Application of Linear Programming for Profit Maximization: A Case of Paints Company, Pakistan

Author's Details:
(1) **Zaman Haider-**Manager NCBA and E WCC Lahore Pakistan
(2) **Rizwan Fareed-**University of Gujrat, Lahore Pakistan
(3) **Muhammad Bilal Tariq -**University of Gujrat, Lahore Pakistan
(4) **Syed Usman-**GC University Lahore Pakistan
(5) **Naeem Ud din-**CIMS Lahore Pakistan
(6) **Miss Sehrish khan-**University of Gujrat, Lahore Pakistan

Abstract
The purpose of the study is to highlight the effectiveness of linear programming in determination of optimal combination of various products that an organization produces to maximize contribution. For this purpose, data for the month of July 2016 of a paint company in Pakistan has been obtained for one of the main products "Plastic Emulsion" that is produced in three different sizes (quarter, gallon and drummy). The simplex method is used to determine the optimal mix of these sizes to be produced to maximize contribution. The results show that the company can earn maximum contribution by only investing its resources (raw material) in the production of gallon and producing 444 units of it thus generating contribution Rs. 162038. This study will recognize the this company and also to the other manufacturing companies, particularly in Pakistan, with the effectiveness of linear programming for making decisions about optimal combination of products to be produced to obtain maximum return.
Key words: *linear programming, paint company, Pakistan*

Introduction

There is a strict shortage of production inputs that the industries are facing now a day that ultimately results in low output. A firm can contribute to the increase production in real sector and to the economic growth through its managerial decisions about maximum output. Thus the managerial decisions are always directed towards finding the right way to maximize profit.

The production-oriented industries require optimal management decisions about production levels due to the present competitive pressure. Due to this pressure, several management theories are promulgated to resolve practical problems e.g. industry specific problems and environmental problems faced by the industries that necessitated the development of several mathematical techniques. Linear programming is one of the most popular that is based on mathematical approach to reach out the optimal solution with constraint resources. There subsist several other mathematical approaches like OLS model, markov analysis, time series analysis and inventory control system.

Linear Programming may be termed as a mathematical approach for identifying the optimum combination of products to be produced to maximize profit or minimize cost as per objective within the given resources. Linear programming is also used in operations management to find optimum solution of several types of problems like allocation of resources, transportation problems and responsibility problems where linear programming works to choose best course of action among several alternatives (Yahya, 2004). Linear programming is a term that emcompases mathematical techniques targeted to optimizing outcome by amalgamating resources (Lucey, 1996).

The term "linear" here implies one of the assumptions of linear programming i.e. "linearity of variables". For being linear by the variables, we mean that they have straight line or proportional relation. However, the term "programming" refers to an iterative process of moving towards the best solution from the state of "Do nothing".

Linear programming model consists of a linear objective function and several constraint functions. The model is formulated to find the optimum product mix with provided resources (constraints). The objective function in LP model is usually finding the product mix for maximizing contribution or profit or minimizing cost or variable cost whereas, constraint functions are set of restrictions that limit the production at specific level. For example, limited quantity of material availability, limited number of labour hours and limited numbers of machines hours are some of the commonly faced constraints. Moreover, optimal product mix is that point that is achieved with the help of linear programming justifying the constraints and achieving the objective function.

Linear programming is extensively used in many areas of study. Most pervasively, it is used to find solution to engineering, economics and business problems. Some organizations where linear programming is used are transportation companies and manufacturing companies etc.

LP modelling solves particular mathematical problems by forming specific rules that deals with the allocation of limited resources under strict technological or practical restrictions when certain course of action has to be chosen (Andrade, 1990). The simplest form of linear programming for profit maximization is:

$$Maximize\ (Y) = B^T Z \qquad subject\ to: \qquad AZ \leq C$$

Where Z denotes vector of variables, B and A are coefficient's vectors of matrices that are known. The expression "$Maximize\ (Y) = B^T Z$" is the objective function that is to be maximize and the expression $AZ \leq C$ is the vector of constraints to be met while optimizing objective function**.** When there are "n" decision variables and "m" constraints, LP can represent the model in mathematical form either for minimization (e.g. cost or labour hours) or maximization (e.g. profit or contribution) (Corrar and Teophilo, 2003), (Salau, 1998). The objective function in LP modelling is:

$$\boldsymbol{Max(Y) = B_1Z_1 + B_2Z_2 + B_3Z_3 + \dots\dots\dots. + B_nZ_n}$$

The constraint functions are:

$$\boldsymbol{A_{11}Z_1 + A_{12}Z_2 + A_{13}Z_3 + \dots\dots\dots\dots. + A_{1n}Z_n\ (\leq or\ \geq)C_1}$$

$$\boldsymbol{A_{21}Z_1 + A_{22}Z_2 + A_{23}Z_3 + \dots\dots\dots\dots. + A_{2n}Z_n\ (\leq or\ \geq)C_2}$$

$$\boldsymbol{A_{m1}Z_1 + A_{m2}Z_2 + A_{m3}Z_3 + \dots\dots\dots\dots. + A_{mn}Z_n\ (\leq or\ \geq)C_m}$$

Non- negative variables are:

$$\boldsymbol{Z_1, Z_2, Z_3, Z_n \geq 0}$$

Objectives of the study

The objectives of this study are:

1. To suggest an optimal product mix that would maximize contribution of "The company"
2. To highlight the distinctiveness of linear programming modelling at firm level as an optimization technique
3. To suggest the manufacturing concerns to use linear programming for determining their optimal product mix.
4. To contribute in literature about linear programming with reference to Pakistan as there is massive research gap in this area.

For the purpose of accomplishing these objectives, data has been collected from a renowned paint manufacturing company. For the purpose of secrecy, we will use "The Company" instead of real name of that company. The company produces various types of paints. The records of company show that the product named "Plastic Emulsion" is the main product in terms of number of units to be sold and thus generating contribution. The product is made in three sizes: Quarter (1 kg), Gallon (3.64 kg) and Drummy (14.56 kg). The data used in the study is for the month of July 2016 obtained from company records and from interviews from Manufacturing department personal and Accounts department personnel. The purpose of this study is to know what number of each size of Plastic Emulsion to be manufactured and sold to earn maximum contribution.

Theoretical Background

There are contrastive views of researchers about relevance of LP modelling to various managerial decisions. These views are indulged over a long time that improved the LP technique applicability in solving business decision making problems. The economic literature recognize the importance of linear programming particularly for the better planning of developing countries that usually have scarce resources where linear programming assist in apportioning these truncated resources in obtaining optimal solution.

Evidencing from the history, LP technique is a mathematical tool that was enrooted by George Dantzig, a mathematician, in 1947 for rationalizing logistics for U.S. Air Force. Later, he argued to use this technique to solve business problems. He also developed simplex method that was improved version of linear programming (Dantzig, 1993).

In case of allocation problems, there are plenty of activities that are required to be done, being various alternative ways of doing them, with limited resources to be allocated among these activities, then management has to confront with the dilemma of how effectively integrate these resources and activities to maximize output. This problem is termed as optimization problem and can be construed with mathematical programming. Charles & Cooper (1963) termed LP modelling as a single objective optimization approach as it exerts to bring out single objective of either maximization (profit or contribution) or minimization (cost or time). Similarly, Gupta and Hira (2009) argue that LP modelling wields to optimize a linear function termed as objective function contingent on aggregation of linear equations known as constraint functions.

Dowing (1992) recognized the superiority of linear programming over many other optimization techniques like Lagrangian method and Graphic method. Lagrangian method can be used with one constraint, Graphical method can be used with two constraints and linear programming can be used with plenty of constraints. Backing this argument, Dwivedi (2008) conceives that as LP modelling assist in obtaining optimum solution in constraint problems, it is of inordinate importance for business decision making. He argued that linear programming requires three particularizations including objective particularization, constraints particularizations and non-negativity condition. Authenticating this view, many other authors gave the general LP model specification for example Dowling (1992), Henderson et al (2003), Dwivedi (2008) and Koutsoyiannis (1979).

Turban and Meredith (1991) also recognize the importance of linear programming as a tool of decision making in management science. Management science techniques are made of three ingredients: (1) decision variables that cannot be controlled, (2) environment factors that are also cannot be controlled and (3) result variables. The LP model is also made up of these ingredients with different terms.

LP modelling is claimed to be an operations research methodology by many researchers for example by Wagner (2007) and Lucey (2002). They posit that LP is one of the most acknowledged techniques of operations research. Other areas where LP can be used include: airlines, energy planning, education, portfolio management, transportation problems and scheduling etc.

According to Sarggeaunt (1965), there are three main problems where linear programming is used including blending and mixing, distribution cost and planning problems. He also stated the other areas of applicability of linear programming for example decision of plant location, and staff management. Identical list of problems have been provided by Emory and Niland (1968) where linear programming can be productively used. These include: concoction of gasoline stocks, inception of chemical products, commingling of fertilizers and production of cement etc. Moreover, Hillier and Lieberman (2001) maintains that apart from allocating resources, there are many other appliances as well.

Emory and Niland (1968) catalogued difficulties in adopting linear programming e.g. identification of essential constraints (row equations) and possible choices (column vectors) and presentation of these rows and columns in linear equations. The authentic definition of the problem requires deep understanding of the LP technique and that of company's operations and problem to be solved. Another problem may arise in necessary data collection as experiences shows that it is always time consuming and costly to collect data for linear programming. Another problem noted by Turban (1993) is that uncertainties about the behaviour of variables like commodity prices make it difficult for the managers to choose the best from several alternatives

According to Kurtz (1992), linear programming cannot be used in several organizational problems such as employee attitudes and ramifications of strikes. Furthermore, the managers who are proficient in its accurate models also refrain from its use a they are unaware of its applicability.

Besides these problems, linear programming has been extensively used in many areas of study. Taha (1992) applied linear programming in poultry farm to determine accurate proportion of calcium, fibre and protein in food. Adam et al (1993) implemented linear programming on Multi-Band Enterprises to determine the optimal combination of citizens band radios and portable radios. Murugan and Manivel (2009) applied linear programming on textile industry with linear interactive optimizer software to determine the optimal numbers of each of three of its products to be produced to maximize profit of the organization. Adeyemo and Otiero (2009) extended the scope of applicability of linear programming from management sciences to environmental sciences and conducted study in South African Rand to maximize revenue where sixteen types of crops were implanted in an area of 2500ha. Kareem and Aderoba (2008) attempted to demonstrate the potency of linear programming by applying it in determination of optimal crew required for maintenance in Akure (cocoa processing company), Nigeria. The application of linear programming is not restricted to the manufacturing industries, it also works for service industry e.g. banks. Balbirer (1981) used linear programming to make financial planning of bank (Central Carolina Bank-CCB) to maximize shareholders' returns. Cohen and Hammer (2013) used linear programming for asset management of bank. Fielitz and Loeffler (2013) implemented linear programming for liquidity management of banks. Guven and Persentili (1997) used linear programming for balance sheet management of a bank i.e. determining the optimal proportion and combination of assets and liabilities.

Methodology

In order to apprehend optimal combination from various categories of Plastic Emulsion, Simplex method is used. Linear programming converts the data into objective function (in terms of contribution per unit) and relevant constraints functions (in terms of material quantity per unit) which are as follows:

$$Max(Y) = 98Z_1 + 365Z_2 + 1380Z_3$$

Where Z_1, Z_2, Z_3represent Plastic Emulsion with size Quarter, Gallon and Drummy respectively. The coefficients represent contribution per unit with respective products. The constraints are formed with major material used to manufacture these products that are:

$0.02Z_1 + 0.06Z_2 + 0.28Z_3 \leq 27$ $\quad$ $np = 6$

$0.05Z_1 + 0.15Z_2 + 0.7Z_3 \leq 68$ $\quad$ $Tylose$

$0.014Z_1 + 0.042Z_2 + 0.196Z_3 \leq 19$	$Amonia$
$0.013Z_1 + 0.039Z_2 + 0.182Z_3 \leq 18$	$Sodium$
$0.034Z_1 + 0.102Z_2 + 0.476Z_3 \leq 46$	$Polyrone$
$0.012Z_1 + 0.036Z_2 + 0.168Z_3 \leq 16$	$Margal$
$0.14Z_1 + 0.42Z_2 + 1.96Z_3 \leq 189$	GP
$0.01Z_1 + 0.03Z_2 + 0.14Z_3 \leq 14$	$Foam$
$0.7Z_1 + 2.1Z_2 + 9.8Z_3 \leq 945$	Tio

Where coefficients represent quantity of material to be used to manufacture each product and constant represents maximum quantity available of each type of material.

The first step in simplex method is the introduction of slack variables and converting inequalities into equalities (fig. 1). In second step, initial simplex tableau is formed (Table 1). The table shows the state of "do nothing". The negative values in the last row under Z_1, Z_2 and Z_3 show the loss of contribution from each product if such product is not produced. From this table, pivot element, pivot row and pivot column is chosen. The column under Z_3 is the pivot column as it has the largest negative value i.e. (-1380). Then, all constants are divided with coefficients of pivot column. Since $(\frac{16}{0.168})$ provides the smallest ratio, its row becomes the pivot row. Finally, coefficient at the intersection of pivot row and pivot column is termed as pivot element i.e. (0.168). In third step, the pivoting process starts in which the pivot row is divided by pivot element (Table 2). Having reduced to pivot element to 1, the next step is to clear the pivot column (Table 3). The initial results in Table 3 show that by producing 95 units of Z_3 i.e. Drummy. But it is not optimal decision as the company is still losing from Z_1 and Z_2 (see values -1.4 and -69.68 in the last row). Therefore, iterative process continues and from table 3, again pivot column is selected that is Column under Z_2 that has maximum negative value (-69.68). Then pivot row is selected with the same process as previous with smallest ratio (i.e.$\frac{95}{0.214}$). Finally, the pivot element is selected again that is now 0.214. Then, pivoting process will be undertaken again (Tables 4 and 5). Then final results obtained in table 5 show that the optimal decision to maximize contribution would be to produce 444 units of Z_2 (Gallon). At this point the overall contribution is Rs. 162,038 that is higher than the previous state of Rs. 131,100. The values of slack can be read from the table 4 and 5 that can be used in other products of the company.

Conclusion

The study is conducted on a paint company in Pakistan. The purpose of the study is to determine the optimal quantity to be produced from three sizes (quarter, gallon and drummy) of one of the main product of the company "Plastic Emulsion". For this purpose, a well known mathematical tool Simplex Method is used. The rationale for the study is to highlight the distinctiveness of linear programming modelling at firm level as an optimization technique, to suggest the manufacturing concerns to use linear programming for determining their optimal product mix and to contribute in literature about linear programming with reference to Pakistan as there is massive research gap in this area. The results of simplex method show that the company can earn maximum contribution Rs. 162038 by producing only Gallon. The values of slack are also important as the company can put these resources left after producing Gallon to other projects of the company. These findings would be helpful for the company to maximize its contribution not only for this month but the company can use this technique throughout its existence. Additionally, this study would insist other companies to utilize this technique to optimize their financial performance.

References

1. Adam, 1993., "Production and Operations Management", Prentice-Hall of India Private Limited, New Delhi

2. Adeyemo, J. and F. Otiero, 2009, "Optimizing Planting Areas using Differential Evolution and Linear Programming" *International Journal of Physical Sciences*, vol.4 pp.212-220

3. Andrade, E. L. (1990): Introdução à Pesquisa Operacional. LTC, Rio deJaneiro.

4. Charles,Cooper & Henderson, 1963, "An Introduction to Linear Programming", John Willy, New York

5. Corrar, Teophilo L., Carlos Renato (2003): Pesquisa Operacional para Decisão em Contabilidade e Administração. Editora Atlas, Rio de Janeiro.

6. Dantzig, G.,1993, "Computational Algorithm of the Revised Simplex Methods", RAND Memorandum RM-1266

7. Dowing, E.T., 1992, "Theory and Problems of Introduction to Mathematical Economics", Schaum's Outline Series, McGraw-Hall Inc., New York

8. Dwivedi, D., 2008, "Managerial Economics", VIKAS Publishing House PVT Limited, New Delhi

9. Emory, W and P. Niland, 1998, "Making Management Decisions", Oxford & IBH, New Delhi

10. Gupta, P.K and D.S. Hira, 2009, "Operations Research", S. Chand Company Limited, New Delhi

11. Henderson, J and R. Quandt, 2003, "Microeconomic Theory: A Mathematical Approach", McGraw-Hill Book Co., Tata, New Delhi

12. Hillier, F and G. Lieberman, 2001, Introduction to Operations Research", McGraw-Hill Book Co., New York

13. Kareem,B and A. Aderoba, 2008, "Linear Programming based Effective Maintenance and Manpower Planning Strategy", *International Journal of the Computer, the Internet and Management*, vol.16 pp.26-34

14. Koutsoyiannis, A.,1987, "Modern Microeconomics", English Language Book Society, Macmillan Press Ltd.

15. Kurtz, D et al, 1992, "Principles of Management", McGraw-Hill Inc., USA

16. Lucey, T., 2002, "Quantitative Techniques", Bookpower, London

17. Murugan, N and S. Manivel, 2009, "Profit Planning of an NGO run Enterprises using Linear Programming Approach*", International Research Journal of Finance and Economics,* Issue 23, 443-454

18. Sargeaunt, M, 1965, "Operation Research – An introduction", Macmillan New York

19. Taha, H., 2008, "Operations Research: An Introduction", 110001 PHI Learning Private Limited, New Delhi

20. Turban, J and T. Meredith, 1991, "Principles of Management Science", Prentice Hall and India Private Ltd.

21. Wagner, H., 2007, "Principles of Operations Research with Application to Managerial Decision, Prince-Hall of India Private Limited, New Delhi

22. Yahya, W. B. (2004); Determination of Optimum Product Mix at Minimum Raw Material Cost, Using Linear programming. *Nigerian Journal of Pure and Applied Sciences, Vol.19:1712-1721.*

APPENDIX

Figure 1

$0.02Z_1 + 0.06Z_2 + 0.28Z_3 + s_1 = 27$	$np = 6$
$0.05Z_1 + 0.15Z_2 + 0.7Z_3 + s_2 = 68$	*Tylose*
$0.014Z_1 + 0.042Z_2 + 0.196Z_3 + s_3 = 19$	*Amonia*
$0.013Z_1 + 0.039Z_2 + 0.182Z_3 + s_4 = 18$	*Sodium*
$0.034Z_1 + 0.102Z_2 + 0.476Z_3 + s_5 = 46$	*Polyrone*
$0.012Z_1 + 0.036Z_2 + 0.168Z_3 + s_6 = 16$	*Margal*
$0.14Z_1 + 0.42Z_2 + 1.96Z_3 + s_7 = 189$	*GP*
$0.01Z_1 + 0.03Z_2 + 0.14Z_3 + s_8 = 14$	*Foam*
$0.7Z_1 + 2.1Z_2 + 9.8Z_3 + s_9 = 945$	*Tio*

Initial Simplex Table **Table 1**

Z_1	Z_2	Z_3	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	C
0.02	0.06	0.28	1	0	0	0	0	0	0	0	0	27
0.05	0.15	0.7	0	1	0	0	0	0	0	0	0	68
0.014	0.042	0.196	0	0	1	0	0	0	0	0	0	19
0.013	0.039	0.182	0	0	0	1	0	0	0	0	0	17.5
0.034	0.102	0.476	0	0	0	0	1	0	0	0	0	46
0.012	0.036	0.168	0	0	0	0	0	1	0	0	0	16
0.14	0.42	1.96	0	0	0	0	0	0	1	0	0	189
0.01	0.03	0.14	0	0	0	0	0	0	0	1	0	14
0.7	2.1	9.8	0	0	0	0	0	0	0	0	1	945
-98	-365	-1380	0	0	0	0	0	0	0	0	0	0

Pivoting (1) **Table 2**

Z_1	Z_2	Z_3	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	C
0.02	0.06	0.28	1	0	0	0	0	0	0	0	0	27
0.05	0.15	0.7	0	1	0	0	0	0	0	0	0	68
0.014	0.042	0.196	0	0	1	0	0	0	0	0	0	19
0.013	0.039	0.182	0	0	0	1	0	0	0	0	0	17.5
0.034	0.102	0.476	0	0	0	0	1	0	0	0	0	46
0.07	0.214	1	0	0	0	0	0	6	0	0	0	95
0.14	0.42	1.96	0	0	0	0	0	0	1	0	0	189
0.01	0.03	0.14	0	0	0	0	0	0	0	1	0	14
0.7	2.1	9.8	0	0	0	0	0	0	0	0	1	945
-98	-365	-1380	0	0	0	0	0	0	0	0	0	0

Second Table **Table 3**

Z_1	Z_2	Z_3	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	C
0.0004	0.0008	0	1	0	0	0	0	-1.68	0	0	0	0.4
0.001	0.0002	0	0	1	0	0	0	-4.2	0	0	0	0.8
0.00028	0.000056	0	0	0	1	0	0	-1.17	0	0	0	0.38
0.00026	0.000052	0	0	0	0	1	0	-1.09	0	0	0	0.21
0.00068	0.000536	0	0	0	0	0	1	-2.86	0	0	0	0.78
0.07	0.214	1	0	0	0	0	0	6	0	0	0	95
0.0028	0.00056	0	0	0	0	0	0	-11.7	1	0	0	2.8
0.0002	0.00004	0	0	0	0	0	0	-0.84	0	1	0	0.7
0.014	0.0028	0	0	0	0	0	0	-58.8	0	0	1	14
-14	-69.68	0	0	0	0	0	0	8280	0	0	0	131100

Pivoting (2) **Table 4**

Z_1	Z_2	Z_3	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	C
0.0004	0.0008	0	1	0	0	0	0	-1.68	0	0	0	0.4
0.001	0.0002	0	0	1	0	0	0	-4.2	0	0	0	0.8
0.00028	0.000056	0	0	0	1	0	0	-1.17	0	0	0	0.38
0.00026	0.000052	0	0	0	0	1	0	-1.09	0	0	0	0.21
0.00068	0.000536	0	0	0	0	0	1	-2.86	0	0	0	0.78
0.327	1	4.67	0	0	0	0	0	28.03	0	0	0	444
0.0028	0.00056	0	0	0	0	0	0	-11.7	1	0	0	2.8
0.0002	0.00004	0	0	0	0	0	0	-0.84	0	1	0	0.7
0.014	0.0028	0	0	0	0	0	0	-58.8	0	0	1	14
-14	-69.68	0	0	0	0	0	0	8280	0	0	0	131100

Third Table **Table 5**

Z_1	Z_2	Z_3	s_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	C
0.000373	0	-.00037	1	0	0	0	0	-1.682	0	0	0	0.3645
0.000934	0	-.00093	0	1	0	0	0	-4.21	0	0	0	0.0088
0.000261	0	-.00026	0	0	1	0	0	-4.205	0	0	0	0.7112
0.000243	0	-.00005	0	0	0	1	0	-1.093	0	0	0	0.1869
0.000504	0	-.00250	0	0	0	0	1	-2.871	0	0	0	0.5420
0.327	1	4.67	0	0	0	0	0	28.03	0	0	0	444
0.00261	0	-.00056	0	0	0	0	0	-11.78	1	0	0	2.55
0.000186	0	-.00004	0	0	0	0	0	-0.84	0	1	0	0.6112
0.013	0	-.0028	0	0	0	0	0	-58.87	0	0	1	12.756
21.38	0	325.40	0	0	0	0	0	10233	0	0	0	162038