# Comparison among different approaches of estimating pore pressure development in liquefiable deposits

S. Rios, M. Millen, J. Quintero & A. Viana da Fonseca
*CONSTRUCT-GEO, Faculty of Engineering of the University of Porto (FEUP), Porto, Portugal*

ABSTRACT: Estimating pore pressure development in liquefiable deposits is very important to predict liquefaction consequences at the surface namely in terms of damages to structures. Several simplified methods have been proposed from which the stress-based method from Seed et al. (1975) is the most widely used, due to its simplicity. Various attempts have been made however to develop an energy based method that could avoid the conversion to an equivalent loading, trying to quantify the liquefaction resistance in terms of a measure that reflects the true nature of seismic shear wave loading. This paper investigates the advantages and limitations of stress-based and energy based methods. For that purpose, effective and non linear dynamic numerical analysis were performed and compared with the simplified methods showing the parameters needed in each method, its assumptions and simplifications and its consequences in terms of the pore pressure prediction.

## 1 INTRODUCTION

Earthquake-induced liquefaction can cause significant damages to buildings as seen by recent events in Christchurch (Diaz, 2016, Bray et al., 2017). Although important technical achievements in understanding and mitigating liquefaction have been accomplished in the last decades, significant damage still occurs in seismic areas around the world. The generation of excess pore water pressure and liquefaction can dramatically change the dynamic response of a soil deposit and interacting structures. Thus the time at which liquefaction occurs, may have a significant influence on the performance of a structure during a seismic event. Most studies have concentrated their attention on predicting liquefaction triggering instead of estimating the pore pressure time series evolution throughout the seismic event. However, there are several advantages of having the whole pore pressure build-up with time. First, this allows the definition of the time to liquefaction (tliq), i.e., the point when there is a change in state from solid to liquid. The information of whether liquefaction happens early or late in a particular ground motion can be invaluable for estimating surface damage. On the other hand, the pore pressure time series will allow the estimation of flow rates between layers and also the extent of pore pressure build up whether it reaches a state of liquefaction or not. In fact, a partially liquefied soil can still experience considerable softening behaviour that can alter the dynamic properties of soil-structure systems as well as modify the upward propagating shear waves. While liquefaction triggering depends on the liquefaction criteria (e.g., exceeding a certain pore pressure ratio defined by the ratio between the excess pore pressure and the initial effective vertical stress) the pore pressure time series shows to what extent liquefaction occurs. In this paper different approaches to estimate pore pressure from the simplified methods to the more complex numerical analysis will be discussed in terms of their advantages, limitations and uncertainties.

## 2 BACKGROUND

### 2.1 *Introduction*

The prediction of pore pressure has been extensively studied in the past decades due to its importance in triggering liquefaction and several simple empirical methods have been developed. These can be divided in three main groups: stress based, strain based and energy based. Stress-based methods were the first to be developed resulting from observations made on stress-controlled cyclic triaxial tests where an uniform shear stress is applied measuring the build-up of pore pressure with increasing number or cycles. Although being generally used, problems have been identified in evaluating the exact state at which liquefaction initiates (Kramer, 1996, Youd, 1972) since the build-up of pore pressure is more accurately predicted by cyclic shear strains and therefore strain controlled cyclic simple shear tests have been used to measure pore pressure build-up. On a different perspective, several energy based methods have been presented, following the assumption made by Nemat-Nasser et al. (1979) that pore water pressure generation can be uniquely related to the cumulative energy dissipation per unit volume of soil up to the onset of liquefaction. Stress-based methods also suffer from several major biases, as explained in the next paragraph, and therefore they should be used in parallel with other methods (e.g. strain or energy-based) that rely on different assumptions. The following approaches are considered in this work:

- 1D nonlinear dynamic analysis performed using the commercial software FLAC® (Itasca, 2016) with the PM4Sand constitutive model (Boulanger et al., 2017)
- Simplified Stress based method from Seed et al. (1975)
- Simplified Energy based method adapted from Kokusho (2013)

### 2.2 *Stress based methods*

One of the first pore pressure models to be developed was the one proposed by Seed et al. (1975) which was then simplified by Booker et al. (1976) - Equation 1:

$$r_u = \frac{2}{\pi} arcsin\left[\left(\frac{N}{N_L}\right)^{1/2\beta}\right] \quad (1)$$

where,
$r_u$ is the pore pressure ratio relating the excess pore pressure and the effective confining stress
$N$ is the equivalent number of uniform cycles
$N_L$ is the number of cycles required to cause liquefaction
$\beta$ is an empirical parameter
$N_L$ and $\beta$, can be determined by cyclic triaxial tests. For a given soil, $N_L$ increases as relative density increases and decreases as the magnitude of loading (or cyclic stress ratio) increases. Booker et al. (1976) proposed a value of 0.7 for $\beta$, while Polito et al. (2008) proposed the following empirical equation:

$$\beta = c_1 FC + c_2 Dr + c_3 CSR + c_4 \quad (2)$$

where $FC$ is the fines content, $D_r$ is the soil relative density, and $c_1$, $c_2$, $c_3$ and $c_4$ are regression constants which vary with the fines content

The number of uniform cycles ($N$) equivalent to an irregular earthquake ground motion can be obtained by the weighting scheme proposed by Seed et al. (1975) which was later used by Idriss (1999), Boulanger et al. (2006), Kishida et al. (2014). The Seed stress based model considers a power relationship between the cyclic stress ratio and the number of cycles – Equation (3):

$$CSR = a.N^{-b} \quad (3)$$

where $a$ and $b$ are fitting parameters

Therefore, for two individual stress cycles with CSRA and CSRB, the relative number of cycles to cause failure at these two stress ratios is easily obtained (Equation 4). Assuming a reference value of uniform cycles for the magnitude of 7.5 (NM=7.5), the obtained ratios of CSR correspond to the definition of a magnitude scaling factor (MSF) used in the Seed simplified procedure to calculate the seismic demand of liquefaction potential. There have been several proposals for the b parameter such as b=0.34 for sands (Idriss, 1999) and b=0.135 for clays and plastic silts (Boulanger et al., 2006).

$$\frac{N_A}{N_B} = \left(\frac{CSR_B}{CSR_A}\right)^{1/b} \Leftrightarrow \quad MSF = \frac{CSR_M}{CSR_{M=7.5}} = \left(\frac{N_{M=7.5}}{N_M}\right)^b \tag{4}$$

For this work this method was implemented using the Equation (1) suggested by Booker (1976). The $N/N_L$ ratio was calculated by Equation (5) and an $N_{ref}$ equal to 15 cycles, which is the value that Idriss (1999) indicates for a magnitude of 7.5. The *CSR* was calculated with Equation (6) where a peak counting method was used to identify the acceleration peaks ($acc_{peaks}$), counting the largest peak between successive zero crossing. The *CRR* was $CSR_{15}$ from the element tests.

$$\frac{N_L}{N} = \sum N_{ref} * \left(\frac{CRR}{CSR}\right)^{1/b} \tag{5}$$

$$CSR = \left|acc_{peaks}\right| * \frac{\sigma_{v0}}{\sigma'_{v0}} * r_d \tag{6}$$

where $r_d$ was calculated by equation (7) being $M$ is the magnitude:

$$r_d = e^{[f(z)+g(z)*M]} \tag{7}$$

$$f(z) = -1.012 - 1.126 * sin\left(\frac{z}{11.73} + 5.133\right)$$

$$g(z) = 0.106 + 0.118 * sin\left(\frac{z}{11.28} + 5.142\right)$$

### 2.3 *Energy based methods*

The energy based methods were mainly developed as a way to avoid the conversion to an equivalent loading, trying to quantify the liquefaction resistance in terms of a measure that reflects the true nature of seismic shear wave loading. The development of an energy-based liquefaction triggering method was first proposed by Davis et al. (1982) and more recently, Kokusho (2013) proposed a simplified liquefaction triggering procedure also based on the dissipated energy because it is closely linked to soil grain movement (Roscoe et al., 1963) and is a core aspect of numerous constitutive effective stress models (e.g. Cubrinovsi & Ishihara, 1998). Dissipated energy has been demonstrated to be approximately constant across different amplitudes of loading and even for irregular loading histories (Azeiteiro et al., 2017). However, methods that adopt dissipated energy have two major drawbacks, one is that the estimation of the dissipated energy within a soil profile from a seismic shear wave is far from trivial and very dependent on soil characteristics and changes as pore pressure increases. Secondly, the dissipated energy rapidly increases as the soil approaches liquefaction, and therefore a small change in the criteria for liquefaction triggering (e.g. change the limiting pore pressure ratio from 0.95 to 0.98), can have a large impact on the evaluated capacity.

In this work the energy based method (EBM) from Kokusho (2013) was adapted to provide the estimation of the pore pressure time series. The demand was estimated by performing equivalent linear analysis using the open-source python package, Pysra v0.3.0 (Kottke, 2018). The clay layers were modelled with the Modified Hyperbolic Soil Type using the expressions from Vardanega et al. (2013) and a minimum damping of 2%. The sand layer was modelled using the Modified Hyperbolic Soil Type where the curvature factor was set to 1.0 and the $\gamma_{ref}$ was set so that the maximum shear stress was reached at a ratio of 20. The strain energy (*W*), as defined by Kokusho (2013), was calculated from the shear stresses and strains obtained in the equivalent linear analysis to better account for the free-surface effect, compared to using a conversion from the upward propagating energy density proposed by Kokusho (2013). The strain energy was then inserted in equation (8) to obtain the dissipated energy ($\Delta W$). To evaluate the influence of Equation (8), this was compared to the hysteretic damping ratio ($\xi$) definition ($\xi = \Delta W/(W*\pi)$) herein assumed 30%. In this method the equivalent linear analysis is used again to calculate the strain energy which is then converted to the dissipated energy by the damping relationship, and so this was called EqLin+Damp.

$$\frac{\Delta W}{\sigma'_{v0}} = 10^{\frac{\log(W/\sigma'_{v0}) - \log(5.4)}{1.25}} \tag{8}$$

In the soil capacity estimation, the relationships between dissipated energy at liquefaction and $CSR_{20}$ proposed by Kokusho (2013) were used for the two liquefaction criteria: 2% and 5% of double amplitude axial strain (DA):

$$\frac{\Delta W}{\sigma'_{v0}} = 0.032 - 0.48 * CSR_{20} + 2.40 * CSR_{20}^{\ 2} \qquad \text{for DA=5\%} \qquad (9)$$

$$\frac{\Delta W}{\sigma'_{v0}} = 0.032 - 0.48 * CSR_{20} + 1.97 * CSR_{20}^{\ 2} \qquad \text{for DA=2\%} \qquad (10)$$

A simple pore pressure model, inspired in Green et al. (2000) model, was used since it does not need any calibrating parameters:

$$ru = \sqrt{\frac{\Delta W}{\Delta W_{liq}}} \qquad (11)$$

## 3 NUMERICAL ANALYSIS DESCRIPTION

Nonlinear dynamic analysis were performed using the commercial software FLAC® as a part of another study investigating the performance of buildings on liquefied soil (Millen et al., 2019). The selected constitutive model was PM4Sand (Boulanger et al., 2017) which was developed for liquefying soils. Two different calculations were performed: effective stress analysis (ESA) and nonlinear analysis (NLA) assuming the bulk modulus of the water to be null. This latter intends to simulate the case where the soil does not liquefy, to evaluate the impact of stiffness and strength degradation due to liquefaction on the assumptions related to estimation of stresses and dissipated energy in simplified methods. The soil profile consisted of three soil layers: two non-liquefiable layers made of hard clay located at the top and at the bottom while the middle layer, was made of sand. The water table was assumed at the interface of the first and second layers. In the numerical analysis the input upward propagating motion was used at the bottom of the model. The 2 metre thick clay on the top has a dry specific weight of 15.6 kN/m$^3$, undrained strength of 50 kPa, maximum shear modulus of 50 MPa, porosity of 0.412 and permeability of 8x10$^{-8}$ m/s. The 6 metre thick liquefiable sand has a saturated weight of 19.6 kN/m$^3$, constant volume friction angle $\phi'$=33º, relative density *Dr*=65%, and permeability *k*=1.6x10$^{-5}$ m/s. The liquefiable layer was also assigned the additional PM4Sand properties of normalised shear modulus of *Go*=782.7, and contraction rate parameter, *hpo*=0.32 (Boulanger et al., 2017), for modelling in FLAC. The contraction rate parameter was calibrated by performing numerical element tests using the subroutines provided by the authors of the PM4Sand model and adjusting the parameter to obtain a cyclic resistance ratio of 0.15 for 15 cycles. The 12 metre thick clay on the bottom has a saturated weight 20.2 kN/m$^3$, undrained strength of 200 kPa, maximum shear modulus of 200 MPa, porosity of 0.375 and permeability of 1x10$^{-9}$ m/s. Only two ground motions, recorded on soft soil sites, were considered for the case study. The first motion was recorded at the Duzce station in Turkey with $V_{s30}$ of 280m/s, during the Kocaeli Earthquake 1999 (*Mw*=7.51) in Turkey, and the second from Dinar station during the Dinar Earthquake 1995 (*Mw*=6.4) in Turkey, and were taken from the NGA2-west strong motion database from Ancheta et al. (2013) numbers 1158 and 1141 respectively. The two components of each ground motion were rotated through 100 angles to obtained the maximum single direction response in terms of Arias Intensity, such that the energy in the record was compatible with the duration. In order to calculate the $CSR_{15}$ and $CSR_{20}$ of the sand (i.e, the cyclic stress ratio that the sand can sustain until it liquefies with 15 or 20 cycles of constant stress amplitude), direct simple shear tests (element tests) were simulated in FLAC® assuming the conditions of the middle sand layer used in the 1D model. This means that for each soil profile of the 1D model, 10 element tests were performed just varying the peak cyclic stress ratio (0.40, 0.30, 0.26, 0.20, 0.18, 0.15, 0.10, 0.06).

## 4 DIFFERENCES BETWEEN THE METHODS

### 4.1 *Comparison of the methods in the estimation of the soil capacity*

The element test results from FLAC provided the estimation of some of the soil parameters used in the different methods, namely $CSR_{15}$, $CSR_{20}$, "*b*" and $\beta$. Figure 1 shows the comparison of the element tests results with the parameters and assumptions used by the methods. In Figure 1a) the element tests results for a liquefaction criterion of $ru=\Delta u/\sigma'_{v0}$=0.98 are plotted against Equation (3) using an upper and lower bound for *b* of [0.2-0.4]. The data obtained by Okamura et al. (2003) show that the densest and strongest sands had *b* values of 0.45, 0.50, and 0.54, whereas the looser and weaker sands had *b* values of 0.13, 0.15, and 0.21. Since the sand of the present case has an intermediate *Dr*=65%, the upper value of the weaker sands and the lower value of the dense sands was used. The b value obtained by the adjustment to the element tests results at a *ru*=0.98 was 0.36. The graph shows the impact of different b values as this parameter is often poorly characterized unless a significant number of laboratory tests is performed. In Figure 1b) the element tests results were plotted in terms of the pore pressure ratio against the number of cycles normalized by the number of cycles required to liquefy that layer. The same plot shows the Booker et al. (1976) Equation (1) with two different β values. The value of β=0.7 is suggested by the author, while the value of 0.99 was obtained by Equation (2) for the sand layer using $CSR= CSR_{15}$=0.19. In Figure 1c) the normalized dissipated energy obtained from the element tests for different liquefaction criteria is compared to the same parameter obtained by Equations 9 and 10 proposed by Kokusho (2013) in terms of $CSR_{20}$=0.18 obtained in the element tests. It seems that the 2% double amplitude axial strain results obtained by Kokusho (2013) agrees well with the data. In Figure 1d) the pore pressure ratio is plotted against the normalized dissipated energy. Although this relation is usually assumed to be hyperbolic, potential or exponential, these laws do not provide a good fit as indicated on Figure 1d) for an hypothetical hyperbolic law, which may be source of error in the simplified methods.

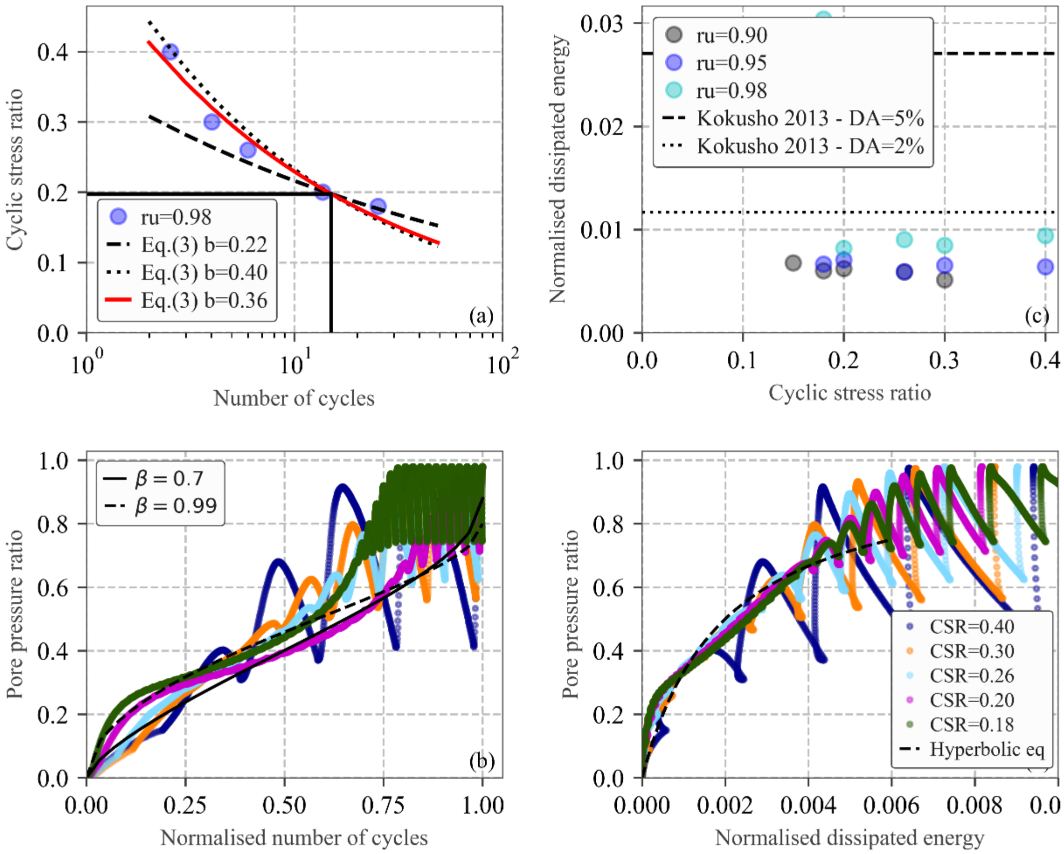


Figure 1 – Element tests results

As mentioned before, the stress based method needs the conversion to an equivalent number of cycles. In Figure 2 the equivalent number of cycles obtained by the simplified stress based method (SBM) is compared to the numerical analysis (ESA and NLA). The equivalent number of cycles from the numerical analysis was calculated by converting the shear stresses into cyclic stress ratios and then applying Equation (5). Both the SBM and the numerical analysis were calculated for the same range of *b* values [0.2-0.4] so that the uncertainty associated to this value could be observed. This equivalent conversion procedure has several uncertainties related to *rd* equation, and to the estimation of surface acceleration. In this work, *rd* was calculated by Equation (7) but there are several other proposals in the literature. Finally, this equivalent cycle procedure assumes that the shear stresses does not reduce due to increased excess pore water pressure which could be expected due to the softening of the soil. As it can be seen in Figure 2 the *b* value of 0.22 gives very high equivalent number of cycles which would reduce the accuracy of the pore pressure prediction. However, it should be noted that a *b* around 0.22 was obtained by several authors for clean sands with relative densities around 65% (Silver et al., 1976, Carraro et al., 2003). For that reason, from now on the SBM will be calculated with *b*=0.36.

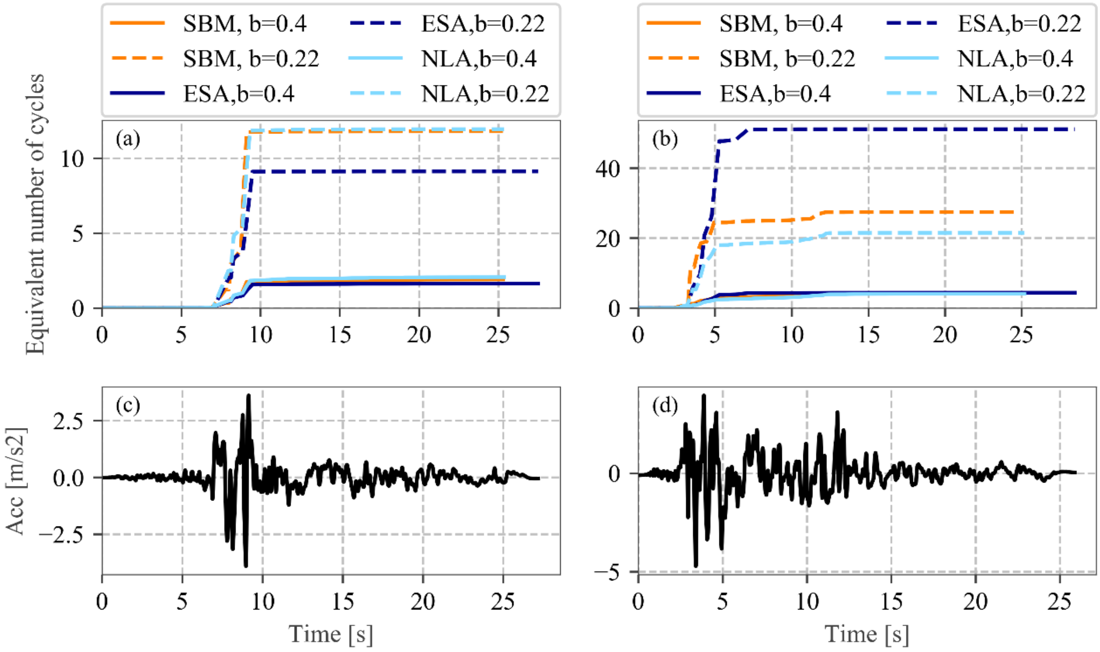


Figure 2 – Equivalent number of cycles obtained by the stress based method and its comparison with the numerical analysis (a and b for motion 1 and 2 respectively), acceleration records (c and d for motion 1 and motion 2 respectively)

Since the simplified energy based methods are generally based on the evaluation of the dissipated energy, it is interesting to compare the dissipated energy from the numerical analyses with the simplified method adapted from Kokusho (2013) (EBM). In the same graph the normalized dissipated energies at liquefaction obtained with equations (9) and (10) are included. The EqLin+Damp, EBM and NLA are very similar for motion 1 with a slight overprediction with EBM. However, none of these methods predicts liquefaction as they are below the DA=2% threshold, conversely to ESA that stands above. The dissipated energy from ESA is similar to other methods up to the onset of liquefaction (approximately 10 seconds according to Figure 3), but increases further due to liquefaction weakening the soil. For motion 2, both EBM and EqLin+Damp predict liquefaction but with much lower dissipated energy values than the ESA. It should be pointed out that these records are quite short in terms of the time where the strong shaking occurs and thus aspects such as change in the rate of energy build up could not be examined. Finally, the simplified methods are compared with the ESA in terms of the estimation of the pore pressure time series (Figure 4). For the SBM the $\beta$ value was calculated with Equation (2) obtaining 0.99 and for *b* the optimum fit was assumed (0.36). For motion 1, the SBM predicted liquefaction at 8.2 s, whereas for ESA it is approximately 10 s. For motion 2, the SBM predicts liquefaction at 3.3 s, EBM at 10.7 s and Eqlin+Damp at 12 s, where for ESA it is 5.4 s.

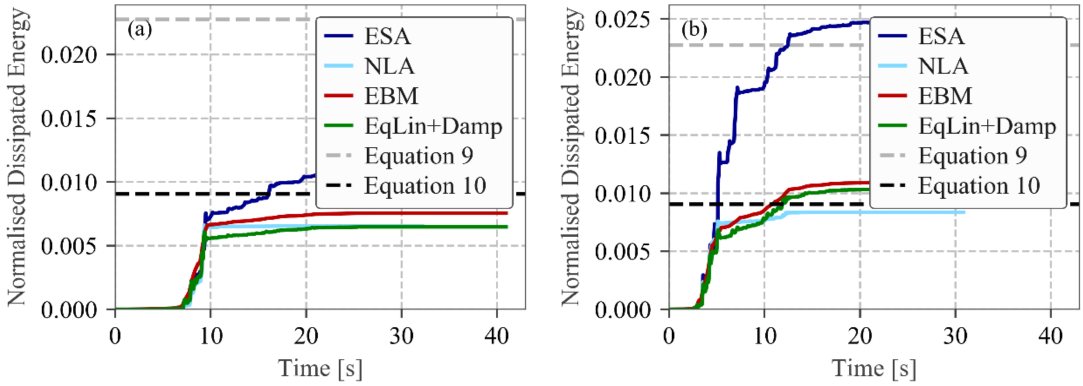

Figure 3 – Normalised dissipated energy time series: a) motion 1, b) motion 2

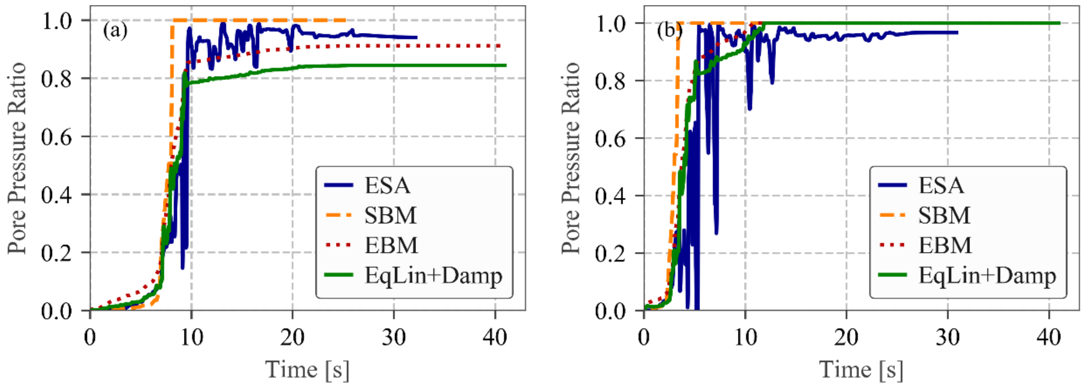

Figure 4 – Pore pressure prediction by the simplified methods and the effective stress analysis (ESA): a) motion 1, b) motion 2

## 5 CONCLUSIONS

This paper compares pore pressure prediction methods in terms of the different steps used in the models: number of equivalent cycles for the stress based method and dissipated energy for the energy based methods. For each case, the parameters assumed and its range of values were discussed in its implications on the pore pressure prediction. Finally, the pore pressure time series predicted by each model is presented and the time to liquefy is compared. The SBM slightly overestimates liquefaction, while EBM underpredicts liquefaction, which means that using them together may point to the correct value. Although the presented data is only for two ground motions and therefore the results are not very conclusive, this parametric analysis highlighted some of the assumptions and simplifications used by each method that cause differences in the results. For that reason, and since all the simplified methods have biases, more than one method should be used at the same time in order to provide more accurate estimates.

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