Vacuum Fluctuations, Zitterbewegung and Klein Paradox(*)

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Abstract. This paper was written to young researches in Physics. We intend to briefly show that the Zitterbewegung is very difficult to be measured in real particles and that, on the other hand, the spectrum of measured masses of neutral vector mesons can be reasonably described taking into account the Klein Paradox effects.

***Key words**: Dirac´s relativistic equation; vacuum fluctuations; vector mesons*.

(I)Introduction.

As is well known paper,[1] Dirac equation successfully merges quantum mechanics with special relativity. It provides a natural description of the electron spin, predicts the existence of antimatter and is able to reproduce accurately the spectrum of the hydrogen atom. The appearance of positive and negative energy states of free electrons (which is the essence of the *vacuum fluctuations*) predicted by the relativistic Dirac´s equation[2-4] can be considered as the natural transition to the Quantum Field Theory (QFT). It also predicts unexpected effects of relativistic quantum particles, such as "Zitterbewegung"[5,6] and "Klein´s Paradox."[7] These and other *vacuum fluctuation* phenomena are key fundamental examples for understanding relativistic quantum effects. Of course, a precise description of the relativistic quantum phenomena can only be obtained with the QFT. Relativistic quantum equations are only able to give general features of the vacuum fluctuations effects. Since the publication of Dirac´s relativistic quantum equation, almost one century ago, a large number of papers on vacuum fluctuation effects can be found in the literature. So, here only few references are cited. In **Section 1**is presented an analysis of the Zitterwbewegung done by Gerritsma et al.[1] using a quantum simulation of the 1-dimensional Dirac equation. In **Section 2** is shown how the spectrum of measured masses of neutral vector mesons can be reasonably described taking into account Klein Paradox effects. Lifetimes estimations of the mesons are also done.

(*) Dedicated "in memoriam "N.C.Fernandes.

(1) Zitterbewegung.

To investigate Zitterbewegung, Gerritsma et al.[1] have recently presented a proof-of-principle quantum simulation of the one-dimensional Dirac equation. In this simulation analysis they have considered a single $^{40}Ca^{+}$ ion trapped in a linear Paul trap.[8] They assumed[1] that the ion obeys a Dirac equation in 1 + 1 dimensions, that is,[1-4]

$$i\hbar\partial\psi/\partial t = H\psi = (cp\sigma_x + mc^2\sigma_z)\psi \qquad (1.1),$$

with only one motional degree of freedom, related to positive- and negative-energies $E = \pm (p^2c^2 + m^2c^4)^{1/2}$. According to Eq.(1.1) the evolution of the electron is described by the operator

$$x(t) = x(0) + pc^2H^{-1}\,t + i\xi(e^{\,2iHt/\hbar} - 1) \qquad (1.2),$$

where the operator $\xi = (\hbar c/2)(\sigma_x - ipcH^{-1})/H$. The first two terms represent the evolution that is linear in time, as expected for free particle, whereas the third, oscillating, term may induce the Zitterbewegung with an amplitude that depends on the expected values of ξ.

They have demonstrated[1] that this one-dimensional Dirac dynamics for a free particle shows Zitterbewegung and several of its counterintuitive quantum relativistic features. They also concluded that their experiment serves as a first step to explain more complex ("real") quantum situations.

(2) Klein Paradox and the Masses of Neutral Vector Mesons.

In few words, the Klein Paradox [2] appears in the framework of a relativistic quantum equation when a relativistic particle with inertial mass m interacts with a potential barrier V_o and $V_o \geq mc^2$. When this occurs, there is an unexpected interference effect between the positive and negative energies states of the particle and it "surpass" the barrier.

Differently to what happens with the Zitterbewegung, we have shown in preceding papers,[9-11] using Dirac´s relativistic equation, that the measured meson masses can be reasonably estimated taking into account the Klein paradox. As these calculations are seen in details elsewhere,[9-11] only a brief review of our results is presented here.

We have assumed that mesons are quark-antiquark systems that interact via a static neutral vector gluon field $V_\nu(x)$, where $\mathbf{V} = 0$ and

$V_4 = V(r) = Kr^2/2 - \Delta$, where K is the harmonic constant and Δ another constant which subsumes, in a simple way, the remaining interaction between quarks. Thus, the radial Dirac´s equation, in the reduced mass system, becomes written as[9]

$$df(r)/dr = (\chi/r)\, f(r) + [\, (\mu c^2 - E^{(\mu)})/\hbar c + V(r)/\hbar c]\, g(r)$$

and (2.1),

$$dg(r)/dr = - (\chi/r)\, f(r) + [\, (\mu c^2 + E^{(\mu)})/\hbar c - V(r)/\hbar c]\, f(r)$$

where g(r) and f(r) are the large and small components, respectively, $\mu = m/2$ is the reduced mass of the two quarks, each one with mass m, $\chi = -(\ell+1)$ if $j = \ell +1/2$ and $\chi = +\ell$ if $j = \ell-1/2$ and $E^{(\mu)}$ is the energy eigenvalue. Note that the total angular momentum J of the meson states is obtained in our scheme by coupling the 1/2 unit of spin to the angular momentum $j = \ell \pm 1/2$. Now, putting $K = \mu\omega^2$, $\xi = (\mu\omega/\hbar)^{1/2} r$ and $E^{(\mu)} = \eta\hbar\omega + \mu c^2 - \Delta$, equations (2.1) become:

$$df(\xi)/d\xi = (\chi/\xi)\, f(\xi) + [\, \varepsilon_- + A\xi^2]\, g(\xi)$$

and (2.2),

$$dg(\xi)/d\xi = - (\chi/\xi) g(\xi) + [\, \varepsilon_+ - A\xi^2]\, f(\xi)$$

where $\varepsilon_- = -\eta\varepsilon$, $\varepsilon_+ = \eta\varepsilon + 2/\varepsilon$, $A = \varepsilon/2$ and $\varepsilon = (\hbar\omega/\mu c^2)^{1/2}$. So, the inertial mass $M^{(\mu)}$ becomes given by

$$M^{(\mu)}c^2 = E^{(\mu)} - \mu c^2 + 2mc^2 = \eta\hbar\omega + 2mc^2 - \Delta. \qquad (2.3).$$

Eqs.(2.2) are solved by expanding the large $g(\xi)$ and small components $f(\xi)$ into power series of ξ.[10] In **Figure 1** are shown, for example, $g(\xi)/\xi$ as a function of ξ for $n = \ell = 0$ and $\varepsilon = 0, 1.0, 2.0$.

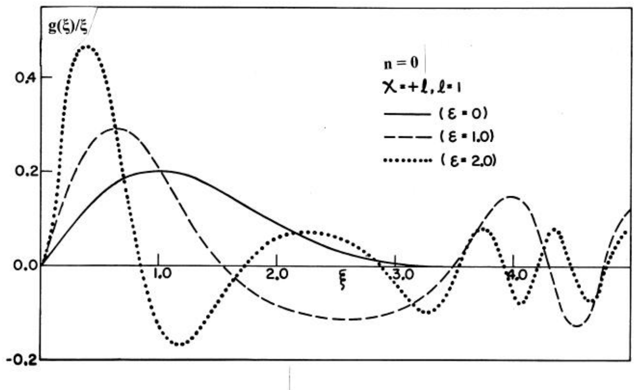


Figure 1. Large component $g(\xi)/\xi$ as a function of ξ for $n = 0$, $\varepsilon = 0,1,2$ and $\ell = 1$.

In the non-relativistic limit, that is, when $\varepsilon \to 0$, we verify that $f(\xi) \to 0$ and that $g(\xi)$ obeys the radial equation for the non-relativistic harmonic oscillator:[2]

$$[d^2/d\xi^2 - \ell(\ell+1)/\xi^2 - \xi^2 + 2\eta]\, g(\xi) = 0 \quad . \qquad (2.4).$$

Solving Eq.(2.4),[2] we verify that total energy of the system is given

$$E_{n\ell} = M^{(n,\ell)} c^2 = \hbar\omega\,(2n+\ell+3/2) + 2mc^2 - \Delta.$$

In this non-relativistic limit, each state is characterized by two numbers n and ℓ. Each state really depends solely on the combination of the two quantum numbers $2n + \ell = \Lambda$. We can,[2] therefore, call $\Lambda = 0,1,2,...$ the principal quantum number. Each Λ larger than 1 can be realized by several combinations of the values n and ℓ, and the energy levels (n,ℓ) with $\Lambda \geq 2$ are, therefore degenerate.

In all cases $\varepsilon \neq 0$, for large ξ values, that is, larger than a critical value ξ_c ,we verify that $g(\xi) = i\, f(\xi) = \varphi \exp[i(\varepsilon\xi^2/6 + \theta)]$ meaning that there is no bound state for the quark- antiquark system (see **Figure 1**). This is exactly the Klein Paradox phenomenon. Note that we are interpreting the amplitude $|\varphi|^2$ to be proportional to probability to find the system in the unbounded state. When $\varepsilon \neq 0$, analyzing the functions $g(\xi)$ and $f(\xi)$, we verify that the decay process occurs only for a few *particular (discrete)* values of η. These values, indicated by $\eta_{critical} = \eta^*_{(n,\ell)}$ are determined numerically as a function of $\varepsilon = (\hbar\omega/\mu c^2)^{1/2}$, n and ℓ. In **Figure 2** is shown[9] $\eta^*_{(n,\ell)}$ as function of ε for $\ell = 0$ and n = 0,1,2,...,6.

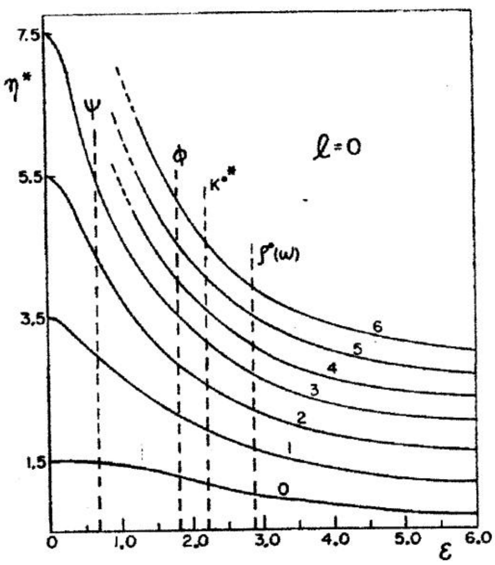


Figure 2.

Figure 2. The parameter $\eta^*_{(n,\ell)}$ of the nth resonance as a function of ε for $\boldsymbol{\ell = 0}$. The labels n = 0,1,2,..,6 indicate the fundamental, first, second state and so on, respectively. The vertical dashed lines correspond to the mesons $\rho^o(\omega)$, Φ, K^{o*} and ψ.

So, the resonant meson masses, according to Eq.(2.3), would be given by

$$M^{(n,\ell)} c^2 = \eta^*_{(n,\ell)}\hbar\omega + 2mc^2 - \Delta. \qquad (2.5).$$

The masses $M^{(n,\ell)}$ of the n^{th} resonances (n = 0,1,2..) with ℓ = 0,1,2,... corresponds, respectively, to the fundamental, first, second excited states and so on of the mesons.

The spectrum of the predicted meson masses (ω, ρ^o, Φ, K^{o*},ψ) compared with the experimental results are shown in **Figures 3,4, 5** and **6.** The values of quark masses, frequencies ω and Δ, for the different mesons, used to estimate the masses $M_{(n,\ell)}$ are seen in reference 9.

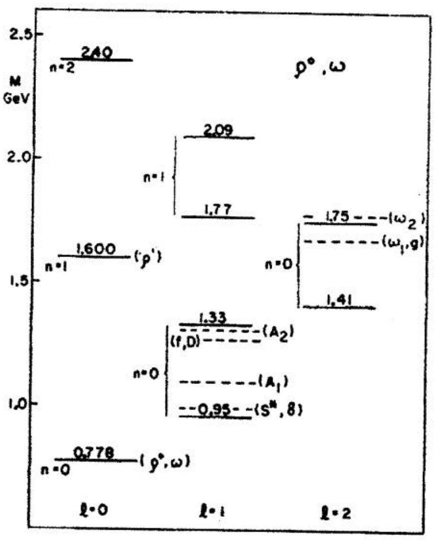


Figure 3

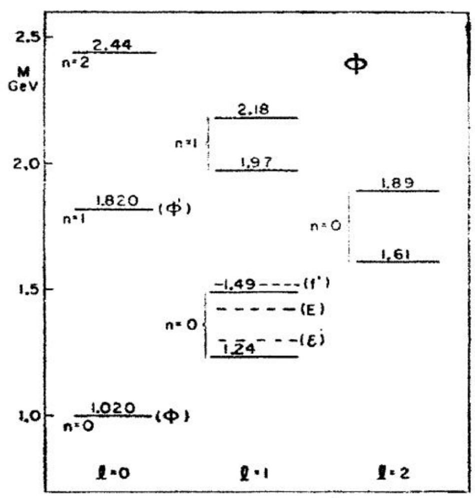
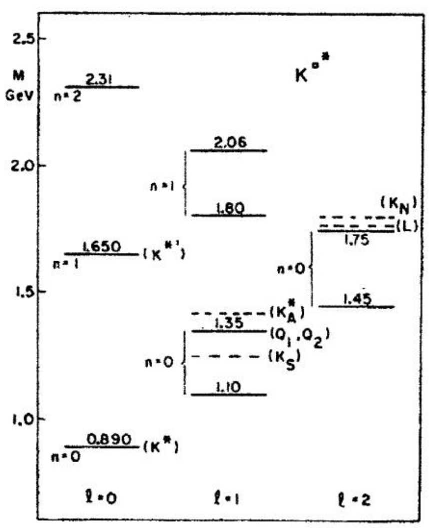


Figure 4

Figure 3.The mass spectrum for the ρ^o and ω mesons. **Figure 4**. The mass spectrum for Φ mesons. Theoretical predictions are indicated by (—) and the experimental results by (-----).



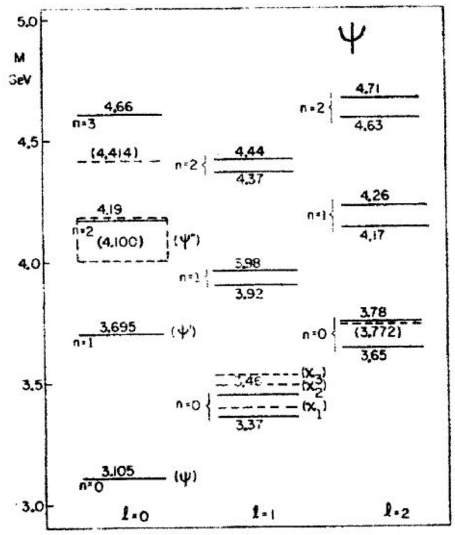


Figure 5. **Figure 6.**

Figure 5. The mass spectrum of the K^{o*} mesons. **Figure 6.** The mass spectrum of the ψ mesons. Theoretical predictions are indicated by (—) and the experimental results by (-----).

Finally, making an analogy with the theory of alpha-decay[11] of nuclei the lifetimes τ of the resonances have been estimated assuming that the probability to find the system in an unbound state is proportional to $|\varphi|^2$. Taking into account the relativistic currents we have shown that τ is given by $\tau \sim \hbar\sqrt{\eta}/(\mu c\varepsilon\backslash|\varphi|^2)$. We have found τ in range from 10^{-23} up to 10^{-20} s, which seem reasonable values, given the crudeness of our approach.

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