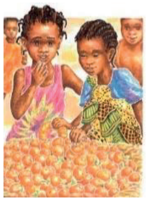


***Figure 2.* Handa's basket full of tangerines at the end of the story *Handa's Surprise*, (Browne, 1995)**

CONCLUSION

The findings of this research indicate the use of picture books is an effective approach to support mathematical discussion and to support children's overall mathematical proficiency in a senior infant classroom. This study also revealed that mathematical discussion supports children's engagement in mathematization and in key mathematical processes such as problem-solving, reasoning, connecting and communicating. Moreover, a range of mathematical content areas such as number, measures and algebra were addressed and it was observed that the children made links between content areas. Within these content areas mathematical concepts were addressed.

Implications for Policy and Curriculum

While the *Primary School Curriculum* (GoI, 1999a) alludes to the importance of mathematical language and the development of mathematical concepts, it does not place the same emphasis on the importance of developing mathematical processes through mathematical discussion. Furthermore, mathematical discussion is not a prominent feature of the *Primary School Curriculum: Teacher Guidelines* (GoI, 1999b) as it does not elaborate on how mathematical discussion can be developed nor does it recommend various pedagogical approaches. This research has depicted one effective pedagogical means of facilitating mathematical discussion. As previously noted, *Aistear* (NCCA, 2009) recommends that teachers and children engage in interactions to develop language which includes mathematical language. This guidance can be adhered to through the use of picture books.

To conclude, the findings clearly establish the fundamental role that mathematical discussion plays in supporting young children's mathematical proficiency. This study highlights the effectiveness of facilitating mathematical discussion in a senior infant classroom through the use of picture books which provides opportunities to develop mathematical language, engage in mathematical processes and support mathematical thinking.

REFERENCES

Ahlberg, A. (2006). *The runaway dinner*. London: Walker Publications.

Anderson, A., Anderson, J., & Shapiro, J. (2004). Mathematical Discourse in Shared Storybook Reading. *Journal for Research in Mathematics Education*, 35 (1), 5-33.

Browne, E. (1995). *Handa's surprise*. London: Walker Publications.

Cheeseman, J. (2015). Mathematical Conversations that Challenge Children's Thinking. In B. Perry, A. MacDonald & A. Gervasoni (Eds.), *Mathematics and Transitions to School: International Perspectives*. New York: Springer.

Cohen, L., Manion, L., Morrison, K. (2011). *Research methods in education* (7th ed.). London: Routledge.

Department of Education and Science. (2005a). *An evaluation of curriculum implementation in primary schools*: *English, mathematics and visual arts*. Dublin: Stationery Office.

Department of Education and Science. (2005b). *Delivering equality of opportunity in schools* (DEIS): *An action plan for educational inclusion*. Dublin: Stationery Office

Dunphy, E., Dooley, T., & Shiel, G. (2014). *Mathematics in early childhood and* *primary education (3-8 years): Definitions, theories, development and progression*. Report No. 17. Dublin: National Council for Curriculum and Assessment.

Dunphy, E. (2015). Transition to school: Supporting children's engagement in mathematical thinking processes. In A. Gervasoni, A. MacDonald & B. Perry (Eds.) *Mathematics and Transition to School: International Perspectives*. London, UK: Springer.

French, V. (2000). *Oliver's milkshake*. London: Hodder Children's Books.

Fuson, K., Kalchman, M. & Bransford, J. (2005). Mathematical understanding: An Introduction. In M. S. Donovan, & J. Bransford (Eds.), *National research council: How students learn: Mathematics in the classroom. Committee on how people learn: A targeted report for teachers*, (pp. 217-256). Washington, DC: The National Academies Press.

Government of Ireland. (1999a). *Primary school curriculum: Mathematics*. Dublin, Ireland: The Stationery Office.

Government of Ireland. (1999b). *Primary school curriculum: Mathematics: Teacher guidelines.* Dublin, Ireland: The Stationery Office.

Hong, H. (1999). Using Storybooks to Help Young Children Make Sense of Mathematics Chapter 18. In J. Copley (Ed.), *Mathematics in the early years,* (pp. 162-168). Reston: VA: National Council of Teachers of Mathematics.

Maykut, P., & Morehouse R. (1994). *Beginning qualitative research: A philosophical and practical guide.* London: The Falmer Press.

Montague-Smith, A., & Price, A. (2012). *Mathematics in early years education*. London, United Kingdom: Routledge.

National Council for Curriculum and Assessment (NCCA) (2009). *Aistear: The early childhood curriculum framework: Principles and themes*. Dublin, Ireland: NCCA

National Research Council (NRC) (2009). *Mathematics learning in early childhood: Paths towards excellence and equity*. Washington, DC: The National Academies Press.

Nic Mhuirí, S. (2011). Look who's talking now- the role of talk in relation to mathematical thinking and problem solving. In T. Dooley, D. Corcoran, M. Ryan, & (Eds.), *Proceedings of the fourth conference on research in mathematics education* (MEI 4) (pp. 320-332). Dublin, Ireland: MEI.

Shiel, G., Cregan, A., McGough, A., Archer, P. (2012). *Oral language in early childhood and primary education (3-8 years)*. Dublin, Ireland: NCCA.

Suggate, J., Davis, A., & Goulding, M. (2010). *Mathematics knowledge for primary teachers*. (4th ed.). London: David Fulton.

van den Heuvel-Panhuizen, M., Elia, I., & Robitzich, A. (2014). Effects of reading picture books on kindergartners' mathematics performance, *Educational Psychology*, 1-24. doi: 10.1080/01443410.2014.963029

Whitin, D. J. (1992). Explore mathematics through children's literature. *School Library Journal*, 38(8), 24-28.

WHAT PISA MAY TELL US ABOUT MATHEMATICAL LITERACY IN AN ERA OF DATA SCIENCE

Anthony E Kelly[1] and Finbarr Sloane[2]

[1]George Mason University, U.S.A. and [2]National Science Foundation, U.S.A.

The demands of mathematical literacy are responsive to the context in which they are used. For many decades, this context has remained largely stable, especially for classroom mathematical practices. Increasingly, changes in science and engineering have begun to redefine mathematical literacy – changes that are most evident in the emerging field of data science. This paper reviews emerging definitions of data science, and their implications for the workplace and scientific research and development. It will use a report on Junior Cycle Project Maths (viewed through the lens of PISA 2016) as an approximate indicator of where Irish students stand on certain elements of data science.

PISA AND TOPICS FOR MATHEMATICS LITERACY

For the purposes of international comparison by the Organisation for Economic Cooperation and Development (OCED), mathematical literacy was defined alongside mathematics, with the latter, not surprisingly, as the centre of gravity of the construct (OECD, 2006, p.12):

> an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen.

At the same time, the definition recognised that the construct of "mathematical literacy" must extend beyond mathematics and be responsive to changes in the world and in individuals' lives (OECD 2006, p. 76):

> A mathematically literate citizen realises how quickly change is taking place and the consequent need to be open to lifelong learning. Adapting to these changes in a creative, flexible and practical way is a necessary condition for successful citizenship. The skills learned at school will probably not be sufficient to serve the needs of citizens for the majority of their adult life.

Even back in 2006, the OCED recognized that changes in technology and the needs of the workplace would place new demands on how "mathematical literacy" would be defined and enacted (OECD, 2006, p. 76):

> The requirements for competent and reflective citizenship also affect the workforce. Workers are less and less expected to carry out repetitive physical chores. Instead, they are engaged actively in monitoring output from a variety of high-technology machines, dealing with a flood of information and engaging in team problem solving. The trend is that more and more occupations will require the ability to understand, communicate, use and explain concepts and procedures based on mathematical thinking. The steps of the mathematisation process are the building blocks of this kind of mathematical thinking.

The OECD has also defined the context for mathematical literacy as including four domains: (a) personal, (b) occupational, (c) societal and (d) scientific (OECD, 2017, pp. 61-62).

A more recent definition of mathematical literacy in PISA defines it as including "[reason] mathematically and [use] mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena" (OECD, 2017, p. 51). In other words, the OCED has recognised the growing value of statistics, science and engineering ideas for the construct of "mathematical literacy."

PISA CORE TOPICS

Those designing PISA framed the core concepts needed for mathematics literacy (OECD, 2006, p. 82), which remain up to PISA 2018 as: (1) space and shape, (2) change and relationships, (3) quantity, and (4) uncertainty.

Under the topic of space and shape, OECD (2006) emphasised (p. 84):

- Recognising shapes and patterns
- Describing, encoding and decoding visual information
- Understanding dynamic changes to shapes
- Similarities and differences
- Relative positions
- 2-D and 3-D representations and the relations between them
- Navigation through space

For change and relationship, the OECD (2006, p. 86) noted that:

> Change and relationships can be represented in a variety of ways including numerical (for example in a table), symbolical, graphical, algebraic and geometrical. Translation between these representations is of key importance, as is the recognition of an understanding of fundamental relationships and types of change. Students should be aware of the concepts of linear growth (additive process), exponential growth (multiplicative process) and periodic growth, as well as logistic growth, at least informally as a special case of exponential growth.

The OECD (2017, p. 59) added:

> *Change and relationships* is evident in such diverse settings as growth of organisms, music, and the cycle of seasons, weather patterns, employment levels, and economic conditions. Aspects of the traditional mathematical content of functions and algebra, including algebraic expressions, equations and inequalities, and tabular and graphical representations, are central in describing, modelling, and interpreting change phenomena.

For quantity, OECD (2006, p. 89) explained that:

> Important aspects of quantity include an understanding of relative size, the recognition of numerical patterns, and the use of numbers to represent quantities and quantifiable attributes of real-world objects (counts and measures). Furthermore, quantity deals with the processing and understanding of numbers that are represented to us in various ways. An important aspect of dealing with quantity is quantitative reasoning. Essential components of quantitative reasoning are number sense, representing numbers in various ways, understanding the meaning of operations, having a feel for the magnitude of numbers, mathematically elegant computations, mental arithmetic and estimating.

The OCED (2017, p. 59) added:

> To engage with the quantification of the world involves understanding measurements, counts, magnitudes, units, indicators, relative size, and numerical trends and patterns. Aspects of quantitative reasoning—such as number sense, multiple representations of numbers, elegance in computation, mental calculation, estimation and assessment of reasonableness of results—are the essence of mathematical literacy relative to quantity . . . Geometry serves as an essential foundation for space and shape, but the category extends beyond traditional geometry in content, meaning, and method, drawing on elements of other mathematical areas such as spatial visualisation, measurement and algebra.

And, for uncertainty, OECD (2006, p. 93) viewed the topic as overlapping significantly with statistics.

- The omnipresence of variation in processes
- The need for data about processes
- The design of data production with variation in mind
- The quantification of variation
- The explanation of variation

To this topic, the OCED (2017, p. 60) added: “The uncertainty and data content category includes recognising the place of variation in processes, having a sense of the quantification of that variation, acknowledging uncertainty and error in measurement, and knowing about chance.”

Shiel and Kelleher (2017), who analysed the PISA 2012 data under the four headings of (1) space and shape, (2) change and relationships, (3) quantity, and (4) uncertainty reported the following findings:

Space and shape (geometry and trigonometry)

A consistent finding in PISA mathematics has been the low performance of students in Ireland on Space & Shape items. In the two cycles in which mathematics has been a major assessment domain in PISA (2003, 2012), students in Ireland have achieved mean scores below the corresponding OECD average scores on Space & Shape, with female students, in particular, struggling on items in this content area. (Shiel & Kelleher, 2017, p. 151)

Change and relationships (algebra)

In PISA 2012, students in Ireland achieved a mean score on Change & Relationships (501.1) that was significantly higher than the corresponding OECD average (492.5), though Ireland lagged behind a number of countries including Canada (525), Belgium (513), and Switzerland (530) as well as Japan (542) and Korea (559) …

Functions. …the Functions content area is integrated into Change & Relationships in the PISA assessment, and into Algebra in TIMSS. Hence, international assessments do not provide separate information on how students perform in this area. (Shiel & Kelleher, 2017, p. 152-153).

Quantity (number)

In PISA 2012, students in Ireland achieved a mean score on Quantity that was significantly above the corresponding OECD average (by 10.1 score points), though students in Ireland at the 90th percentile achieved a score that was not significantly different from the OECD average at that marker (Shiel & Kelleher, 2017, p. 149).

Uncertainty and data (statistics and probability)

Students in Ireland perform relatively well in the corresponding area in PISA mathematics (Uncertainty & Data), with mean scores that were significantly above the corresponding OECD averages in 2003 and 2012. Indeed, Uncertainty & Data was an area of strength for students in Ireland on PISA, even before the implementation of Project Maths, though there was a significant drop of 8.5 score points between 2003 and 2012. Students in Ireland also performed well on Data & Chance in TIMSS 2015, with a mean score that was significantly higher than on average across OECD countries in the study. (Shiel & Kelleher, 2017, p. 149).

ENTER DATA SCIENCE

Do these PISA results provide a basis to assess how well Irish students are prepared to engage a version of mathematical literacy that includes new demands from science, engineering and the changing workplace? Why is this important?

The "flood of information" anticipated by OCED in 2006 has become a tsunami of data due to ubiquitous data-gathering technologies and exponential growth in computational power. Primary among data-gathering technologies is the emergence of the "internet of things."

The internet of things is the network of devices with sensors that permeate our homes, healthcare, manufacturing, transportation, agriculture, and places of work. According to Forbes magazine, the internet of things has "six core building blocks (hardware, connectivity, cloud platform & analytics, applications, cybersecurity, and system integration) and six supporting technologies (additive manufacturing [3D printing], augmented and virtual reality [AR & VR], collaborative robots, connected machine vision, drones / UAVs, self-driving vehicles [SDVs])" (https://www.forbes.com/sites/louiscolumbus/2018/12/13/2018-roundup-of-internet-of-things-forecasts-and-market-estimates/#a2c5df67d838).

Data from these devices can inform better use of a myriad of objects, tools and resources, including smart phones, smart homes, smart hospitals, smart manufacturing, smart agriculture, and smart workplaces.

Not surprisingly, this deluge of data is transforming the nature and practice of science and engineering. At the US National Science Foundation, the importance of data science is recognised by connected funding across a set of "big ideas" that will guide billions of dollars of future investment. While each of the big ideas differs in contexts (e.g., navigating the new Arctic, biology, the future of work, or astronomy), it is clear that advances in data and data analytics are the primary drivers (https://www.nsf.gov/news/special_reports/big_ideas/).

Implications for the definition of mathematical literacy

Leading academic institutions are responding to this data challenge by developing an emerging interdisciplinary effort called "data science." Data science is a combination of inferential thinking (i.e., statistics and mathematics), computational thinking (i.e., elements of computer science), and an identified content or knowledge area of application (Blei & Smyth, 2017; Cao, 2017; Wing, Janeja, Kloefkorn & Erickson, 2018).

Importantly, while data science includes mathematical and statistical foundations, it adds concepts from computer science and emphasises how data are collected, managed, and analysed. In a consensus report of a panel of experts, the US National Academies of Science, Engineering and Medicine add elements to mathematics and statistics in order for data scientists to develop "data acumen" (National Academies of Sciences, Engineering, and Medicine, 2018, p. 22):

- Computational foundations,
- Data management and curation,
- Data description and visualization,
- Data modeling and assessment,
- Workflow and reproducibility,
- Communication and teamwork,
- Domain-specific considerations, and
- Ethical problem solving.

PISA, PROJECT MATHS AND DATA SCIENCE

Could Ireland's "Project Maths," which was tailored in large part to the prior PISA test performance (Kirwan, 2015), provide a basis to teach and assess mathematical literacy if it were extended to more elements of data science?

To partially answer this question, we note that Project Maths was at least partially based on Freudenthal's realistic mathematics education (RME) (Conway & Sloane, 2005).

Leavy and Sloane (2011a) argued, based on their assessment of preservice primary school teachers' understanding of statistics, that statistics was mostly understood procedurally, making it difficult for these beginning teachers to conceptualize and engage statistics instruction in real-world contexts. However, Sloane (2005) showed that through the use of intensive Lesson Study, teachers could engage student learning of mathematics that went beyond procedural fluency. Leavy and Sloane (2011b), working with preservice teachers in

Ireland, again showed that, with intensive support, preservice teachers can move beyond procedural applications of statistics.

Based on the analyses by Shiel and Kelleher (2017), Irish students at the Junior Cycle have relatively solid foundations in: (a) change and relationship, (b) quantity, and (c) uncertainty and data. We see from the above analyses that Irish students have a reasonably good preparation in the mathematical and statistical elements posed by PISA. Yet, the framing of mathematical literacy by PISA continues to view mathematics and statistical foundations as mostly stand-alone constructs that are yet to be integrated with computational, data management, and domain-specific considerations (not to mention ethical or privacy concerns).

Especially in the case of uncertainty and data, work by Leavy and Sloane (2011a, 2011b) described the fragility of this inference in the population of pre-service primary school teachers and explored the training efforts needed for such an inference to be real. Extra effort will be required there and in space and shape to round out the mathematics literacy aspects of data science.

LOOKING FORWARD

We acknowledge the limitations of this analysis. First, definitions of data science extend well beyond traditional mathematical topics. Indeed, we limited our analysis to overlap with the mathematical and statistical foundations of data science. We further limited our analysis to those elements of mathematics and statistics that bore resemblance to ideas assessed in the PISA framework. And even here, there is (especially at the Junior Cycle) limited overlap with topics such as (a) set theory and basic logic, (b) multivariate thinking via functions and graphical displays, (c) basic probability theory and randomness, (c) matrices and basic linear algebra, (d) networks and graph theory, and (e) optimization.

Nonetheless, the transformations in science and engineering and the workplace that are already in progress, and the investments in research and development exemplified by the US National Science Foundation, suggest serious attention may be warranted to expanding definitions of mathematical literacy to incorporate data science in Ireland.

REFERENCES

Blei, D., & Smyth, P. (2017). Science and data science. *Proceedings of the National Academies of Sciences*, *114* (33), 8689-8692.

Cao, L (2017). Data science: A comprehensive overview. *ACM Computing Surveys (CSUR)*. *50*(3). Doi: 10.1145/3076253.

Conway, P. & Sloane, F. (2006). *International trends in post primary mathematics education.* Dublin: National Council for Curriculum and Assessment.

Kirwan, L (2015). Mathematics curriculum in Ireland: The influence of PISA on the development of Project Maths. *International Electronic Journal of Elementary Education*, 2015, *8*(2), 317-332.

Leavy, A. & Sloane, F. (2011a). Understanding and modelling pre-service teacher's knowledge of statistics. In T. Dooley and D. Corcoran (Eds.), *Proceedings of the Third National Conference on research in mathematics education: MEI3*. Dublin: St. Patrick's College of Education.

Leavy, A. & Sloane, F. (2011b). Applying results of statistics education research to teaching statistics in Irish primary schools. Author: *Proceedings of the World Congress of Statistics (ISI-2011)*. Amsterdam: IOS Press.

National Academies of Sciences, Engineering, and Medicine (2018). *Data science for undergraduates: Opportunities and options*. Washington, DC: The National Academies Press. https://doi.org/10.17226/25104.

OECD. (2006). *Assessing scientific, reading and mathematical literacy: A framework for PISA 2006*. Paris: OECD Publishing.

OECD. (2016). *PISA 2015 assessment and analytical framework*. Science, reading, mathematics and financial literacy. Paris: OECD Publishing.

OECD (2017). *PISA for development assessment and analytical framework: Reading, mathematics and science*. Paris: OECD Publishing.

Shiel, G., & Kelleher, C. (2017). *An evaluation of the impact of Project Maths on the performance of students in Junior Cycle mathematics*. Dublin: Educational Research Centre. Accessed at: http://www.erc.ie/wp-content/uploads/2018/07/PM_EvaluationStrand1.pdf

Sloane, F. (2005). Lesson study in Japanese mathematics classrooms: An exemplar of design research. In S. Close, T. Dooley and D. Corcoran (Eds.), *Proceedings of the first national conference on research in mathematics education: MEI1*. St. Patrick's College of Education, Dublin: Ireland.

Wing, J. M., Janeja, V. P., Kloefkorn, T. & Erickson, L. C. (2018). *Data science leadership summit: Summary report*. Technical Report: Columbia University, NY.

INVESTIGATING THE LONGITUDINAL IMPACT OF PARTICIPATING IN SCHOOL-BASED LESSON STUDY ON MATHEMATICS TEACHERS' PROFESSIONAL COMMUNITY

Emily Lewanowski-Breen[1], Aoibhinn Ní Shúilleabháin[1] and Maria Meehan[1]

[1]School of Mathematics and Statistics, University College Dublin

Teacher professional communities have recently figured among the most influential factors for supporting teachers in their learning and in enacting educational change in schools. While lesson study has been documented as a means to support the development of such communities, previous studies have not addressed the sustainability of the professional communities which emerge. In this study, we follow-up with six mathematics teachers from two post-primary schools in the Republic of Ireland, who engaged in school-based lesson study in 2012/13, in order to investigate the long-term impact on their teacher professional community. Our findings indicate that the mathematics teachers in both schools had developed a predominantly mature professional community during their participation in lesson study in 2012/13. Moreover, we find that six years on, the community has been sustained in one school and further strengthened in the other. These findings suggest that lesson study may be a viable model to develop and sustain mathematics teachers' professional communities in the long-term.

INTRODUCTION

Growing trends have emerged in recent years which position professional communities at the forefront of attempts to support teachers in becoming life-long learners and in enacting educational change in schools (Stoll, Bolam, McMahon, Wallace, & Thomas, 2006; Vescio, Ross, & Adams, 2008). As outlined by Vescio and colleagues (2008), such communities are rooted in the premise of improving both teacher practice and student learning and can thus provide a means to connect teacher learning to the lived realities in the classroom (Sargent & Hannum, 2009). This is framed against a national backdrop of increasing emphasis on teacher learning and collaborative practice (Teaching Council, 2016).

Within this arena, lesson study (LS) has emerged as a valuable means to cultivate teacher professional community development (Baricaua Gutierez, 2016; Lewis, Perry, & Hurd, 2009; Lieberman, 2009) as it provides a context in which teachers can collaborate with one another and is rooted in both teacher learning and student learning (Lewis et al., 2009). However, previous studies have not examined the long-term impact of LS on teacher professional communities. Specifically, research is needed to provide insight into whether the emerging communities are sustained in the years following participation in LS. This research attempts to address this gap in the literature by conducting a longitudinal study with six mathematics teachers, from two post-primary schools in the Republic of Ireland (ROI), who engaged in school-based LS as part of a comparative case study in 2012/13 (see Ní Shúilleabháin, 2016). This study will specifically address the following research question: How do post-primary mathematics teachers perceive the long-term impact of participating in school-based LS on their teacher professional community?

LITERATURE REVIEW

Teacher professional community

Within the last decade, a growing body of research has emerged which documents the development of professional communities and their impact on teaching and learning practices (see Vescio et al., 2008 for review). As argued by Fullan and Hargreaves (1992), teachers must be encouraged to engage with their colleagues in the context of a learning community in order to cultivate more progressive practices within the profession. This view is supported by empirical research which has shown that professional communities provide teachers with a fertile ground for new learning, whilst also having a positive impact on students' learning (Grossman, Wineburg, & Woolworth, 2001; Vescio et al., 2008). Such communities can thus serve as a valuable context for school-based professional development (PD), instructional improvement, and school reform (Stoll et al., 2006).

Teacher community development

While the concept of teacher community has been extensively referenced in recent years, the defining characteristics of such communities are not simply delineated as evidenced by the multitude of interpretations offered by scholars in the field. A 'professional learning community', for example, is regarded as a cohesive group of teachers who work together in an effort to collectively enhance both teacher and student learning (Vescio et al., 2008); also commonly referred to as a 'teacher professional community' (Grossman et al., 2001). A 'community of practice', on the other hand, is broader in its conception and can involve members from a diverse range of professional backgrounds (Wenger, 1998). Despite the variations in conception, however, common characteristics have emerged within the research literature. As posited by Westheimer (1999), teacher-based communities are anchored in shared values and understandings, mutual engagement, interdependence, meaningful relationships, and a concern for minority and individual views. Vescio et al. (2008) similarly contend that teacher communities must be rooted in the premise of improving both teacher practice and student learning. These attributes are reflected in an empirical framework developed by Grossman et al. (2001, p. 94) for teacher community formation.

Lesson study and teacher professional community

While different models and approaches to support community development have been documented in the research literature (see Grossman et al., 2001 for example), numerous international studies have pointed to the value of LS in cultivating teacher professional communities (Baricaua Gutierez, 2016; Lewis et al., 2009; Lieberman, 2009). As documented by Lewis et al. (2009), LS not only supports teachers in developing a shared goal as part of the research lesson, but it also increases the visibility of their pedagogical ideas and beliefs; an important aspect of professional community formation. Moreover, a study conducted by Baricaua Gutierez (2016) emphasised the value of the reflective process in encouraging teachers to collectively analyse emerging instructional practices, thus supporting them in forming a genuine community, whereby all contributions are valued. This can subsequently encourage teachers to become more open to sharing their ideas and opinions with each other within the context of a professional community.

METHODOLOGY

Participants

In order to investigate the long-term impact of participating in LS on mathematics teachers' professional community, a longitudinal study was conducted with mathematics teachers from two post-primary schools in the ROI. Both schools, Doone and Crannóg (pseudonyms), had previously engaged in school-based LS as part of a study in 2012/13 (see Ní Shúilleabháin, 2016). While both schools are urban-based, they differ in their student cohorts. Doone, for example, is a single-sex boys' school comprising 550 students, whereas Crannóg has a mixed gender population of around 900 students. Although some members of the LS groups have left their respective schools since the initial study, three teachers from each school, who took part in the LS cycles, volunteered to participate in this follow-up research. These participating teachers varied in their years of experience teaching mathematics (see Table 1), with one teacher, Nora, working as a volunteer resource teacher in Doone.

Table 1: Participating teachers' current years of experience teaching mathematics (N=6)

Crannóg		Doone	
Name	*Years of experience*	*Name*	*Years of experience*
Eileen	9	Kate	9
Fiona	37	Lisa	13
Walter	18	Nora	41

Data collection and qualitative analysis

Data for this study were generated through one-to-one semi-structured interviews with the six participating teachers, which varied in length from 20 to 35 minutes. The questions were informed by the dimensions outlined in Grossman et al.'s (2001, p. 94) framework for teacher community formation: (D1) Formation of group identity and norms for interaction, (D2) Navigating group conflict, (D3) Negotiating the essential tension (D4) Communal responsibility for individual growth, which will henceforth be referred to as D1, D2, D3 and D4 respectively. These interviews explicitly addressed both the teachers' recollections of LS and their post-LS experiences in order to investigate the long-term impact on their community. Qualitative analysis of the transcribed audio files was conducted in NVivo according to the four dimensions in Grossman et al.'s (2001) framework. Each dimension was further divided into codes which aligned with the descriptors under the respective dimension e.g. "identification with subgroups" (Grossman et al., 2001, p. 94) The individual interviews were examined for evidence of the main themes, with the responses being coded using the codes included therein. Once coded, the responses were divided into three time periods (pre-LS, LS 2012/13, and post-LS). Each row of codes from the framework was then analysed and the most prevalent code was identified in each case as related to the associated stage of community development; beginning, evolving and mature. Each theme was subsequently assigned the most dominant stage of development based on the prevalence of the codes included therein (e.g. D1 = mature). This was repeated for each time period.

FINDINGS

Analysis of the interview data revealed that mathematics teachers in both schools felt their participation in LS has had a long-term positive impact on their relationship with their colleagues. Our findings also suggest that while the teachers in the two schools did not engage with one another in the context of a professional community prior to their participation in LS, both teacher communities were predominately in the mature stages of development during the LS cycles in 2012/13. Furthermore, we find that the professional community which emerged in Crannóg has been further strengthened since the teachers' participation in LS, while the professional community in Doone has been sustained in the years following the initial study. In this paper, we expand on the findings for Crannóg as the teacher professional community in this school showed evidence of strengthening in the years following LS.

Crannóg Pre-LS

Prior to their participation in LS, the mathematics teachers in Crannóg did not engage with one another in the context of a professional community. While they had timetabled department meetings, these were primarily concerned with matters relating to administration rather than teaching and learning. Moreover, the teachers only occasionally exchanged resources in an informal and would not have openly provided feedback or discussed the shared resources as their brief encounters were "not conducive to constructive criticism" (Eileen). Nevertheless, they got on well as a "group of teachers" (Grossman et al., 2001, p. 4) and reported a sense of collegiality within the mathematics department prior to LS.

Crannóg LS 2012/13

Based on the mathematics teachers' recollections of LS, there is evidence to suggest that they had developed a mature teacher professional community during the LS cycles in 2012/13. As indicated in the analysis, the most dominant stage of development for D1 and D2 was mature, while D4 was classified as evolving/mature. However, there was no evidence of D3 or its associated codes as this dimension was not discussed in the context of LS 2012/13 (see Table 2 for summary). We expand on D1 and D4 below as these dimensions were most widely discussed during the teachers' recollections of LS and also featured references to the beginning and/or evolving stages of development, although these were much fewer in number and were not evident in all three interviews.

Table 2: Summary of responses for LS showing the most dominant stage of development for each dimension, in addition to the presence of beginning, evolving and mature codes across each dimension

Dimension	Beginning codes present	Evolving codes present	Mature codes present	Dominant stage of development
D1	X	-	X	Mature
D2	-	-	X	Mature
D3	-	-	-	No evidence
D4	X	X	X	Evolving/Mature

In relation to D1, it was clear that the teachers in Crannóg identified with the group as a whole during LS as evidenced by Eileen's recollection that "[LS] made us all work together as a really good team". The teachers also continually pointed to their communal sense of responsibility for the LS work and recognised the value of having diversity in perspectives within the group, thus aligning with the features of a mature professional community. While there were codes relating to the beginning stage of development evident in D1, these were primarily attributed to the presence of informal subgroups within the larger LS group. However, the teachers acknowledged that these subgroups were not formally maintained, and they continued to identify with the group as a whole throughout the LS cycles.

With regards to D4, the teachers were keenly aware of the obligations of group membership such as contributing to the LS meetings, sharing resources and contributing pertinent information. However, Eileen admitted that she initially held back during the LS meetings due to her inexperience, which aligns with the beginning stage of development: "I probably gave the least because I had just qualified". Additionally, there were more accounts of teachers acknowledging their colleagues as a resource for learning (Evolving) rather than demonstrating an active commitment to each other's professional growth (Mature):

> It was interesting just working with colleagues and hearing their different ideas, even if it's on a different methodology, just hearing their different ideas that was interesting as well. So yeah, whenever you have a discussion with somebody you always learn something (Fiona).

Crannóg Post-LS

Based on the analysis of the post-LS data, our findings suggest that the mathematics teachers' professional community which emerged during LS 2012/13 has been further strengthened in the years following the initial study. While D2 remained mature and unchanged, both D1 and D4 showed evidence of further development, which we outline below. Although there was still evidence of codes relating to the beginning and/or evolving stages of development in D1 and D4, these were much fewer in number in comparison to the data for LS 2012/13 and were not present in all three interviews. In addition, references to D3 were also reported in the responses relating to post-LS and this dimension was classified as mature (see Table 3 for summary of results).

Table 3: Summary of responses for post-LS showing the most dominant stage of development for each dimension, in addition to the presence of beginning, evolving and mature codes across each dimension

Dimension	Beginning codes present	Evolving codes present	Mature codes present	Dominant stage of development
D1	X	X	X	Mature
D2	-	-	X	Mature
D3	-	-	X	Mature
D4	-	X	X	Mature

In relation to D1, it is evident that the mathematics teachers continue to identify with the group as a whole and they report a stronger sense of unity within the department. The group are also welcoming of new members and they typically become assimilated into their culture of shared learning and collaboration: “We are good team, so I think anyone that comes in realises that and gets involved, and kind of realises what our ethos is” (Eileen).

Moreover, the teachers have developed new interactional norms since participating in LS, which is a further indication that their professional community has strengthened in recent years. For instance, the mathematics department now meets for 40 minutes every week to discuss teaching and learning approaches. The use of the Microsoft ecosystem has also facilitated the exchange of resources amongst the mathematics teachers and supported greater levels of engagement with one another. Matters relating to administration and planning are now commonly discussed online through Microsoft Teams, allowing more of the weekly meetings to be dedicated to the teaching and learning of mathematics. It is also worth noting that all other subject departments in the school have similarly begun to meet on a weekly basis to discuss teaching and learning approaches, in addition to administration:

> We would have been one of the first departments to have met as a subject every week and then all the departments in the school meet as a subject every week. So, I think our participation in lesson study did sort of spawn off a number of benefits for the whole school as well (Walter).

With regards to D4, the teachers reported that their participation in LS formalised the exchange of ideas and it has now become a norm within their community. While the teachers initially recognised their colleagues as a resource for learning, they have come to acknowledge a sense of commitment to their colleagues’ learning and openly share ideas with one another: “I think in order to develop as maths teachers you have to work together and share your ideas otherwise it’s just static” (Fiona). Although Eileen admitted to holding back during the LS cycles, she now recognises the obligations of community membership and ensures to seek for clarification and contribute ideas during their weekly meetings. These examples provide evidence to suggest that the mathematics teachers’ professional community in Crannóg has been further strengthened in the years following their participation in LS.

DISCUSSION

The findings for this study support empirical research, which has shown that LS can serve as a valuable means to foster teacher professional community development (Baricaua Gutierez, 2016; Lewis et al., 2009; Lieberman, 2009). As indicated in our analysis, the mathematics teachers in both schools developed a predominantly mature professional community during their participation in LS in 2012/13. Moreover, we find that the community which emerged in Crannóg has been further strengthened since the teachers’ participation in LS, while the community in Doone has been maintained in the years following the initial study. These findings suggest that school-based LS may be a viable model to develop and sustain mathematics teachers’ professional communities in the long-term.

Following from the research literature, our findings also substantiate the value of professional communities in fostering teacher learning (Grossman et al., 2001; Lieberman, 2009; Vescio et al., 2008) and connecting it to the lived realities in the classroom (Sargent & Hannum, 2009). For example, the mathematics teachers in this study reported that their engagement in a professional community has exposed them to a diverse range of pedagogical ideas and methodologies and contributed to their development as mathematics teachers. They particularly value the opportunity to discuss the teaching and learning of mathematics as evidenced by the fact that the mathematics teachers in Crannóg continue to meet on a weekly basis to discuss their practice and share ideas.

However, given that the mathematics teachers in both schools continue to engage with one another in the context of a professional community, it is pertinent to consider the factors which promote and sustain teacher professional community development. Consistent with previous studies, our findings suggest that shared values (Westheimer, 1999), strong working relationships (Stoll et al., 2006) and positive attitudes towards self-development are all key enablers of professional community development. However, continued support from school management (Stoll et al., 2006) and time provisions (Grossman et al., 2001) may account for the differences between the two teacher professional communities. As reported during the interviews, the teachers in Crannóg have been provided with on-going support from school management following the LS cycles and provisions have been put in place to ensure that they can continue to meet as a group. In contrast, the teachers in Doone have not been given time allocations since due to timetabling issues, which subsequently limits their engagement with one another during the school day. This may have hindered the further strengthening of their mature teacher professional community in the years following LS (Westheimer, 1999).

As a limitation to this research, not all mathematics teachers who participated in the LS cycles in 2012/13 volunteered to take part in this follow-up research. While the participating teachers reported that all members have positive perceptions of LS and believe it contributed to the development of their community, it is possible that those who chose not to participate in this follow-up study may have a different perspective on the LS experience. Nevertheless, the findings for this study provide ground for future research in the field. For example, research could be conducted to examine the factors that promote and sustain professional communities. Longitudinal case studies documenting the impact of such communities on student learning and achievement similarly await further research as this will help build our understanding of professional communities and their impact on teaching and learning practices.

CONCLUSION

Taken together, the findings of this research have important implications for teacher practice and educational policy in the ROI. First, this study provides empirical evidence to suggest that LS could serve as a sustainable model for school-based PD and support mathematics teachers in their professional growth. By developing and sustaining professional communities through LS, teachers could also establish a sustainable "culture of shared learning" (Teaching Council, 2018, p. 6), which embodies the values and principles unpinning the Cosán National Framework for Teacher Learning (Teaching Council, 2016). Moreover, professional communities have the potential to provide teachers with a greater sense of autonomy over their learning, especially as the community can address matters relevant to their own subject area(s) and school background. This is particularly relevant to the current educational climate in the ROI, which seeks to continue building the teaching capacity in Science, Technology, Engineering and Maths education (STEM Education Review Group, 2016). In turn, teacher professional communities may help shift educational change in the ROI toward a more sustainable, teacher-led approach, that is grounded in the lived realities of the profession.

REFERENCES

Baricaua Gutierez, S. (2016). Building a classroom-based professional learning community through lesson study: Insights from elementary school science teachers. *Professional Development in Education*, *42*(5), 801–817.

Fullan, M., & Hargreaves, A. (1992). Teacher development and educational change. In M. Fullan & A. Hargreaves (Eds.), *Teacher development and educational change* (pp. 1–9). London: RoutledgeFalmer.

Grossman, P., Wineburg, S., & Woolworth, S. (2001). Toward a theory of teacher community. *The Teachers College Record, 103*, 942-1012.

Lewis, C., Perry, R., & Hurd, J. (2009). Improving mathematics instruction through lesson study: A theoretical model and North American case. *Journal of Mathematics Teacher Education, 12*(4), 285-304.

Lieberman, J. (2009). Reinventing teacher professional norms and identities: The role of lesson study and learning communities. *Professional Development in Education, 35*(1), 83-99.

Ní Shúilleabháin, A. (2016). Developing mathematics teachers' pedagogical content knowledge in lesson study: Case study findings. *International Journal for Lesson and Learning Studies, 5*(3), 212-226.

Sargent, T., & Hannum, E. (2009). Doing more with less: Teacher professional learning communities in resource-constrained primary schools in rural China. *Journal of Teacher Education, 60*(3), 258-276.

STEM Education Review Group. (2016). *STEM education in the Irish school system.* Dublin.

Stoll, L., Bolam, R., McMahon, A., Wallace, M., & Thomas, S. (2006). Professional learning communities: A review of the literature. *Journal of Educational Change, 7*(4), 221-258.

Teaching Council. (2016). *Cosán framework for teachers' learning.* Dublin.

Teaching Council. (2018). *Teaching council strategic plan 2018-2020.* Dublin.

Vescio, V., Ross, D., & Adams, A. (2008). A review of research on the impact of professional learning communities on teaching practice and student learning. *Teaching and Teacher Education, 24*(1), 80-91.

Wenger, E. (1998). *Communities of practice : Learning, meaning, and identity*. Cambridge: Cambridge University Press.

Westheimer, J. (1999). Communities and consequences: An inquiry into ideology and practice in teachers' professional work. *Educational Administration Quarterly, 35*(1), 71-105.

POST-PRIMARY TEACHERS' MOTIVATIONS FOR FLIPPING, AND CONTINUING TO FLIP, THE MATHEMATICS CLASSROOM

Caoimhe McDonnell and Maria Meehan

School of Mathematics and Statistics, University College Dublin

The "flipped classroom" model is being implemented in educational settings internationally. In this model, the in-class and out-of-class activities of the traditional classroom are "flipped" – students might watch videos out of class, and work on tasks in class. Despite its popularity, research is mainly focused at higher education, and little is done in the Irish context. In this study, through one-to-one interviews with six post-primary teachers who have flipped their mathematics classrooms, we explore their initial motivations for doing so, and examine what motivates them to continue or discontinue the practice. Anticipated benefits, external factors, and being inspired by another, were given as motivations for initially flipping, while an improved classroom culture and perceived pedagogical benefits were reasons given for continuing the practice. Lack of time was a factor in teachers' decisions to discontinue. For the four who continue to "flip", their practice has evolved in a number of ways: a transition from using others' videos to making one's own; a decrease in frequency of flipping; and, an increase in the use of active learning in class. The teachers express a desire to belong to community of flipped classroom practitioners, in order to share experiences and resources, particularly mathematical tasks.

INTRODUCTION

The book by high-school science teachers Bergman and Sams (2012) has done a great deal to promote the methodology of "flipping the classroom" internationally. The authors were first motivated to create videos to enable students who missed class, to catch-up, and then realised that they could ask students to watch the videos at home, and use the in-class time to support students while they worked on problems. In essence, they flipped the order in which these activities are traditionally done. They describe how, over time, the flipped classroom evolved to the "flipped mastery classroom" (p. 51) where students work at their own pace, and demonstrate mastery of learning outcomes in their own time. Similarly, the much-viewed video of Salman Khan (2011) describes how, what started out as uploading mathematics videos for his cousins to watch remotely on YouTube, lead to the development of Khan Academy, which aims to use technology to remove the "one-size-fits-all lecture" (6:17) from the classroom. Both can be credited with popularising the idea of the flipped classroom.

In terms of research on flipped classrooms, Akçayir and Akçayir (2018) carried out a review of existing research from 2000 to 2016. They noted that 79% of the studies included were published in 2015-2016, with the increasing availability of technology suggested as a reason for the recent increase. Interestingly they found that most of the existing research was conducted at higher education, with only 16% of the 71 studies focusing on the primary and post-primary levels. In addition, much of the research focuses on the advantages and disadvantages of flipping the classroom, along with the nature of the in-class and out-of-class activities used. Given the current lack of research that examines the flipping of mathematics classrooms in Ireland, we are interested in exploring post-primary mathematics teachers' experiences of flipping the classroom in Ireland. In this paper we narrow the focus to specifically address the following research questions (RQ):

RQ1. What initially motivates teachers to flip their mathematics classroom within the Irish post-primary system?

RQ2. What motivates these teachers to continue/discontinue to flip their mathematics classrooms, and for those who continue, how does their practice evolve over time?

LITERATURE REVIEW

One feature of using the flipped classroom model is that more in-class time is available for activities. Akçayir and Akçayir (2018) identified the following learning activities as the most common in flipped classrooms: discussion; small group activities; feedback to students; and, engaging students in problem-solving activities. Other activities carried out by students during in-class time include working on questions that would typically be completed as traditional homework, and engaging in active learning, such as peer instruction/interaction (Abeysekera & Dawson, 2015). During the in-class component, the teacher may act as a facilitator or tutor (Akçayir & Akçayir, 2018; Bergmann & Sams, 2012). The use of technology is a prominent feature of many flipped classrooms. The review by Akçayir and Akçayir (2018) found that videos were used in the out-of-class setting in 78.87% of flipped classroom, with the next most common uses comprising readings, quizzes and online discussion. Students may also be required to engage in note-taking while watching videos (Bergmann & Sams, 2012; de Araujo, Otten, & Birisci, 2017). The instructor must choose between using videos available online or making videos of their own. Bergmann and Sams (2012) recognise that it may be easier in the beginning to use videos made by others.

The most common advantages of the flipped classroom model that appear in the literature are the following: increased learner performance; flexible learning; improved student satisfaction and engagement; and, positive student feedback and perceptions (Akçayir & Akçayir, 2018). Interestingly, several studies have found that the flipped model benefits lower-performing students the most (Bhagat, Chang & Chang, 2016; Day, 2018; Ryan & Reid, 2016). Ryan and Reid (2016) found no difference in student performance when analysing the class as a whole. However, by splitting the students into groups using their pre-test scores, they found an increase in the performance of students belonging to the lowest third. Similarly, Day (2018) found that students in the lower two quartiles improved their grades when learning using the flipped model. The flexibility of the model allows students to work at their own pace and those who miss lessons can catch up more easily (Bergmann & Sams, 2012).

The most common disadvantages of the flipped classroom are the following: the time investment for the teacher; inadequate student preparation out-of-class; poor video quality; and, time investment out of class for the student (Akçayir & Akçayir, 2018). Other disadvantages of the model include the need for the teacher to engage in pre-class preparation in order for the in-class session to run as intended (Akçayir & Akçayir, 2018; de Araujo et al., 2017), and the need for the teacher to monitor student engagement in the out-of-class activities. Placing the responsibility on the students to do this work can make ensuring that they watch videos difficult (Bergmann & Sams, 2012; de Araujo et al., 2017).

A study similar to the one reported in this paper was conducted by de Araujo et al. (2017) at the high-school and pre-college level in the United States. They present two case studies of mathematics teachers who flipped their classrooms, examining their motivations for doing so, and their experiences with the flipped model. They found that both teachers were motivated to flip after hearing about the perceived advantages of the flipped model, in particular its potential to increase student-teacher interactions and deepen student understanding.

METHODOLOGY

One-to-one semi-structured interviews were conducted with seven post-primary mathematics teachers who had experience of flipping the classroom. Participants were sought via Twitter, a "Flipped Classroom Newsletter" (www.practisewhatyouteach.ie/flipped-newsletter), and through suggestions made by family, friends, and colleagues. Teachers were contacted via email and those who agreed to participate were interviewed during February and March 2019.

The interviews varied in length from 24 minutes to 1 hour and 31 minutes. One of the seven teachers had a misconception of the flipped model and therefore this interview was not included in the analysis. The other six interviews were transcribed and coded using thematic analysis (Braun & Clarke, 2006), taking an inductive approach. The six phases of thematic analysis as outlined in Braun and Clarke (2006, p. 87) were followed. Firstly, all six interviews were listened to repeatedly and read through. Secondly, initial codes were generated for two of the interviews. Thirdly, these codes were collated into themes. The remaining four interviews were then coded using the initial codes, and additional codes were added when required. In the fourth phase, thematic maps were generated, and in the fifth phase themes were named and described. Reporting the findings represents the sixth phase.

Table 1: Information on participating teachers. *H = Higher Level, O = Ordinary Level, M = Mixed Level **Infrequent Flipping

Teacher (pseudonyms)	Time spent teaching maths	Time spent flipping	Level of year groups where class flipped*						School Gender	Currently flips
			1	2	3	4	5	6		
Alice	37 years	1.3 years	M			M	H		Mixed	Yes
Conor	8 years	2 months	M						All-boys	No
Frank	8 years	1.5 months	M	O	H		H	H	Mixed	No
John	15 years	5-6 years	M	H	H				Mixed	Yes
Karl	20 years	6-7 years**	M	H	H	M			Mixed	Yes
Rachel	3 years	1 year		M					Mixed	Yes

The six teachers' experiences with flipping their classrooms varied considerably (see Table 1.) Alice, John, Karl, and Rachel currently engage in it, while Conor and Frank do not, and only engaged in it for a few months when they did. Alice trialled flipping two years ago for several months and then reverted to a non-flipped classroom. However, this year she is flipping her fifth-year class consistently, and her first- and fourth-year classes, regularly. John has the most experience and has been implementing a flipped mathematics classroom for five to six years. However, his experience is limited to junior year groups as he has not taught senior classes during that time. Karl has used the flipped model infrequently since implementing it for approximately three months, six to seven years ago. Rachel flipped the classroom last year consistently. She is on leave this year but plans to reimplement the model when she returns to teaching. All six teachers work in public schools in the greater Dublin area, one of which is a Delivering Equality of Opportunity in Schools (DEIS) school.

FINDINGS

We first present the teachers' motivations for initially engaging in the flipped classroom, followed by their reasons for continuing/discontinuing the practice. We then examine how their practice has evolved over time.

Teachers' initial motivations to flip the mathematics classroom

Three main themes were identified in relation to what initially motivated our teachers to flip their mathematics classrooms. These are anticipated benefits, external factors and being inspired by others who had engaged in the practice.

Teachers spoke about the anticipated benefits of implementing a flipped classroom, which fall into three categories: use of out-of-class time; use of in-class time; and, the development of students. All six teachers assigned videos for students to view out-of-class, and the flexibility this provided to students was noted as a factor in the decision to flip. The teachers felt that videos would allow students to work at their own pace - they could pause, rewind and re-watch videos as needed. Karl noted: "I would always joke with them: 'With this thing you have got total control. You can mute me, you can fast forward me, you can rewind, you can do whatever you want'". It was also thought that videos would provide flexibility for the teacher when students were absent and allow them to catch up: "I could do things like when students are absent, they could still engage with my lesson which they wouldn't be able to do in my old style of teaching" (John). Teachers commented that the videos could be used as a revision tool and felt there was potential for the teacher to build a bank of video resources over time. It was anticipated that the out-of-class tasks would be manageable for all students, in contrast to the frustration they expressed with students not completing homework in the non-flipped model. As Alice commented:

> I would say: "Why didn't you ask me in the class?" "I thought I understood it at the time but when I got home I didn't." I definitely felt it was the whole thing of getting past the place where students were coming in without their homework done.

The teachers anticipated that the extra in-class time, created by having students watch videos at home, would enable them to support students while working on questions: "The main advantage I hoped would be that I would be more available to the students to help them with their daily problems" (Rachel). They also felt it would allow students to spend more time doing mathematics: "Well I liked the idea of having more time in class to spend on problems" (Conor). Finally, the teachers hoped that flipping the classroom would enable them to foster the development of certain student attributes. They expected student attitude and enjoyment to improve, as well as their sense of ownership of mathematics, and it was hoped that flipping could assist in the development of students as independent learners.

External factors that prompted the teachers to flip the classroom include: feeling under time pressure; meeting an expectation of varying one's teaching methodologies and effectively incorporating technology in the classroom; and, in one instance, an unplanned absence of the teacher. Rachel noted: "There is a bit of a push to encourage teachers to change their methodologies". In terms of being inspired by others who had engaged in the practice, the teachers heard of the flipped model from several sources, including YouTube videos, Initial Teacher Education, and Bergmann and Sams' (2012) book. Several teachers mentioned hearing Niall O'Connor (a mathematics teacher and flipped model advocate in Dublin) speak about his practice and Sal Khan's TED Talk (2011) as their inspirations for flipping.

Teachers' motivations for continuing/discontinuing the practice

Two themes were identified when analysing the reasons that the four teachers in our study, who continue to flip their mathematics classrooms, gave: classroom culture, and pedagogical benefits. In terms of classroom culture, a relaxed in-class setting, and more teacher enjoyment was cited as a motivation to continue to flip: "I certainly enjoy it much more, I interact more with the kids … The kids now that I teach who are using the flipped classroom seem more relaxed in the classroom" (Alice). A perceived improvement in student attitude and enjoyment was also a motivation. Rachel noted that "Everybody was in much better form. The students were much more willing to engage … I think students were more willing to do the problem-solving work in class". Most of the teachers spoke about receiving positive parental feedback. Karl continues to flip as he aims to use a variety of methodologies, including the flipped model, to engage students and improve their attitude and enjoyment.

Perceived pedagogical benefits motivate the teachers to continue flipping the classroom. They feel the model encourages students to become independent learners and hope it will better prepare students for life after school: "Students need to be independent learners. They need to [remove] the teacher as a crutch and be able to go into college and work on their own" (John). They believe that it helps students to develop valuable skills, as Rachel noted: "It encourages students to be a bit more autonomous and to take responsibility for their own learning, which in turn leads to development of skills" (Rachel). They feel that aspects of the flipped classroom help to foster independent learning in students, for example, placing more responsibility for learning from the videos onto the students.

The "extra" in-class time as a result of having students watch videos outside class is another reason to continue flipping. All four teachers feel that this time allows them to have a better awareness of student understanding: "I am spending time with them and there is a lot of like verbal questioning and so I would be learning a lot more about where they are at" (John). Karl observed that "you can see that they need help because you are just circling all the time". Alice emphasised the importance of this:

> We are probing a little or chatting a little or asking them "Why did you do that?" or whatever, it's amazing the lack of understanding … you realise that students can get to a very high level in maths and not really understand some stuff.

Finally, two teachers referred to the potential of the flipped classroom to help with differentiation and personalisation. Alice utilises the flipped model to differentiate in one of her classes, with students of different abilities watching different videos and working on different problems. It is important to note that many of the teachers, during in-class sessions, do not expect all students to reach the same level or question and accommodate their varying abilities and pace. John's focus is on personalisation which he recognises as "learner led" and the "holy grail of education". He acknowledges that personalisation is an idealised vision that is yet to be fully achieved and he is committed to working towards it: "It's kind of big picture thinking but personalisation is, for me, what it is all about".

Time was the main factor that affected the two teachers' decisions to discontinue flipping the classroom. Frank feels that he lacks the time to create the videos, although he acknowledges that he would like to use the model again. Conor feels the time he had to invest into sourcing videos online is better spent "writing appropriate problems and finding good resources".

Evolution of teachers' practice

With regards to the evolution of the four teachers' practices of flipping, our findings include: a transition from using others' videos to making one's own videos; an increase in confidence in making videos; improved video quality; a decrease in frequency of flipping; and, an increase in the use of active learning and problem-solving strategies.

Interestingly, all four teachers who continue to flip now make their own videos. They have become less focused on making the perfect video: "This is not a YouTube production. This is just a classroom" (Alice), and are less self-conscious about uploading their videos onto YouTube. They feel that their videos have improved in quality and they are better at excluding irrelevant material resulting in shorter versions. Half of teachers also upgraded their equipment over time. For Karl and John, the frequency of their flipping has changed. John remarked: "When I first started flipping, I thought I had to be doing it five days a week, but I felt like I was missing out on other things as well". He now flips his classroom three days a week, using the other two days to implement other strategies. Similarly, Karl likes to use a variety of methodologies. Finally, how most of the teachers use the in-class time has changed, with some now investing more time in finding and creating mathematical tasks to encourage active learning, and promote problem-solving strategies.

DISCUSSION AND CONCLUSION

The motivations of teachers to flip their classrooms initially, and their reasons for continuing to implement the model, largely mirror the advantages of the flipped classroom outlined in the literature (Akçayir & Akçayir, 2018). Time was cited by Conor and Frank as their reason for discontinuing the practice, and the time investment required to implement the model, especially initially, is evident in the literature (Akçayir & Akçayir, 2018). The teachers in both de Araujo et al's (2017) and this study who continue the practice of flipping feel that the benefits of flipping outweigh the time requirements. Conor and Frank flipped for the least amount of time which may have resulted in them not experiencing such benefits. Their practice may not have sufficiently evolved, resulting in a less time-efficient flipping in comparison to the others. For example, Frank would spend thirty minutes making one video, whereas John could do so in fifteen.

There are similarities between the teachers' motivations to flip initially, and then to continue the practice. These include: enabling students to become independent learners; improved student attitude and enjoyment; and, "extra" in-class time. An initial motivation to flip was the availability of teacher support to students, while they are working on problems in class. By supporting the students during class, teachers had a better awareness of student understanding which is one of the motivations for them to continue to flip the classroom. The ability to build a bank of videos was an anticipated benefit that motivated teachers to implement the model in the first instance. However, it appears that this may not be as useful or beneficial as expected. Alice, Rachel, and John felt that building a bank of videos was not optimal, and that videos should be made by the teacher to adapt them for their own students: "I have heard other people … using other teachers' flipped videos but I don't see how that could work. I think flipped has to be tailor made for your classroom" (Rachel). They also felt videos could be improved by providing up-to-date context: "I can build in more context and I can talk about how Tottenham won last night … whereas if it's from last year's video I can't mention that" (John).

Interestingly, the external factors that motivated teachers to flip the classroom initially were not reflected in the literature. These included the desire to effectively incorporate technology into the classroom and the need to vary one's teaching methodologies. Recent initiatives in the Irish Education System may have impacted teachers' awareness of such factors. For example, the Department of Education and Skills have published a digital strategy for schools (DES, 2015a) and schools have been receiving grants to incorporate technology, such as tablets or laptops, into teaching and learning practices. Therefore, teachers are keenly aware of the expectation to integrate technology, which in turn provides motivation for using the flipped model. John noted: "I went to a one-to-one iPad school. I was given a tablet and I had no textbooks … I wanted to find a way that I could use the technology that could enhance what I do". Similarly, the recent Junior Cycle reform, which has more of a focus on the development of student skills than before, has challenged teachers to move away from traditional teaching methods (DES, 2015b).

Knowing another teacher who flips the classroom appears to be essential. It often contributes to the initial decision to flip and to the subsequent decision to continue the practice. When asked what supports or resources would be useful for teachers deciding to flip, being in contact with experienced practitioners was discussed. Alice and Karl felt that having someone to guide a teacher through the process and answer their questions would be highly beneficial. John recognised that having a community of practice would support a teacher about to flip:

> Having a community practice does make it easier of course because you can say 'Have you tried this?' or 'Have you tried that?' or, you can share ideas and say: 'Oh have you tried this, this is really good'.

The teachers felt that sharing examples of a flipped classroom specifically in the Irish context was important:

> If they were given examples of what a flipped classroom looks like in their educational setting and not some magical school in England or in the States where everything looks fantastic and the kids are wearing blazers … it would work quite well. (Frank)

Providing opportunities for teachers to meet each other and share their experiences would contribute to the successful implementation of flipped classrooms. We therefore recommend the establishment of a flipped classroom community of practice.

REFERENCES

Abeysekera, L., & Dawson, P. (2015). Motivation and cognitive load in the flipped classroom: Definition, rationale and a call for research. *Higher Education Research & Development*, *34*(1), 1–14.

Akçayır, G., & Akçayır, M. (2018). The flipped classroom: A review of its advantages and challenges. *Computers and Education*, *126*(July), 334–345.

Bergmann, J., & Sams, A. (2012). *Flip your classroom: Reach every student in every class every day*. Eugene, OR: International Society for Technology in Education.

Bhagat, K. K., Chang, C. N., & Chang, C. Y. (2016). The impact of the flipped classroom on mathematics concept learning in high school. *Educational Technology & Society*, *19*(3), 124–132.

Braun, V. & Clarke, V. (2006). Using thematic analysis in psychology. *Qualitative Research in Psychology, 3(*2), 77-101.

Day, L. J. (2018). A gross anatomy flipped classroom effects performance, retention, and higher-level thinking in lower performing students. *Anatomical Sciences Education*, *11*(6), 565–574.

De Araujo, Z., Otten, S., & Birisci, S. (2017). Mathematics teachers' motivations for, conceptions of, and experiences with flipped instruction. *Teaching and Teacher Education*, *62*, 60–70.

Department of the Education and Skills, Ireland. (2015a). *Digital Strategy for Schools: Enhancing Teaching Learning and Assessment.* Dublin: Department of the Education and Skills.

Department of the Education and Skills, Ireland. (2015b). *Framework for Junior Cycle*. Dublin: Department of the Education and Skills.

Khan, S. (2011, March 15). *Let's use video to reinvent education* [Video File]. Retrieved from https://www.ted.com/talks/salman_khan_let_s_use_video_to_reinvent_education

Ryan, M. D., & Reid, S. A. (2016). Impact of the flipped classroom on student performance and retention: A parallel controlled study in general Chemistry. *Journal of Chemical Education, 93*(1), 13–23.

UNEXPECTED CONSEQUENCES OF PROVIDING ONLINE VIDEOS IN A SERVICE MATHEMATICS MODULE: WHAT ONE LECTURER NOTICED

Maria Meehan

School of Mathematics and Statistics, University College Dublin

This study describes how brief-but-vivid accounts of incidents kept over three consecutive offerings of a large, first year, service mathematics module, were analysed and three unexpected consequences of making online videos available to students were identified in the BBV accounts written during the third offering of the module. It was in this offering that students, for the first time, were given the option of attending lectures or watching online videos or both. The first unexpected consequence relates to how the lecturer struggled to view poor attendance at lectures differently; the second to a difference, observed by the lecturer, in the way that students engaged with tasks during lectures; and, the third, to how the lecturer realised that videos could be used to encourage students to take more responsibility for their learning. Though these changes could be described as subtle, they have not been reported in the literature relating to the provision of online lectures as an extra resource to university students. The study also provides an example of how the Discipline of Noticing can be used as a professional development tool for those implementing initiatives in their teaching.

INTRODUCTION

In MEI5, a paper describing how I introduced a selection of short online videos as a supplementary resource for students in a large, first-year, *Maths for Business* module in 2012/13 (Meehan, 2013) was presented. The paper describes how the introduction of these videos had been prompted, in part, by my reflections on a collection of brief-but-vivid accounts (Mason, 2002), that were written while teaching the module during the previous academic year, 2011/12. For the next offering in 2013/14, I made the entire content of the module available in the form of 67 short, online videos and gave students the option of attending lectures or viewing videos or both. During this third offering, I continued to write brief-but-vivid accounts. In total, over the three consecutive years of teaching the module, I had written 72 accounts. In this paper I describe how the accounts over the three years were analysed in order to address the following research question: What changes did I notice during the third offering of the module when students were given the option to attend lectures or watch videos or both?

LITERATURE REVIEW

Online Videos and Live Lectures at the University Level

In the past decade technological advances have made it possible for lecturers to easily record lectures (live or otherwise) and provide them as an online resource for students. With the availability of online lectures at the university level, there has been a growing body of research examining their impact in a number of areas. Due to space restrictions, we highlight some of the main areas of research here along with some findings and provide a selection of references. For a more comprehensive review see for example, Meehan and McCallig (2019).

Students express satisfaction with the availability of online lectures and perceive them to be beneficial to their learning (Danielson, Preast, Bender, & Hassall, 2014; Traphagan, Kucsera, & Kishi, 2010). In general, students tend to use online lectures to replace, review, and/or revise the live lecture (Danielson et al., 2014; Howard, Parnell, & Meehan, 2017; Leadbeater, Shuttleworth, Couperthwaite, & Nightingale, 2013). Most studies address the key question of how the provision of online lectures impacts student performance, although it has been noted that research to date yields inconclusive results (e.g. Danielson at al., 2014; Inglis, Papliana, Trenholm, & Ward, 2011; Owston, Lupshenyuk, & Wideman, 2011). The impact on lecture attendance of making videos available has also been examined with some studies finding that while attendances drops, performance remains unaffected (Owston et al., 2011; Traphagan et al., 2010; Yoon, Oates, & Sneddon, 2014). The frequency with which students view online lectures and/or their viewing patterns, has also been studied (Leadbeater et al., 2013; Owston et al., 2011). Finally, there are studies that examine students' preferences for lectures and/or videos, the reasons behind the preferences, and the impact of resource usage on performance (Bassili, 2006; Inglis et al., 2011; Howard et al., 2017; Meehan & McCallig, 2019).

Online Videos and Live Lectures in *Maths for Business*

The study reported in Meehan and McCallig (2019) is based on data collected on lecture attendance, videos accessed, and mathematical achievement, prior to, and at the end of, the module, for the cohort of students taking *Maths for Business* in 2013/14 – the same cohort as in Offering 3 of this paper. The authors found that when videos are embedded in a structured manner in the module, students who engage with the content fall into one of four main categories – those that: mainly attend lectures; mainly use videos; use both videos and lectures to cover the same content; or, use both videos and lectures but rarely to cover the same content. They also found that time spent using either or both resources has a significant impact on learning. Howard, Meehan and Parnell (2017) conducted a further study examining the 2015/16 *Maths for Business* cohorts' use of live lectures and/or videos, and their reasons for choosing one or both of the resources. Not surprisingly, they identified patterns of resource-usage similar to that in Meehan and McCallig (2019). The labelled as "Switchers" those students who used both resources but rarely to cover the same content. Interestingly some of these students attended lectures for the first few weeks and then switched to viewing videos for the remainder of the semester. In terms of achievement, those who primarily attend lectures (with or without viewing videos) achieved on average the highest marks. Reasons for students' choice of resource(s) are also presented in Howard et al. (2017).

The Discipline of Noticing

Practitioners develop habits or routines that enable them to deal efficiently with daily practice. However, once formed, one may not think about whether these habits continue to be effective, or if there is a better way to act. Mason (2011) suggests that: "the mark of professional development is that participants can imagine themselves in the future acting responsively and freshly rather than habitually" (p. 38). He proposes the Discipline of Noticing as one such professional development tool and describes it as "a collection of practices which together can enhance sensitivity to notice opportunities to act freshly in the future" (Mason, 2002, p. 59).

A key practice advocated by Mason (2002) is to record an incident for the purpose of analysing it at a later date by writing a "brief-but-vivid" (BBV) account of it. A BBV account is brief in the sense that it does not contain extraneous information that detracts from the main incident, and vivid in the sense that it contains enough detail that someone who may have been there could easily recognize the incident. In addition, it should be an "account-of" (p. 40) the incident as opposed to an "account-for" (p. 40) it.

> To *account-for* something is to offer interpretation, explanation, value-judgement, justification, or criticism. To give an *account-of* is to describe or define something in terms that others who were present (or who might have been present) can recognize (p. 41).

The challenge of writing BBV accounts has been noted by both Mason, and by others who engaged in using the Discipline of Noticing as a professional development tool (see Breen, McCluskey, Meehan, O'Donovan, & O'Shea, 2014 and references within for studies of university mathematics educators reflecting on practice).

Mason (2002) provides several suggestions for working with a collection of BBV accounts. One of these is "threading themes" (p. 119) which involves examining the set of accounts for similarities or common threads. By "labelling" a theme or practice, it may be easier to notice and recognize it in the future, and consequently respond freshly rather than habitually.

METHODOLOGY

This study relates to accounts kept during three offerings of *Maths for Business.* Offering 1 (O1) was in Semester 1 of 2011/12, while Offering (O2) and Offering 3 (O3) were in Semester 1 of 2012/13 and 2013/14 respectively. Students who had taken Ordinary Level Mathematics or who had achieved a D1 or less in Higher Level Mathematics in the Leaving Certificate Examination (or equivalent) were assigned to my class. The approximate number of students in my cohort for O1, O2, and O3 was 300, 200, and 170 respectively. In O1, the module was assessed by a midterm MCQ and a final examination. In O2 this changed to ten weekly quizzes, a midterm MCQ, and final examination, with the MCQ removed in O3.

Throughout O1 and O2 of *Maths for Business* I kept BBV accounts of incidents that occurred, usually during lectures. Although every effort was made to write accounts that were brief, vivid and provided an "account-of" an incident, occasionally I lapsed into "accounting-for". Having kept BBV accounts for the duration of the first two offerings, I noticed themes developing and became sensitised to some issues. Consequently, during O3 I often felt the need to "account for" incidents when writing accounts and many of them might not strictly qualify as BBV accounts. However I included them in the analysis as the context, "actors" and nature of the incidents were clear and made them suitable for analysis in this research.

Table 1 gives the number of accounts written for each of the three offerings, and the number of lectures for which an account was written. Some accounts, while describing an incident or incidents during a lecture, were divided into smaller, self-contained sections which I call segments. These segments formed the units of analysis.

Table 2 describes the context of each segment. In *Maths for Business*, I used a combination of "traditional" lecturing (During Lecture) and Inclass Exercises (IE). The latter describes the context where I asked students to work on tasks during lectures, by themselves or in small groups, while I walked around. Some segments relate to incidents before the lecture or immediately after the lecture, or to thoughts I had while writing the accounts.

Table 1: For each offering, the number of accounts written and associated segments, and the number of lectures about which accounts were written.

	Number of Accounts	Number of Segments	Number of Lectures
2011/12	22	48	19
2012/13	24	44	20
2013/14	26	38	26
Total	72	130	65

Table 2: The number of segments relating to each context

	Before Lecture	During Lecture	Inclass Exercise	After Lecture	Writing Accounts	Student Forum	Total
2011/12	6	18	9	8	5	2	48
2012/13	4	17	13	5	7	1	47
2013/14	1	17	19	1	5	0	43
Total	11	52	41	14	17	3	138

The segments were uploaded to Nvivo where each was coded for the context to which it referred, and also the "actors" involved: self; individual student; small group of students; and, entire class. Additionally, each segment was coded for its content. It was possible for a segment to be given multiple codes. Some codes were then arranged under a category. An as example, a category named "Student Progress" had as its codes: Monitoring Student Progress; Positive Student Progress; "I haven't got a clue"; and, Lack of Student Progress. Using Nvivo it was possible to identify which codes were prevalent in each of the three offerings, and in this way, identify any changes in what was noticed over the three offerings.

RESULTS

In this section, three changes that I noticed during O3 are presented. While attendance at lectures was poor in the three offerings, the first change relates to my own reaction to low attendance – my frustration in O1 and O2, and how during O3, I had to remind myself to react differently to it. The second change noted was a difference in students' on-task behaviour during IEs in O3 when compared with O1 and O2. The third change relates to how the availability of the videos during O3 helped me address the challenge of students who struggled to become independent mathematical learners in the transition to university.

Reacting to Low Attendance in Lectures

While attendance at lectures during O1 and O2 was not recorded, there are eight and seven segments respectively from the accounts of each offering where attendance is noted. As each semester proceeded, it is usually poor attendance that was noted. For example: “I approximate that I had less than one third of the class attending last week. […] There are approximately 80 students out of 300 when I start the lecture” (L22, W8, O1). Similar examples occurred during O2. The morning after Halloween: “I count 38 [out of 200] present when I start the lecture a few minutes after 10am, although some more enter in ones and twos after that” (L23, W8). One lecture time seemed more popular than others: “The 2pm lecture on a Monday always has a bigger crowd than the two 10am lectures” (L19, W7, O1).

Some consequences of students missing lectures were noted. For example, lack of progress by some on an IE could be explained by previous absences:

> It is a business example, identical to one I did last Tuesday but with the numbers changed. I walk up the right hand side of the theatre. One student has a blank page. He tells me he doesn’t know what to do. I ask him if he was at last Tuesday’s lecture and he says no. I throw my hands up and say “Engagement” and walk away. A similar interaction occurs with two other students (L24, W9, O1).

From this account and some of the actions I take in response to poor attendance noted in other accounts, it is apparent that poor attendance frustrated me. In W9, O1 and W5, O2 I sent an email to the class voicing my concerns over poor attendance and engagement. In L25, W9, O1, on noticing that “I have a much bigger crowd than usual for a Tuesday morning lecture”, I (pointlessly?) proceeded to lecture those present on the importance of attending lectures. In L15, W5, O2, on finding 30 students present when I arrived at the lecture just before 10am, “I decide to take attendance and pass three make-shift attendance sheets around the theatre”. When the lecture ended, 94 students were present.

The availability of the videos meant that even if attendance at lectures remained poor, what had to change was my reaction to it. The first indicator that the videos might have implications for how I react actually occurred during O2 when the supplementary videos were available. On the morning after Halloween described above, I introduced the technique of Gaussian Elimination. I asked the class to try one:

> As there are now approximately 60 people present I get to look at what most people are doing. Most are making progress and getting through the problem in the systematic way I suggested. What about the 140 who weren’t here today? I am glad I have my videos. Otherwise I really think I’d have to go over it all again in detail on Monday (L23, W8).

On accessing the daily views of a video example of Gaussian Elimination in the days before it was examined on the weekly quiz, I noted that there were 23, 19, 5, 20, 17 and 107 views (W11, O2). I realised that nonattendance at lectures did not necessarily imply that students were not engaged. And rather than feel obliged to (perhaps begrudgingly) help those who missed a lecture on a critical topic to catch up, I felt relieved that I could proceed guilt-free.

However old habits die hard and I had to remind myself during O3 that nonattendance did not automatically equate to nonengagement. From L15, W6, I note: “There were 67 present today – the usual faces I see”. In writing the aims for the next lecture I thought about how I would make my point to class the importance of attending lectures: “I plan to go in today and ask them to do a marginal analysis example. I just want to show the people who didn't attend yesterday what they missed". And then I caught myself: "Ah! But what about the videos?"

On-Task Behaviour During Inclass Exercises

As outlined above in the account from L24, W9, O1 some students, who were unable to make any progress during an IE, admitted to having missed a previous lecture. Whether it was because of poor attendance or some other factors, it was not unusual for accounts to note a lack of progress during IEs, even on exercises that should have been revision. In O2, I asked the class to construct a total cost function from some information given:

> As this was revision, I told the class I'd give them a minute to write down the function. I went up the right hand side of the lecture theatre. I looked in about 20 notebooks and approximately 5 had it perfectly right. Some of the others just smiled at me and said "Sorry, totally lost" or "Sorry, no idea" (L16, W6, O2).

During O3, I noted a distinct difference in the atmosphere during IEs:

> Everything I asked them to do, they just put their heads down and did it. I remember from previous years that as I would walk up one side of the theatre you would hear the noise level rise from the other side and if you looked over, people would be turned in their seats talking to those around them (L10, W4, O3).

This behaviour continued throughout the semester, and was noted again in L23, W9, O3. Of course this may have been a particularly diligent cohort of students, however in week 9 I did wonder if the industry displayed in lectures might be due to the availability of videos:

> The other thing that I have noticed is that unlike previous years, when I am going around [during an IE] I rarely come across someone who says "I wasn't here the last day". I used to find that so frustrating. I want to do an exercise in class, I walk around, and there always seemed to be lots of people who couldn't even start because they hadn't been at a previous lecture. Maybe the videos have eliminated the "odd-timers" from the picture – the people who knew they should be coming and would show up the odd-time? Maybe the videos have salved their consciences! (L24, W9, O3).

Taking Responsibility for Learning in the Transition to University

In O1-O3, the students in my class had taken OL Mathematics or equivalent, and it was not unusual for some to experience anxiety about studying mathematics at university. This in turn could make some students reticent to even attempt problems. During the first week of an offering, one student told me that she was not working on a task because she was "no good at maths" while another stated that she did not know what to do because she had taken OL Mathematics. Since this was the first week, nonattendance at lectures could not have been a factor, in the same way that it *may* have been in the segment above, where students declared during an IE that they were "totally lost" or had "no idea". When writing the account of an IE task where students were asked to compute the gross national product of an economy at two time-intervals given the function that modelled it, I wondered if some students were accustomed to a post-primary environment where the teacher took responsibility for *making* them learn mathematics, which resulted in a type of learned helplessness:

> "What do I substitute for?" "I don't know what you want us to do?" "Do you want us to sub that in there?" "What is *t*?" About two people had done it correctly. I know I am not supposed to "account for" but some don't seem to be reading the questions or stopping to think before they attempt to work something out. Are behaviours they have learned before arriving here, preventing them from engaging appropriately? (L9, W3, O1)

When students say they are "totally lost" or "haven't got a clue" or seem at a loss as to how to tackle a problem, I feel an onus to help. But "How do I fix this in a class of 300?" (L9, W3, O1). At the start of O3, the manager of the Maths Support Centre (MSC) informed me that a small number of students arrived at the centre before the first quiz claiming they were (mathematically) lost. I thought it a poor use of the MSC tutors' time to explain a topic from scratch to individual students when the videos were already there.

> I told him to instruct his tutors to respond to a student who comes in saying "I haven't got a clue", by suggesting they watch a relevant video there and then, rather than have the tutors give their own explanations from scratch (Segment 6.3, W2, O3).

I further suggested to students that they could use the videos to be more specific about where they were getting stuck, and in this way the tutors and I could help them more effectively:

> I started the lecture with a little speech about how I didn't want to hear the phrase "I haven't got a clue about …" from anyone in the class. […] I said that there was no reason for anyone not to have a clue as the videos were there and if stuck, I wanted them to watch them and instead say: "I am stuck at this exact part" (L7, W3, O3).

In this way, students could not "get off the hook" by declaring themselves "totally lost". They had to take the first steps - find and watch a relevant video and identify the point of confusion.

DISCUSSION AND CONCLUSION

A weakness of this study is that it relates to what one lecturer noticed in one module. However, in writing and examining the BBV accounts, I observed some subtle consequences of making videos available that have not appeared in other research in this area. Indeed, using the methodologies of other studies, it is difficult to see how they might. For example, the study by Meehan and McCallig (2019) identifies a cohort of students who consistently attend lectures (with or without using videos as complements). This study, specifically the second change described above, complements their quantitative findings, by shedding light on the difference that having students who choose to consistently attend lectures has on the atmosphere in the lecture, particularly one in which students are expected to work on tasks.

Using videos to encourage students to take more responsibility for their mathematical learning in the transition to university, is also a technique that others could use in their modules. It seems counterintuitive that by giving students the extra resource of videos, one is encouraging them to take control of their learning. For weaker students in particular, videos have more explanatory power than notes for example and hence may be more effective in achieving this.

Finally, as has been argued before (Breen et al., 2014) this study also shows how the Discipline of Noticing can be used in professional development. By keeping and analysing BBV accounts, I identified areas of my practice that could be improved. I also realised that a belief I have as a lecturer is that it is my duty to help all students learn even if they only attend sporadically, or struggle to engage. No wonder I was frustrated by poor attendance and overwhelmed by lack of progress during IE! The videos have provided me with another way to help these students, and in doing so, have made for a much more contented lecturer.

REFERENCES

Bassili, J. N. (2006). Promotion and prevention orientations in the choice to attend lectures or watch them online. *Journal of Computer Assisted Learning, 22,* 444-455.

Breen, S., McCluskey, A., Meehan, M., O'Donovan, J., & O'Shea, A. (2014). A year of engaging with the discipline of noticing: Five lecturers' reflections. *Teaching in Higher Education, 19*(3), 289-300.

Danielson, J., Preast, V., Bender, H., & Hassall, L. (2014). Is the effectiveness of lecture capture related to teaching approach or content type? *Computers and Education, 72,* 121-131.

Howard, E., Meehan, M., & Parnell, P. (2017). Live lectures or online videos: students' resource choices in a first-year university mathematics module. *International Journal of Mathematical Education in Science and Technology, 49*(4), 530-553.

Inglis, M., Papliana, S., Trenholm, J., & Ward, J. (2011). Individual differences in students' use of optional learning resources. *Journal of Computer Assisted Learning, 27,* 490-502.

Leadbeater, W., Shuttleworth, T., Couperthwaite, J., & Nightingale, K. P. (2013). Evaluating the use and impact of lecture recording in undergraduates: Evidence for distinct approaches by different groups of students. *Computers and Education, 61,* 185-192.

Mason, J. (2002). *Researching your own practice: The discipline of noticing.* London: RoutledgeFalmer.

Meehan, M. & McCallig, J. (2019). Effects on learning of time spent by university students attending lectures and/or watching videos. *Journal of Computer Assisted Learning, 35*(2), 283-293.

Meehan, M. (2013). Video killed the radio star: Will the university lecturer be its next victim? In T. Dooley, S. NicMhuirí, M. O'Reilly & R. Ward (Eds.), *Proceedings of the Fifth Conference on Research in Mathematics Education* (pp. 212-223). Dublin: MEI.

Owston, R., Lupshenyuk, D., & Wideman, H. (2011). Lecture capture in large undergraduate classes: Student perceptions and academic performance. *Internet and Higher Education, 14*, 262-268.

Yoon, C., Oates, G., & Sneddon, J. (2014). Undergraduate mathematics students' reasons for attending live lectures when recordings are available. *International Journal of Mathematical Education in Science and Technology, 45*(2), 227-240.

Traphagan, T., Kucsera, J. V., & Kishi, K. (2010). Impact of class lecture webcasting on attendance and learning. *Educational Technology Research and Development, 58*(1), 19-37.

THE INFLUENCE OF HOME FACTORS ON MATHEMATICS OUTCOMES IN MULTIGRADE CLASSROOMS

Breed Murphy and Aisling Leavy

Mary Immaculate College, University of Limerick, Ireland

Multigrade classrooms are a significant feature of the Irish educational landscape in primary schools, particularly in rural settings. Conflicting results are reported from studies analysing academic achievements of students in multigrade classrooms. While it is widely reported that students in single-grade and multigrade classes achieve similar results in academic achievement tests, recent research has suggested that this is not true in all situations. It has been noted that longitudinal studies have revealed unstable outcomes in student achievement. The aim of this study is to explore the academic outcomes for children in multigrade settings in small schools in Ireland drawing on data from two waves of the 'Growing Up in Ireland' (GUI) study. In 2007, the first wave involved a nationally representative sample of 8568 nine-year old children, 1,250 of whom were being educated in multigrade classrooms in small schools. Four years later, over 1100 of these multigrade children participated in the second wave of the study. In this paper, measures of mathematics norm-referenced tests undertaken by the children at age 9 and age 13 are analysed. In addition, home factors are also explored. The outcomes for children in multigrade classes in small schools are compared with their single-grade counterparts. The data provide significant insight into the academic achievements of the students involved in the study as well as home factors which influence their achievement.

INTRODUCTION

Multigrade education is a form of education which receives little acknowledgement in educational policy, research or curricula to the point which it is sometimes considered 'invisible' (Little, 2001). In multigrade classes, students from two or more separate grades are taught in the same setting (Quail & Smyth, 2014). Just under 30% of primary-school children in Ireland are taught in multigrade classes (Eivers & Chubb, 2017). There has been little research on the impact of multigrade classes on student outcomes, particularly which considers the influences of home factors such on student attainment (Fan & Chen, 1999).

This study is focused on children's mathematics attainment in multigrade classes in small schools and aims to offer a perspective on the role of home-factors. It sets out to establish if children in multigrade classes in small schools have similar levels of attainment to their single-grade counterparts. Secondly, it aims to advance research on student achievement by considering aspects of children's backgrounds such as family characteristics and parental involvement in their children's education as partial explanations for influences on attainment. Thirdly, we aim to explore if the influence of background factors on achievement varies according to the age of the child.

REVIEW OF THE LITERATURE

Impact of multigrade classes on student mathematics outcomes: International studies

Research investigating mathematics outcomes for students in multigrade classes compare test results of these students against results of students of a similar age in a single grade class. In the majority of studies, the test instrument is a standardized achievement test. Studies of mathematics outcomes for children in multigrade classes reach a diverse range of conclusions. Adams (1953) examined the arithmetic reasoning and arithmetic fundamentals of fifth grade students in multigrade and combination classes with fourth grade students. Adams concluded that children were not held back by being grouped with students in a lower grade level. Two decades later, a study by Veenman (1995) provided support for the findings of Adams (1953). This synthesis of previous studies examining student outcomes in multigrade and multiage classes reported that the multigrade classes are simply no worse and no better than single grade classes (Veenman, 1995). Indeed, a more recent study by Thomas (2012) converges on the same findings as those by Adams and Veenman.

However, a number of studies challenge the findings of these studies. A year after his 1995 study, Veenman (1996) reconsidered his conclusions and stated that upon further analysis there was some support for the possibility that student achievement in mathematics may suffer in multigrade classes. Similarly, Russell, Rowe and Hill (1998) reported that their examination of mathematics assessments results suggested that being educated in a multigrade class had a negative, albeit non-significant, effect on student mathematics outcomes. Mariano and Kirby (2009) concluded that overall, 3rd, 4th and 5th grade students in multigrade classrooms experienced consistently small and negative effects and achieved lower scores than expected had they been in a monograde classroom, even when teacher characteristics were taken into account. For most of the students involved in the study, being in a multigrade classroom appeared to have a statistically significant effect, with the exception of 4th graders who were in a combined multigrade with 5th grade pupils.

Impact of multigrade classes on student mathematics outcomes: Irish studies

The National Assessments of Mathematics and English Reading (NAMER) carried out by the Educational Research Centre since 1972 offer insights into student attainment in Ireland. Eivers, Close, Shiel, Millar, Clerkin, Gilleece and Kiniry (2010) found no significant difference between the achievement levels of students in single-grade and multigrade schools in mathematics in NAMER 2009. Although both second class and sixth class pupils in multigrade classes achieved a higher mean score that their single-grade counterparts, the differences were not statistically significant (Eivers et al, 2010).

An analysis of the mathematics scores of nine-year old Irish students in a separate study carried out by Quail and Smyth (2014), using the norm-referenced Drumcondra Mathematics achievement test, similarly concluded that being in a multigrade class had little impact. However, when the outcomes were disaggregated according to gender, girls in classes with older children scored significantly worse in maths than those in single-grade classes.

Factors influencing outcomes

Some studies extend beyond comparison of mathematics achievement scores and focus on identifying factors which may influence outcomes While Thomas (2012) examined variables such the influence of prior academic achievement on future outcomes, he also acknowledged that one potential variable omitted from his analyses was parental involvement. The selection of background characteristics in our study is informed by the pertinent literature and focuses predominantly on the relationship between parental involvement and academic outcomes.

Parental factors and student educational attainment

The relationship between *parental employment* and children's educational attainment has been explored in a several studies (Ermisch & Francesconi, 2002). According to Schildberg-Horisch (2016), the effect of parental employment status on educational achievement is not obvious as parental employment reduces the time available to parents to spend with their child, while simultaneously increasing the availability of resources within the family. *Socio-economic status* is a relevant factor at the individual level (Capraro, Capraro & Wiggins, 2000). Coleman (1968) claimed that socio-economic factors bear a strong relation to academic achievement, to the point that differences between schools account for only a small fraction of student achievement. *The level of parents' education* is an important influence in the achievement of their children (Smyth, Whelan, McCoy, Quail & Doyle, 2009). The involvement and *engagement of parents* with schools, particularly where parents support learning in the home has been noted as bring a significant influence in raising achievement (Harris & Goodall, 2007). Studies have also examined *parental expectations* and found associations between parents' educational expectations and children's attainment (Seginer, 1986). Studies have raised questions about the direction of the relationship between expectations and attainment (Spera, Wentzel & Matto, 2009). Finally, *family structure* is reported as being an influence in academic success (Ermishch & Francesconi, 2001). Although research frequently focuses on both family structure and marital status of parents, it is also acknowledged that the estimated effect of childhood family structure on achievement may be spurious (Mayer, 1997).

METHODOLOGY

This research involves secondary analysis of data collected from a national longitudinal study of children in Ireland called the 'Growing Up in Ireland' study. The 'Growing Up in Ireland' (GUI) study is funded by the Irish government and is being undertaken in a joint collaboration between the Economic and Social Research Institute and Trinity College Dublin (ESRI, 2010). In 2007, data collection commenced on a nationally representative sample of 8568 nine- year- old children. In excess of 2700 of children were in multigrade classes at age 9. Of these, 1253 attended small schools where all children would generally expect to spend the duration of their primary schooling in multigrade classes. This paper reports on the analysis of data supplied by the children and parents in their questionnaires, along with academic outcomes in mathematics at two time points (ages 9 and 13). The academic outcomes data reported in this study are gleaned from results on norm-referenced mathematics assessments (Drumcondra Numerical Ability test) administered as part of the study. The study uses descriptive and inferential statistics to analyse the data including regression models.

Data collection and analysis

The outcome variable, mathematics achievement logit score, is measured at two time-points: age 9 and age 13. The logit score is used in this study as it takes account of variations in the level of difficulty in the test questions, as well as adjusting the results according to the grade level of the child. The home factors explored in this study were (i) the employment status of the primary caregiver, (ii) the highest level of education achieved by the primary caregiver, (iii) social class, (iv) parental involvement, (v) parental expectations and (vi) family structure.

Both descriptive and inferential analyses are carried out on the GUI data. The distributions of mathematics scores achieved by children in multigrade classes in small schools are compared with their single-grade counterparts. Independent samples t-tests investigate if differences in mean scores for both groups are significant at p=.05 level. In order to explore the impact of home factors on mathematics outcomes for children, family and parental measures are included in a regression model. Although multiple regression can predict the value of the outcome variable, in this study the multiple regression determines the proportion of the variation in mathematics achievement explained by the background factors.

FINDINGS

First, we investigate the mathematics outcomes from wave 1 of the GUI study for 9-year old children in small multigrade classes (n=1248) and compare them to their peers in single-grade classes (n=5114). Analysis of the logit score indicates that while there were small differences in the mean scores, in favour of those children in multigrade classrooms, they were not significantly different. Figure 1 illustrates the distribution of mathematics logit scores. For the students in multigrade classes, there was a smaller range of scores, with the minimum score in a multigrade class being -3.1 and the maximum score being 1.9. In single-grade classes, while the maximum score was the same, the minimum score was lower at -3.62. The mean scores for both group were similar (multigrade mean= -.779; single-grade mean= -.726). This is not a statistically significant difference (t=-1.793, p=.073).

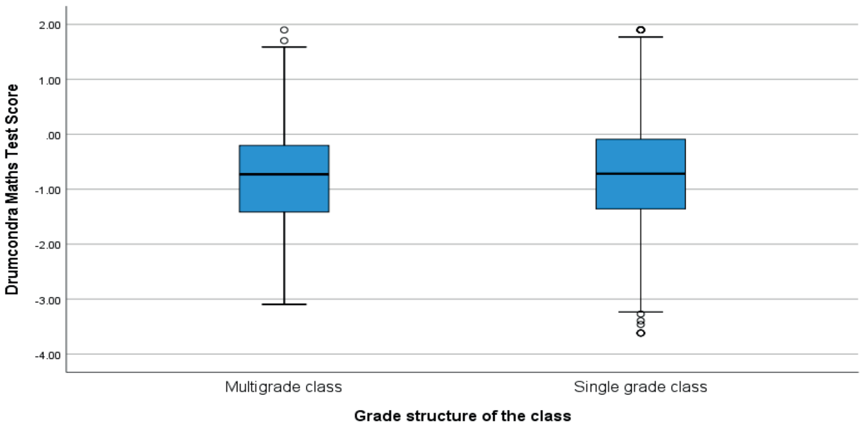


Figure 1: Comparison of logit scores for children in multigrade and single-grade classes at age 9

Our second analysis investigates the mathematics outcomes from wave 2 of the GUI study. Wave 2 focused on the same children (as in Wave 1) when they progressed in their education to age 13. Figure 2 reveals differences in mathematics outcomes between children who are likely to have been in multigrade classes for the duration of primary school (n=1177) and those in single-grade classes (n=4247). Again, we see that the mean logit score for children in multigrade classes was slightly higher. This difference of .06064, although small, was statistically significant indicating that among students at age 13, children from multigrade classes performed better than those in single-grade classes (t=1.982, p<.05).

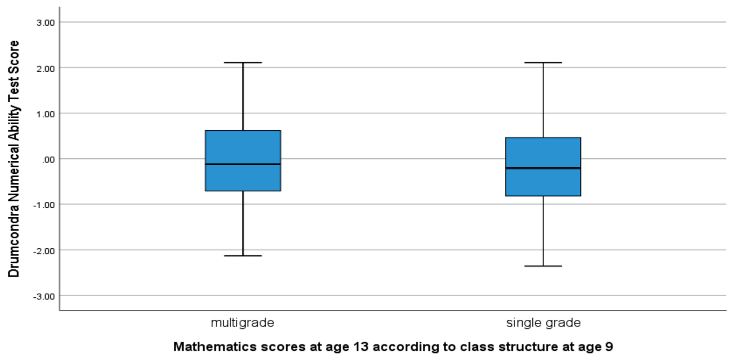


Figure 2: Distribution of logit scores for children in multigrade and single- grade classes at age 13

Home factors influencing mathematics achievement: Descriptive Statistics

The specific home factors were gleaned from questionnaires completed by parents. The questionnaire required the primary and secondary caregiver to identify their employment status, family structure and occupation which was used to determine social class. In addition they responded to a series of questions. Three questions provide relevant data in this study: What is the highest level of education (full-time or part-time) which you have completed? How often do you/your spouse provide help with homework? How far do you expect study child to go in education/training?

Table 1 presents the descriptive data for parental expectations at ages 9 and 13 and shows the highest level of education they expect their children to attain. Table 2 presents the descriptive data for five home factors: employment status, family type, level of education for the primary caregiver social class (as measured by parental occupation), and frequency of homework help. Table 2 presents the descriptive data for parental expectations at ages 9 and 13. As can be seen from analysis of tables 1-2, the employment status of the primary caregivers of both groups of children are very similar. A smaller proportion of children in multigrade classes come from households with a single adult. A greater proportion of parents of students in single-grade classes have achieved a degree or post-graduate qualification. A larger proportion of parents of children in single-grade classes expect their children to attain post-graduate degrees or higher degrees. Parents of children in multigrade classes also appeared to help with homework more frequently. Regarding social class, twice as many parents of children in single-grade classes are in professional social class and a greater proportion of parents of children in multigrade classes being in skilled or semi-skilled manual labour.

Table 1: Parents' expectations for their children at ages 9 and 13

	Multigrade		**Single-grade**	
	Age 9	Age 13	Age 9	Age 13
Junior Certificate or Equivalent	0.4%	0.3%	0.8%	0.4%
Leaving Certificate/ Equivalent	12.4%	5.3%	10.5%	7.0%
An apprenticeship or trade	10.7%	5.9%	5.3%	3.8%
Diploma or Certificate	12.7%	9.4%	10.2%	9.1%
Degree	45.5%	53.4%	50.3%	47.3%
Postgraduate/higher degree	18.2%	23.8%	23.0%	30.7%
Don't know (option at age 13)		2.1%		1.6%

Table 2: Home factor variables

	Multigrade	**Single Grade**

Employment Status		
Work or in training	58%	59.9%
Home duties or retired	35.7%	32.8%
In education	1.6%	1.6%
Unemployed	3.1%	3.5%
Family structure		
Single person and one or two children	7.2%	12.9%
Single person and three or more children	4.5%	7.1%
Couple and one or two children	32.6%	35.5%
Couple and three or more children	55.36%	44.5%
Level of education – primary caregiver		
No education or a primary education	4.7%	6.6%
Secondary or vocational	41.2%	35.3%
Degree or post-graduate	14.5%	18.9%
Family Social Class		
Professional workers	6.8%	12.7%
Managerial and technical	33.7%	36.2%
Non-manual	19.8%	18.4%
Skilled manual	17%	13.1%
Semi-skilled	13.8%	8.7%
Unskilled	1.4%	1.6%
Validly no social class	7.4%	9%
Frequency of homework help		
Always/Nearly Always	54.2%	50.3%
Regularly	21.0%	19.5%
Now and again	17.0%	18.5%
Rarely	6.1%	8.8%
Never	1.8%	3.0%

Table 3: R^2 value explaining the proportion of variance explained by parental factors at age 9

	Multigrade	Single-grade
Employment status of primary caregiver	.025	.001
Family structure	.037	.011
Primary caregiver's highest level of education	.071	.057
Secondary caregiver's highest level of education	.093	.076
Family's social class	.093	.078
Parent's expectations	.122	.133
Parental involvement with homework	.144	.157

A multiple linear regression model incorporating these background factors was constructed to investigate their influence on mathematics outcomes at age 9. Among 9 year old children in small schools (Table 3), the model accounted for 14.4% in the variation in students' mathematics outcomes. There is a statistically significant result F (6, 1095) =21.528, p<.0005. The same factors accounted for 15.7% of the variation in mathematics outcomes among children in single-grade classes. Similarly, the regression model indicates a statistically significant result F(6, 4378)= 114.371, p<.0005.

Table 4: R^2 value explaining the proportion of variance explained by parental factors at age 13

	Multigrade	Single grade
Employment status of primary caregiver	.016	.004
Family structure	.035	.011
Primary caregiver's highest level of education	.078	.083
Secondary caregiver's highest level of education	.087	.106
Family's social class	.087	.107
Parent's expectations	.117	.179
Parental involvement with homework	.132	.201

At age 13, parental factors continued to explain variance in mathematics attainment. The proportion of variance explained by home factors decreased to 13.2% among students in small schools, but increased to 20.1% for their single-grade counterparts (Table 4).

At age 9, the mathematics attainment of children in multigrade classes and children in single-grade classes was similar. At age 13, children who previously studied in a multigrade class achieved higher scores than their single-grade counterparts. Multiple factors influence academic achievement outcomes. In this study, parental characteristics, involvement and expectations explain a proportion of variance in the mathematics scores of their children. The influence of home factors varies according to the classroom structure and the age of the child. It is interesting to note that while parents' expectations and mother's education play an increasing role in the attainment of children in single-grade classes as they get older, home factors explain a smaller proportion of variance in achievement among students in multigrade classes in small schools as they get older. This suggests that there are other influencing factors which are not captured in the design of this study.

REFERENCES

Adams, J.J. (1953). Achievement and social adjustment of pupils in combination classes enrolling pupils of more than one grade level. *Journal of Educational Research, 47,* 151-155.

Capraro, M. M., Capraro, R. M., & Wiggins, B. B. (2000). An investigation of the effect of gender, socioeconomic status, race and grades on standardized test scores. Paper presented at the meeting of the Southwest Educational Research Association, Dallas, TX.

Coleman, J.S., (1968). Equality of Educational Opportunity. Equity and Excellence in Education, 6(5), 19-28.

Eivers, E. & Chubb, E. (2017). *Shaping schools: What TIMSS tells us about education systems. ERC Research Series (1)*. Dublin, Educational Research Centre

Eivers, E., Close, S., Shiel, G., Millar, D., Clerkin, A., Gilleece, L., & Kiniry, J. (2010). *The 2009 National Assessments of Mathematics and English Reading.* Dublin: Educational Research Centre. Accessed at: http://www.erc.ie/documents/na2009_report.pdf

Ermisch, J., & Francesconi, M. (2001). Family structure and children's achievements. *Journal of Population Economics, 14*(2), 249–270.

Ermisch, J., Francesconi, M. (2002). The effect of parents' employment on children's educational attainment. *ISER working paper*, 2002-21.

Fan, X., & Chen, M. (2001). Parental involvement and students' academic achievement: A meta-analysis. *Educational Psychology Review, 13*(1), 1-22.

Harris, A. & Goodall, J. (2007). *Engaging parents in raising achievement: do parents know they matter?* London: Department for Children, Schools and Families.

Little, A. W. (2001). Multigrade teaching: towards an international research and policy agenda. *International Journal of Educational Development, 21*, 481-497.

Mariano, L.T. & Kirby, S.N. (2009). Achievement of Students in Multi-Grade Classrooms: Evidence from the Los Angeles Unified School District. Santa Monica, CA: RAND Corporation. https://www.rand.org/pubs/working_papers/WR685.html.

Mayer SE (1997). *What money can't buy: Family income and children's life chances.* Harvard University Press, Cambridge.

Quail, A. & Smyth, E. (2014). Multigrade teaching and age composition of the class: The influence on academic and social outcomes among students. *Teaching and Teacher Education, 43,* 80-90.

Russell, V. J., Rowe, K. J., & Hill, P. W. (1998). *Effects of Multigrade Classes on Student Progress in Literacy and Numeracy: Quantitative Evidence and Perceptions of Teachers and School Leaders.* Annual Conference of the Australian Association for Research in Education.

Schildberg-Hoerisch, H. (2010). Does parental employment affect children's educational attainment? *Economics of Education Review, 30*(6), 1456-1468.

Seginer, R. (1986). Mothers' behavior and sons' performance: An initial test of an academic achievement path model. *Merrill-Palmer Quarterly, 32,* 153–166

Smyth, E., C.T. Whelan, S. McCoy, A. Quail, and Doyle, H. (2010). Understanding parental influence on educational outcomes among 9-year-olds in Ireland: The mediating role of resources, attitudes and children's own resources. *Child Indicators Research, 3,* 85–104.

Spera, C., K. Wentzel, & Matto, H. (2009). Parental aspirations for their children's educational attainment: Relations to ethnicity, parental education, children's academic performance, and parental perceptions of school climate. *Journal of Youth and Adolescence, 38*(8): 1140–52.

The Economic and Social Research Institute (ESRI). (2010). *Growing up in Ireland Child Cohort Wave 1 - 9 years, 2008*. [dataset]. Version 1. Irish Social Science Data Archive. SN: 0020-01. URL http://www.ucd.ie/issda/data/GUIChild/GUIChildWave1

Thomas, J.L. (2012). Combination classes and educational achievement. *Economics of Education Review, 31*(6), 1058–66.

Veenman, S. (1995). Cognitive and Noncognitive Effects of Multigrade and Multi-Age Classes: A Best-Evidence Synthesis. *Review of Educational Research, 65*(4), 319-381.

Veenman, S. (1996). Effects of multigrade and multi-age classes reconsidered. *Review of Educational Research, 66*(3), 323-340.

PLANNING FOR TEACHING EARLY MATHEMATICS: NEGOTIATION OF SHARED INTENTIONS

Siún Nic Mhuirí[1], Thérèse Farrell[1], Córa Gillic[1,2] and Mary Kingston[1]

[1]Dublin City University, [2]Millview Childcare

This paper uses thematic analysis to investigate how shared intentions for the Maths4all project were negotiated. Individuals or pairs prepared seven mathematical activity guides for preschool and primary school groups. These plans were then reviewed in team meetings using the Teaching for Robust Understanding framework (Schoenfeld, 2013) as a conversation guide. Thematic analysis of field notes taken at these meetings shows that the framework acted as a catalyst for discussions in which the ideological focus of the project became more defined. Other key themes that informed this development included looking across primary and preschool contexts; consideration of teacher interpretation of project output; the curricular context; and interrogation of frequently used language.

INTRODUCTION

This paper details the early phases of the Maths4all project funded by Science Foundation Ireland (SFI). The project will develop a website hosting continuous professional development (CPD) resources to support the teaching of early mathematics. The research team comprises of practicing teachers and academic staff from Dublin City University. Four of the academics are primarily involved with mathematics education while one specialises in Early Childhood Education. Team members who are practicing teachers have extensive teaching experience, one in preschool-settings and one in the primary school system. Both are pursuing postgraduate studies and have contributed to the development of this paper. Here, we analyse our approach to the first phase of the project. This involved planning and reviewing activities that would later be filmed in primary and preschool settings. We will discuss how review of plans using the Teaching for Robust Understanding (TRU) framework (Schoenfeld, 2013) facilitated a negotiation of shared intentions for the project.

THEORETICAL FRAMEWORK

First, we outline Wenger's (1999) theory on communities of practice. Then we present an overview of the Teaching for Robust Understanding (TRU) framework (Schoenfeld, 2013).

Communities of Practice

The three defining features of a community of practice (CoP) are mutual engagement, joint enterprise and a shared repertoire (Wenger, 1999). Engagement with the joint enterprise requires negotiation and "creates among participants relations of mutual accountability that become an integral part of the practice" (Wenger, 1999, p. 78). Our joint enterprise is defined by the structure of the SFI project. We intend to create resources for a website which will support high-quality early mathematics teaching. Within this remit much remains to be negotiated, for example, the teaching practices that we wish to foreground in CPD materials. This paper charts our first engagement with the joint enterprise. For this reason, the repertoire of resources for negotiating meaning was evolving. This is discussed further below.

Wenger (1999) contends that meaning is negotiated in the interplay between participation and reification. Participation refers to the process of taking part in social practice as well as the relationships arising from the process (Wenger, 1999). In our case, participation involved individual planning and reflection as well as collective participation in team meetings. Reification is understood as both process and product and is concerned with abstractions that reify something of the practice of a community in "congealed form" (Wenger, 1999, p. 59). Meeting notes, agreed plans for teaching, even this research paper can be considered a reification around which the negotiation of meaning was organised.

We recognise that it could be fruitful to work at the *overlap* between an academic CoP and a teaching CoP (figure 1, i). However, the teacher-members of our team operate in two distinct communities and research highlights discontinuities across primary school and preschool settings (Dunphy, 2017; O'Kane, 2016). Our CoP might also be theorized as engaged in work at the *periphery* of a teaching community (figure 1, ii) but we choose to conceive of our work as an example of a *boundary practice*. Wenger's (1999) elaboration of boundary practices draws from only two communities (figure 1, iii). We locate our CoP somewhere between an academic CoP, the CoP of our primary-teacher member and the CoP of our preschool teacher member (figure 1, iv). Positioning our team as a distinct CoP in its own right, acknowledges the expertise of all individuals. It also highlights the complexity of what we are hoping to do in drawing from and reinterpreting the practices of the original communities.

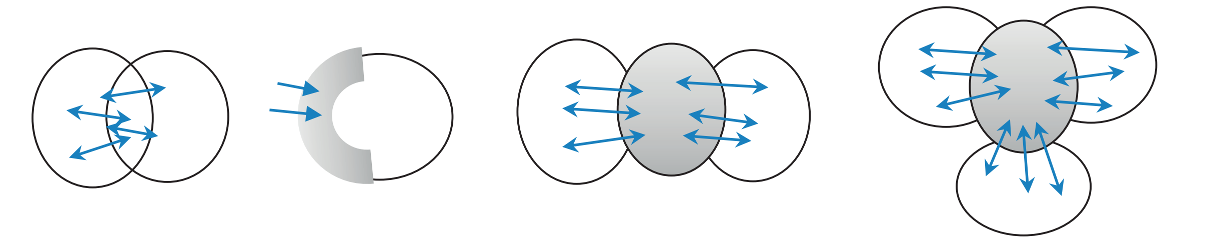


Figure 1: Practices at borders of CoPs. Images (i), (ii), and (iii) are based on Wenger (1999, p. 144). Image (iv) shows our boundary practice drawing from, and contributing to, three distinct communities.

The Teaching for Robust Understanding (TRU) framework

The TRU framework (Schoenfeld, 2013) describes five dimensions of classrooms which have been identified by research as critical for children's mathematics learning. The dimensions are: the *mathematics*; *cognitive demand*; *access to content*; *agency*, *authority and identity*; *uses of assessment*. The *mathematics* involves the disciplinary concepts and practices made available for learning. *Cognitive demand* aims to capture the extent to which children have opportunities to engage in 'productive struggle'. *Access to content* addresses the extent to which activity structures support the active engagement of all children. *Agency, authority and identity* refers to the extent to which children have opportunities to contribute to discussions in ways that build agency, mathematical authority and positive identities. *Uses of assessment* relates to how classroom activities elicit and build on student thinking. Use of the framework had been written into the SFI application by the lead author at the project outset and team members had varying degrees of familiarity with it. The need to appraise the suitability of the framework for early mathematics teaching was recognised (further details below) but the TRU conversation guide (Baldinger, Louie and the Algebra Teaching Study and Mathematics Assessment Project, 2014) was adopted for use as a way to structure coherent conversations about planning for mathematics teaching. This paper focuses on the first stage of the project where we were creating and reviewing plans for teaching.

METHODOLOGY

Seven plans for teaching were prepared by individual team members or pairs and four review meetings took place with three or four team members present each time. The lead author was present at all meetings. The introduction to each meeting involved discussing queries that had arisen previously. Two to three plans were then considered in each session. Four of the plans were edited in minor ways, if at all, after the initial meetings. The remaining three plans, which were discussed at a second meeting, were altered in more comprehensive ways.

The data considered here consists of field notes taken by the first author during meetings. These notes consisted of introductory notes on general issues and sections dealing with each of the five dimensions of the TRU framework. The notes were circulated to attending members after each meeting for comments and corrections. We wanted to investigate in what way, if any, the review meetings facilitated development of shared intentions for the project. We decided not to focus on individual contributions because the research interest was in the evolving practice of the community not the practices or beliefs of individuals (Grundén, 2019). This aligns with our aim of working as co-researchers rather than interrogating the experience of teacher team-members and follows a constructionist perspective where meaning and experience are understood to be socially produced and it is not appropriate to “focus on motivation or individual psychologies” (Braun & Clarke, 2006, p. 85).

Data was shared on Google Drive as coding software that would allow collaboration was not available. Interesting segments were highlighted and the comment function was used to name codes. This allowed for data to be coded with multiple codes. We tracked through the phases of thematic analysis outlined by Braun and Clarke (2006): familiarisation with the data; generating initial codes; searching for themes; reviewing themes; defining and naming themes; producing the report. All authors, academics and practicing teachers, engaged in stage 1 and the first author lead on the second two stages. All collaborating authors reviewed themes and contributed to the remaining phases. This analysis was not undertaken in a linear manner. Instead, initial codes led to consideration of possible themes which in turn lead to refining of codes and a reconsideration of themes. We recognise that themes are constructed by researchers rather than ‘discovered’ in the data (Braun & Clarke, 2006). It was decided that tests of inter-rater reliability were not warranted for this small data corpus. Instead, we note that the quality of qualitative research is largely connected with notions of trustworthiness and rigor (Golafshani, 2003). For this reason, the quality of our analysis rests on our efforts to make explicit and justify the decisions we have made (Braun & Clarke, 2006).

We used a semantic approach to generating inductive codes where codes were identified within the explicit meanings of the data and only at later stages was there an attempt to theorize the broader meanings. When searching for and reviewing themes (stages 3 and 4), we recognized that a number of codes were pervasive across the data. We tested whether these codes could be considered as themes by tracking, in the data and theoretically, their relationship with other codes and each other. We also referred to Braun and Clarke (2006, p. 82) who state that a theme "captures something important about the data in relation to the research question" and is indicative of some level of patterned meaning within the data". The analysis has resulted in identification of a cluster of major and minor themes (shown in grey and white respectively on figure 2). We have chosen to use this terminology rather than 'subtheme' as no hierarchy is obvious and the minor themes appear densely connected to each other and to the overarching themes. This is likely to be due to the limited quantity and nature of the source data where we returned to central questions at the start of each meeting. Our discussions were furthered structured by the TRU conversation guide.

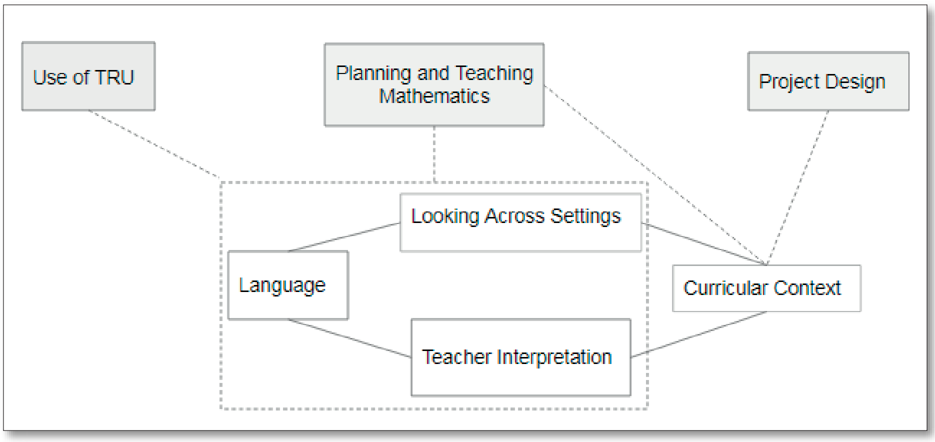


Figure 2: Overview of themes. Overarching themes shown in grey, minor themes in white.

RESULTS

We begin by discussing the minor themes and conclude by relating these to the overarching themes of: *Planning and Teaching Mathematics; Project Design* and *Use of TRU*.

Curricular Context and Looking across Settings

Curricular context, in particular interrogating the expectations of the draft specification for the new primary mathematics curriculum (NCCA, 2017), became a key focus. For example, challenges arose in how to pitch a tangram activity for first class due to a perceived jump in expectations of the shape strand (meetings 3 and 4). We were also cognisant of the recommended practices in the research reports underpinning the redeveloped curriculum. For example, we aimed to create meaningful contexts for learning and selected play-based and picture book contexts for early years settings (Dooley et al., 2014) noting that these activities could be extended to make them suitable for an infant classroom (Meeting 1).

The curricular context for preschool is a notably different space (Dunphy et al., 2014). *Looking across settings* and interrogating affordances and constraints of primary and preschool contexts became a feature of our meetings. We noted that play-based approaches are recommended in both settings as outlined by *Aistear, The Early Childhood Curriculum Framework* (NCCA, 2009) but a tension exists for primary teachers who also have a duty to teach the content specified in the primary curriculum (Gray & Ryan, 2016) (Meetings 1 and

2). There is still an expectation that primary mathematics activities should be structured and comprehensive assessment records collated (Meetings 2, 3, 4). Teachers in preschool settings may have greater pedagogical scope than infant teachers in primary schools which can lead to a more responsive approach to young children's thinking. For example, the affordances of smaller group numbers in preschool settings was noted (Meeting 1) and we discussed how it may be more feasible for teachers to orchestrate equitable access to content and opportunities to develop children's agency and identity in small group settings.

Opportunities for learning exist in having teachers look across early years and primary settings to make curricular connections explicit. The use of cognitively demanding tasks is one of the metapractices recommended in the research reports underpinning the redeveloped primary curriculum (Dooley et al., 2014). In our discussions of how such tasks may play out with young children, we made connections to the skills and dispositions outlined in Aistear, in particular the notion of perseverance (Meeting 1). Aistear, Síolta (CECDE, 2006) and the new draft primary curriculum have something meaningful to offer teachers across settings. Síolta standard 7, component 7.6, indicates that curriculum planning should be "based on a child's individual profile, which is established through systematic observation and assessment for learning" (CECDE, 2006, p. 56). This approach to planning is in line with the new draft primary curriculum, where progression continua charting key stages in the development of children's mathematical thinking are provided. Teachers are expected to use the continua to create "appropriately challenging" and playful learning experiences for children at different levels of learning (NCCA, 2017, p.13). In practical terms, we noted that it is possible to use the lower levels of the progression continua for the draft new primary curriculum to consider the development of children's thinking in early years settings (Meeting 1).

Language and Teacher Interpretation

The *Language* theme incorporates attention to the meaning of particular terms, some of which might be considered to be associated with either teachers or researchers. We have chosen the term language rather than terminology because this theme relates to essential aspects of meaning and communication rather than technical discussions of definitions. There were a number of terms that provoked debate across the meetings. These included: cognitive demand/problem solving; lesson plan/activity guide; mathematize; prior understandings; enrichment/extension. Our deliberations on these terms might be understood as the CoP developing a repertoire of shared meanings (Wenger, 1999). For example, the following notes were taken in meeting 2 when we discussed the terms 'problem-solving,' and 'cognitive demand' (which is a TRU framework dimension).

> ...many infant teachers will claim that they are not doing problem-solving because of associations with word problems. Many are actually doing cognitively-demanding tasks so it was felt that 'cognitive challenge' was preferable to 'problem-solving'

This extract also has significance to the theme of *Teacher Interpretation.* This refers to our consideration of how teachers may interpret the products of this project, i.e., teaching plans and CPD materials. Consideration of teacher interpretation was also evident in our discussion of the terms 'lesson plan' and 'activity guide'. Consider the following extract from meeting 1.

> The preferred term for the early years setting is 'activity guide'. It was felt that in general, practitioners may have negative associations with the more formal connotations of 'lesson plan' while 'activity guide' positions the resources as more in line with a play-based approach. We spoke about the opportunities of adopting this language for the primary school lessons, not only to encompass the possibilities of incorporating play-based approaches but also to signal the need for flexibility and the importance of being responsive to student thinking

When we returned to discuss this issue in meeting 3, it was stated that "student teachers tend to see lesson plans as a 'finished product' which they could enact verbatim. Suggestion that we have no control over how our end products will be interpreted so should operate on ideological grounds". This highlights the tight connections between themes as our discussion of particular language (provoked in part by the TRU framework) led to questions about teacher interpretation which in turn fed into the evolving project design.

Major Themes

Use of TRU was identified as an overarching theme because of the way in which it underpinned our discussions. At times, we explicitly discussed how and why we were using the framework and appraised its suitability in the context of early mathematics (meetings 1 and 2). While there was agreement that using TRU was worthwhile for moderating planning, there was concern about how teachers in early years settings might interpret the language of the framework (meeting 1). There was also suggestions about how the conversation guide could be clarified to support observations of early mathematics learning. Under the 'Access to Content' dimension of the TRU conversation guide, one of the questions is:

> What is the range of ways that students can and do participate in the mathematical work of the class (talking, writing, leaning in, listening hard; manipulating symbols, making diagrams, interpreting text, using manipulatives, connecting different ideas, etc.)? (Baldinger et al., 2014, p. 9)

It was suggested that the examples in brackets do not pay sufficient attention to how children may engage in mathematical work in play-based approaches (meeting 1) and that we must remain cognisant of this when we use the TRU framework to structure our observations in real settings (meetings 1 and 3). Using the TRU framework to structure our review of plans meant that we viewed fine-grained planning decisions through a research lens, evaluating and refining plans according to whether the dimensions of the framework were evident or not. This was significant for choices we made in specific activities but using TRU also acted as a catalyst for us to consider broader issues in the teaching of mathematics, e.g., the use of cognitively demanding tasks with young children (meeting 1 and 2). As detailed below, these conversations became vital, not just in relation to the original proposed activities, but also in terms of how they impacted our sense of purpose in project design and how they connected with more generalized ideas about the planning and teaching of mathematics.

Planning and Teaching Mathematics, an overarching theme, can be traced to a code which originally sought to attend to fine-grained decisions about the proposed plans. This code was refined to capture issues relevant across all contexts and activities. In this guise, it became so fundamental that it was eventually recognised as a theme. Captured here were ideas about planning and teaching such as; making connections when selecting and sequencing tasks; anticipating and preparing for student responses; how to assess and build on prior understandings; choosing representations; choosing and supporting children's understandings of contexts in mathematics problems; how to support young children's recording strategies; developing accurate terminology while respecting students' own language and thinking and ensuring all learners are catered for. The literature supports the contention that these ideas are of high significance in mathematics teaching (c.f., Dooley, Dunphy & Shiel 2014). The added import here stems from the fact that we were experiencing these issues from the 'inside' and the 'outside', operating on both sides of a boundary at once (Wenger, 1999). This boundary is described with reference to children's prior understandings in the following extract.

> Very difficult to consider prior knowledge for a class we don't know. This is not a problem for a teacher in general but is for the teacher in this research context. (Meeting 2)

We were planning mathematical activities as teachers might but this was still a theoretical undertaking as we were planning for children that we could not know.

Project Design, the final overarching theme, underpinned all of our discussions. *Looking across settings* and planning specific details according to the *curricular context* was important on a technical or practical level. Our attention to *Language* and *Teacher Interpretation* led to an expansion from attention to practical issues in earlier meetings to more explicit consideration of project purpose and attendant possibilities and limitations. For example, this extract from meeting 3, discusses the cognitive demand of a proposed task:

> A note that this relates as much to how tasks are mediated as to the lesson plans themselves. An acknowledgement that the CPD element is very important in this. Discussion of the insignificance of a single lesson for both child and teacher.

Our boundary practice created opportunities for us to engage in teacher practices such as planning. Considering how these activities might play out highlighted the centrality of the teacher's role which in turn led to a recognition of the need to foreground this in supporting CPD documentation. The intricate analysis of the possibilities of different options in planning mathematical activities was balanced with a realization of the limitations of individual planning guides for student and teacher learning. Despite awareness of the constraints of the project, there was also a growing sense of purpose as evidenced in the first extract above under *Language and Teacher interpretation*, where ideological rather than practical grounds were identified as way of selecting terminology. Similarly, in later meetings, we explicitly discussed the need to foreground inclusive practices so as to "empower (student) teachers to address diversity" (meeting 3) and decided to mandate mixed-ability groups for all activities (meeting 4). We also discussed how we could present extra follow-on activities (meeting 1, 3, 4) so that they would not be "understood as suggestions for higher achievers only…Need to consider how to present this so as be clear that all children are capable of engaging" (meeting 4). Our boundary practice was also influenced by our research orientation and noting issues worthy of further research was a regular occurrence across all meetings. This feeds into our vision for how the project, and how this CoP, may evolve over a longer timescale.

Conclusion

This paper details only the first steps of a multi-layered, dynamic project. Limitations include the small data set and lack of attention to individual participation trajectories (Wenger, 1999). To date, the project has opened a discursive space for team members. Whether the artefacts produced by our CoP will have impact on the wider constellation of CoPs engaged in early mathematics education in Ireland remains to be seen.

REFERENCES

Baldinger, E. M., Louie, N., & the Algebra Teaching Study and Mathematics Assessment Project. (2014). *The TRU Math conversation guide: A tool for teacher learning and growth*. Berkeley, CA & E. Lansing, MI: Graduate School of Education, University of California, Berkeley & College of Education, Michigan State University. Retrieved from: http://TRU.berkeley.edu.

Centre for Early Childhood Development and Education (CECDE). (2006). *Síolta: The national quality framework for early childhood education*. Dublin: Centre for Early Childhood Development and Education.

Dooley, T., Dunphy, E., & Shiel, G. (2014). *Mathematics in early childhood and primary education: Teaching and learning (NCCA Report no. 18).* Dublin: NCCA.

Dunphy, E. (2017). Transition from preschool to primary school optimising opportunities for mathematics learning. In B. Mooney (Ed.) *Education Matters Yearbook 2017-2018* (pp. 105 -110). Retrieved from: http://educationmatters.ie

Dunphy, E., Dooley, T., & Shiel, G. (2014). *Mathematics in early childhood and primary education: Definitions, theories, development and progression (NCCA Report no. 17)*. Dublin: NCCA.

Gray, C., & Ryan, A. (2016). Aistear vis-à-vis the primary curriculum: The experiences of early years teachers in Ireland. *International Journal of Early Years Education*, *24*(2), 188-205. doi: 10.1080/09669760.2016.1155973

National Council for Curriculum and Assessment (NCCA). (2009). *Aistear: The early childhood curriculum framework*. Dublin: NCCA.

NCCA (2017). *Primary mathematics curriculum: Draft specification junior infants to second class*. Dublin: NCCA.

O'Kane, M. (2016). *Transition from preschool to primary school* (NCCA Report no. 19). Dublin: NCCA.

Schoenfeld, A. H. (2013). Classroom observations in theory and practice. *ZDM*, *45*(4), 607-621.

Wenger, E. (1999). *Communities of practice: Learning, meaning, and identity*. Cambridge: Cambridge university press.

INVESTIGATING COGNITIVE DEMAND OF HIGHER-LEVEL LEAVING CERTIFICATE MATHEMATICS EXAMINATION TASKS PRE- AND POST- CURRICULUM REFORM

Rachel O'Connor, Aoibhinn Ní Shúilleabháin and Maria Meehan

University College Dublin

In 2010 the phased introduction of the new Project Maths curriculum began in post-primary schools in Ireland. This new curriculum aimed to enable students to develop problem-solving skills by providing relevant, contextual applications of mathematics, while simultaneously increasing the levels of cognitive demand required of students. This research aims to investigate whether the levels of cognitive demand required to complete tasks in the Leaving Certificate Higher-level mathematics examinations changed as a result of the curriculum reform. The methodology of this research includes the systematic analysis of Leaving Certificate examination tasks, from 2007 to 2017, using an adaptation of the Stein and Smith (1998) task analysis framework. Using this framework, tasks were classified as being of high-level or low-level cognitive demand. Analysis of the data collected suggests that a statistically significant increase in the levels of high-cognitive demand tasks did occur following the curriculum reform. Our findings are discussed in relation to two recent studies that used different frameworks to examine the cognitive demand of tasks in post-primary mathematics.

INTRODUCTION

The Project Maths (PM) reform of the mathematics curriculum in Ireland aimed to provide students with contextual, problem-solving based tasks in order to move the focus away from abstract, procedural mathematics, thus increasing the levels of cognitive demand, or the levels of thinking, required by students. In this study, we aim to analyse the levels of cognitive demand required of students in the Leaving Certificate (LC) Higher-level mathematics examinations before and after the PM reform. The task analysis framework of Stein and Smith (1998) is applied to classify LC mathematics tasks as being of high-level, or low-level, cognitive demand. This study will endeavour to answer the research question: in what ways, if any, were the levels of cognitive demand required in the Leaving Certificate Higher-level mathematics examinations influenced by the Project Maths reform?

LITERATURE REVIEW

Cognitive demand

Cognitive demand can be defined as "the kind and level of thinking required of students in order to successfully engage with and solve the task" (Stein, Smith, Henningsen & Silver, 2016, p. 1). The type of thinking required of the student depends on the nature of a particular task or learning objective (Stein & Smith, 1998) and thus the importance of cognitive demand is seen in its relationship to student learning. While there are a number of frameworks for analysing cognitive demand in the literature, we focus on the work of Stein and Smith (1998) who divide cognitive demand into two levels: low-level and high-level demand. Low-level cognitive demand tasks include: memorisation tasks; and procedural tasks without connections to concepts. High-level cognitive demand tasks include: procedural tasks with connections to the underlying concept; and, tasks that require students to 'do mathematics' in contrast to applying a practiced procedure. Stein and Smith's framework (1998) further includes descriptor-based subcategories of each of these four categories of tasks. (Their framework, which has been adapted for use in this study, is given in Figure 2.)

Analysis of Irish examination papers

In recent years, two studies examining cognitive demand of Irish mathematics examination papers have been carried out. The first study views the contexts, content, and processes underpinning the Junior Certificate (JC) mathematics examinations before and after the PM reform (Cunningham, Close, & Shiel, 2017). The data comprised the JC mathematics examinations from 2003 and 2015 and analysis was conducted using the TIMSS and PISA frameworks (Cunningham et al., 2017). Their findings suggest that there was some movement over time towards placing more emphasis on higher-level cognitive demand tasks in the JC mathematics examinations. However, the study found that this movement was not at a level that would be expected following such a broad reform.

The second study comprised an empirical review of the intellectual skills and knowledge domains in the LC examinations from 2005 to 2010 (Burns, Devitt, McNamara, O'Hara, & Brown, 2018). They used the presence of key words and their context to analyse the levels of cognitive demand in twenty-three LC subjects, including mathematics. The study found that the intellectual skill of 'apply', of low-level cognitive demand, had an occurrence of 90.6% in the mathematics examinations. This finding suggests that a high status is attributed to performance of procedural techniques in the mathematics examinations. The research concluded that the general emphasis on knowledge recollection and lack of emphasis on high-level cognitive demand in the written examinations was detached from the aims of the LC.

Two other studies conducted with the use of Stein and Smith's (1998) framework will be mentioned here. The first study found that LC Higher-level maths papers in 2009 and 2010 contained approximately 25% questions of high-level cognitive demand (Aysel, O'Shea, & Breen, 2011). The second study suggests that further effort is needed to increase the levels of high cognitive demand tasks within Irish LC mathematics textbooks (O'Sullivan, 2017).

METHODOLOGY

Data collection

The LC Higher-Level mathematics papers (paper 1 and paper 2) were collated from the years 2007 to 2017 inclusive. This timeframe was chosen so that there would be an adequate amount of data from before and after the PM reform. Due to the phased introduction of the PM syllabus, additional papers were set between 2010 and 2013. In total, twenty-seven papers were collected and included in the study. From the old syllabus, paper ones were collected from 2007 to 2012 and paper twos were from 2007 to 2011. From the PM syllabus, paper ones were collected from 2012 to 2017 and paper twos from 2010 to 2017. Two paper ones (2011 PM and 2013) contained elements from both the old syllabus and the PM syllabus. In addition to this, the 2013 paper one and 2013 PM paper one had 75% of their questions in common. Therefore, the 2013 paper one was not included in the analysis of the dataset. The paper one examinations contained eight questions prior to the syllabus reform and nine questions following the reform. Regarding paper two, each paper prior to the PM reform contained eleven questions, and nine questions after the reform. However, given the element of choice in paper two prior to the reform, and due to the small proportion of students (5%) attempting questions nine, ten and eleven (SEC, 2005), we included only the first eight questions from these papers in this study. For the purpose of this research, the unit of analysis is part of a question, for example, (a)(i) or (b)(ii). These units of analysis will be referred to as tasks. In total, 1018 tasks were analysed.

Data analysis: framework and procedures

Each task was analysed using an adapted version of Stein and Smith's task analysis framework (1998), seen in Figure 2. Each descriptor within the framework was given a label to identify it within the papers. These labels can be seen in Figure 2. The types of tasks were colour-coded to distinguish them during the coding process. As the task analysis guide was initially designed as a framework for in-class tasks (Stein & Smith, 1998), it was necessary to adapt the framework to ensure it was suitable for examination tasks. For example, while many of the examination tasks could be classified as high-level cognitive demand if it had been the students' first time engaging with those concepts, they were instead classified as low-level cognitive demand because the students' previous experience with those concepts in the classroom was acknowledged.

The tasks in each paper were coded manually by the first author using the framework below. The coding was done with reference to each examination's marking schemes in order to assess the levels of cognitive demand required to receive full marks in each task. Individual tasks were analysed to determine which descriptors depicted the task. Descriptor M1 was applied to every task because every task requires some element of producing previously learned rules or facts. Hence M1 was not included in the analysis. The following is an example of a task and how it was coded:

(c) A and B are two helicopter landing pads on level ground. C is another point on the same level ground. $|BC| = 800$ metres, $|AC| = 900$ metres, and $|\angle BCA| = 60^\circ$.
A helicopter at point D is hovering vertically above A.
A person at C observes the helicopter to have an angle of elevation of 30°.

(i) Find $|AD|$, in surd form.

(ii) Find $|BD|$.

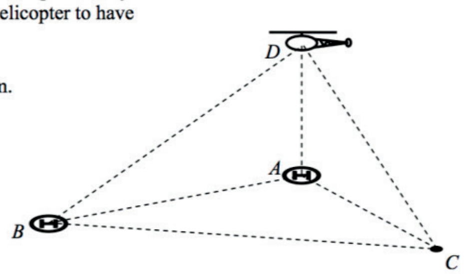


Figure 1: Task taken from 2011, paper two, question (5), part (c)(i).

This task was labelled with descriptors P1, P2, P5 because the use of a procedure to calculate a length in a triangle given such information should be evident to students as a result of their prior experience with previous tasks and would therefore require limited cognitive demand to complete the procedure. The task was also coded with the descriptor PC3 because in order to complete this procedure, students must first make the connection between the worded-representation and the diagrammatic-representation of this task. Some tasks contained descriptors from only one classification hence they were categorised as that type of task.

However, some tasks contained descriptors from multiple classifications. In these cases, the task was classified by the highest level of cognitive demand present. This method of classification was chosen because, while a task may be primarily 'procedures without connections', a connection to the underlying concepts must be made to complete that task, and thus obtain full marks in the examination question. In this particular example, the mathematical procedures required to complete the task were straightforward and could be completed with limited cognitive demand. However, the fact that the students were required to make connections between multiple representations ensured that a higher-level of cognitive demand was needed to complete the task fully. Therefore the task was classified as 'procedures with connections' due to the descriptor with the highest level of cognitive demand present.

Low-Level Cognitive Demands	High-Level Cognitive Demands
Memorisation Tasks **M1**. Involve either producing previously learned facts, rules, formulae, or definitions or committing facts, rules formulae, or definitions to memory. ***This descriptor was automatically assumed to be present in each task because every task requires the production of previous knowledge.*** **M2**. Cannot be solved using procedures because a procedure does not exist or because the time frame in which the task is being completed is too short to use a procedure. **M3**. Are not ambiguous- such tasks involve exact reproduction of previously seen material and what is to be reproduced is clearly and directly stated. **M4**. Have no connection to the concepts or meaning that underlay the facts, rules, formulae, or definitions being learned or reproduced.	***Procedures with Connections Tasks*** **PC1**. Focus students' attention on the use of procedures for the purpose of developing deeper levels of understanding of mathematical concepts and ideas. ***This descriptor was applied if the task highlighted a link to students between procedures and underlying concepts, or if the task required students to notice a concept based on the repeated use of procedures.*** **PC2**. Suggest pathways to follow (explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas as opposed to narrow algorithms that are opaque with respect to underlying concepts. **PC3**. Usually are represented in multiple ways (e.g. visual diagrams, manipulatives, symbols, problem situations). Making connections among multiple representations helps to develop meaning. ***This descriptor was applied if the task included any additional representations of the initial question.*** **PC4**. Require some degree of cognitive effort. Although general procedures may be followed, they cannot be followed mindlessly. Students need to engage with the conceptual ideas that underlie the procedures in order to successfully complete the task and develop understanding. ***In this descriptor, the idea of not following the procedure mindlessly was focused upon. This descriptor was used if a student was required to use familiar procedures but it was not obvious that the procedure was required from the task or prior experience.***
Procedures without Connections Tasks **P1**. Are algorithmic. Use of the procedure is either specifically called for or its use is evident based on prior instruction, experience, or placement of the task. ***In this descriptor experience was taken to be the students' previous experience of completing fundamentally similar tasks in class or previous exam papers.*** **P2**. Require limited cognitive demand for successful completion. There is little ambiguity about what needs to be done and how to do it. ***In this descriptor it was taken that the lack of ambiguity could come from the fact that the student would have completed many similar questions in class.*** **P3**. Have no connection to the concepts or meaning that underlie the procedure being used. ***This descriptor was used infrequently as very few tasks were completely unrelated to any concept or meaning.*** **P4**. Are focused on producing correct answers rather than developing mathematical understanding. **P5**. Require no explanations or explanations that focus solely on describing the procedure that was used.	***Doing Mathematics Tasks*** **DM1**. Require complex and non-algorithmic thinking (i.e. there is not a predictable, well-rehearsed approach or pathway explicitly suggested by the task, task instructions, or a worked-out example). **DM2.** Require students to explore and to understand the nature of mathematical concepts, processes or relationships. **DM3.** Demand self-monitoring or self-regulation of one's own cognitive processes. **DM4**. Require students to access relevant knowledge in working through the task. ***This descriptor was used when a task contained multiple elements or topics and students were required to use relevant knowledge from a variety of areas in mathematics.*** **DM5**. Require students to analyse the task and actively examine task constraints that may limit possible solution strategies and solutions. **DM6.** Require considerable cognitive effort and may involve some level of anxiety for the student due to the unpredictable nature of the solution process.

Figure 2: Adaptation of Task Analysis Guide cited in Boston and Smith (2009). Descriptors labelled with relevant codes e.g. 'P3'. Adaptations highlighted in bold and *italics*.

A random sample of ten tasks was given to two other mathematics teachers to code. The framework was shared with them and they were asked to use the descriptors to classify the tasks as a particular type. Both teachers matched the first author's classifications for nine out of ten tasks. Once the tasks had all been classified, the number of tasks in each category was counted for every year to assess the levels of cognitive demand required to complete each paper. The proportion of each type of task was compared for every year before and after the PM reform in order to assess if changes to the levels of cognitive demand had occurred. A significance test (two tailed t-test with 95% confidence interval) was then conducted to analyse if the levels of cognitive demand were significantly different as a result of the PM reform.

FINDINGS

Percentage of task-types before and after the PM reform

The percentage of tasks under each of the four task-types in Paper 1 and 2 combined from 2007 to 2017 is given in Figure 3.

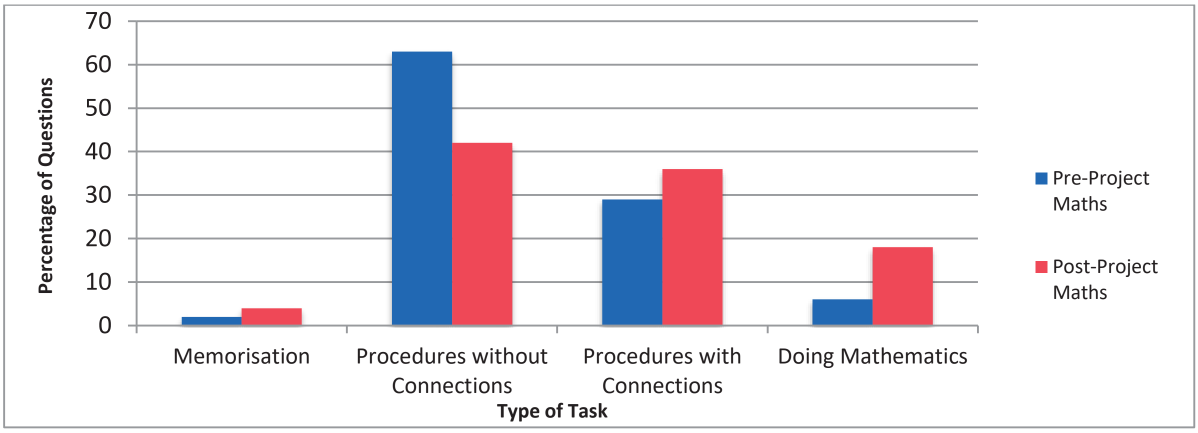


Figure 3: Percentage of tasks under each of the four task-types in Paper 1 and 2 together, before and after the complete PM reform. This data does not contain the 2013 paper one due to a 75% overlap with 2013 PM paper one.

From Figure 3, one can see that the most notable difference between the types of tasks before and after the PM reform is the percentage of 'procedures without connections' tasks. Before the reform the average percentage of 'procedures without connections' tasks was 63%, and this fell to 42% following the reform. This difference was significant within a 95% confidence interval. Another noticeable difference is the increase in high-level cognitive demand tasks ('procedures with connections' and 'doing mathematics') after the reform. This is again significant within a 95% confidence interval.

Distribution of task-types

In Figure 4 and 5 we see the percentage of tasks under each of the four task-types in the paper one and paper two examinations. We notice that in both papers the majority of tasks are procedural, with the papers consisting, on average, of 85% procedural tasks, both 'with connections' and 'without connections'. 'Procedures without connections' emerged as the dominant task type, with papers consisting on average of 52.5% of these tasks. We see in

Figure 4 that in the majority of paper one examinations, the low-level cognitive demand tasks were more frequent than the high-level cognitive demand tasks. As can be seen in Figure 5, the distribution of low-level and high-level cognitive demand tasks is more even across the paper two examinations.

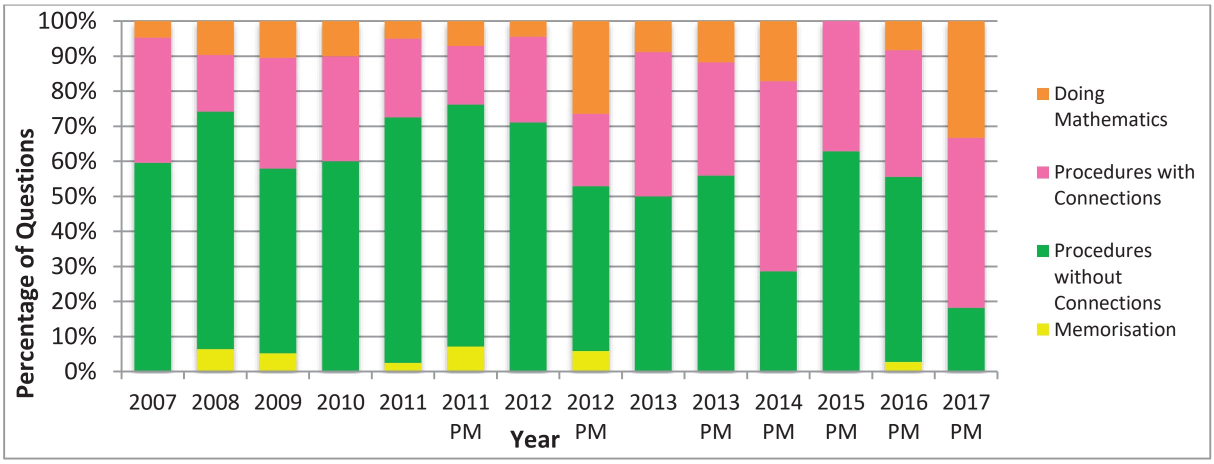


Figure 4: Percentage of tasks under each of the four task-types in LC Higher-level paper one examinations from 2007-2017. Note that 2013 and 2013 PM contained 75% of the questions in common.

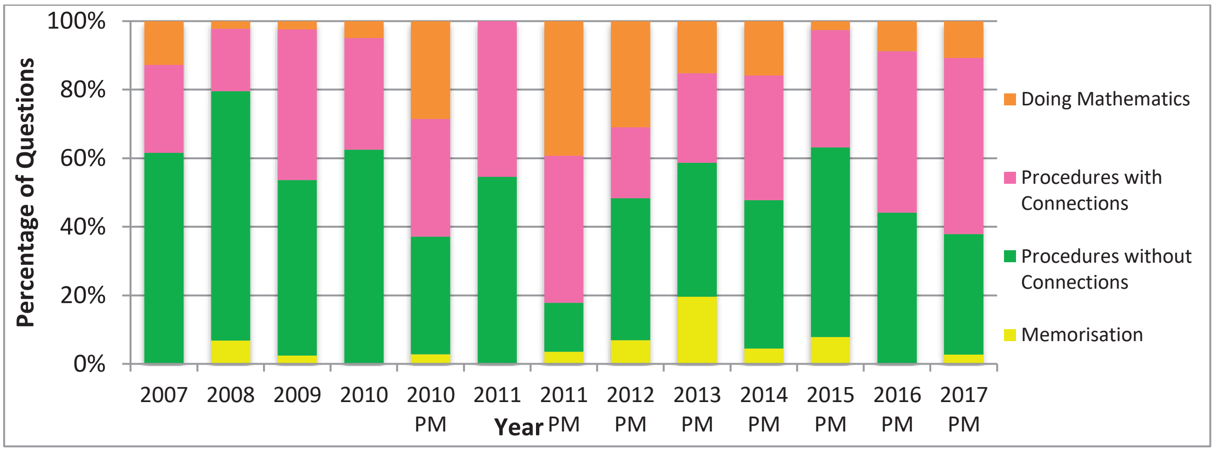


Figure 5: Percentage of tasks under each of the four task-types in LC Higher-level paper two examinations from 2007-2017.

DISCUSSION AND CONCLUSIONS

Consistency with the aims of PM

These results are consistent with the aims of the PM curriculum, which were to increase the levels of cognitive demand by involving students in problem-solving and providing contextual applications of mathematics. The findings show that "procedures without connections" tasks was the dominant task-type, both before and after PM. This may be explained by the number of tasks within pre- and post-PM syllabi that are procedure based, such as solving a quadratic equation or differentiating a function. It is necessary for these topics to be assessed within the examination as they form a core part of the syllabus. It is also important to note, that many of these tasks are considered as 'procedures without connections' because of the prior experience students had with these procedures when engaging with them in class. These procedures can require high-levels of cognitive demand during the initial knowledge acquisition phase, with the levels of cognitive demand decreasing as students gain experience practicing these procedures.

The increase in ‘doing mathematics’ tasks reflects the aim of PM to increase the levels of problem solving required of students and decrease the levels of abstract, practiced procedures. However, while providing ample opportunities for students to ‘do mathematics’ in the classroom can provide challenging and rewarding learning experiences, it could be argued that by the time the students attempt their examinations, the majority of the ‘doing mathematics’ tasks should be complete. It may be more appropriate that students have the opportunity to apply the knowledge they have mastered in a summative assessment situation.

Topics relating to task-types

Following a brief review of the topics on each paper and the task-types with which they were classified, it was found that the proportion of task-types per paper can often be linked to the types of topics that paper assesses. For example, the ‘memorisation’ tasks appeared most frequently in paper two examinations. A reason for this could be the regularity with which the topic of geometry appears in paper two. Many of the tasks classified as ‘memorisation’, were those that asked students to reproduce a proof from the geometry strand.

As previously discussed, the PM reform aimed to provide relevant contextual applications of mathematics for students. In many cases, these applications appeared in the form of a word-problem with corresponding diagram, requiring students to make a connection between multiple representations of a concept. In such cases, the tasks required students to complete straightforward, practiced procedures, and so would initially be classified as ‘procedures without connections’. However, the addition of the diagram ensured that students were required to make connections between representations, resulting in them being classified as ‘procedures with connections’.

Comparison of marks awarded per task-type

One question that arose from this research was whether or not the PM reform would place a higher value on cognitively demanding tasks in examinations, thus awarding them higher marks than lower cognitively demanding tasks. When comparing the percentage of marks available to the percentage frequency of each task-type in the 2007 and 2017 examinations, it was found that these percentages were approximately even. While the overall levels of cognitively demanding tasks increased, these tasks were not awarded a disproportionate amount of marks.

Comparison of findings with current Irish research

When analysing the results of this study in relation to comparable Irish studies, similarities occur in the findings. Our findings correspond with those of Cunningham et al. (2017) who also found increased levels of cognitive demand, albeit in the JC mathematics examinations, after the PM reform. In the empirical review of the LC mathematics papers from 2005 to 2010, Burns et al. (2018) found that 97.5% of tasks investigated were procedural. This study found that prior to the PM reform (2007-2012), procedural tasks comprised 92% of the examination papers. These results are comparable to the results of the Burns et al. study, strengthening the validity of these findings.

In conclusion, this research has suggested that the aims of the PM reform to increase levels of cognitive demand are being met in relation to the LC Higher-level examination papers. While this should be seen as a positive result, the suitability of having more 'doing mathematics' tasks in the examinations must be considered. Asking students to engage in complex and non-algorithmic thinking with an unpredictable solution process under the constraints of time-limitations has the potential to cause anxiety and stress for students in an already highly pressurised situation. While decreasing the levels of 'procedures without connections' tasks can be seen as a positive outcome, a corresponding increase in 'procedures with connections' rather than 'doing mathematics' tasks may be a fairer substitution for examination students.

ACKNOWLEDGEMENTS

Sincere thanks to Dr Emma Howard for reviewing parts of this research.

REFERENCES

Aysel, T. (2012). *An exploration of the effects of high-stakes examinations on the teaching and learning of mathematics in post-primary education in Ireland and Turkey*. PhD thesis, National University of Ireland Maynooth, Ireland.

Boston, M.D., & Smith, M.S. (2009). Transforming secondary mathematics teaching: Increasing the cognitive demands of instructional tasks used in teachers' classrooms. *Journal for Research in Mathematics Education, 40*(2), 119-156.

Burns, D., Devitt, A., McNamara, G., O'Hara, J., & Brown, M. (2018). Is it all memory recall? An empirical investigation of intellectual skill requirements in leaving certificate examination papers in Ireland. *Irish Educational Studies, 37*(3), 351-372.

Cunningham, R., Close, S., & Shiel, G. (2017). Ireland's junior certificate mathematics examination through the lens of the PISA and TIMSS frameworks: Has Project Maths made any difference? In T. Dooley and G. Gueudet (Eds.). *Proceedings of the Tenth Congress of the European Society for Research in Mathematics Education* (*CERME10)* (pp. 3444-3451). Dublin: DCU Institute of Education and ERME.

O'Sullivan, B. (2017). *An analysis of mathematical tasks used at second-level in Ireland.* PhD thesis. DCU, Ireland.

State Examinations Commission (SEC). (2005). *Chief Examiner's report: Higher, ordinary and foundation levels – mathematics Leaving Certificate examination 2005.* Dublin: SEC.

Stein, M.K., & Smith, M.S. (1998). Mathematical tasks as a framework for reflection: From research to practice. *Mathematics Teaching in the Middle School, 3*(4), 268-275.

Stein, M.K., Smith, M.S., Henningsen, M.A., & Silver, E.A. (2016). *Implementing standards-based math instruction: a casebook for professional development.* New York: Teachers College Press.

"MODERN MATHS" AND "PROJECT MATHS": POLAR OPPOSITES OR MIRROR IMAGES?

Elizabeth Oldham

Trinity College Dublin

The "Project Maths" curriculum initiative, affecting post-primary mathematics education in Ireland from 2008 and fully established only by 2018, has attracted much attention. Unlike reforms in the preceding 30 years, it involved a fundamental critique of the nature and purpose of mathematics education and addressed both junior cycle and senior cycle at the same time. The resulting curriculum has been contrasted with that reflecting so-called "Modern Maths", introduced in Ireland in the 1960s and 1970s. However, in this paper, it is argued that the two initiatives (while differing in some key respects) had many features in common, and that the vision and excitement round the earlier development has been lost – and its purpose misunderstood – over the intervening years. As well as aiming to re-establish the historical narrative, the paper addresses the issue of faithfully implementing curricula, especially those infused by a vision of their subject area not necessarily shared by teachers.

INTRODUCTION

The decade from 2008 to 2018 has been one of notable activity in Irish mathematics education. A major curriculum initiative at post-primary level, "Project Maths", was introduced at the beginning of that period, and was fully established – in the sense that all students completing post-primary education experienced it in its final form from First Year through to Sixth Year – only in 2018. The initiative has been the subject of lively debate. In discussions, the style of curriculum introduced by Project Maths is often contrasted with an earlier movement, "Modern Maths" ("Modern Mathematics", or, in America, "New Math"), which affected Irish mathematics education from the 1960s. In this paper, it is argued that *the perception of contrast is based in part on a serious misunderstanding of the aims and legacy of the Modern Maths period*, and that *such misunderstanding has masked some of the difficulties in making radical changes to the culture of mathematics education in Ireland*.

Two reasons can be given for presenting such an argument now. First, the recent initiation of further junior cycle reform provides a natural starting point for reflection on Project Maths and on the changes that it has brought about. While this paper does not offer a critique of the project, it raises issues that may be relevant to such a critique. Secondly, with the passage of time, the number of people who can recall the Modern Maths movement is decreasing, and first-hand recollections are in danger of being lost. The author was a school teacher in the Modern Maths era, and subsequently worked with international studies of curriculum and attainment dealing with issues that it raised; hence, the paper has the strengths – and the limitations – of presenting documentary evidence framed by relevant personal experience.

In the paper, the context and frameworks that underpin the discussion are outlined. The story of the Modern Maths period is then told in some detail; the era that followed is also described, and a short account is given of the Project Maths initiative. Comparisons and contrasts are identified and conclusions drawn, noting implications for the future.

CONTEXT AND FRAMEWORKS

The context is provided by a brief overview of international trends in mathematics education over the fifty years from around 1960. While there are differences in emphasis and timing between countries, a typical pattern involves four periods: first, the introduction of "Modern" mathematics (identified here by the initial capital letter, and described more fully below); secondly, a swing back towards emphasis on basic computational skills; thirdly, a reaction to this, involving priority given to problem solving; and finally, a focus also on contexts and applications in what are known as "Reform" curricula (Herrero & Owens, 2001; Lampiselkä, Ahtee, Pehkonen, Meri, & Eloranta, 2007; Walmsley, 2007). This simple outline conceals some variations. A more nuanced analysis of the theories affecting the first two periods in the years up to 1980, as regards both curriculum content and pedagogy, is provided by Howson, Keitel and Kilpatrick (1981). By 1990, a move away from domination by any specific theory is noted in Howson's (1991) study of the content of (mainly) European curricula. The subsequent emergence of Reform curricula again reflects theoretical underpinnings: in particular, social constructivist beliefs about learning, and an understanding of mathematics itself as a dynamic subject built up by human beings rather than as a body of knowledge that is already existing and close to absolute truth (Ernest, 2014). Similarities and contrasts between the Modern Maths and the Reform periods in the USA have already been identified (Herrero & Owens, 2001; Walmsley, 2007); the present paper addresses the issues in an Irish context. Overall, the four periods provide a historical framework for the paper.

A second framework is drawn from the large-scale studies of mathematics education conducted by the International Association for the Evaluation of Educational Achievement (IEA) (http://timssandpirls.bc.edu) and by the Organisation for Economic Cooperation and Development (OECD) via its Programme for International Student Assessment (PISA) (http://www.oecd.org.pisa). The IEA's Second International Mathematics Study (SIMS), a cross-national study of curriculum and achievement carried out in the 1980s, introduced the now familiar three-level model of curriculum: *intended* (decreed typically by a State department of education or equivalent), *implemented* (taught by teachers in school classrooms), and *attained* (learnt by students). Originally applying to mathematical content, the model was developed into a 3×3 grid, used by SIMS and in a slightly expanded version by PISA (Travers & Westbury, 1989; Shiel, Cosgrove, Sofroniou, & Kelly, 2001). The rows indicated the three levels, while the columns represented *content* (the original version), *contexts* (such as the structure of education systems and school and classroom conditions) and *antecedents* (representing factors such as level of economic development and characteristics of participating teachers and students). Over time, the model has been refined in different ways, especially with regard to influences on the implemented (or "enacted") curriculum: for example, including textbooks and examinations explicitly as important determinants of teaching, and taking account of teacher and student knowledge, beliefs and practices (Thompson & Usiskin, 2014). Criticisms of such models include the fact that they may seem to view teachers as agents who should carry out official intentions faithfully, rather than as professionals with their own agency (Looney, 2014). In this short paper, reference is made chiefly to the original model dealing with content, though bearing other aspects in mind, and there is no intention to cast teachers in a passive role.

THE MODERN MATHS STORY

The origins of the 1960s curriculum changes in Ireland are not readily available in official documents. However, evidence does exist, notably in the papers written in the 1970s and early 1980s for Ireland's participation in SIMS. The Irish National Committee for SIMS included an inspector from the Department of Education and several teachers who had taught the new courses in the 1960s, hence providing well-informed insights into the developments. The present author was the research coordinator. An article based on one of the Committee's major submissions to the curriculum element of SIMS (Oldham, 1980) is the source of material not otherwise referenced in this section.

The reforms of the 1960s had their roots in the 1950s; they reflected the international movement towards Modern mathematics that aimed to bring school subject-matter more in line with the subject as developed at third level – in particular emphasising mathematics as the *study of structures* and presenting it via uniform and precise language (Organisaton for Economic Cooperation and Development (OECD), 1961; Fey, 2016; Howson et al., 1981). The third-level focus was mainly on content. However, at school level the changes were also intended to enhance *learning for understanding*: emphasising the relationships between mathematical concepts and topics, and in some cases focusing on concept development via discovery or from intuitive starting points (OECD, 1961; Howson et al., 1981; Walmsley, 2007). In Ireland, the revised curricula of the 1960s and early 1970s resulted from conscious engagement with these trends. The aim was not only to update mathematical content, but also – an important point for this paper – to counteract a perceived over-emphasis on procedural fluency at the expense of conceptual understanding.

The changes were phased in over a decade. The first revisions were made to the Leaving Certificate (senior cycle) courses in 1964 for examination from 1966; they brought in typically Modern elementary material – sets, relations, functions as special relations, and number bases other than ten. Changes not associated with the Modern movement also took place, introducing statistics, probability at what is now called Higher level, and differential calculus and coordinate geometry at what is now called Ordinary level. In 1966, revised Intermediate Certificate (junior cycle) courses were initiated, for examination from 1969; they included the elementary Modern material and statistics, together with informal transformation geometry alongside the familiar deductive treatment in the style of Euclid. This necessitated a further revision of the Leaving Certificate courses for 1969. A major feature at Higher level was the introduction of matrices and (as an option) groups, together with enhanced treatment of complex numbers and vectors, giving the study of *algebraic structures* a very prominent place. The Ordinary course likewise featured algebraic structures: complex numbers, vectors, and groups (again as an option), and a more formal study of transformation geometry. It also included some integration. The final revision of the period, in 1973, was to the Intermediate courses; a major feature was the replacement of the hybrid approach to geometry by a unified Modern system based on sets and transformations. That curriculum was the first to contain a "Preamble" setting out objectives. Notably, these referred to *understanding* of concepts and logical structure and of the nature of proof; *discovery* of *generalisations*; and *awareness* of *applications from everyday life* (Department of Education, n.d.).

The early developments were highlighted in the media and were generally greeted with excitement. However, they preceded much of the growth in post-primary school participation that was a feature of the late 1960s and 1970s, changing the nature of the target population. The curricula catered poorly for less-academic learners and those unready for abstractions, and this led in time to considerable disenchantment. When revised Leaving Certificate courses were introduced in 1976 (to build on the 1973 Intermediate curriculum), the Ordinary course was made less abstract, in particular by excluding groups and integration. Overall, the revision marked the beginning of a move away from Modern material, for instance changing the placement of options in the Higher course so that the strong focus on algebraic structure could be avoided (Department of Education, n.d.).

Description so far has focused on the *intended* level. *Implementation* in the classroom was supported initially by (for the time) extensive courses for teachers, introducing them to the new content. However, it can be argued that these courses did not address the underlying philosophy of the Modern movement; that teachers' beliefs about mathematics and mathematics education were not necessarily in accord with the Modern philosophy; and that – partly as a result – the intentions were never fully implemented in the majority of classrooms. In terms of *attainment*, dissatisfaction with the level of procedural fluency displayed by students developed over time. The 1973 and 1976 revisions placed some additional emphasis on this aspect by reflecting it more strongly in the examination papers, though without embracing the spirit of the back-to-the-basics movement that was dominant in the USA [1].

BETWEEN THE MODERN MATHS AND PROJECT MATHS PERIODS

The pace of curriculum change in Ireland slowed after the 1976 revisions. However, in the 1980s and 1990s, further reviews led to introduction of revised curricula for the Intermediate Certificate (1987 – re-designated for the Junior Certificate in 1989), Leaving Certificate (1992) and Junior Certificate (2000). The rationale for and details of these revisions are described in a paper that is the source for material not otherwise referenced in this section (Oldham, 2007). The briefs for all three changes were limited, so the reviews were reactive rather than proactive: addressing perceived problems with implementation of their predecessors and with resulting student attainment, rather than rethinking mathematics education for the future – or indeed engaging deeply with either the back-to-the-basics approach or the subsequent focus on problem solving. However, it is relevant to note that the preambles or introductions again emphasised understanding and (to some extent) applications.

The effect of the changes was to greatly reduce the amount of material reflecting the Modern period. In terms of philosophy, the curricula were consciously eclectic: typical of the 1980s (Howson, 1991), but rather less so of the 1990s as the international community engaged with the Reform period. The Leaving Certificate Higher course, in particular, remained formal and abstract – *but* scarcely more so, if at all, than the curricula of the 1950s and earlier in dealing with traditional content: a point that has been overlooked in discussions of abstraction in Modern curricula. The legacy of the 1960s was reduced chiefly to the retention of some set theory (though no longer as an explicit foundation of as much content as possible); the presence of complex numbers, matrices and (as an option) groups, all treatable in a Modern spirit if teachers so chose; and continued use of Modern terminology and symbolism.

THE PROJECT MATHS STORY

The story of Project Maths has been well documented, for example in the comprehensive report by Shiel and Kelleher (2017): a source for otherwise unreferenced material in this section. A brief account suffices here. Dissatisfaction with student attainment sparked a critique by the National Council for Curriculum and Assessment (NCCA), beginning in 2002. The critique was eventually formalised in a discussion document (NCCA, 2005); circulation of the document, together with a questionnaire, to all post-primary schools initiated a consultation period with stakeholders. The NCCA also commissioned a research report on international trends in mathematics education, focusing on teaching, learning and assessment (Conway & Sloane, 2006) [2]. Thus, the context was set for beginning a substantial review.

Development of the revised curriculum took place over the next couple of years. At the *intended* level, it was envisaged that teaching and learning would emphasise, not only conceptual understanding, but also problem solving and applications set in real-life contexts – thus reflecting the Reform tradition – with State examinations mirroring those emphases. The eventual outcome was a curriculum innovation project introducing the revised curriculum on a phased basis. It began in 2008 in 24 schools ("Phase 1"), first involving only two out of five curriculum "strands" ("Probability and statistics" and "Geometry and trigonometry"); the strands "Number", "Algebra" and "Functions" were introduced in Phase 1 schools over the following two years. Content changes included a further decrease in abstract algebra (groups, matrices and even vectors being eliminated) and also a reduction in calculus, both to accommodate extra probability and to allow time for constructivist learning. Implementation in all other schools followed the same pattern after a time-lag of two years. Atypically, the curriculum was introduced simultaneously into the first year of both junior and senior cycle.

For *implementation*, unusually extensive support for teachers addressed the intended philosophy and focused largely on pedagogy. It highlighted investigative approaches that had not previously taken root in a classroom culture which – despite all intentions – still featured over-emphasis on procedures (Oldham, 2001). A factor conducive to culture change was the altered style of examination papers, but the co-existence of older (including pre-Modern) and newer approaches, notably for algebra, was a confounding influence (Prendergast & Treacy, 2018). Overall, implementation was accompanied by animated discussions, written and oral, "pro" and "anti": some based on the erroneous assumption that the outgoing curriculum content was still Modern and was intended to be taught without understanding (see note [2]).

Studies of *attainment* still reflect a curriculum in the process of implementation. However, there is anecdotal evidence of a decrease in procedural fluency: perhaps a result of general time pressures together with more focus on conceptual understanding and problem solving.

COMPARISONS AND CONTRASTS

Attention is now drawn to major comparisons and contrasts between the Modern Maths and Project Maths curricula, designated here for brevity as MM and PM respectively. The three curricular levels – intention, implementation and attainment – are considered in turn.

As regards *intention*, both MM and PM originated at system level, notably in response to international trends: the first constituting a rapid and maybe uncritical adoption of Modern approaches (Oldham, 1980), the second involving a rather belated reaction to some twenty years of Reform-oriented debate. In contrast, the intervening revisions (with limited briefs) were driven more by national issues and teacher-generated pressure. Both MM and PM reflected identifiable philosophies of mathematics education, though these philosophies differ: that for MM being more static and absolutist, and that for PM being in a more dynamic tradition (Ernest, 2014) – neither, it may be said, uniformly popular with mathematicians. As noted, intervening revisions produced more eclectic curricula. Both PM and MM, like all curricula discussed here, strongly emphasised teaching for and learning with understanding; however, in the case of MM there was particular emphasis on mathematical coherence, while PM stressed student construction of meaning and benefited from evolution in theories of learning since the earlier period. The aims formulated for both MM and PM also made reference to applications, though in PM their role especially with regard to real-life contexts is very much greater. It is ironic that some of the Modern topics seen in the past as remote from student experience would be ripe for application in the era of the Internet.

For *implementation*, both reforms were introduced with considerable media publicity, and provoked discussion on a scale not experienced in the intervening period. Both MM and PM were phased in, albeit in notably different ways, allowing teachers to come to terms gradually with new content and approaches. In both cases, more professional development was provided than was usual at the time, though perhaps inevitably it was insufficient to overcome the challenge to teachers whose own explicit or tacit philosophies of mathematics education differed from those of the curricula. In both cases also, problems arose when older and newer content or approaches were combined in the curriculum: notably in MM for geometry and in PM for algebra. However, for PM – in contrast to the case for MM – intended change in classroom culture was supported by the radically altered style of State examination papers.

With respect to *attainment*, it is difficult to compare meaningfully across the fifty-year period. Proficiency naturally reflects curricular emphases at the time, and so change is inevitable. However, both MM and PM gave rise to perceptions of decreased procedural fluency, perhaps because such fluency does not result automatically from enhanced focus on understanding.

DISCUSSION AND CONCLUSIONS

The question posed in the title can now be answered: Modern Maths and Project Maths are indeed mirror images in many respects; moreover, since mirror images are laterally inverted, the metaphor can be extended to embrace the ways in which they are polar opposites. The similarities between the two, and their stories as told above, closely echo those observed for New Math and Reform curricula in the USA (Herrero & Owens, 2001). Telling the Irish stories has had two chief aims: first, historical accuracy, and secondly, lessons for the future.

The first aim was addressed by outlining the vision and excitement that accompanied introduction of the Modern Maths curricula and highlighting the extent to which these curricula (and indeed those succeeding them) were intended to promote *meaningful learning*. This is not to overlook the extent to which some aspects were mismatched to the student body, especially as the size of the participating cohort grew and included more learners whose strengths were not in the field of abstract study. The reformers of the Modern Maths era may have been "incredibly naïve" in trying to bring about change in mathematics curricula and teaching without taking account of "complex institutional, political and cultural factors that shape school values and practices" (Fey, 2016, p. 422), but they were well intentioned. In fact, *all* changes discussed here reflected high ideals and were carried out with great care.

However, misinterpreting the goals of the Modern Maths period unintentionally masks the difficulty of introducing a curriculum based on a philosophy of mathematics education not widely shared by teachers, whose participation with conviction is so crucial in implementing official intentions, and indeed in moderating any that are intrinsically unworkable. When Modern curricula were taught by teachers who did not grasp – or who disagreed with – their underlying philosophy, teaching focused unduly on procedural fluency without developing conceptual understanding; analogously for Reform curricula, activities intended to facilitate construction of meaning can become hands-on but not minds-on busywork. Of note also is that abstraction and formalism did not start with the Modern period, but were features of mathematics education long before it. By over-identifying these features with a movement that – despite its virtues – was deemed to have failed (Herrero & Owens, 2001), we have minimised debate on where in mathematics education they have an appropriate place.

We are less naïve now than in the 1960s with regard to the challenges. Nonetheless, if we underestimate the difficulties experienced in changing classroom culture in the past, we may be paving the way for disappointment with regard to doing so in the future. If this paper helps in framing suitable discussion of the issues, its second aim will have been achieved.

NOTES

1. An assertion by Oldham (1980) that the courses represented the start of the "back to the basics" movement in Ireland was based on her incomplete understanding of that movement at the time.

2. This otherwise admirable book contains an error in attributing to the author (Oldham, 2001) the view that curricula at the time were still in the Modern tradition. The error has been replicated in several papers; some even suggest that Modern work was *intended* to be learnt without understanding. Oldham's paper noted the *withdrawal* from most Modern content, and (of course) made no claim that procedural emphasis was "Modern".

REFERENCES

Conway, P., & Sloane, F. (2006). *International trends in post-primary mathematics education: Perspectives on learning, teaching and assessment*. Dublin, Ireland: National Council for Curriculum and Assessment.

Department of Education (n.d. [1976]). *Rules and programme for secondary schools 1976/77*. Dublin, Ireland: Stationery Office.

Ernest, P. (2014). What is mathematics, and why learn it? In P. Andrews & T. Rowland (Eds.), *MasterClass in mathematics education* (pp. 3–14). London, England: Bloomsbury.

Fey, J. (2016). A review of the new math: A political history. *Journal for Research in Mathematics Education*, *47*(4), 420–422.

Herrera, T. A., & Owens, D. T. (2001). The "new new math"?: Two reform movements in mathematics education. *Theory into Practice*, *40*(2), 84–92.

Howson, G. (1991). *National curricula in mathematics*. Leicester, England: The Mathematical Association.

Howson, G., Keitel, C., & Kilpatrick, J. (1981). *Curriculum development in mathematics*. Cambridge, England: Cambridge University Press.

Lampiselkä, J., Ahtee, M., Pehkonen, E., Meri, M., & Eloranta, V. (2007). Mathematics and science in Finnish comprehensive school. In E. Pehkonen, M. Ahtee, & J. Lavonen (Eds.), *How Finns learn mathematics and science* (pp. 35–48). Rotterdam, Netherlands: Sense Publishers.

Looney, A. (2014). Curriculum politics and practice: From "implementation" to "agency." *Irish Teachers' Journal*, *2*(1), 7–14.

National Council for Curriculum and Assessment. (2005). *Review of Mathematics in post-primary education: A discussion paper*. Dublin, Ireland: National Council for Curriculum and Assessment.

Oldham, E. (1980). Post-primary mathematics in Ireland: An analysis of patterns in curriculum change. In D. J. Murphy & V. Rice (Eds.), *Mathematics, science and the gifted child: Conference proceedings and occasional papers* (pp. 49–59). Dublin, Ireland: Department of Higher Education and Research, Trinity College Dublin.

Oldham, E. (2001). The culture of mathematics education in the Republic of Ireland: Keeping the faith? *Irish Educational Studies*, *20*, 266–277.

Oldham, E. (2007). A lot done, more to do? Changes in mathematics curriculum and assessment 1986-2006. In D. Corcoran & S. Breen (Eds.), *Second International Science and Mathematics Conference, Drumcondra, Dublin, September 2006* (pp. 161–174). Dublin, Ireland: Dublin City University.

Organisation for Economic Cooperation and Development. (1961). *New thinking in school mathematics.* Paris, France: Organisation for Economic Cooperation and Development.

Prendergast, M., & Treacy, P. (2018). Curriculum reform in Irish secondary schools – a focus on algebra. *Journal of Curriculum Studies, 50*(1), 126–143.

Shiel, G., Cosgrove, J., Sofroniou, N., & Kelly, A. (2001). *Ready for life? The literacy achievements of Irish 15-year olds with comparative international data.* Dublin, Ireland: Educational Research Centre (ERC).

Shiel, G., & Kelleher, C. (2017). *An evaluation of the impact of Project Maths on the performance of students in junior cycle mathematics*. Dublin, Ireland: ERC. Accessed at:

Thompson, D. R., & Usiskin, Z. (Eds.). (2014). *Enacted mathematics curriculum: A conceptual framework and research needs*. Greenwich, CT: Information Age Publishing.

Travers, K. J., & Westbury, I. (1989). *The IEA Study of Mathematics I: Analysis of mathematics curricula*. Oxford, England: Pergamon.

Walmsley, A. L. E. (2007). *A history of mathematics education during the twentieth century*. Lanham, MD: University Press of America.

WHAT'S THE POINT: EVALUATING THE IMPACT OF THE BONUS POINTS INITIATIVE FOR MATHEMATICS

Niamh O'Meara[1], Mark Prendergast[2] and Paraic Treacy[3]

[1] EPISTEM, University of Limerick, [2] University College Cork, [3] University of Brighton

Since 2012 mathematics has been assigned a special status within Irish post-primary education with the introduction of a Bonus Points initiative (BPI). Students are now awarded an extra 25 CAO points in their upper post-primary school state examination results if they achieve a passing grade at higher-level. These extra points will increase the likelihood of these students getting a place on the course of their choice at third level. This incentive was introduced to encourage students to study the subject at higher-level. Anecdotally there have been many mixed reviews about the success of the BPI. While the numbers taking HL mathematics have steadily increased, there have been concerns expressed that many students who are not mathematically capable of performing up to the standard required are now opting for the HL paper and that the difficulty of this examination and the marking schemes have been adjusted accordingly (Treacy, 2018). This paper reports on a national study, the first of its kind in Ireland, that was conducted to investigate teachers' perspectives (n = 266) on the BPI. The authors will investigate if the increase in the number of students studying higher-level mathematics in Ireland has occurred in tandem with an increase in the mathematical proficiency of post-primary students and will ascertain the impact of the BPI on the profile of higher-level mathematics classes. It will report on findings from a national study.

INTRODUCTION

The benefits of studying advanced/higher-level mathematics, henceforth referred to as higher-level mathematics, have been well documented in the literature. According to Chinnappan, Dinham, Herrington and Scott (2008) higher-level mathematics facilitates the development of a variety of skills that underpin a scientifically literate workforce. Kennedy, Lyons and Quinn (2014, p. 35) add that higher-level mathematics courses in high school are critical if we are to produce graduates who are capable and confident in making informed decisions about "…issues such as renewable energy production…or climate change". Furthermore, a study by Wolfe (2002) found that mathematics is the only A-level subject in the UK that positively influences potential future earnings. Many researchers also hypothesise that there is a correlation between participation rates in higher-level mathematics and participation rates in other science subjects such as physics (Chinnappan et al., 2008; Kennedy et al., 2014). This is a cause of concern due to the low participation rates reported in physics in the Western world (De Witt, Archer & Moot, 2018).

Despite the importance of mathematics and the necessity for a mathematically literate workforce for economic growth, many countries worldwide report low numbers of students studying higher-level mathematics at upper secondary level. In Australia, Goodrum, Druhan and Abbs (2011) found that all high school science subjects, mathematics included, were experiencing dramatic declines. Similarly, in the UK participation in higher-level mathematics, that is mathematics post GCSE level (age 16), has been a cause of concern for many years. According to Noyes (2012) only 10-15% of 16-year-old students choose to continue their study of mathematics and he reports that this figure is low when compared with other developed countries. Similar problems have been reported internationally, in the USA (National Commission on Mathematics and Science Teaching, 2000); India (Garg & Gupta, 2003); and France (Charbonnier & Vayssettes, 2009). In Ireland, mathematics is not strictly a compulsory subject for the Leaving Certificate examinations, however it is treated as such by schools since it is a gatekeeper for the vast majority of third-level courses. Thus, studying mathematics for Senior Cycle[1] is typically expected of all students and this is reflected in the numbers completing these examinations each year (SEC, 2018).

Due to the importance of higher-level mathematics and the issues in relation to uptake that the authors have just discussed, it is unsurprising that improving mathematics participation and achievement at upper secondary level is an area of considerable focus amongst education systems and policy makers worldwide (Hodgen, Foster, Marks & Brown, 2013; Noyes, 2013). According to Brown, Brown and Bibby (2008, p.3) "Improving participation rates in specialist mathematics after the subject ceases to be compulsory at age 16 is part of government policy in England". Internationally, although advocated for, it appears as though very few policies or strategies have been introduced to increase participation rates in higher-level mathematics and Noyes (2013) outlines how there is currently very little consensus about how to tackle the issue of low participation rates in certain subjects. However, in recent years, Ireland has adopted a policy which is hoped will address the shortage of students studying higher-level mathematics.

In 2011, the proportion of students studying higher-level mathematics in their final two years of secondary schooling was 15.8%. In 2012, the Government of Ireland introduced the Bonus Points Initiative [BPI], which sought to encourage more students to opt to study mathematics at higher-level for Senior Cycle during their secondary education (Treacy, 2017). In Ireland, students must sit a summative examination, known as the Leaving Certificate at the end of upper secondary school. The Leaving Certificate acts as a gatekeeper to tertiary education with students awarded points based on their six best subjects. Prior to 2012, the maximum points that could be awarded for the top grade in a subject studied in its most advanced form (higher-level) was 100. Since 2012, mathematics has been assigned a special status within Irish schools with the introduction of the BPI. Students are now awarded an additional 25 points if they achieve a pass grade at higher-level ($\geq$ 40%) in their mathematics Leaving Certificate examination. Many people have cited that the perceived level of difficulty is one of the principal causes for poor uptake of higher-level mathematics (Brown et al., 2008) and the additional points offered is seen as a way of acknowledging the level of difficulty associated with higher-level mathematics while simultaneously increasing the uptake of higher-level mathematics. The DES (2017) are now considering expanding this initiative to other subjects but prior to this the authors believe it is critical that the BPI is critiqued and this paper will

[1] In Ireland, post-primary education is divided into two cycles. Junior Cycle is made up of the first three years of post-priamry education when students are aged between 12/13 and 15/16. Senior Cycle is a two year cycle that follows the Junior Cycle, with an optional "gap year", known locally as Transition Year, offered to students between Junior and Senior Cycle.

present findings in relation to teachers' perspectives on the BPI and the impact it has had on the profile of higher-level mathematics classes and students' proficiency in mathematics.

RESEARCH QUESTIONS

Following on from the extensive literature review, the authors derived the following research questions that will underpin this study:

1. Since the introduction of the BPI, do teachers believe there has been a notable improvement in the mathematical capabilities of post-primary students?
2. What are teachers' experiences of the impact of the BPI on the student profile in higher-level mathematics classes?

METHODOLOGY

To address these research questions a mixed method approach was adopted. Such an approach combines both qualitative and quantitative methods of data collection. It was important to get a high response rate and the authors felt that the response rate would be increased if they used a research tool that would be easy to distribute and collect and one that the participants did not find too time consuming to complete. As a result, all data within this study was gathered through a questionnaire. The questionnaires were designed with the help of a Teacher Research Advisory Group (TRAG), which consisted of five teachers. The teachers involved in this group were experienced in their positions and were recruited using a purposive sampling method. Members of the TRAG were invited to participate on the basis of the expertise they could bring to the research and the contemporary experiences they have in similar peer groups to the research participants (Murphy, Lundy, Emerson & Kerr, 2013). Their remit was to assist the authors in refining the items on the questionnaire and providing initial insights into expected responses to each item.

The sampling frame for the study was a list of all 723 post primary schools in Ireland (DES website, February 2015) and stratified sampling was used. Around 11.1% of these schools are community schools, 35% are vocational schools, 1.9% are comprehensive schools and the remaining 52% are secondary schools. These school types were the four strata used when selecting the sample. The targeted sample size was 800 teachers. Based on advice from the TRAG, a stratified random sample of 400 schools was selected: 44 schools (11.1%) were community schools; 140 (35%) were vocational schools; 8 schools (1.9%) were comprehensive schools; and 208 (52%) were secondary schools.

The questionnaires were distributed in April 2018 via post and were addressed to the Head of Mathematics at each school. It was requested in the accompanying information sheet that the two copies of the questionnaire enclosed should be completed by two teachers of higher-level senior cycle mathematics in the school and returned in the stamped addressed envelopes. 266 teachers completed and returned the surveys, a response rate of 33.3% which is within the 20%–30% range recommended by Veal and Flinders (2001) for mailed surveys. The quantitative data was recorded, summarized and analysed using the computer package SPSS. The open-ended questionnaire responses were transcribed and analysed using NVivo. The authors employed thematic content analysis. A coding scheme was generated based on a mixed deductive and inductive approach. On the one hand, codes were derived theoretically, taking into account the research questions, the literature review and the results emanating from the quantitative analysis. On the other hand, themes were identified from the open-ended questions, providing the basis for generating new codes or modifying the existing codes. Each of the authors worked separately on the data, to derive their own codes. The coding allocated by each researcher was then compared and any discrepancies were discussed and resolved by the authors before the coding scheme was finalized.

RESULTS

Statistics, released by the State Examinations Commission, show that the proportion of students opting to study higher-level mathematics for their Leaving Certificate has increased from 15.8% in 2012 to 31.5% in 2019. In this study teachers were asked if they believed that this 15.7 percentage point increase was as a direct result of the BPI or whether other factors such as the introduction of a revised curriculum, which was introduced around the same time as the BPI, played a role. The results are presented in Figure 1.

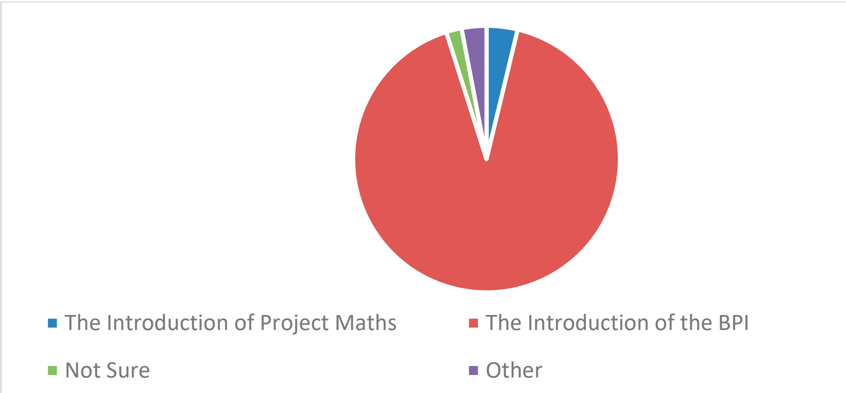


Figure 1. Teachers' perceptions of factors influencing the increased uptake of higher-level mathematics

In total 266 teachers responded when asked which initiative they believed was most influential in increasing the numbers taking the higher-level mathematics exam. Figure 1 shows that the vast majority of these teachers (n = 243) believed that the BPI was responsible for the increased uptake while only 10 teachers believed the new curriculum to be a factor. In addition to this, teachers were also asked to rate their level of agreement with the statement *"More students are now studying higher-level mathematics at Junior Cycle as a direct result of the Bonus Points Initiative."*. As shown in Figure 2, a vast number of teachers (57.4%) agree or strongly that the BPI has an impact on the uptake of higher-level at Junior Cycle, while only 16.0% disagreed or strongly disagreed with this viewpoint. This highlights the impact of the BPI beyond Senior Cycle mathematics.

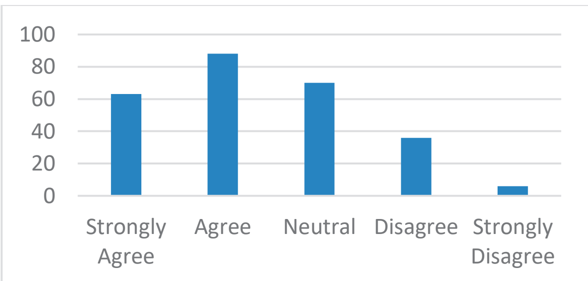


Figure 2. Teachers' perceptions of the influence of BPI at Junior Cycle

Given that the BPI is perceived to have had such an influence on the uptake of higher-level mathematics, the authors were keen to investigate whether teachers believe there has been a corresponding improvement in students' mathematical aptitude. Teachers in this study did not believe this to be the case. When asked if they believed that the increased number taking higher-level resulted in improved standards in mathematics among post-primary graduates 265 teachers offered a response with 155 (58.5%) believing this not to be the case. On the other hand, 50 teachers (18.9%) did see an improvement in students' mathematical competencies while 60 teachers (22.6%) were unsure.

The second research question underpinning this study required the authors to analyse both quantitative and qualitative data. First, the authors conducted thematic analysis on the responses offered by teachers to the question *"What impact (if any) has the Bonus Points Initiative had on the student profile of your Senior Cycle mathematics groupings?"* All 266 teachers in the study offered a response to this question and the majority believed that the BPI had a significant impact on the student profile in their classroom with only 8 teachers (3.0%) reporting that the BPI had no impact on the student profile in their classroom. The most common change reported by teachers was that the BPI resulted in people not suited to higher-level mathematics now persevering with it to the detriment of some.

T152: "Higher numbers trying higher [level] though [they] are not at all suited and many of these struggle from the outset."

T391: "More of the students who struggle with higher-level mathematics stay and do the exam. They stay purely to earn bonus points. Many stay who would be better served at ordinary level. Our failure rate has increased at higher-level because of this"

T168: "Bonus points have encouraged more students to try higher-level maths which is great. However, some of the students deciding to do higher-level do not have the required standard of maths to enable them to do so. It is putting enormous pressure on teachers."

A total of 81 teachers (30.5%) alluded to this type of change in student profile. This finding was echoed in the quantitative literature when 266 teachers ranked their level of agreement with the statement *"Many students who are struggling at higher-level persist due to the provision of Bonus Points."* 199 teachers (74.8%) strongly agreed with this statement while a further 62 (23.3%) agreed.

Another change in student profile, possibly a direct consequence of previous findings, reported by a number of teachers (n = 61) was in relation to more mixed ability classes. The large number of less able students doing higher-level mathematics has resulted in a much wider range of abilities than would have been the case prior to 2012.

T431: "The range in abilities is far too great. There are students attempting [higher-level] for the sake of trying to achieve more points, when they are simply not capable and end up doing poorly in their exams."

T52: "More students are doing HL and remaining in higher-level despite the lack of progress in some cases. The average ability of HL students has decreased."

This change in student profile, as these responses indicate, presents teachers a series of new challenges to contend with.

Finally, another change in student profile reported by teachers relates to less ambitious students now selecting higher-level mathematics. 35 teachers reported that students in higher-level now have lower expectations of themselves with many aiming to just reach, rather than exceed, the score required to be awarded bonus points. Teachers also report that such students are not as hardworking as those that would have selected higher-level in the past

> T383: "Students are hanging on at higher-level to gain bonus points. A lot of students now have the attitude '40% will do"

> T217: "Students who would have taken ordinary level prior to the introduction of BPI are now attempting the higher-level paper and are willing to settle for a low grade"

> T373: "Definitely have a lot more students taking it on, that probably wouldn't have before. You also have a lot of students who hang in there and aren't willing to do the work involved and just try and pass it."

CONCLUSION

The findings of this study have shown that the BPI has achieved one of its goal, in that it increased the number of Irish students studying higher-level mathematics. There has been a significant increase, from 15.8% to 31.5%, in the seven years since the BPI was introduced. On the surface, this may appear to indicate that Ireland has found an incentive, as suggested by Brown et al. (2008), to increase the participation levels in higher-level mathematics. However, increased participation rates was only one of the aims of the BPI. An additional objective of this initiative was to enhance students' mathematical skills (Treacy, 2018). Many researchers, such as Hodgen et al. (2018), have called for a simultaneous increase in participation and attainment but this study reveals that while the BPI is successful in the former, it may not be having the desired effect on the latter. Only 18.9% of teachers surveyed believe that the BPI has resulted in an overall improvement in students' mathematical ability, despite many more students studying mathematics in its most advanced form. This belief is also reinforced when one compares students' results pre and post-BPI. In 2018, 37.7% of higher-level students attained 70% of more in their Leaving Certificate mathematics examination, compared with 47.2% in 2011. This is despite many believing that the difficulty level of the Leaving Certificate mathematics examination decreasing in this time period (Treacy, 2018). One possible reason for this is that the students now taking mathematics are doing so solely to obtain the 25 additional bonus points and not because of any renewed interest or motivation for the subject. Instead, as reported by teachers in this study, current higher-level students are happy to study higher-level mathematics without investing the time and effort required to improve their skills or excel in the subject. As such, the authors recommend that a campaign to highlight the importance of mathematics in almost every career and in a multitude of daily tasks is undertaken. Such a campaign would allow these additional students studying higher-level mathematics to see the importance of the subject, as discussed by Chinnappan et al. (2008) and Kennedy et al. (2014), and this may in turn provide an incentive to dedicate the time and effort needed to improve their mathematical skillset.

Another possible reason for the increase in participation, but not in competence may be due to the changing profile of higher-level mathematics classes. Teachers in this study reported that many students who they deem unsuitable for higher-level are now opting for this course of study and as a result, there is a much greater range of abilities in higher-level mathematics classes than was the case prior to 2012. According to Linchevski and Kutscher (1998) mathematics is one of the more difficult subjects for working with mixed ability groupings while Harem and Ireson (2003) suggest that mixed ability grouping is inappropriate for mathematics. The BPI was introduced without any consideration for the impact it may have on class profiles and as such, teachers received no training in dealing with the knock-on effects of the BPI, including guidance on how to develop teaching strategies to cater for more mixed ability students. The authors are not proposing that such mixed ability groups have a negative effect on student learning, in fact some studies have shown that such diversity can have a positive impact on student learning (e.g. Davidson & Kroll, 1991). On the other hand, Boaler, William and Brown (2000), and more recently Taylor, Francis, Archer, Hodgen, Pepper, Tereshchenko and Travers (2017), state that there is not enough conclusive evidence to make a judgement about the impact of mixed ability grouping on student learning. Instead, the authors argue that a drastic change from more streamed or tracked classes to a mixed ability setting, without any formal training was a very difficult task for teachers and something they are struggling to deal with. As such, the authors recommend that continuous professional development is made available to teachers in the immediate future that focuses on developing the skills needed to teach and assess in mixed ability settings.

Overall, the authors conclude that while the BPI has been successful in attracting more students to higher-level mathematics, such increases in uptake have not occurred in tandem with improvements in students' mathematical ability. The recommendations proposed in this paper may help to improve students' competency in mathematics and if this was the case the authors believe that the BPI could be considered a success and used as a model for improving mathematics participation and attainment internationally. However, without some additional changes and revisions the BPI will simply serve to attract students, in an exam-driven system, to study a subject that they do not value and force teachers to engage in teaching styles that they may not be familiar with or have any training in.

REFERENCES

Boaler, J., Wiliam, D., & Brown, M. (2000). Students' experiences of ability grouping-disaffection, polarisation and the construction of failure. *British educational research journal*, 26 (5), 631-648.

Brown, M., Brown, P., & Bibby, T. (2008). "I would rather die": Reasons given by 16-year-olds for not continuing their study of mathematics. *Research in Mathematics Education*, 10 (1), 3-18.

Charbannier, É., & Vayssettes, S. (2009). PISA 2009 Presentation Note (France). Organisation for Economic Cooperation and Development. Available: www.oecd.org/pisa/46624019.pdf

Chinnappan, M., Dinham, S., Herrington, A.J. & Scott, D. (2008). Year 12 students' and higher mathematics: Emerging issues. In P. Jeffreys (Ed.), AARE 2007 International Educational

Research Conference Proceedings (pp. 10-20). Freemantle: Association for Active Educational Researchers.

DeWitt, J., Archer, L., & Moote, J. (2018). 15/16-Year-Old Students’ Reasons for Choosing and Not Choosing Physics at a Level. *International Journal of Science and Mathematics Education*, 1-17.

Garg, K. C., & Gupta, B. M. (2003). Decline in science education in India–A case study at+ 2 and undergraduate level. *Current Science*, 84 (9), 1198-1201.

Goodrum, D., Druhan, A., & Abbs, J. (2012). The status and quality of year 11 and 12 science in Australian schools. Canberra, ACT: Australian Academy of Science.

Hallam, S., & Ireson, J. (2003). Secondary school teachers' attitudes towards and beliefs about ability grouping. *British Journal of Educational Psychology*, *73*(3), 343-356.

Hodgen, J., Foster, C., Marks, R., & Brown, M. (2018). Evidence for review of mathematics teaching: Improving mathematics in key stages two and three. London: Education Endowment Foundation.

Kennedy, J., Lyons, T., & Quinn, F. (2014). The continuing decline of science and mathematics enrolments in Australian high schools. *Teaching Science*, 60 (2), 34-46.

Linchevski, L., & Kutscher, B. (1998). Tell me with whom you're learning, and I'll tell you how much you've learned: Mixed-ability versus same-ability grouping in mathematics. *Journal for Research in Mathematics Education*, 533-554.

Murphy, C., Lundy, L., Emerson, L., & Kerr, K. (2013). Children's perceptions of primary science assessment in England and Wales. *British Educational Research Journal*, 39 (3), 585-606.

National Commission on Mathematics, & Science Teaching for the 21st Century (US). (2000). *Before it's too late: A report of the National Commission on Mathematics and Science Teaching for the 21st Century*. Washington, DC: Diane Publishing Company.

Noyes, A. (2013). The effective mathematics department: Adding value and increasing participation?. *School Effectiveness and School Improvement*, 24 (1), 1-17.

State Examinations Commission (SEC) 2018. State examinations statistics. Published: State Examinations Commission (SEC). Available from: https://www.examinations.ie/statistics/

Taylor, B., Francis, B., Archer, L., Hodgen, J., Pepper, D., Tereshchenko, A., & Travers, M. C. (2017). Factors deterring schools from mixed attainment teaching practice. *Pedagogy, Culture & Society*, *25*(3), 327-345.

Treacy, P. T. (2018). Incentivizing advanced mathematics study at upper secondary level: the case of bonus points in Ireland. *International Journal of Mathematical Education in Science and Technology*, 49 (3), 417-436.

Veal, W. R., & Flinders, D. J. (2001). How block scheduling reform effects classroom practice. *The High School Journal*, 84 (4), 21-31.

Wolf, A. (2002) Does education matther? Myths about education and econdomic growth. London, UK: Penguin.

TIPPING THE SCALES: AN EXAMINATION OF TEXTBOOK TASKS IN THE CONTEXT OF CURRICULUM REFORM

Brendan O'Sullivan

School of Mathematical Sciences, Dublin City University

This paper is concerned with the analysis of mathematical textbook tasks at second-level in Ireland, in the context of the introduction of the revised curriculum initiative entitled 'Project Maths'. A total of 7635 tasks on the topics of Pattern, Sequences and Series and Differential Calculus contained in three textbook series for senior cycle, in editions available before and those available after the curriculum change, were analysed. The analysis presented here was informed by the use of a framework: Usiskin's multidimensional model of mathematical understanding (Usiskin, 2012). The research question considered is: what kind of understanding (using Usiskin's dimensions) is being promoted in the tasks analysed? The use of high quality tasks that promote understanding helps to maintain a high level of mathematical literacy. My findings suggest that the post-'Project Maths' textbook tasks offer greater opportunities in the area of mathematical understanding when compared to those in the older textbooks, but that there is still scope for further development. Based on my analysis, it would appear that all three textbook series have neglected important aspects like reasoning-and-proving and real life applications. Furthermore, the findings indicate that there is a need for more balance in tasks to ensure greater proficiency in mathematical literacy.

INTRODUCTION

This paper reports on part of a study concerned with the analysis of mathematical textbook tasks at second level in Ireland, in the context of the introduction of the revised curriculum entitled 'Project Maths'. The aim of the study was to gain greater insight into the nature of tasks that students and teachers work with in Irish classrooms by using five different frameworks. This paper will focus on reporting the results found using one of these frameworks: Usiskin's multidimensional model of mathematical understanding (Usiskin, 2012).

Textbooks have always been regarded as having an important role within the mathematics curriculum (Fan, Zhu & Miao, 2013). They can be seen as a link between the intended and the implemented curriculum, serving as the potentially implemented curriculum (Valverde, Bianchi, Wolfe, Schmidt & Houang, 2002). Research in relation to the implemented curriculum has shown that in several countries, mathematics textbooks can influence classroom instruction. Teachers often follow a sequence of topics as suggested by textbooks and much of the work completed in class is drawn from textbooks (Lepik 2015, Eisenmann & Even 2011, Haggarty & Pepin 2002). The situation is no different in Ireland, where it is reported that a lot of the time in the classroom appears to be related to the textbook and very often it is the only resource which students have access to during the lesson aside from the teacher, while most of the problems assigned for classwork and homework come from the textbook (Project Maths, 2017). As questions from textbooks would normally be assigned as classwork or given to students for homework (Hourigan & O'Donoghue, 2007, p. 471), these tasks provide an insight into the teaching and learning taking place in Irish classrooms. In this paper, the following research question is considered. What kind of understanding (using Usiskin's dimensions) is being promoted in the textbook tasks analysed? The framework is used to classify tasks on the topic of Pattern, Sequences and Series and Differential Calculus from three popular Irish textbook series.

CONTEXT: THE IMPACT OF 'PROJECT MATHS'

A number of reports have been published in relation to 'Project Maths' and its impact. The National Council for Curriculum and Assessment (NCCA) (NCCA, 2012) in its response to the debate on 'Project Maths' notes that textbooks have traditionally supported practising routine questions with solutions based on illustrative examples. The report calls for more emphasis to be given to students engaging in problem-solving approaches and justifying or explaining their solutions (NCCA, 2012, p. 18).

The NCCA also commissioned a report (Jeffes, Jones, Wilson, Lamont, Straw, Wheater and Dawson, 2013) exploring the impact of 'Project Maths' on student learning and achievement in the initial pilot schools that introduced the syllabus in 2008 and the remaining schools in the country that introduced it in 2010. The new syllabuses were introduced on a phased basis. There are five strands at senior cycle in total: 1) Statistics and Probability, 2) Geometry and Trigonometry, 3) Number, 4) Algebra and 5) Functions. Strands 1 and 2 were introduced in 24 Pilot Schools in 2008 and in all other schools in 2010. Strands 3 and 4 followed in 2009 for the pilot schools and in 2011 for the remaining cohort, while the final strand was introduced in 2010 and 2012 respectively.

One of the main findings of this report (Jeffes et al., 2013, p. 3) is that more traditional approaches like using textbooks and copying from the whiteboard continue to be widespread. The report suggests that students need to be regularly given high quality tasks that require them to engage with the processes promoted by 'Project Maths', including: problem-solving; drawing out connections between mathematics topics; communicating more effectively in written form; and justifying and providing evidence for their answers.

A Chief Examiner's Report (State Examinations Commission, 2016) in Leaving Certificate mathematics was published in 2016, the first of its kind published after the introduction of the 'Project Maths' syllabus. It reviewed candidates' performance in the 2015 examinations and set itself the goal of identifying strengths and challenges in order to provide guidance for teachers and students in the future (SEC, 2016). It noted that the syllabus expectations are more ambitious than previously and are not always easy to achieve; the authors commented that there has been a deliberate attempt to emphasise higher order thinking skills but acknowledged that this presents difficulties for both students and teachers alike (SEC, 2016, p. 8). The report recommended that students should become more familiar with describing, explaining, justifying and providing examples. It noted that these skills assist with improving understanding (SEC, 2016, p. 9). Teachers were also reminded to encourage students to practise solving problems involving real-life applications of mathematics. As part of this process, students should be asked to model these situations by constructing algebraic expressions or equations and/or representing them differently by drawing diagrams (SEC, 2016, p. 30). Similarly the Organisation for Economic Co-operation and Development (OECD) produced a report to offer advice to teachers in relation to the findings from the Programme for International Student Assessment (PISA) 2012 and strategies for teaching and learning. The report recommends that teachers should encourage students to think more deeply about what has been learned and encourage the establishment of connections with real-world problems (OECD, 2016, p. 38).

Textbook Studies

O'Keeffe and O'Donoghue (2012) conducted a study of the textbooks published in response to 'Project Maths' that were available at the time (ten in all). The study found that all textbooks analysed fell short of the standard needed to support the 'Project Maths' (intended) curriculum effectively, as outlined in the 'Project Maths' syllabus documents for junior cycle and senior cycle, but that some of the new textbooks were better aligned to 'Project Maths' expectations than others.

Davis (2013) examined the prevalence of reasoning-and-proving in the topic of complex numbers in six Irish textbooks and one teaching and learning plan produced for teachers during the introduction of the 'Project Maths' curricular initiative. His study uses a framework consisting of five main components: namely pattern identification, conjecture development, argument construction, technological tools, and reasoning-and-proving objects. Only 1.4% of tasks in Ordinary Level textbooks and 1.3% of tasks in Higher Level textbooks involved pattern identification or conjecture development. There were no opportunities to test conjectures, construct counterexamples or develop proof subcomponents in any of the materials examined. The results from Davis' study suggest that the six textbook units do not align with the syllabus introduced as part of 'Project Maths' (Davis, 2013, p. 54).

THEORETICAL FRAMEWORK

Usiskin (2012) deals with the understanding of a concept in mathematics from the standpoint of the learner and how the learner interacts with it. Usiskin's multi-dimensional framework is not limited to a consideration of instrumental and relational understanding as categorised by Skemp (1976). A number of other dimensions are introduced to achieve this. Five dimensions of this understanding are outlined in his framework: the Skill-Algorithm dimension, the Property-Proof dimension, the Use-Application (modelling) dimension, the Representation-Metaphor dimension and the History-Culture dimension. The term 'dimension' is used because each element can be accessed independently of the others. These dimensions are not presented in a hierarchy; it is possible for each to co-exist and one aspect is not meant to precede another. As a framework, it can provide a comprehensive examination of learning.

The first dimension Skill-Algorithm looks at the algorithms that are necessary for the learning of a concept and the choice of a particular algorithm because it is more efficient than other algorithms known. Property-Proof understanding identifies the mathematical properties that underlie a concept. This aspect of learning goes deeper, looking beyond arbitrary rules and considering the mathematical theory behind them. Use-Application understanding focuses on the applications of a concept or how it can be used in some way. The Representation-Metaphor dimension encourages the representation of a concept in some way. Finally History-Culture understanding concerns itself with the how and why of the development of a mathematical concept over time. The History-Culture dimension is an aspect that is identified by Usiskin as necessary for the 'real true' understanding. The premise is that those who study the history of mathematics or cross-cultural mathematics obtain an understanding of mathematical concepts that is different from the other dimensions.

Example of classification of a task using the framework

> An artificial ski slope is described by the function
> $h = 165 - 120s + 60s^2 - 10s^3$
> where s is the horizontal distance and h is the height of the slope. Show that the ski slope never rises.

Three of Usiskin's dimensions are evident here.

- Skill-Algorithm: the student can use a model to find the derivative and to prove that the function is always decreasing.
- Property-Proof: Proving that the ski slope never rises. (Function always decreasing).
- Use-Application: The ski slope is a real life situation.

METHODOLOGY

For this work, a task is considered to be an activity where a student interacts with a mathematical topic by attempting to solve a question either as homework or within the classroom. This is in keeping with Mason and Johnston-Wilder's definition (2006, p.4) that a task is what learners are asked to do in the mathematics classroom.

It was decided to focus on the senior cycle material in particular because of the high-stakes (Leaving Certificate) examination that accompanies it. Textbook tasks on the topics of Pattern, Sequences and Series, and Differential Calculus are considered here. These topics were chosen because they are present on both Higher and Ordinary Level Leaving Certificate Mathematics syllabuses and both were also present on the previous syllabus. Also the topic of Pattern, Sequences and Series was introduced in the first phase of the syllabus implementation while Differential Calculus was in the final stage. I analysed tasks from three textbooks series available on the Irish market: referred to as Textbook A, Textbook B and Textbook C. These three textbook series were selected because they were the first to be published in response to the new curriculum, while they have also traditionally been the most popular in Irish classrooms.

From the six pre-'Project Maths' and six post-'Project Maths' textbooks, each chapter relating to Patterns, Sequences and Series and Differential Calculus was analysed. A total of 7635 tasks (3584 pre-'Project Maths' and 4051 post-'Project Maths') were classified from the chapters chosen. It was necessary to discard 16 tasks from the analysis when ambiguity was encountered (for example, where a misprint made it difficult to interpret what a task required). Questions sometimes consisted of several parts and it was necessary to break these up and treat them as multiple tasks. Several checks were made over time to ensure that what was treated as a task in one textbook was consistent with the other five textbooks regardless of how exercises were structured or presented.

After I had coded the textbook tasks, at least one of my two PhD supervisors also looked at each task separately and we compared our classifications after each framework analysis was complete. We then discussed any of the classifications that we had differences on and gave our perspective on why we analysed them as we did, coming to agreement on how the coding should be applied. Having clarified and resolved our coding, we made any necessary revisions and reviewed the existing classifications of previous tasks in light of these revisions, in order to ensure consistency throughout the analysis. This led to a final set of classifications. It should be noted that a single task can have multiple dimensions present and the percentages in the tables in the following section will not total to 100%.

RESULTS

Table 1 contains the results of the classification of the exercises on the topic of Pattern, Sequences and Series and Differential Calculus in the three textbook series under consideration. It shows the number and percentage of tasks falling into each classification for both the pre- and post- ‘Project Maths’ eras. In Table 2, each of the three textbook series at Higher and Ordinary level published for the post-‘Project Maths’ era are classified in terms of the five dimensions of understanding.

Table 1: Classification of tasks using Usiskin’s Multidimensional Model in pre- and post- ‘Project Maths’ textbooks

	Pre-‘Project Maths’ Textbook Series average	Post-‘Project Maths’ Textbook Series average
Skill-Algorithm	3556 (99.22%)	3909 (96.49%)
Property-Proof	181 (5.05%)	382 (9.43%)
Use-Application	237 (6.61%)	655 (16.17%)
Representation-Metaphor	107 (2.99%)	706 (17.43%)
History-Culture	0 (0%)	0 (0%)

DISCUSSION AND CONCLUSIONS

All three series have a high incidence of the Skill-Algorithm dimension at both Higher and Ordinary level in Usiskin’s dimensions of mathematical understanding. Of the remaining dimensions, the textbook A series has the greatest incidence of the Property-Proof and Representation-Metaphor dimensions at Higher and Ordinary level. The textbook C series has the greatest number of tasks corresponding to the Use-Application category at both Higher and Ordinary level.

Table 2: Classification of tasks using Usiskin's Multidimensional Model for post- 'Project Maths' textbooks

	Skill-Algorithm	Property-Proof	Use-Application	Representation-Metaphor	History-Culture
Textbook A Higher Level	846 95%	117 13.1%	133 14.9%	187 21%	0 0%
Textbook A Ordinary Level	630 97.4%	67 10.4%	103 15.9%	180 27.8%	0 0%
Textbook B Higher Level	605 95.9%	55 8.7%	66 10.5%	81 12.8%	0 0%
Textbook B Ordinary Level	457 97%	35 7.4%	91 19.3%	84 17.8%	0 0%
Textbook C Higher Level	822 98.9%	58 7%	136 16.4%	53 6.4%	0 0%
Textbook C Ordinary Level	549 94.7%	50 8.6%	126 21.7%	121 20.9%	0 0%

With Usiskin's multidimensional model, an increase was recorded in the three dimensions of Representation-Metaphor, Property-Proof and Use-Application yet a much greater incidence would be desirable. Despite modest improvements, it would appear that students would benefit from greater exposure to more varied tasks. The Chief Examiner's Report has recommended that students should become more familiar with the processes of description, explanation, justification and the provision of examples. It noted that 'these are skills that are worth practising, as they will improve understanding' (SEC, 2016, p. 30). The NCCA in its report responding to the debate on the 'Project Maths' curriculum and its introduction called for more emphasis to be given to students engaging in problem-solving approaches and justifying or explaining their solutions (NCCA, 2012, p. 18). Similarly the report on the impact of 'Project Maths' in pilot schools (Jeffes et al., 2013) observed that students are building up expertise with the use of procedures. The report also noted that students are problem-solving and making mathematical representations but to a lesser extent than using procedures. An absence of engagement with reasoning and proof, communicating mathematically, or making connections between mathematics topics was also observed. It would appear that the textbooks do not support this goal adequately and teachers will need to augment existing tasks to achieve it. The Chief Examiner's Report has acknowledged that the syllabus expectations are more ambitious than previously and that they are not necessarily easy to achieve; 'there has been a deliberate attempt to increase the emphasis on higher-order thinking skills. These are skills that students find difficult to master and teachers may find difficult to instil' (SEC, 2016, p. 9).

A key concern arising from my analysis is the lack of balance in the textbook tasks analysed. Swan and Burkhardt (2012) have suggested principles for task design suitable for use as assessment. These principles could also be used when designing tasks suitable for use in the classroom and/or for homework. They suggest that a balanced series of tasks should meet all the different goals and objectives that the curriculum aspires to. This is something that the textbook tasks do not currently achieve in relation to the 'Project Maths' curriculum according to my analysis. It is also recommended that tasks should be viewed by students as something interesting or having a potential use outside of the classroom. The lack of Usiskin's History-Culture dimension and the relatively low incidence of the Use-Application dimension in my analysis would suggest that the Irish textbook tasks do not currently achieve this.

The area of reasoning-and-proving is worthy of attention, I found that there was a low incidence of tasks classified in the property-proof dimension of Usiskin's framework and in general, very few of the tasks classified required the explanation of findings or the justification of conclusions. These results are also supported by Davis' (2013) analysis. It appears that more tasks are required in order to encourage students to engage in creative reasoning, explain findings, justify conclusions and communicate their mathematical thoughts. Jeffes et al. (2013) also highlight this when they call for tasks that involve students in 'communicating more effectively in written form; and justifying and providing evidence for their answers'. In fact, they called for high quality tasks 'to engage with the processes promoted by the revised syllabuses, including: problem-solving; drawing out connections between mathematics topics' (p.32). Teachers will have to take care in the classroom, if not doing so already, to encourage students to verify their solutions, solve tasks using several methods and to explain their mathematical thinking when completing tasks.

REFERENCES

Davis, J. D. (2013). Examining reasoning-and-proving in the treatment of complex numbers in Irish secondary mathematics textbooks. *Bulletin of the Irish Mathematical Society, 71*, 41-57.

Eisenmann, T., & Even, R. (2011). Enacted types of algebraic activity in different classes taught by the same teacher. *International Journal of Science and Mathematics Education*, *9*(4), 867-891.

Fan, L., Zhu, Y., & Miao, Z. (2013). Textbook research in mathematics education: Development status and directions. ZDM: *International Journal on Mathematics Education, 45*(5), 633-646.

Haggarty, L., & Pepin, B. (2002). An investigation of mathematics textbooks and their use in English, French and German classrooms: Who gets an opportunity to learn what? *British Educational Research Journal*, *28*(4), 567-590.

Hourigan, M., & O'Donoghue, J. (2007). Mathematical under-preparedness: the influence of the pre-tertiary mathematics experience on students' ability to make a successful transition to tertiary level mathematics courses in Ireland. *International Journal of Mathematical Education in Science and Technology*, *38*(4), 461-476.

Jeffes, J., Jones, E., Wilson, M., Lamont, E., Straw, S., Wheater, R. & Dawson, A. (2013). Research into the impact of Project Maths on student achievement, learning and motivation: Final report. Slough: NFER.

Lepik, M. (2015). Analyzing the use of textbook in mathematics education: The case of Estonia. *Acta Paedagogica Vilnensia, 35*, 90-102.

Mason, J. & Johnston-Wilder, S. (2006). Designing and Using Mathematical Tasks. St. Albans:Tarquin Publications.

Moffett, P. (2009). Reform via textbooks: Lessons from a cross-national collaboration. In D. Corcoran, T. Dooley, S. Close, R. Ward (Eds.), Proceedings of Third National Conference on Research in Mathematics Education, (pp. 262-272). Dublin: MEI.

National Council for Curriculum and Assessment (NCCA). (2012). Project Maths responding to current debate. Retrieved from http://www.ncca.ie/en/Publications/Other_Publications/Project-Maths-Responding-to-Current-Debate.pdf

Organisation for Economic Co-operation and Development (OECD). (2016). Ten questions for mathematics teachers ... and how PISA can help answer them. PISA, OECD Publishing, Paris.

O'Keeffe, L., & O'Donoghue, J. (2012). *A Review of School Textbooks for Project Maths*. National Centre for Excellence in Mathematics and Science Teaching and Learning (NCE-MSTL), Limerick.

Project Maths. (2017). Retrieved from https://www.projectmaths.ie/for-teachers/explanation-of-resources/textbooks/

Skemp R. R. (1976). Relational understanding and instructional understanding. *Mathematics Teacher*, *77*(1):20–26.

State Examinations Commission. (2016). Leaving Certificate Examination 2015 Mathematics: Chief Examiner's report. Retrieved from https://www.examinations.ie/misc-doc/EN-EN-53913274.pdf

Swan, M., & Burkhardt, H. (2012). Designing assessment of performance in mathematics. *Educational Designer*, *2*(5).

Usiskin, Z. (2012). What does it mean to understand some Mathematics? Retrieved from http://www.amesa.org.za/AMESA2013/Files/P01.pdf

Valverde, G. A,. Bianchi, L. J., Wolfe, R. G., Schmidt, W. H. & Houang, R. T. (2002). *According to the book: Using TIMSS to investigate the transition of policy into practice through the world of textbooks.* Springer Science & Business Media

THE "N" FRAMEWORK – A POTENTIAL SOLUTION TO NUMERACY ACROSS THE CURRICULUM

Kathy O' Sullivan, Niamh O' Meara, Merrilyn Goos and Paul F. Conway

Epistem, School of Education, University of Limerick

Numeracy is often referred to as an essential skill that all people should possess in order to engage fully in society. Governments and policymakers around the world are encouraging teachers to teach numeracy across the curriculum. This paper proposes a theoretical framework of teacher knowledge for the integration of numeracy across the curriculum in post-primary schools in Ireland. Teacher knowledge is complex and consists of many different facets of knowledge. The proposed framework integrates theories from existing models of general teacher knowledge (Shulman, 1986), with models of subject specific teacher knowledge (Ball, Thames and Phelps, 2008), and with a numeracy model developed by Goos, Geiger and Forgasz (2014). This enabled the authors to develop an integrated framework of numeracy knowledge and skills, subject-specific knowledge and pedagogical content knowledge which are all essential components of knowledge for teaching in the 21st century. Teachers need to have a deep understanding of these different types of knowledge to teach students effectively in any subject across all subjects.

INTRODUCTION

Being numerate is an essential part of daily life and it involves much more than being able to complete basic mathematical operations (Goos, Geiger, Dole, Forgasz, & Bennison, 2019). Governments and policymakers have noted that numeracy, referred to as mathematical literacy in some countries, is a lifelong skill which needs to be addressed. However, the term numeracy often carries different meanings. Internationally, the concept of numeracy/ mathematical literacy is defined by OECD Programme for International Student Assessment (PISA) (2016, p.5) as:

> …an individual's capacity to formulate, employ, and interpret mathematics in a variety of contexts. It includes reasoning mathematically and using mathematical concepts, procedures, facts and tools to describe, explain and predict phenomena. It assists individuals to recognise the role that mathematics plays in the world and to make the well-founded judgments and decisions needed by constructive, engaged and reflective citizens.

Frejd and Geiger (2018) discovered that mathematical literacy had different meanings in different countries. While there is not a broadly-accepted definition of numeracy /mathematical literacy, the focus of developing people's mathematical literacy is to "use mathematics to participate effectively in society and to contribute in a productive and critical manner" (Frejd & Geiger, 2018, p.3). In order to improve and develop students' levels of numeracy, teachers' knowledge of numeracy must first be addressed and their knowledge of how to embed numeracy across the curriculum also needs to be considered.

FACTORS INFLUENCING THE TEACHING OF NUMERACY ACROSS CURRICULUM

Ireland's poor performance in PISA 2009 and the decline in students' mathematical literacy led to the implementation of new teaching strategies to ensure that the young people of Ireland are numerate by the time they complete compulsory schooling. The introduction of *The National Strategy to Improve Literacy and Numeracy among Children and Young People 2011-2020* (DES, 2011) was presented as a response to improve students' literacy and numeracy levels in Ireland. The strategy states that all teachers should embed numeracy in their lessons to improve the numerate abilities of the students. To implement this teaching strategy effectively, teachers must be aware of the various approaches, methodologies and interventions that they can use to teach numeracy across all areas of the curriculum (DES, 2011). Teachers need to be able to help students understand the use and application of numeracy across a variety of subjects if this goal is to be achieved.

Researchers in Australia have shown, that in order for teachers to improve the numerate abilities of their students, teachers must first equip themselves with the necessary skills to develop their own understanding of how mathematical concepts and numeracy affect their own lives and their subject area (Leder, Forgasz, Kalkhoven, & Geiger 2015; Goos, Geiger, & Dole 2013). Bennison (2015) acknowledges that within every subject, teachers can exploit numeracy learning opportunities; however, the teacher first needs to identify numeracy opportunities within their specific subject area. Westwood (2008) found that teachers play a key role in enhancing the teaching and learning of numeracy throughout the students' school experience, which in turn will help young people to appreciate and enjoy the mathematics they encounter throughout their lives. The Literacy and Numeracy Strategy, implemented in schools and communities across Ireland, emphasised that the teaching and learning of numeracy skills is not only the responsibility of the mathematics teacher but instead should be a priority across all post-primary subjects (DES, 2011). For teachers to identify numeracy opportunities of learning within their subject area, they must first understand the concept of numeracy and possess a knowledge of numeracy along with other components of knowledge required for teaching.

IMPORTANCE OF TEACHER KNOWLEDGE

Teacher knowledge can be described in many ways. In general, it is the knowledge that teachers possess and how they convey their knowledge to their students. It is evident from the literature on teacher education that teacher knowledge is essential for effective teaching. Hence, in order for teachers to teach effectively in the 21st Century, it is crucial that they develop high levels of proficiency in the areas of Numeracy Knowledge and Skills, Subject Specific Knowledge and Pedagogical Content Knowledge.

Fennema and Franke (1992) use the example of the mathematics teacher needing mathematical knowledge to enable them to teach their students effectively. Using this example, the same can be said for numeracy knowledge and skills, in that a teacher needs to have the knowledge and understanding of numeracy in order to develop their students' numerate abilities in their specific subject area. Teachers need to possess a good knowledge of numeracy and be able to identify numeracy opportunities within their subject area in order to teach it effectively. In order to create a teacher knowledge framework for numeracy across the curriculum, an in-depth analysis of frameworks of teacher knowledge was conducted. Many researchers have sought to identify what knowledge a teacher should possess in order to teach effectively in the classroom (Shulman 1986; Fennema & Franke 1992; Van Driel, Verloop, & De Vos 1998; Mishra & Koehler 2006; Rowland 2007; Ball et al 2008). It was found that many researchers have different beliefs on what makes a good model of teacher knowledge, with some academics believing some aspects of knowledge to be more important than others.

It was concluded that the frameworks proposed by Shulman (1986) and Ball et al. (2008) were the most suitable to use as the foundation of the new framework for teaching numeracy across the curriculum. It is important to include Shulman's (1986) theoretical framework as it is not specific to teaching one subject, whereas all other frameworks analysed were specific to mathematics, science or technology teaching. Ball et al. (2008) further refined Shulman's categories of teacher knowledge by dividing the subject matter knowledge proposed by Shulman into common and specialised content knowledge. This is very important for teaching as it identifies the different types of knowledge a person such as an economist or scientist should have, while also encompassing the knowledge a teacher of economics or science should possess specifically for teaching. Ball et al. (2008) also created a separate section for pedagogical content knowledge which included curricular knowledge as put forward by Shulman (1986). The models of teacher knowledge for the "N" framework are discussed below in detail.

Models of Teacher Knowledge

Shulman (1986) emphasises the need for teachers to develop three categories of knowledge in their teaching; subject matter knowledge, pedagogical content knowledge and curricular knowledge.

Subject matter knowledge is "the amount and organization of knowledge per se in the mind of the teacher" (Shulman, 1986, p.9). Subject matter knowledge is the cornerstone upon which all other domains are developed. Shulman (1986) suggested that the teacher must not only understand the subject matter themselves but also be in a position to explain the content in a way that the student understands. Pedagogical content knowledge is the knowledge required to convey the subject matter knowledge to students through the practice of teaching. Shulman (1986) highlights that pedagogical content knowledge represents the "blending of content and pedagogy into an understanding of how particular topics, or issues are organised, represented and adapted to the diverse interests and abilities of learners" (Shulman 1987, p.8). Shulman (1987) also recognises that pedagogical content knowledge is the domain which will differentiate a person who is a specialist in a particular subject area from a teacher in that subject area. Furthermore, Shulman (1986) discusses curricular knowledge, which is the knowledge required to teach the subject matter and relate it to other topics in other subject areas. By doing this, it gives the student a better learning experience and also reiterates the fact that many topics and subjects are interrelated.

Ball et al. (2008) developed a model of teacher knowledge based on the theories put forward by Shulman (1986) but specific to teaching mathematics at primary level. Subject matter knowledge and pedagogical content knowledge were underlying features of this model. Ball et al. (2008) further defined subject matter knowledge and pedagogical content knowledge by developing different subcategories within each domain. The subcategories for subject matter knowledge were common content knowledge, specialised content knowledge and knowledge at the mathematical horizon. Ball et al. (2008) explain common content knowledge as the knowledge that teachers should have in order to solve problems mathematically, using their mathematical knowledge. Following on from common content knowledge, Ball et al. (2008) discuss knowledge at the mathematical horizon, which is an awareness of how mathematical concepts taught in early childhood will have an effect on students' understanding in later years and how this knowledge is crucial in the teaching of mathematics. Ball et al. (2008) explain that while specific mathematical concepts may not be relevant to the student now, the teacher must remember that they are preparing the students for mathematical topics they will encounter in the future. Ball et al. (2008) describe the scenario where teachers need knowledge beyond what they are teaching. This is described as specialised content knowledge and is a very important domain in any model of teacher knowledge. It is the difference between a scientist or a mathematician teaching the content and a science or mathematics teacher teaching the content. A teacher needs to be able to explain the reasoning behind a procedure, whereas it is sufficient for a scientist to only know how to carry out the procedure; he/she does not need to know the reason why they are carrying out such procedures. Ball et al. (2008) describe this domain as a unique domain; it is the type of knowledge that is not needed for anything else other than teaching.

Ball et al. (2008) then focus on pedagogical content knowledge, which is again split into three aspects: knowledge of content and students, knowledge of content and teaching and knowledge of content and curriculum. When teachers are competent in the domain of knowledge of content and students, they are able to predict/anticipate students' answers and misconceptions. Ball et al. (2008) describe this domain as knowing the students and knowing the mathematics being taught. The next aspect in the pedagogical domain, knowledge of content and teaching, is described by Ball et al. (2008, p. 401) as "knowing about teaching and knowing about mathematics". This is the knowledge teachers use to determine how to sequence the topics they teach from the syllabus. The reason teachers need to have knowledge of content and teaching is because the teacher knows that the students must understand a certain topic before moving onto a more complex area in the subject. This decision making by the teacher requires the teacher to have a knowledge of the subject (in which case Ball et al. (2008) use mathematics as the subject) and a knowledge of pedagogy which they believe will affect the students' learning. Finally Ball et al. (2008) emphasise the need for knowledge of content and curriculum which has been discussed in detail by many other researchers in the field and was first introduced by Shulman (1986). Again Ball et al. (2008) highlight the need for teachers to gain the knowledge of the content and the curriculum, requiring the teacher to understand fully what resources they will use and how they will utilise the resources to enable an optimal learning experience for their students without straying too far from the curriculum.

Model of Numeracy

While the models of teacher knowledge discussed above are important, numeracy knowledge did not appear in any of the models of teacher knowledge. Numeracy is a new concept and not easily defined (Jablonka 2015; Goos et al. 2019). There are many different interpretations of numeracy from basic mathematics to a much broader definition put forward by Goos et al. (2014) when they developed a model for Numeracy in the 21st century. For the purpose of this research, we analysed many different definitions of numeracy (Crowther 1959; Cockroft 1982; Goos et al. 2014; Organisation of Economic Co-operation and Development [OECD] 2016) and found Goos et al. (2014) to be the most appropriate for this study as it was developed specifically to support teachers in embedding numeracy across the curriculum. The model based on the multidimensional nature of numeracy. Goos et al. (2014) firstly describe the different *contexts* in which people will encounter the need to use numeracy i.e. personal and social lives, in work and employment. The next element described in the model is *mathematical knowledge*. This encompasses all the mathematical skills, knowledge and problem solving strategies one will require (in terms of both knowledge and application) to participate fully in society and the workplace. *Dispositions* are the next element defined as an essential part of model. A person's confidence and willingness to use mathematical skills to engage fully in society is a vital component of being numerate. To be able to use different materials and digital *tools* to help gain an understanding of numeracy was another element. All of the elements *contexts, mathematical knowledge, dispositions* and *tools,* are rooted in a *critical orientation*, which is the ability to use all of the skills and knowledge to make informed decisions and judgements in life.

THE "N" FRAMEWORK

As described in the previous sections, many researchers have put forward different models of knowledge a teacher should possess in order to teach effectively in the classroom. From the literature review, it became apparent to us that while a number of frameworks for teacher knowledge exist, none of these frameworks address numeracy knowledge, subject specific knowledge, and pedagogical content knowledge together. We argue that being an effective teacher of numeracy within any subject area requires all three components of knowledge and therefore have developed a new three-component model for teacher knowledge that we refer to as the "N" framework.

As discussed in the previous section, it is essential that teachers have a clear understanding of the term numeracy and what it means to teach numeracy within their subject area. If teachers are expected to incorporate numeracy into their teaching, then it is vital that teachers gain a deep understanding of the concept of numeracy along with their subject matter knowledge and pedagogical content knowledge.

We have discussed in detail the different models of teacher knowledge and therefore have decided that Shulman (1986) and Ball et al. (2008), along with Goos et al.'s (2014) numeracy model in the 21st century are the most comprehensive models to base the "N" Framework on. Teachers need to have a deep understanding of three main categories to teach students effectively in any subject. The categories numeracy knowledge, subject specific knowledge and pedagogical content knowledge are presented in the "N" Framework in Figure 1.

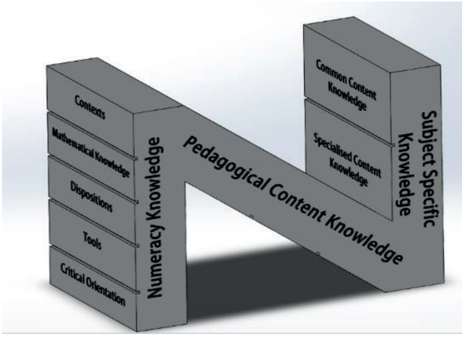


Figure 1: The "N" Framework

The purpose of developing the "N" Framework is to establish a framework consisting of the three essential categories of knowledge required by teachers to teach numeracy in any subject across the curriculum effectively. We believe that the "N" framework can be used to guide teachers in their teaching of numeracy across the curriculum. It enables teachers to understand numeracy and how it can be embedded in the teaching of their subject.

Numeracy Knowledge Pillar

The first pillar of the "N" Framework is the numeracy knowledge block. This pillar represents the nature of numeracy in the 21st century. This pillar contains five different dimensions Contexts, Mathematical Knowledge, Dispositions, Tools and Critical Orientation. These components of numeracy knowledge, as discussed in the previous section are critical for a teacher in the teaching of numeracy within their subject area. If a teacher understands the concept of numeracy, they will be able to embed this in their teaching and this will further enhance the students' development of numeracy skills for the future.

Subject Specific Knowledge Pillar:

The next pillar of the "N" Framework that we developed is the subject specific knowledge pillar. The authors looked at many different theories and frameworks for teacher knowledge, but the two that emerged the most suitably aligned to integrate with numeracy for us were Shulman's Knowledge Growth in Teaching (1986) and more recently, Ball et al's Content Knowledge for Teaching (2008). Both Shulman (1986) and Ball et al. (2008) highlight the importance of specific subject knowledge, with categories of common content knowledge and specialised content knowledge. We chose to exclude the knowledge at the mathematical horizon domain as this is specific to the teaching of mathematics and cannot be utilised in a general teacher knowledge framework. While we do not believe in a hierarchy of knowledge, we do believe that all teachers must possess a proficient level of subject-specific knowledge as well as numeracy knowledge.

Pedagogical Content Knowledge Bridge

Pedagogical content knowledge is the bridge and connection between the pillars of numeracy knowledge and subject specific knowledge within the proposed "N" framework. This is the connection between a numerate person and a person who specialises in a particular subject area such as science, business studies or mathematics etc. Being able to take both categories of knowledge and convey this knowledge through their teaching is at the core of this framework and is known as pedagogical content knowledge. Pedagogical content knowledge is the bridge and link between a numerate teacher being able to find the numeracy learning opportunities within their subject and develop this knowledge as part of the specific subject which in turn enables their students in becoming numerate citizens for the 21st century. The three pillars complement each other in such a way that numeracy knowledge is linked through the bridge of pedagogical content knowledge and embedded in the subject specific knowledge which enables a teacher to develop the numeracy knowledge of his/her students in the different subject areas across the school curriculum.

CONCLUSION

The field of numeracy in Ireland is complex. It is characterised by policy, a lack of resources/ curricular materials and the absence of a unified definition of numeracy for teachers. Likewise, teacher knowledge is multifaceted in nature and thus is difficult to define. However, if teachers are expected to embed numeracy in all aspects of their teaching, it is critical that a numeracy framework for teaching is developed to guide teachers in the most effective way on how best to implement numeracy teaching within the school curriculum. Teacher knowledge matters and in order for students to learn effectively. It is essential that the teachers have a good understanding of all types of knowledge. The "N" framework addresses the issues of numeracy knowledge and teacher knowledge combined and we believe that this framework will help pre- service and in-service teachers become aware of the different knowledge domains required for teaching numeracy in any subject across the curriculum in post-primary schools in Ireland.

REFERENCES

Bennison, A. (2015). Supporting teachers to embed numeracy across the curriculum: a sociocultural approach. *ZDM - International Journal on Mathematics Education, 47(4),* pp.561–573.

Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special. *Journal of teacher education, 59*(5), 389-407.

Cockcroft, W. (1982). *Mathematics counts*. London: HMSO.

Department of Education and Skills. (DES). (2011). *Literacy and numeracy learning for life: The national strategy to improve literacy and numeracy among children and young people 2011-2020*. Dublin: Author.

Fennema, E., & Franke, M. L. (1992). Teachers' knowledge and its impact. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 147-164). New York, NY: Macmillan.

Frejd, P., & Geiger, V. (2017). Exploring the notion of mathematical literacy in curricula documents. In G. Stillman, G. Kaiser & W. Blum. (Eds.), *Mathematical modelling and applications: Crossing and researching boundaries in mathematics education* (pp. 255-264). Cham: Springer. DOI: https://doi.org/10.1007/978-3-319-62968-1_22

Geiger, V., Goos, M., & Forgasz, H. (2015). A rich interpretation of numeracy for the 21st century: A survey of the state of the field. *ZDM*, *47*(4), 531-548.

Goos, M., Geiger, V., & Dole, S. (2013). Designing rich numeracy tasks. Task design in mathematics education. *Proceedings of ICMI Study, 22(1),* pp. 589–597

Goos, M., Geiger, V., & Dole, S. (2014). Transforming professional practice in numeracy teaching. In Y. Li, E. Silver, & S. Li (Eds.), *Transforming mathematics instruction: multiple approaches and practices* (pp. 81–102). New York: Springer.

Goos, M., Geiger, V., Dole, S., Forgasz, H., & Bennison, A. (2019). *Numeracy across the curriculum: Research-based strategies for enhancing teaching and learning.* Sydney: Allen and Unwin.

Jablonka, E. (2015). The evolvement of numeracy and mathematical literacy curricula and the construction of hierarchies of numerate or mathematically literate subjects. *ZDM*, *47*(4), 599-609.

Leder, G., Forgasz, H., Kalkhoven, N., & Geiger, V. (2015). Pre-service teachers and numeracy in and beyond the classroom. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 349–356. Sunshine Coast: MERGA.

Ministry of Education (1959). *15 to 18: A report of the Central Advisory Council for Education*. London: HMSO. Retrieved from http://www.educationengland.org.uk/documents/ crowther/crowther1959-1.html

Mishra, P., & Koehler, M. J. (2006). Technological pedagogical content knowledge: A framework for teacher knowledge. *Teachers College Record*, *108*(6), 1017-1054.

Organisation for Economic Co-operation and Development (OECD). (2016). *PISA 2015 assessment and analytical framework: Science, reading, mathematics and financial literacy*. Paris: Author.

Rowland, T., & Turner, F. (2007). Developing and using the 'Knowledge Quartet': A framework for the observation of mathematics teaching. *The Mathematics Educator*, *10*(1), 107-124.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*, 1-22.

Van Driel, J. H., Verloop, N., & De Vos, W. (1998). Developing science teachers' pedagogical content knowledge. *Journal of Research in Science Teaching*, *35*(6), 673-695.

Westwood, P. S. (2008). *What teachers need to know about numeracy*. Camberwell, Australia: Australian Council for Educational Research (ACER) Press.

AN INVESTIGATION INTO THE PROBLEM-SOLVING CAPACITIES OF PRESERVICE POST-PRIMARY MATHEMATICS TEACHERS: IMPLEMENTATION OF TAUGHT STRATEGIES

Emma Owens and Brien Nolan

CASTeL and School of Mathematical Sciences, Dublin City University

In this study, we report on the ability of nine pre-service post-primary mathematics teachers on a concurrent, initial teacher education programme in an Irish university to apply the 'Rubric Writing' approach to solving mathematical problems of (Mason, Burton, & Stacey, 2011). The conceptual framework of the study draws on (Chapman, 2015), who identifies different characteristics that underpin the effective teaching of mathematical problem-solving. Included here is the capacity to solve problems effectively. The participants in the study had previously received instruction on problem-solving in a formal university module, focussing on the 'Rubric Writing' approach. Each participant undertook two mathematical problems in a 'Think Aloud' manner in recorded interviews. The interviews were then analysed for evidence of implementation of the 'Entry' phase of this approach. We report on this analysis and on how it will be embedded in the ongoing research project.

PROBLEM SOLVING AND TEACHERS OF PROBLEM SOLVING

Mathematical problem solving occupies a privileged position in the Irish post-primary mathematics syllabus. Problem solving is identified as one of the six elements of the Unifying Strand of the Junior Cycle syllabus that over-arches the four content strands (Number, Geometry & Trigonometry, Algebra & Functions, Statistics & Probability). Likewise, mathematical problem solving is highlighted under the 'Being Numerate' heading of the Junior Cycle Key Skills, and constitutes one of the 24 'Statements of Learning' of the Junior Cycle (NCCA, 2017). So mathematical problem solving is recognized and valued as a central part of post-primary mathematics education, nationally and internationally (Conway & Sloane, 2005).

According to Kilpatrick (1985), while problems have held a fundamental role in the school mathematics curriculum, problem solving has not. Kilpatrick comments that with the focus on the idea of developing problem solving skills, there is confusion regarding the actual definition of problem solving. Indeed, Schoenfeld (1992) identifies that there are various meanings for the terms "problems" and "problem solving": see also (Lester, 2013). Nowhere is this lack of agreement clearer than in the work of Chamberlin (2008). In this study, Chamberlin applied a Delphi technique protocol (Cohen, Manion & Morrison, 2007, p.309) to attempt "to ascertain what mathematical problem solving is in the primary and secondary mathematics classroom" (Chamberlin, 2008, p. 1). After three rounds of analysis, a group of twenty participants (experts on mathematical problem-solving in the classroom) reached consensus on just 21 of 38 statements on the relevance of certain issues to the components and characteristics of problem solving. This lack of consensus must be set against the widespread acknowledgement of the importance of problem solving in the mathematics curriculum (Conway & Sloane, 2005).

In recognition for the need for clarity of our use of the phrase *mathematical problem solving*, we will highlight three items. Firstly, Lester (2013) notes that among the many different perspectives on problem solving, there appears to be agreement that there must be a goal, a problem solver and the lack of a means of immediately attaining the goal. Next, we consider the role of problem solving within the statement of key learning outcomes as presented in the NCCA syllabus document:

> Students should be able to investigate patterns, formulate conjectures, and engage in tasks in which the solution is not immediately obvious, in familiar and unfamiliar contexts (NCCA, 2017, p.10).

Finally, we mention the characterization offered in (Lester & Kehle, 2003):

> Successful problem solving involves coordinating previous experiences, knowledge, familiar representations and patterns of inference, and intuition in an effort to generate new representations and related patterns of inference that resolve some tension or ambiguity (i.e., lack of meaningful representations and supporting inferential moves) that prompted the original problem-solving activity (Lester & Kehle, 2003, p.510).

This third characterization includes the key elements of problem solving that are the focus of the larger research project of which this study is a part. The problems and problem solving activities of this project reflect the three items noted above. The project in question addresses the development of capacities for teaching problem solving among pre-service, post-primary mathematics teachers (we discuss this in more detail below). A key concern is the question of what these capacities are. That is; what do mathematics teachers need to know, what skills do they need to have, and what are the attitudes that they should hold in order to be effective teachers of mathematical problem solving? The question of how to prepare teachers for the task of teaching problem solving has attracted much attention. Chapman (2015) discusses six capacities that teachers need in order to teach problem solving effectively:

Knowledge of problems

This describes the teacher's ability to select and design mathematical problems. Chapman maintains that when selecting problems, the teacher must be aware of the potential impact of the characteristics of the problems on their students. For example, Silver and Thompson (1984) show that students experience greater difficulty with multistep problems than one-step problems. Other researchers find that secondary school students find abstract problems considerably more difficult than concrete problems (Caldwell & Goldin, 1987), and that students have difficulty in understanding the text of a word problem more than the actual solution (Lewis & Mayer, 1987).

Knowledge of problem solving

Teachers should be proficient in problem solving and in understanding the nature of approaches to problem solving. Chapman (2015) outlines that teachers' own proficiency in problem solving is essential for them to be able to understand students' approaches and predict the implications of these approaches. The teachers' proficiency also underpins their ability to decipher students' unusual solutions, whether or not these will be beneficial, and what makes them so. Problem solving proficiency is also needed so that the teachers may make connections between the mathematics in different solutions for the same problem and the mathematics among different problems. Chapman (2015) suggests that to teach for problem solving proficiency, the teacher must be aware of the many problem solving models that exist and understand the thinking that must occur during the process to achieve a solution.

Knowledge of Problem Posing

This capacity includes a teacher's ability to generate new problems and to adapt existing problems for their students' needs. It is important for teachers to have this skill as it can have a positive influence on students' mathematical thinking and therefore improve their problem solving ability (English, 1997, cited in Chapman, 2015). Chapman outlines findings by Silver, Mamona-Downs, Leung and Kenney (1996) which found that both practicing and preservice teachers created problems that were either predictable, undemanding, ill-formulated or unsolvable, when trying to make expansions on given problems.

Knowledge of students as problem solvers

Chapman notes that this is important for the teacher to have to help improve students' problem solving proficiency. Chapman advises that teachers should be aware of the characteristics of 'good' problem solvers and the heuristics employed and dispositions of these solvers to help them promote these behaviours in their students.

Knowledge of problem solving instruction

This is a trait that is vital for teachers to have to teach for problem solving proficiency. Chapman outlines the drawbacks of showing the students a method and then the students practising similar methods compared to the advantages of the teacher employing a constructivist role in the classroom (Kilpatrick, 1985).

Affective Factors and Beliefs

These could both impact students' problem solving. Chapman refers to Polya (1962) who stated the importance of a teachers' positive attitude in aiding students in problem solving. The understanding of the students' own beliefs is required by the teacher and the teacher must try to develop the students'; belief in their ability, appreciation of understanding concepts, the necessity of word problems as part of mathematics, and a belief that effort will improve their mathematical ability (Kloosterman & Stage, 1992).

THE STUDY

Participants and Mason's Rubric Writing

The participants in this study are pre-service mathematics teachers (PSMTs) undertaking a concurrent initial teacher education programme. Graduates of the relevant programmes typically go on to careers teaching mathematics in Ireland, and so preparing the PSMTs for the task of teaching problem solving is a key concern of the programme team. Participants are drawn from two cohorts of students, both of whom were taking a module that includes the study (and practice) of mathematical problem solving. The participants all went through the Leaving Certificate mathematics curriculum and had reduced exposure to problems since there is a lack of problems in Leaving Certificate textbooks (O'Sullivan, 2017). Along with this there is not a high mathematics entry requirement for either programme from which the participants were drawn. Thus, it is possible to classify the participants as novice problem solvers which implies that structured approaches are useful in their solving of problems (Schoenfeld, 1992). This module adopted the Rubric Writing approach to problem solving (Mason et al., 2011). The participants in the study were introduced to this approach in a series of 10 lectures and 8 workshops. These provided instruction in applying Rubric Writing, supported by worked examples of problem solving, and with the opportunity to apply the technique to unseen problems taken from (Mason et al., 2011). This Rubric Writing approach (which may be described as a problem-solving heuristic) provides structured guidelines to promote the introduction of diagrams and notation, to draw upon prior knowledge, and focus on metacognition through the reviewing of work.

The present study focusses knowledge of problem solving as described above. We focussed on their problem-solving proficiency: "*what is necessary for one to learn and do genuine problem solving successfully" (*Chapman,2015, p.20). In particular, we examine the students' capacity to use the *Entry Phase* of Mason's rubric writing approach. Mason et al. (2011) define the *Entry Phase* as beginning when first encountering the question and ending when the problem-solver is involved in attempting to solve the problem. They note the importance of this phase in progressing and ultimately succeeding in finding a solution. They state that without satisfactory entry to a problem, the next phase, the *Attack Phase,* cannot come about. Mason et al. outline that since the *Entry Phase* is important for the preparation of a successful *Attack Phase*, it is crucial to dedicate sufficient time to it. To create an effective *Entry Phase*, Mason et al. suggest the inclusion of the following three questions in the problem-solvers' rubric writing; *What do I know?*, *What do I want?*, and *What can I introduce?*

Methodology

Participants were volunteers from the cohorts undertaking the module mentioned above (this group comprised a total of 40 students). Nine students participated in the study. They were provided with information relating to data protection, with a plain language description of the project and with a consent form explicitly offering the opportunity to opt out of the study at any stage. The PSMTs were interviewed on a one-on-one basis by one of the researchers (EO). They were given two problems: Problem One dealing with probability and Problem Two with geometry and trigonometry. Both problems were taken from the NRICH website (NRICH, 2019) where the problems are organised by age categories with the difficulty of the problems measured on a scale of 1-3 stars (3 stars being most difficult). Problem One is classified as a 2-star short probability problem appropriate for students age 14-16. Problem Two is classified as a 3-star short trigonometry problem appropriate for students aged 14-16. The participants were asked to solve the problems following a 'Think Aloud' protocol (Salkind, 2010). While the 'Think Aloud' method may alter the participants' problem-solving approach due to the environment it is a method which is widely used in education research to gather data on working memory of participants (Charters, 2003). Interviews were audio-recorded and transcribed. The interviews ranged in duration from 04:54 to 31.36 minutes. The written work produced by the PSMTs during the protocol was retained for analysis.

Data Analysis

The interview transcripts have been analysed in two different ways. An approach based on the general inductive analysis of Thomas (2006) has been reported on elsewhere (Owens & Nolan, 2019). We report here on the second approach, where we rated the combined interview data and written work in terms of the degree to which these evidenced implementation of the Rubric Writing approach of Mason et al., (2011). This was done by both researchers independently and then compared and revised as necessary. To identify evidence of applying the *Entry Phase* we used the following characterizations (Mason et al., (2011)):

Mason describes *'I Want'* as directing attention to the task at hand or deciding what is needed to be done in order to solve the problem. Evidence that we looked for was that the participant (re)stated precisely the goal of the problem (the value of a positive integer N in Problem 1, and the value of a distance x in Problem 2). *'I Know'* refers to selecting all the relevant information given in the problem and identifying any associated mathematical concepts that are likely to be relevant. For this study, we looked for evidence of *I Know* early on in the attempt where the participant recorded relevant information given in the problem and or stating related mathematical facts. '*Introduce'* includes the introduction of notation, organizing the *I Know* elements, and representing information in the question through the use of tables, charts, and diagrams. Evidence of *Introduce* in this study was identified as the drawing of diagrams, the introduction of notation, and constructions within the given diagram in Problem 2.

RESULTS

The analysis of the *Entry Phase* of Mason's Rubric Writing Approach was done by implementing the following grading system; 0 points (no evidence); 1 point (limited evidence); 2 points (strong evidence). This grading was carried out for both questions for each of the nine participants and for each of the three elements of the *Entry Phase* namely, *Introduce*, *I Know,* and *I Want*. This generated 54 data items. Both researchers rated the interview transcripts separately and initially agreed on the scoring of 42 of the 54 items (78%). For the remaining 12 items, the scores differed by at most one point. No disagreements remained following a discussion. Table 1 summarises the grading of the *Entry Phase* for both problems.

Table 1: Entry Phase count for the cohort for both problems.

Problem One		
Introduce	*Want*	*Know*
0	8	9
Problem Two		
Introduce	Want	Know
10	6	9

Examples that were categorized as participants' use of the *Entry Phase* are given below.

Introduce

Question 2 included a diagram, but many participants opted to redraw it themselves:

P9: "I'll draw it out in front of me so there's the well, so I have 5, 10 and x".

Within this category, extensions of given diagrams or constructions based on them also feature, as did diagrams drawn multiple times and with different sizes. Participants also drew diagrams with the view to then manipulating them:

P8: "I'm going to draw them down and make right angled triangles",

P6: "We could make a right angle here…OK so I'll draw it".

Participants also introduced notation which is acknowledged by both Polya (1962) and Mason et al., (2011) as a feature that frequently underpins successful problem solving. Examples of notation include labelling angles or sides (P8):

P8: "I'm just drawing...five Y plus P" … "The sides so that's A and that's B".

I Know and *I Want*

Statements in these categories scored as +1 or +2 only when they clearly appeared in the *Entry Phase* of the problem solving activity – prior to the *Attack Phase*. Many such statements involved students repeating or highlighting the information given in the problem. This is discussed as being important in Mason et al. (2011), and was emphasized during instruction. In some cases, there were instances of *I Want* and *I Know* statements being made *after* the *Entry Phase* of the problem. For example, when Participant 1 got stuck, they then went back and stated what he knows and wants but these were more of a rationale for the approach and work that he did, rather than used as a starting point. Many of the participants jumped straight into an approach without stating what they know about the problem from the information given or stating what they want to find. Participant 5 was an example of this whereby they did not demonstrate any of the *Entry Phase* and specialized straight away.

Some participants excluded different approaches based on their *'I know'* statements. For example, Participant 9 stated that they do not have adequate information to use either the Sine Rule or the Cosine rule. This demonstrates that they know that both of these approaches are applicable to triangles, and also know what is required to use them.

Stating what 'I know' or 'I want' sometimes led participants to an 'Introduce' element:

P7: "I was going to try and do Pythagoras…try and turn one of the triangles into a right-angled triangle by drawing a straight line from the well to the side".

Similarly, Participant 4 states that they know that Pythagoras is only for right-angled triangle and introduced lines breaking up the given triangles into right-angled triangles. Both at the beginning of the attempt and throughout, Participant 4 repeatedly questioned themselves:

P4: "What else do I know?".

Although participants made statements in relation to the three components of the *Entry Phase*, none of them wrote down explicitly '*I know'* or *'I want'* as they had been advised.

CONCLUSIONS AND NEXT STEPS

The interview protocol described allowed us to study PSMTs' implementation of Mason's Rubric Writing approach. The use of the *Introduce* component of the *Entry Phase* was the most commonly used element, particularly in Problem Two. This problem, although given with a diagram, required *Introduce* to solve the problem. For example, in one approach, right-angled triangles and rectangles need to be constructed to solve the problem. This was done to various degrees by several of the participants. However, such were wholly absent in relation to Problem 1 (see Table 1) – where only one student successfully solved the problem.

Our reading of the data is that none of the participants explicitly and purposefully implemented Mason's Rubric Writing approach. No student showed evidence of implementing each of the three elements of the *Entry Phase*, and of the 54 items, only 8 scored a maximum of 2 points. This indicates that continued review of the current module is needed. Despite lectures and practical tutorials emphasizing the effectiveness of Mason's Rubric Writing approach, there was a clear lack of application of it. This is concerning as the participants are not deemed to be expert problem solvers and should therefore rely on a guided approach such as Mason's Rubric Writing when stuck in a problem. An encouraging feature was that although none of the participants wrote down the components of the *Entry Phase,* there were verbal referrals to them. There were many examples of the participants introducing diagrams, stating information that they knew to help them progress or get a starting point, and some participants stated what they were looking for after reading the problem.

Although the number of participants is small [N=9], we see implications for our work with PSMTs in developing their problem solving capacities, recognising this as a central part of their formation as teachers of mathematical problem solving (Chapman, 2015). Deeper engagement over a longer duration is indicated to improve problem-solving proficiency in prospective teachers. Future research work will focus on the other elements of Chapman's framework along with a revised repetition of this study (Chapman, 2015).

REFERENCES

Caldwell, J,H., & Goldin G.A. (1987). Variable affecting work problem difficulty in secondary school mathematics. *Journal for research in mathematics education, 18,* 187-196.

Chamberlin, S. A. (2008). What is problem solving in the mathematics classroom? *Philosophy of mathematics education journal, 23*(1), 1-25.

Chapman, O. (2015). Mathematics teachers' knowledge for teaching problem solving. *LUMAT* (2013–2015 Issues), *3*(1), 19-36.

Cohen, L. Manion. L., & Morrison, K. (2007). *Research methods in education*. London: Routledge.

Conway, P. F., & Sloane, F. C. (2005). *International trends in post-primary mathematics education: Perspectives on learning, teaching and assessment.* NCCA.

English, L. (1997). The development of fifth-grade children's problem-posing abilities. *Educational Studies in Mathematics, 34*, 183-217.

Kilpatrick, J. (1985). A retrospective account of the past twenty-five years of research on teaching mathematical problem solving. In E. A. Silver (Ed.), *Teaching and learning mathematical problem solving Multiple research perspectives* (pp. 1-15). Hillsdale, NJ: Erlbaum.

Kloosterman, P., & Stage, F. K. (1992). Measuring beliefs about mathematical problem solving. *School science and mathematics, 92*(3), 109-115.

Lester Jr, F. K. (2013). Thoughts about research on mathematical problem-solving instruction. *The mathematics enthusiast, 10*(1), 245-278.

Lester, F. K., & Kehle, P. E. (2003). From problem-solving to modeling: The evolution of thinking about research on complex mathematical activity. In R. Lesh, & H. Doerr, (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching* (pp. 501-518). Mahwah, NJ: Lawrence Erlbaum Associates.

Lewis, A.B., & Mater, R.E. (1987). Students' misconceptions of relational statements in arithmetic word problems. *Journal of Educational Psychology, 79,* 363-371.

Mason, J., Burton, L., & Stacey, K. (2011). *Thinking mathematically*. Pearson Higher Ed.

NCCA (2017). *Junior Cycle Mathematics.* https://curriculumonline.ie/Junior-cycle/Junior-Cycle-Subjects/Mathematics

O'Sullivan, B. (2017). *An analysis of mathematical tasks used at second-level in Ireland* (Doctoral dissertation, Dublin City University).

Owens, E.M. and Nolan, B.C. (2019). An investigation into the problem-solving capacities of preservice post-primary mathematics teachers. Accepted for publication in *Proceedings of the British Society of Research into Learning Mathematics.*

NRICH (2019). *Problem Solving.* https://nrich.maths.org/10334

Polya, G. (1962). *Mathematical discovery: On understanding, learning and teaching problem solving: Volume I.* New York: John Wiley and Sons, Inc.

Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition, and sense-making in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 334-370). New York: MacMillan.

Silver, E. A., Mamona-Downs, J., Leung, S. S., & Kenney, P. A. (1996). Posing mathematical problems: An exploratory study. *Journal for research in mathematics education*, 293-309.

Silver E., & Thompson, A. (1984). Research Perspective on Problem Solving in Elementary School Mathematics. *The elementary school journal, 84*(5), 529-545.

Thomas, D. R. (2006). A general inductive approach for analyzing qualitative evaluation data. *American journal of evaluation, 27*(2), 237-246.

OPPORTUNITIES FOR YEAR-ONE CHILDREN TO ACQUIRE FOUNDATIONAL NUMBER SENSE: COMPARING ENGLISH AND SWEDISH ADAPTATIONS OF THE SAME SINGAPORE TEXTBOOK

Jöran Petersson[1], Judy Sayers[2], Eva Rosenqvist[1] and Paul Andrews[1]

[1]Stockholm University, Sweden and [2]Leeds University, UK.

We compare adaptations of a Singaporean year-one mathematics textbook for use in England and Sweden respectively. The texts were analysed in two different ways against the eight dimensions of Foundational Number Sense (FoNS), a set of core competences that the literature has shown to be necessary for year-one children's later mathematical learning. The first analysis, based on frequencies, showed that neither adaptation incorporated any opportunities for children to acquire the two FoNS competence relating to estimation and number patterns respectively. They also showed that the English adaptation comprised significantly more tasks than the Swedish, particularly with respect to systematic counting, where the former comprised 26% more tasks than the latter. The second analysis, based on moving averages, showed that across five of the six FoNS categories for which there were data, the temporal location and emphases of FoNS-related learning were comparable, with, in particular, no such opportunities after the mid-point of the school year in either book. However, the English adaptation's presentation of systematic counting, occurring at various points throughout the school year, was substantially different from the Swedish adaptation, highlighting differences due, we speculate, to interpretations of local didactical traditions.

INTRODUCTION

For many teachers of mathematics, irrespective of where they work, the textbook they use is not only the major resource for lesson planning and the provision of tasks for students but also the means by which the curriculum within which they work is realised and the determinant of what students learn (Tarr, Cháves, R. Reys & B. Reys, 2006). That said, the analysis of textbooks is probabilistic in the sense that teachers make decisions as to how they use any book, leaving the analytical question "what would students learn if their mathematics classes were to cover all the textbook sections in the order given? What would students learn if they had to solve all the exercises in the textbook?" (Mesa, 2004, pp. 255–256). Moreover, in those cultures in which textbooks are unregulated, typically leading to a plethora of choice for teachers, students may receive very different opportunities to learn (Huntley & Terrell, 2014; Tarr et al., 2006). Thus, the reasons for analysing textbooks are varied and include, acknowledging the huge sums of money spent on producing and purchasing them, concerns about value for money (Harel & Wilson, 2011) and their being fit for purpose (Huntley & Terrell, 2014; Tarr et al., 2006). More recently, in part motivating this paper, research has been driven by scholars' desires to better understand the functioning of educational systems more successful than their own (Ding, 2016; Li, Chen & An, 2009; Yang, R. Reys & Wu, 2010). That said, again part motivating this study, while some "effort has been put into content analysis and exploring the ways in which textbooks are used in classrooms… very few mathematics education researchers have taken a really close look at what is in the textbooks, with the focus on how the material is presented and what kind of learning may be implied" (Kajander & Lovric, 2009, p.174). Moreover, while textbook analysis is an increasingly popular undertaking, studies focusing on year-one children are rare. In England, these children are aged 5 and in Sweden 7.

In this paper we compare two adaptations of a popular Singaporean mathematics textbook written for year-one children. These are English adaptation, Maths – No Problem (hereafter MNP), and the Swedish adaptation, Singma. The analyses are framed theoretically by the lens of foundational number sense (FoNS), a set of eight number-related competences, based solely in the integer range 0-20), that research has shown to underpin year-one children's later mathematical learning (Andrews & Sayers, 2015). Acknowledging that all humans (and many other species) are born with number-related insights concerning quantity discrimination (Lipton & Spelke 2005) and that curricula typically expect students to develop the number sense "required by all adults regardless of their occupation" (McIntosh, B. Reys & R. Reys, 1992, p. 3), FoNS, which requires instruction, is intended to provide the foundations of the bridge between the two. The initial aim of the project team, by means of a systematic review of the literature, was to identify a set of curriculum independent competences that would be simple to operationalise in different cultural contexts. Moreover, its origins in the international literature makes the FoNS framework an appropriate tool for comparing textbooks and their presentation of key number-related competences.

Earlier FoNS-related analyses have compared the English version of the Singaporean textbook, MNP, with other texts used in England (Petersson, Sayers, Rosenqvist & Andrews, under review) and the Swedish version, Singma, with other texts used in Sweden (Sayers, Petersson, Rosenqvist & Andrews, under review). The results of these studies have highlighted the extent to which the Singapore import differs in its emphases from books authored by English and Swedish colleagues respectively. Indeed, both analyses allude to the problematic nature of textbook importation and the didactical challenges teachers must face in order to use them successfully. Moreover, since the production of textbooks is unregulated in both England and Sweden, there is no official expectation that textbooks should explicitly address the particular expectations of the two countries' curricula. Thus, assuming that an importer would wish to retain the integrity of the original work, it would seem reasonable to expect the two adaptations to match each other closely. This paper, therefore, is framed by the following question: How are FoNS-related learning opportunities manifested in the two independent translations of the same textbook? Each adaptation is subjected to two analyses, each drawing on different forms of task distribution. In so doing, we acknowledge Rezat's (2006, p. 482) position that a mathematics textbook "is historically developed, culturally formed, produced for certain ends and used with particular intentions". That is, despite English publishers' expectations that purchasers of their Singapore adaptations should attend induction courses, any textbook is clearly a product of the culture and curriculum in which it was written with no obvious guarantee that it would function adequately in another context.

METHODS

Two adaptations of the same Singaporean textbook, one from England (MNP) and one from Sweden (Singma), were identified for analytical purposes. With respect to both adaptations, all materials intended for the use of year-one children were coded, each by at least two members of the project team, for FoNS-related learning opportunities. In this way, each task was coded as a series of 1s and 0s, according to the presence or absence of the eight FoNS categories. Throughout, the focus of the analyses was solely on tasks that expected action on the part of the student. Thus, explanatory worked examples were included but all tasks in teacher guides were excluded. Other studies have counted the number of pages devoted to the content under scrutiny, arguing that since "pages consisting of tasks for the students to solve contain many similar tasks… the result of counting the number of tasks… would probably not differ much from the result obtained by counting whole pages" (Bråting, Madej & Hemmi, 2019). Our view is that because textbooks differ greatly in the ways in which mathematics is presented, some comprising very dense pages and others not (Haggarty & Pepin, 2002), counting tasks is more likely to yield an accurate representation of the opportunities given to children, particularly when we are comparing adaptations of the same book.

In addition to simple frequency analyses, whereby each occurrence of each category was counted, a moving average was calculated for each code as it occurred in each book. This approach is typically used to analyse trends in, for example, temperature over time, while eliminating any undue influence of outliers (Fan & Yao, 2003, p. 9). In similar vein, the use of moving averages with textbooks, whereby data are successive tasks, should offer a clear indication of a textbook's sequential emphases. In this way, single data points are replaced by the arithmetical mean of a sequence of data points, drawn from before, including, and after the point in question. This process smooths out short-term fluctuations in time series so that longer-term patterns become more visible and the influence of outliers is eliminated. Mathematically, a moving average means substituting a single data point (t_k, y_k) with $(t_k, \hat{y}_k)$, where $\hat{y}_k$ is the arithmetic mean of its neighbouring data points y_j as in equation 1. Importantly, if the time period selected for the moving average is too short, then its associated graph becomes noisy and trends may be lost. Similarly, if the time period is too long then important details may be lost (Wakaura & Ogata, 2007). Thus, the choice of time interval is key to the successful use of the approach.

$$\hat{y}_k = \sum_{j=k-n}^{k+n} \frac{1}{2n+1} y_j$$

Equation 1

Of particular interest to the analyst is the size of the divisor, $2n + 1$, which represents the total number of data points included in the calculation and is dependent on the time period chosen for the calculation. That is, $2n+1$ refers to the original point, y_k, and its $2n$ neighbouring data points, n before and n after. In the context of a mathematics textbook, the width $2n+1$ of this window could be the number of tasks that an average student is expected to cover each day, or each week or each month and this choice depends on the research question. Thus, one moving average window could be $\frac{all\ items\ in\ a\ book\ series\ for\ one\ year}{40\ school\ weeks}$, roughly corresponding to a single week's workload across the school year. This means that wherever the moving average diagram shows 'over zero', then the pupil would have met that coded property during that week. In this paper, we have selected a window to represent the likely material a student would encounter during one week.

RESULTS

Table 1 summarises the eight FoNS categories and presents the frequencies of each category in each of the two books. Interestingly, despite the research-led identification of the FoNS codes, the figures show that neither book includes any opportunities for children to engage with estimation or number patterns. Also, were the two adaptations to be exact replicas of the original the two sets of figures would be the same. This is clearly not the case, with, in general terms, MNP comprising more than 15% more tasks overall than Singma. Indeed, across the six FoNS categories for which evidence is available, MNP has more tasks than Singma, ranging from almost 26% more tasks focused on systematic counting to just under 2% for tasks focused on simple arithmetic operations. A chi square test confirmed ($p<0.0005$) the statistical significance of the differences between the two sets of frequencies.

Table 1: Summaries of the eight FoNS categories and the frequencies for each in each book

	FoNS Characteristic	Pupils are encouraged (in the range 0-20) to	Singma	MNP	% change
1	Number recognition	Identify, name and write particular number symbols	614	685	11.6
2	Systematic counting	Count systematically, forwards and backwards, from arbitrary starting points	214	269	25.7
3	Number and quantity	Understand the one-to-one correspondence between number and quantity	335	371	10.7
4	Quantity discrimination	Compare magnitudes and deploy language like ‘bigger than’ or ‘smaller than’	110	120	9.1
5	Different representations	Recognise and make connections between different representations of number	346	370	6.9
6	Estimation	Estimate, whether it be the size of a set or an object	0	0	
7	Simple arithmetic	Perform simple addition and subtraction operations	415	423	1.9
8	Number patterns	Recognise and extend number patterns, identify a missing number	0	0	
		Total tasks per book	1694	1955	15.4

Of course, frequencies alone offer but one perspective on the content of a textbook, typically offering no indication as to the location of different forms of task in the narrative of the whole year’s study. To address this, we turn to moving averages based on a one-week time period.

The graphs shown in figure 1 are, effectively, indistinguishable. Both begin the school year with repeated emphases on tasks involving number recognition, followed by a fallow period and a second, equally strong emphasis ending around four months into the school year. After this, neither book offers any further number recognition-related opportunities.

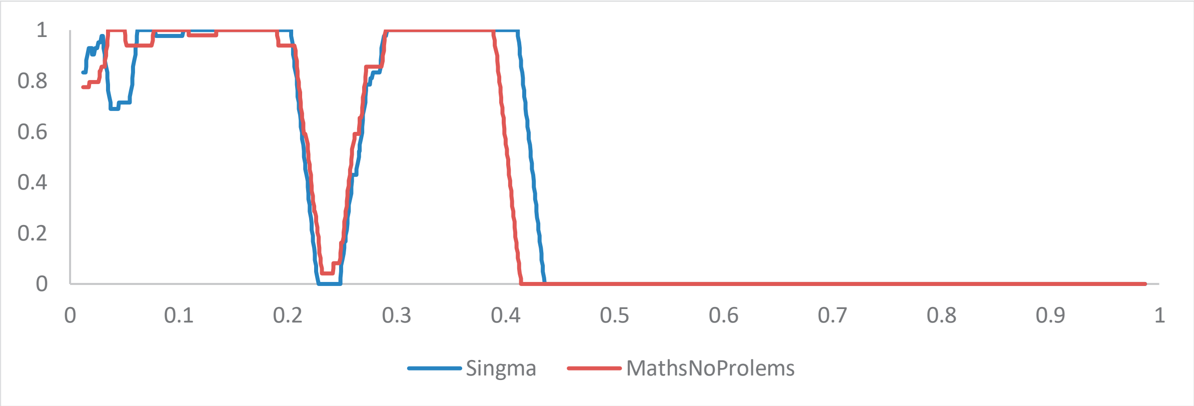


Figure 1: Graphs of the moving averages for FoNS 1 number recognition

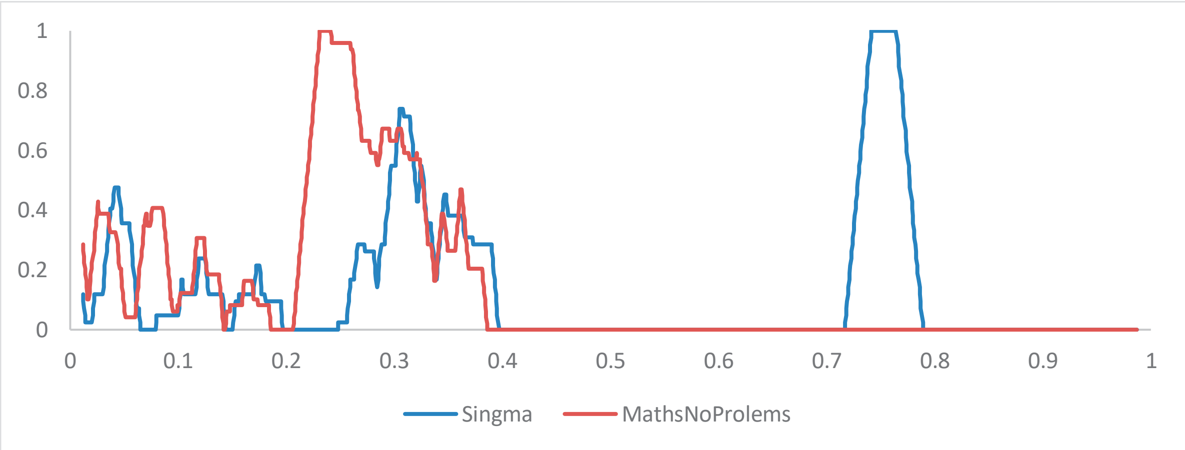


Figure 2: Graphs of the moving averages for FoNS 2 systematic counting

Unlike the close resonance of the graphs for FoNS 1, number recognition, the two books offered different emphases with respect to systematic counting. On the one hand, MNP begins with four short periods of limited emphasis before, after around two months, a final strong emphasis that gradually diminishes towards the four-month mark. On the other hand, the first four months of Singma mirror those of MNP, albeit with consistently lower emphases. The major difference is the spike during the eighth month, whereby a strong emphasis, stronger than at any other time of the year, emerges. Indeed, it is the only occasion that either of the two books offers any FoNS-related opportunities after the midpoint of the school year. What makes these differences particularly interesting is that the two strong spikes reflect when the two books introduce the vocabulary of ordinality; early in MNP and late in Singma.

The graphs of figure 3 show broadly similar trends. Both books end any opportunities for tasks related to the relationship between number and quantity around four months into the school year. That said, the broad patterns are similar, with early high levels of emphasis followed by a second period with slightly lower emphases. The remaining three figures, 4, 5 and 6, show similar trends with respect to the opportunities presented to children concerning quantity discrimination, different representations of number and simple arithmetical operations respectively.

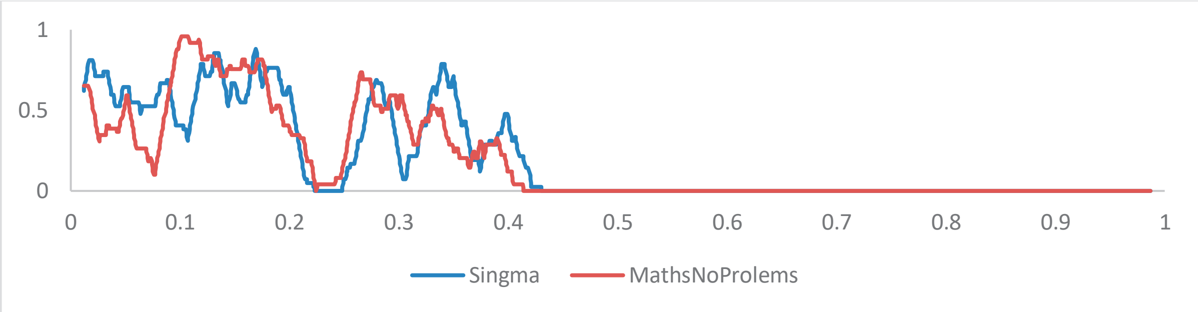


Figure 3: Graphs of the moving averages for FoNS 3 relationship between number and quantity

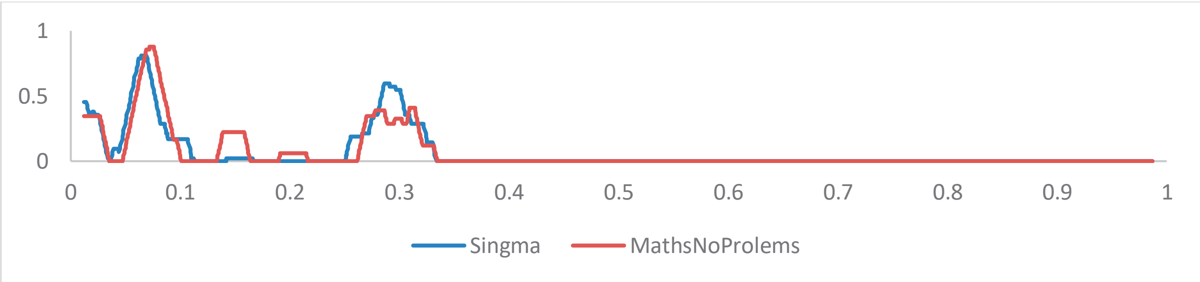


Figure 4: Graphs of the moving averages for FoNS 4 quantity discrimination

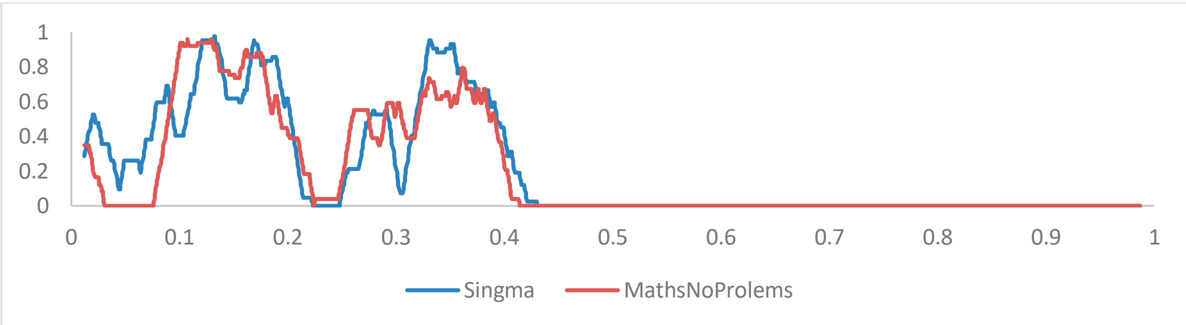


Figure 5: Graphs of the moving averages for FoNS 5 different representations of number

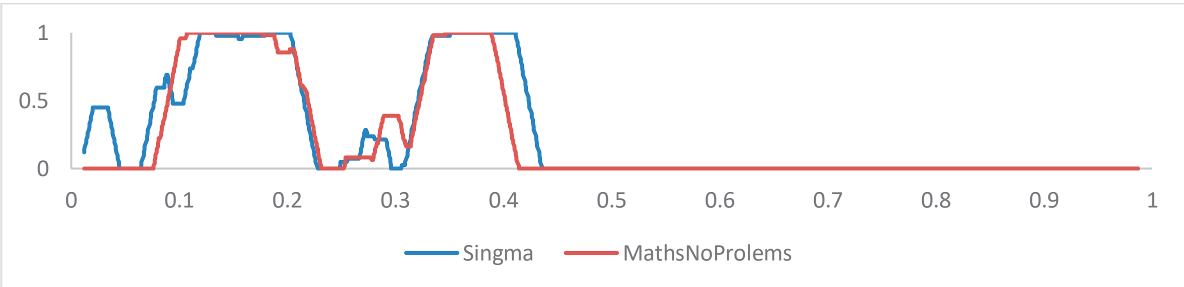


Figure 6: Graphs of the moving averages for FoNS 7 simple arithmetical operations

DISCUSSION

Our goal for this paper was to compare how two adaptations, one English and one Swedish, of the same Singaporean textbook structure year-one children's opportunities to acquire foundational number sense (FoNS). FoNS, which literature has shown to form the basis of later mathematical learning, is an eight dimensional set of competences necessary for year-one children, irrespective of their cultural or curricular traditions. If the two adaptations were merely translations, then it would be reasonable to expect the English and Swedish versions to comprise the same tasks. The two analyses presented above offer some interesting insights into nature of these two adaptations. First, neither book acknowledges the importance of two FoNS categories, omissions that may compromise later mathematical learning. These are estimation (Libertus, Feigenson & Halberda, 2013) and number patterns (Lembke & Foegen 2009).

Table 2. Order systematic counting-related content in Singma and MNP

Singma	MNP
Number track, range [0 - 10]	Number track, range [0 - 10]
Add or subtract by counting [0 - 10]	Add or subtract by counting [0 - 10]
Number track, range [11 - 20]	Ordinal vocabulary
Add or subtract by counting [11 - 20]	Number track, range [11 - 20]
Ordinal vocabulary	Add or subtract by counting [11 - 20]

Second, with the exception of simple arithmetical operations, MNP comprises significantly more tasks across all FoNS categories than Singma, which is interestingly odd in light of our earlier analyses showing that MNP comprised 29% more tasks than the English-authored textbook with which it was compared (Petersson et al. under review) and Singma comprised 36% fewer tasks than the Swedish-authored textbook with which it was compared (Sayers et al., under review). That is, the adapters seem to have very different views, in relation to the typical textbooks of their country, with regard to the sufficiency of the tasks presented in their adaptations. Third, with a single exception, although on this occasion it was systematic counting, the moving averages showed that despite differences in frequencies, the structures of the two textbooks were remarkably similar, with almost identical emphases over the course of the school year. Fourth, with respect to systematic counting, sub-topics were ordered differently in the two books, as shown in table 2. Here the vocabulary of ordinality occurs at different times; after all counting-related material in Singma and at the midpoint, of all the counting material in MNP. Indeed, the spike shown in the second half of the Singma school year was due to the introduction of numbers greater than 10, which all occurred in the first half the year in Singma. To conclude, the two adaptations, while broadly adopting the same structure, differ in a number of respects due, we speculate, to authors' culturally situated interpretations of the number-related curriculum requirements of the two countries' curricula and expectations of learner readiness (Rezat, 2006).

REFERENCES

Andrews, P., & Sayers, J. (2015). Identifying opportunities for grade one children to acquire foundational number sense: developing a framework for cross cultural classroom analyses. *Early Childhood Education Journal*, *43*(4), 257-267.

Bråting, K., Madej, L., & Hemmi, K. (2019). Development of algebraic thinking: opportunities offered by the Swedish curriculum and elementary mathematics textbooks. *Nordic Studies in Mathematics Education*, *24*(1), 27-49.

Ding, M. (2016). Opportunities to learn: Inverse relations in U.S. and Chinese textbooks. *Mathematical Thinking and Learning, 18*(1), 45-68.

Haggarty, L., & Pepin, B. (2002). An investigation of mathematics textbooks and their use in English, French and German classrooms: who gets an opportunity to learn what? *British Educational Research Journal*, *28*(4), 567-590.

Harel, G., & Wilson, W. S. (2011). The state of high school textbooks. *Notices of the American Mathematical Society*, *58*(6), 823-826.

Huntley, M. A., & Terrell, M. S. (2014). One-step and multi-step linear equations: A content analysis of five textbook series. *ZDM*, *46*(5), 751-766.

Kajander, A., & Lovric, M. (2009). Mathematics textbooks and their potential role in supporting misconceptions. *International Journal of Mathematical Education in Science and Technology*, *40*(2), 173-181.

Lembke, E., & Foegen, A. (2009). Identifying early numeracy indicators for kindergarten and first-grade students. *Learning Disabilities Research & Practice, 24*(1), 12-20.

Libertus, M., Feigenson, L., & Halberda, J. (2013). Is approximate number precision a stable predictor of math ability? *Learning and Individual Differences, 25*, 126-133.

Lipton, J. S., & Spelke, E. S. (2005). Preschool children's mapping of number words to nonsymbolic numerosities. *Child Development*, *76*(5), 978-988.

McIntosh, A., Reys, B., & Reys, R. (1992). A proposed framework for examining basic number sense. *For the Learning of Mathematics*, *12*(3), 2–8, 44.

Mesa, V. (2004). Characterizing practices associated with functions in middle school textbooks: An empirical approach. *Educational Studies in Mathematics*, *56*(2), 255-286.

Petersson, J., Sayers, J., Rosenqvist, E., & Andrews, P. (Under review). Teaching Mastery Mathematics: Comparing year one English mathematics textbooks through the lens of foundational number sense. *British Educational Research Journal*.

Rezat, S. (2006). The structures of German mathematics textbooks. *ZDM The International Journal on Mathematics Education*, *38*(6), 482-487.

Sayers, J., Petersson, J., Rosenqvist, E., & Andrews, P. (Under review). Opportunities to learn foundational number sense in three Swedish year one textbooks: Implications for the importation of overseas-authored materials. *Educational Studies in Mathematics*.

Tarr, J., Chávez, Ó., Reys, R., & Reys, B. (2006). From the written to the enacted curricula: The intermediary role of middle school mathematics teachers in shaping students' opportunity to learn. *School Science and Mathematics*, *106*(4), 191-201.

Wakaura, M., & Ogata, Y. (2007). A time series analysis on the seasonality of air temperature anomalies. *Meteorological Applications*, *14*(4), 425-434.

Yang, D.-C., Reys, R., & Wu, L.-L. (2010). Comparing the development of fractions in the fifth- and sixth-graders' textbooks of Singapore, Taiwan, and the USA. *School Science and Mathematics, 110*(3), 118-127.

HIGH ACHIEVING STUDENTS IN LEAVING CERTIFICATE MATHEMATICS: WHY HAS THE GENDER GAP WIDENED?

Aidan Roche[1], Gavin Duffy[1] and Aoibhinn Ní Shúilleabháin[2]

[1]Technological University Dublin and [2]University College Dublin

Research has identified a gender gap in the mathematical attainment of post-primary students around the world, favouring male students. In Ireland, following a review of the outcomes of a high-stakes examination taken by students at the end of post-primary schooling over an 18-year period, a similar such gap has been identified here and is widening. Data are presented to show that this gender gap widened with the introduction of a revised post-primary mathematics curriculum, colloquially known as Project Maths. This paper explores potential reasons behind the widening gap. Problem solving appears to be the pivotal issue and spatial ability may be a contributory factor. Addressing students' spatial ability is explored as way to address the gender gap and enable students to reach their full mathematical potential.

INTRODUCTION: THE GENDER GAP IN POST-PRIMARY MATHEMATICS

This paper represents an initial phase of research establishing the existence of a widening gender gap at the highest attainment level of post-primary mathematics in Ireland. Investigation is needed to understand the underlying issues: Why is there a discrepancy? What has caused the relative situation for female students to worsen? What solutions and actions might help address this imbalance?

High-achieving students and the gender gap at Leaving Certificate

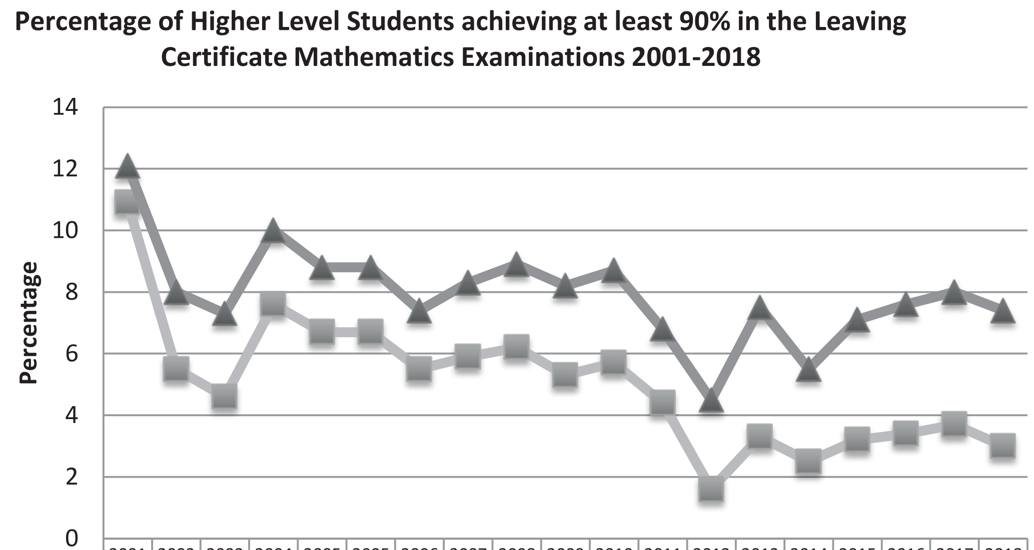
The Leaving Certificate (LC) examination in Ireland constitutes the end of post-primary education assessment, and also acts as university matriculation examination. In the LC mathematics examinations there are three different levels that students may study over a two-year, senior cycle course and sit the examination: Foundation, Ordinary or Higher Level (HL). The curriculum and final examination for these courses vary in breath, depth and difficulty. The State Examinations Commission has published annual statistical reports since 2001 categorising the attainment of 55,000 or so students who sit the LC examinations each year (SEC, 2019).

Inspecting SECs statistics for 2018, we see that approximately one in every three students sat the HL mathematics papers, with one in twenty of these students achieving the highest grade (90%-100%). Within this there are three notable gender differences: More male than female students sat the LC HL mathematics examination; more males achieved the highest grade; and, of those who sat the HL examination, a greater proportion of males than females achieved the highest grade (Table 1).

Table 1: Gender participation in the Leaving Certificate Mathematics Examination 2018

	Male	Female
Mathematics students at all levels	26,429	26,953
Higher Level students	8,741	8,096
% Gender taking Higher Level	33%	30%
Achieving at least 90%	646	245
% Total student cohort achieving at least 90%	2.4%	0.9%
% HL students achieving at least 90%	7.4%	3.0%

These three findings have been constant in the years 2001-2018 (n ≈ 1 million students). That more male students always sit the HL LC examination might be a little surprising given that every year from 2003 to 2016 a greater number of female students completed the preparatory HL Junior Cycle examination (Shiel & Kelleher, 2017, p. 75). Allowing for the weighted difference in participation when one compares the percentage of males and females who sit the LC HL examination who achieve the highest grade (SEC, 2019) we can see that a prevailing gender gap still exists and that this gap has widened (Figure 1).



	2001	2002	2003	2004	2005	2005	2006	2007	2008	2009	2010	2011	2012	2013	2014	2015	2016	2017	2018
% Female	10.9	5.5	4.6	7.6	6.7	6.7	5.5	5.9	6.2	5.3	5.7	4.4	1.6	3.3	2.5	3.2	3.4	3.7	3.0
% Male	12.1	8	7.3	10	8.8	8.8	7.4	8.3	8.9	8.2	8.7	6.8	4.5	7.5	5.5	7.1	7.6	8.0	7.4

Figure 1: Comparing the percentage of Higher Level Students achieving at least 90% in the Leaving Certificate Mathematics Examinations 2001-2018 by gender.

The gap appears to change at the time of the phased introduction of the new post-primary mathematics curriculum 2012-2015, colloquially known as *Project Maths*. Prior to this (2001-2012), of those who took the HL paper, on average 44% more males than females achieved the highest grade. Since the new LC HL examination was fully implemented (2015-2018), that ratio has considerably worsened from a gender equality viewpoint (k = 2.9) with 127% more males than females who took the examination achieving this grade.

International context of a gender gap in mathematics

The existence of a gender gap in students' achievement in mathematics is not unique to Ireland. Research in the US found that while there are no mean differences between boys and girls upon entry to primary school, girls lose one-quarter of a standard deviation relative to boys over the first six years of school (Fryer & Levitt, 2010). Robinson and Lubienski (2011) also established that the gender gap in mathematics widens at post-primary level.

There is international evidence that it is among the highest achieving students that the distinction between the sexes is most pronounced. In the TIMSS 2015 (Trends in International Mathematics and Science Study) comparative assessment of Grade 8 student achievement in Advanced Mathematics, for each of the three assigned cognitive domains of knowing, applying and reasoning, males achieved a significantly higher mean score than females (Mullis, Martin, Foy & Hooper, 2016). Similarly, in PISA (Programme for International Student Assessment), a triennial international survey testing the knowledge and skills of 15-year-old students, more male students than female students on average across Organisation for Economic Co-operation and Development (OECD) countries performed at the highest levels in 2015 (Perkins & Shiel, 2016).

Irish students have performed well in these international post-primary mathematics tests. In TIMSS 2015 Irish students ranked 9th out of 39 countries in Grade 8 Mathematics, and in

PISA 2012 Irish students achieved a mean score significantly above the OECD average score and ranked 18th out of 70 participating countries/economies (Perkins & Shiel, 2016). However, at the more advanced levels of mathematics Irish students do less well. In TIMSS 2015 only 7% of Irish students reached the Advanced Benchmark Ireland compared to 43-54% in some of the highest-ranking regions of Singapore, Chinese Taipei and Korea (Mullis, Foy & Hooper, 2016; Clerkin, Perkins & Cunningham, 2016). This is similar to the results of PISA 2012 when just 9.8% of students in Ireland performed at the highest proficiency levels (Levels 5-6) – below the OECD average of 10.7%. A number of countries have significantly higher percentages of students performing at Levels 5-6, including Japan, Korea and Singapore (20.3% -34.8%). This relatively poorer performance of our highest achieving students along with a "topic of concern: space and shape" were identified by Perkins and Shiel (2016, p.51) as having particular implications for teaching and learning.

Focusing on gender differences, in PISA 2015 Irish male students achieved a mean score of 511.6 while females achieved a mean score of 495.4. This difference of 16.1 points is larger than the corresponding OECD average difference of 7.9 (Perkins & Shiel, 2016). In addition, almost twice as many male students in Ireland perform at Proficiency Levels 5-6, compared with female students (12.4% and 6.5% respectively). In this regard, the mean score of male students was similar to the OECD average, but it is of particular concern that our female students were 0.27 SD below the international mean for females. This was similar to TIMSS 2015 where 5% of Irish female students compared to 8% of male students achieved the advanced benchmark (Clerkin, Perkins & Cunningham, 2016). This suggests that gender-gap between the highest achieving males and females is more pronounced in Ireland than in other countries.

POTENTIAL REASONS FOR THE GENDER GAP AT LEAVING CERTIFICATE FOR HIGH-ACHIEVING STUDENTS

The widening of the gender gap in LC HL attainment might be due to several reasons including: affective factors, changes to the nature of the examination, synchronous changes within the educational system, and perhaps gender differences in spatial reasoning.

What has changed?

Beginning in 2012 students passing HL LC mathematics received 25 bonus Third Level entry points. This is recognised as being a major factor in doubling the number of students sitting the LC HL examination from 15.8% in 2011 to 31.5% in 2018 (SEC, 2019). It is likely the attraction of a bonus point reward has increased the proportion of students with lower mathematical ability in the HL population. It is also probable that, on average, HL mathematics class sizes have increased. It could be argued that the effect of these changes on high-achieving students might be gender neutral.

With the phased introduction of the new mathematics curriculum (NCCA, 2013) teachers across the country engaged in substantial professional development focused on methodologies, use of dynamic software, and teaching through problem solving (Shiel & Kelleher, 2017). In parallel, hundreds of out-of-field teachers (Ní Ríordáin & Hannigan, 2011) were up skilled through universities. However, in TIMSS 2015 one fifth of students were still taught by teachers whose main area of study was something other than mathematics, which was considerably larger than corresponding proportions in the highest achieving countries (Clerkin, Perkins & Chubb, 2018).

Perhaps the most influential change was in the HL LC mathematics examination itself. The new LC HL examination phased in between 2012 and 2015 became far less predictable than the 'old' papers as more questions required solving problems in unfamiliar contexts. Also, topics that would have previously been contained in stand-alone questions are now interconnected in expansive, layered questions. Commenting on the overall percentage decrease in the A-rate, the Chief Examiner's Report (SEC, 2015) noted a substantial increase in the number of candidates taking HL and a deliberate attempt to increase the emphasis on problem solving and higher order thinking skills, "Skills that students find difficult to master and teachers may find difficult to instil" (SEC, 2015, p. 9). We might reasonably conjecture that a student's problem solving ability is now more important than ever for examination success, but why are female students in Irish classrooms finding mastery more elusive than their male counterparts?

Affective measures relating to gender performance mathematics

Research has demonstrated that affective measures impact students' performance in mathematics. Many studies have considered the influence of mathematical-gender stereotypes (Good, Aronson, and Harder, 2008; Song, Zuo & Wen, 2017), the gender attitudes of parents and teachers (Gunderson, Ramirez & Levine, 2012; Hyde et al. 2008), and how these negatively impact on female students' performance in mathematics. Other affective factors such as: confidence and grit (Flanagan & Einarson, 2017); self-efficacy, attitude and self-concept (Erturan & Jansen, 2015; Franceschini et al., 2014; Nosek & Smyth, 2011); the role of competition (Niederle & Vesterlund, 2010); and the role of culture (Nollenberger, Rodriguez-Planas & Sevilla, 2016) affect female students' performance in mathematics across many countries. Fryer and Levitt (2010) observed a gender gap that was evident across every strata of society and could not be explained by either less investment by girls in mathematics or low parental expectations. While these considerations may be worthy of further exploration in the Irish context, it is not apparent how any of these effects would have changed considerably between 2012 and 2015, leading to a widening of the gender gap for high-achieving students in LC mathematics.

Gender differences in spatial ability

Researchers have frequently found gender differences in spatial ability in favour of males (Reilly & Newman, 2013). Flaherty's (2005) study, which included Irish participants, found that while gender gaps in spatial ability in favour of males were global phenomena, they are not stable because culture and experience influence these gender differences. Early stage research in Ireland suggests that this spatial ability gap widens as students move through post-primary school (Harding, 2018). This is not unique to Ireland. Mix and Cheng (2012, p. 219) found that for children with visuospatial defects the gap in spatial ability widens over time.

Gender differences in spatial ability might be a contributory factor to the mathematical gender gap being discussed in this paper. Spatial ability is an aspect of intelligence that "depends on understanding the meaning of space and using the properties of space as a vehicle for structuring problems, for finding answers and for expressing solutions" (National Research Council, 2006, p. 3). Mix and Cheng (2012, p. 5) argue: "The relation between spatial ability and mathematical performance is so well established that it no longer makes sense to ask if they are related." Spatial reasoning is a strong predictor of success in mathematics (Casey, Nuttall, Pezaris & Benbow, 1995; Cheng & Mix, 2014; Lowrie, Logan & Ramful, 2016; Moè, 2015; Newcombe, 2013; Wai, Lubienski & Benbow, 2009).

Spatial ability and problem solving ability are related. Spatial reasoning can reduce working memory load and increasing success in solving mathematical 'word problems' (Duffy, 2017). Hill, Laird and Robinson (2014) identified gender differences in working memory in favour of males. When dealing with novel tasks and problems set in unfamiliar contexts, such as the new HL LC examination, students are rewarded less by rote-learning and algorithmic practice and more by spatial ability and application of working memory (Ma, Husain, & Bays, 2014). Students with high levels of spatial ability tend to be much more adept at mentally representing word problems in mathematics which leads to significantly higher success rates in problem solving (Boonen, van Wesel, Jolles & van der Schoot, 2014; Kozhevnikov, Motes & Hegarty, 2007).

Teaching and learning characteristics of the Irish mathematics classroom

Clerkin, Perkins adn Chubb (2018) identify that working on problems for which there was no immediately obvious solution was more common internationally than in Ireland.
Also teachers' ratings of confidence were slightly lower than the TIMSS average in relation to showing students a variety of problem-solving strategies (*Ibid*). This contrasts with the cultural script in Japanese classrooms (Stigler & Hiebert, 1999).

Considering the amount of time students experience mathematics in school, the average annual instructional hours devoted to mathematics, reported by teachers, was 109 in Ireland compared with a TIMSS average of 138. In particular, time spent teaching geometry in Ireland was significantly lower than TIMSS average (Clerkin, Perkins & Chubb, 2018) and this may explain why Irish students underperform in this topic in international tests. In PISA 2012 students in Ireland performed significantly less well on the Space and Shape (i.e. geometry) subscale (Perkins & Shiel, 2013). This weakness in Shape and Space is line with Ireland's relative underperformance on the geometry subscale in TIMSS (Clerkin, Perkins & Cunningham, 2016). More research is required to determine if Irish mathematics students are missing out on activities that would develop their spatial thinking skills and the consequences of this shortcoming.

Participation rates in subjects requiring high levels of mathematical and spatial skills

Spatial skills instruction is found to improve spatial cognition (Sorby, Veurink & Streiner 2018) and improve spatial performance (Uttal, Meadow, Tipton, & Hand, 2013), thereby improving students' problem solving skills. Furthermore, participating in spatial skills instruction has been found to advance outcomes for gifted STEM students (Miler & Halpern, 2012) and eliminate the spatial ability gender gap (Tzuriel & Egozi 2010). Research on spatial skills has demonstrated that exposure to subjects that stimulate spatial thinking result in indirect but long-lasting skills in mental rotation (Moè, 2015), a commonly-assessed aspect of spatial ability. Gender differences in mental rotation skills can disappear if females have the opportunity engage with spatial tasks and are frequently exposed to spatial thinking.

In Ireland, there is a persistent gender imbalance in subjects that may have consequences relating to mathematical achievement including applied mathematics, physics, engineering, technical graphics, woodwork and metalwork (McGrath, 2016). Even though the number of students completing the HL examination in subjects such as applied mathematics, physics

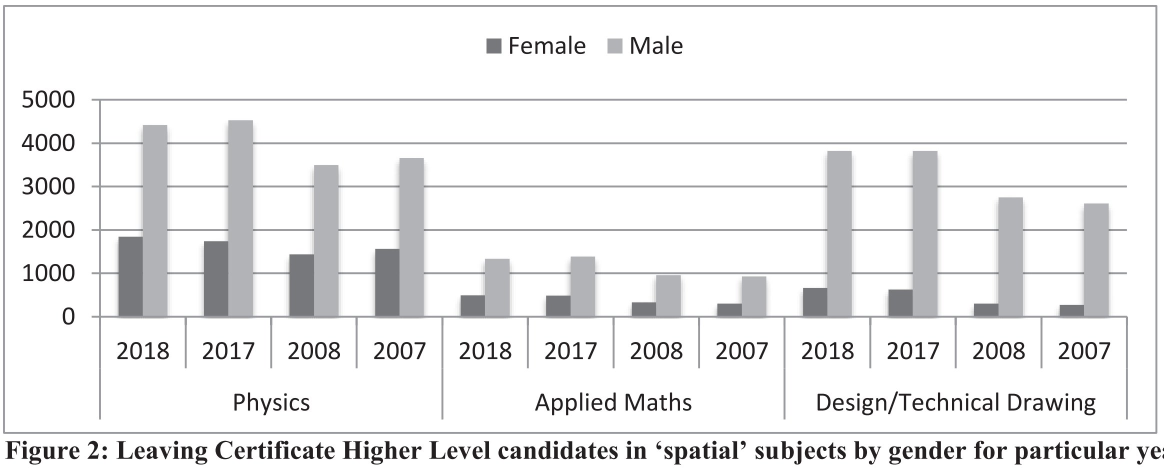


Figure 2: Leaving Certificate Higher Level candidates in 'spatial' subjects by gender for particular years.

and technical drawing has increased in recent years (SEC, 2019), the proportion of female students has not notably changed (SEC, 2019; Figure 2).

Further research on the opportunities for Irish female students to engage in spatial skills development and the impact of such experiences on female students' outcomes in mathematics may be worthwhile.

CONCLUSION

A gender gap at the highest level of attainment in LC HL mathematics seems to have always existed but this paper highlights that it has widened with the phased introduction of the new mathematics examinations in 2012-2015. This widening of the gender gap seems to be connected to changes in the exam, which now requires greater problem solving proficiency.

It is notable that, at the advanced level of mathematics in PISA 2012 and TIMSS 2015, the gender gap disadvantaging Irish female students is larger than average. Yet attainment is higher and the gender gap in mathematics is less pronounced in countries that facilitate more geometry and seem to give greater support for problem solving in their classrooms. An underlying issue of the gender gap for high-achieving students in LC HL mathematics may be cognitive differences in spatial ability between male and female students.

Spatial ability can be developed through spatial skills training and exposure to spatially rich learning experiences in subjects including mathematics (Reilly, Neumann & Andrews, 2017). This has implications for classroom practice (Ontario, 2014) and for teachers whose own comfort level with spatial reasoning is related to students' growth in this area (Gunderson, Ramirez, Beilock & Levine, 2013). Further research is required to support the claim that spatial ability is contributing to a widening gender gap in attainment at LC HL mathematics and to consider appropriate interventions to address this imbalance, thereby supporting equality in gender representation at all levels of mathematics.

REFERENCES

Boonen, A. J. H., van Wesel, F., Jolles, J., & van der Schoot, M. (2014). The role of visual representation type, spatial ability, and reading comprehension in word problem solving: An item-level analysis in elementary school children. *International Journal of Educational Research*, *68*, 15-26.

Casey, M. B., Nuttall, R., Pezaris, E., & Benbow, C. P. (1995). The influence of spatial ability on gender differences in mathematics college entrance test scores across diverse samples. *Developmental Psychology*, *31*(4), 697.

Cheng, Y. L., & Mix, K. S. (2014). Spatial training improves children's mathematics ability. *Journal of Cognition and Development*, *15*(1), 2-11.

Clerkin, A., Perkins, R. and Chubb, E. (2019). *Inside the post-primary classroom: Mathematics and science teaching in Second Year*. Dublin: Educational Research Centre.

Clerkin, A., Perkins, R., & Cunningham, R. (2016). *TIMSS 2015 in Ireland: Mathematics and science in primary and post-primary schools*. Dublin: Educational Research Centre.

Duffy, G. (2017). *Spatial thinking in the engineering curriculum: an investigation of the relationship between problem solving and spatial skills among engineering students*. Doctoral thesis, DIT.

Erturan, S. and B. Jansen. (2015). An investigation of boys' and girls' emotional experience of math, their math performance, and the relation between these variables. *European Journal of Psychology of Education, 30*(4), 421-435.

Ferguson, A., Maloney, E., Fugelsang, J. and Risko, E. (2015). On the relation between math and spatial ability: The case of math anxiety. *Learning and Individual Differences* 39: 1-12.

Flaherty, M. (2005). Gender differences in mental rotation ability in three cultures: Ireland, Ecuador and Japan. *Psychologia*, *48*(1), 31-38.

Franceschini, G., Galli, S., Chiesi, F. & Primi, C. (2014). Implicit Gender–math Stereotype and Women's Susceptibility to Stereotype Threat and Stereotype Lift. *Learning and Individual Differences* 32: 273-277.

Fryer, R. & Levitt, S. (2010). An Empirical Analysis of the Gender Gap in Mathematics. *American Economic Journal* 2 (2): 210-240.

Ganley, C. & Lubienski, S. (2016). Current Research on Gender Differences in Math. Retrieved March 19, 2019 https://www.nctm.org/Publications/Teaching-Children-Mathematics/Blog/Current-Research-on-Gender-Differences-in-Math/.

Ganley, C. & Vasilyeva, M. (2014). The Role of Anxiety and Working Memory in Gender Differences in Mathematics. *Journal of Educational Psychology* 106 (1): 105-120.

Good, C., Aronson, J., & Harder, J. (2008). Problems in the Pipeline: Stereotype Threat and Women's Achievement in High-Level Math Courses. *Journal of Applied Developmental Psychology* 29 (1): 17-28.

Gunderson, E. A., Ramirez, G., Beilock, S. L., & Levine, S. C. (2013). Teachers' spatial Anxiety relates to 1st and 2nd graders' spatial learning. *Mind, Brain, and Education*, 7(3), 196-199.

Harding, R. (2018) What's The Main Reason For The Gender Gap In STEM Careers?. Retrieved March 18, 2019 https://www.rte.ie/eile/brainstorm/2018/0904/991483-whats-the-main-reason-for-the-gender-gap-in-stem-careers/.

Hegarty, M. (2018). Ability and Sex Differences in Spatial Thinking: What does the Mental Rotation Test really Measure? *Psychonomic Bulletin & Review* 25 (3): 1212-1219.

Hill, A., Laird, A. & Robinson, J. (2014). Gender differences in working memory networks: A BrainMap meta-analysis. *Biological Psychology*, 102, 18-29.

Hyde, J., Lindberg, S, Linn, M., Ellis, A. & Williams, C. (2008). Diversity. Gender Similarities Characterize Math Performance. *Science (New York)* 321 (5888): 494.

Kenney-Benson, G., Pomerantz, E., Ryan, A. & Patrick H. (2006). Sex Differences in Math Performance. *Developmental Psychology* 42 (1): 11-26.

Kozhevnikov, M., Motes, M., & Hegarty, M. (2007). Spatial visualization in physics problem solving. *Cognitive Science*, 31(4), 549-579.

Levine, S., Foley, A., Lourenco, S., Ehrlich, S. and Ratliff K. (2016). Sex Differences in Spatial Cognition: Advancing the Conversation. *Wiley Interdisciplinary Reviews. Cognitive Science* 7 (2): 127.

Linn, M. & Petersen, A. (1985). Emergence and Characterization of Sex Differences in Spatial Ability: A Meta-Analysis. *Child Development* 56 (6): 1479.

Lowrie, T., Logan, T., & Ramful, A. (2016). Spatial Reasoning Influences Students' Performance on Mathematics Tasks. *Mathematics Education Research Group of Australasia*.

Ma, W., Husain, M., & Bays, P. (2014). Changing concepts of working memory. *Nature Neuroscience*, 17(3), 347-356.

McGrath, B. (2016). A Report on Science, technology, Engineering and mathematics (STEM) Education. Retrieved from March 18, 2019 https://www.education.ie/en/Publications/Education-Reports/STEM-Education-in-the-Irish-School-System.pdf.

Moè, A. (2016). Does Experience with Spatial School Subjects Favour Girls' Mental Rotation Performance? *Learning and Individual Differences* 47: 11-16.

Mullis, I., Martin, M., Foy, P., & Hooper, M. (2016). TIMSS Advanced 2015 International Results in Advanced Mathematics and Physics. Retrieved March 19, 2019 http://timssandpirls.bc.edu/timss2015/international-results/advanced/

NCCA (2013). Leaving Certificate Mathematics Syllabus. Retrieved from March 19, 2019 https://curriculumonline.ie/getmedia/f6f2e822-2b0c-461e-bcd4-dfcde6decc0c/SCSEC25_Maths_syllabus_examination-2015_English.pdf.

National Research Council. (2006). *Learning to think spatially*. Washington, D.C: National Academies Press.

Newcombe, N. S. (2013). Seeing relationships: Using spatial thinking to teach science, mathematics, and social studies. *American Educator*, 37(1), 26.

Ní Ríordáin, M. & Hannigan, A. (2011). Who teaches our students mathematics at post-primary education in Ireland? *Irish Educational Studies*, 30 (3):289-304.

Niederle, M. & Vesterlund, L. (2010). Explaining the Gender Gap in Math Test Scores: The Role of Competition. *The Journal of Economic Perspectives* 24 (2): 129-144.

Nollenberger, N., Rodríguez-Planas, N. & Sevilla, A. (2016). The Math Gender Gap: The Role of Culture. *The American Economic Review* 106 (5): 257-261.

Nosek, B. & Smyth, F. (2011). Implicit Social Cognitions Predict Sex Differences in Math Engagement and Achievement. *American Educational Research Journal* 48 (5) 1125-1156.

Nosek, B., Frederick, L. Sriram, N. Nicole Lindner, M., Thierry Devos, Alfonso Ayala, Yoav Bar-Anan, et al. (2009). National Differences in Gender-Science Stereotypes Predict National Sex Differences in Science and Math Achievement. *Proceedings of the National Academy of Sciences of the United States of America* 106 (26): 10593-10597.

Ontario. (2014). Paying attention to spatial reasoning: Support document for paing attention to mathematics education K-12. Retrieved June 15, 2019, http://www.edu.gov.on.ca/eng/literacynumeracy/lnspayingattention.pdf.

Perkins, R & Shiel, S. (2016). *A Teacher's Guide to PISA Mathematics and Problem Solving: Findings from PISA 2012*. Dublin: Educational Research Centre.

Perkins, R., Shiel, G., Merriman, B., Cosgrove, J., & Moran, G. (2013). Learning for life: The achievements of 15-year-olds in Ireland on mathematics, reading literacy and science in PISA 2012. Dublin: Educational Research Centre.

Reilly, D., & Neumann, D. (2013). Gender-role differences in spatial ability: A meta-analytic review. *Sex Roles*, *68*(9-10), 521-535.

Reilly, D., Neumann, D. L., & Andrews, G. (2017). Gender differences in spatial ability: Implications for STEM education and approaches to reducing the gender gap for parents and educators. In *Visual-spatial ability in STEM education* (pp. 195-224). Springer.

Robinson, J. & Lubienski, S. (2011). The Development of Gender Achievement Gaps in Mathematics and Reading during Elementary and Middle School: Examining Direct Cognitive Assessments and Teacher Ratings. *American Educational Research Journal* 48 (2): 268-302.

State Examinations Commission. (SEC). (2019). State Examination Statistics. Retrieved March 19, 2019 https://www.examinations.ie/statistics/

Selin, E. & Brenda, J. (2015). An Investigation of Boys' and Girls' Emotional Experience of Math, their Math Performance, and the Relation between these Variables. *European Journal of Psychology of Education* 30 (4): 421-435.

Shiel, G. & Kelleher, K. (2017). An Evaluation of the Impact of Project Maths on the performance of students in Junior Cycle Mathematics. Dublin: Educational Research Centre/NCCA.

Stigler, J. W., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving education in the classroom*. Free Press.

TIMSS. (2015). Achievement in Advanced Mathematics Cognitive Domains By Gender. Retrieved March 10, 2019 http://timssandpirls.bc.edu/timss2015/international-results/advanced/timss-advanced-2015/mathematics/achievement-in-content-and-cognitive-domains/achievement-in-cognitive-domains-by-gender/.

Tzuriel, D. & Egozi, G. (2010). Gender Differences in Spatial Ability of Young Children: The Effects of Training and Processing Strategies. *Child Development* 81 (5): 1417-1430.

Uttal, D., Meadow, N., Tipton, E. & Hand, L. (2013). The Malleability of Spatial Skills: A Meta-Analysis of Training Studies. *Psychological Bulletin*139 (2): 352-402.

MEI 7
Institute of Education,
Dublin City University
2019

dcu.ie/IoE/MEI

ISBN 978-1-873769-95-9
9 781873 769959