Symbolic analysis of linear electric circuits with Maxima CAS

Dejan V. Tošić, Milka M. Potrebić
University of Belgrade – School of Electrical Engineering, Belgrade, Serbia
tosic@etf.rs, milka_potrebic@etf.rs

Abstract: Symbolic analysis of electrical circuits, implemented in software, SALECx, is presented. SALECx is developed in Maxima CAS and it is offered to students, educators, engineers, and others, as free/libre open-source software. SALECx operation is exemplified by several distinct circuits.

Keywords: electric circuit; symbolic analysis; Maxima; SALECx.

I. Introduction

Symbolic simulation is a formal technique to calculate the behavior or a characteristic of a system (e.g. digital system, electronic circuit, or continuous-time system) with an independent variable (sample index, time, or frequency), the dependent variables (sample values, signals, voltages, and currents), and (some or all) the element values represented by symbols [1].

A symbolic simulator is a computer program that receives the system description as input and can automatically carry out the symbolic analysis and thus generate the symbolic expression for the desired system characteristic [2].

Symbolic computation has been used for modeling, simulation, and synthesis of analog circuits and VLSI systems [3] [4] [5].

This paper presents a novel symbolic simulator, SALECx, implemented in the Maxima CAS (**C**OMPUTER **A**LGEBRA **S**YSTEM) [6] programming language.

The underlying theory of the SALECx operation can be found in many good textbooks, e.g. classical [7] [8], contemporary [9] [10] [11] [12] [13], on computer-aided analysis [14], with network synthesis [15], with advanced topics [16], for power engineering [17] [18], on transmission lines [19].

Several recent books present various Maxima CAS applications [20] [21] [22] [23].

In addition, Maxima CAS has been used and recommended to students at the University of Belgrade, School of Electrical Engineering, Electric Circuit Theory course [24].

In this paper we assume that the circuit simulated by SALECx is linear, time-invariant, and finite. All basic circuit elements are contained in the SALECx element catalog. The ABCD element implements an arbitrary two-port element with known *a*-parameters (chain parameters, transmission parameters).

The electric circuit graph is assumed to be connected. If the graph is not connected then one should (1) identify the disconnected components, (2) choose one node in each component, and (3) connect the chosen nodes to make the graph connected.

As a free/libre open-source software package SALECx can be directly recommended and distributed to students, which are a price-sensitive community willing to minimize their expenses. On the other hand, SALECx can be included in Electric Circuit Theory teaching and learning, at initial learning stages, to motivate and encourage students (1) to solve their homework and numerous circuit problems by automated computer-aided symbolic analysis, and (2) to verify their designs and confirm circuit analyses obtained traditionally by hand, i.e. by paper and pencil. Finally, the SALECx free open-source code reveals the underlining algorithm in full detail, promotes a better understanding of the corresponding circuit analysis method, and might prompt some students to edit the code and add their on extensions and contributions.

SALECx can help students to solve much more real-life circuit example problems compared to the relatively smaller number of problems they are willing to solve by hand. Therefore, the example-problem-based learning paradigm can be supported, which allows novice students to grasp concepts and phenomena from circuit theory with higher learning performance and lower mental effort, until they reach expert levels. Consequently, the role of a student might change from passive to active learner under the new learning paradigm.

Mastering circuit analysis requires some degree of practice and one must be adept in algebraic manipulation. Often, the burden of algebraic manipulation causes the student to lose sight of the wood from the trees. In the classic method of study a student must overcome the difficult barriers of mathematics, which makes the subject very unattractive.

When doing mathematics, instead of burdening the brain with the repetitive job of redoing numerical operations which have already been done before, it's possible to save that brainpower for more important situations by using symbols, instead, to represent those numerical calculations (Ernst Mach, 1883). Today, with computer algebra systems, such as Maxima/ Macsyma, it is possible to calculate in minutes or hours the results that would (and did) years to accomplish by paper and pencil. Accordingly, SALECx can help students acquire a "functional understanding" of Electric Circuit Theory and foster mastery of the MNA (**M**ODIFIED **N**ODAL **A**NALYSIS) equation formulation.

Symbolic circuit response generated by SALECx, i.e. closed-form analytic expressions for circuit voltages and currents, can provide better insight than numerical solutions, e.g. obtained by SPICE. By inspection of the symbolic response, it might be immediately clear how a parameter (or an element value) contributes to the performance and behavior of the electric circuit.

II. SALECx in a Nutshell

SALECx is a Maxima program for solving linear time-invariant electric circuits in the complex domain of the Unilateral Laplace Transform or Phasor Transform. SALECx stands for **S**YMBOLIC **A**NALYSIS of **L**INEAR **E**LECTRIC **C**IRCUITS with Ma**x**ima.

SALECx has been developed by Dr. Dejan Tošić, Full Professor, tosic@etf.rs, at the University of Belgrade – School of Electrical Engineering, Belgrade, Serbia.

Reserved symbols and circuit specification:

s – complex frequency [radian/second], symbol, the Laplace variable

I[label] or **I**[label, node] – MNA current variables, symbols

V[0], **V**[1], **V**[2], **V**[3] … – MNA voltage variables, symbols, node voltages, V[0] is set to zero, 0

SALECxPrint – verbose variable

The circuit to be analyzed is specified as a netlist [circuitElement_1, circuitElement_2 …].

A circuit element is specified as a list of the form:

[type, label, a, b, p]
[type, label, a, b, p, IC]
[type, label, [a1,a2], b]
[type, label, [a1,a2], [b1,b2], p]
[type, label, [a1,a2], [b1,b2], p, IC]

type – string that specifies the element type: "R", "L", "C", "I", "V", "Z", "Y", "OpAmp", "VCVS", "VCCS", "CCCS", "CCVS", "IT", "K", "T", "ABCD".

label – string that uniquely identifies circuit element, e.g. "Vgen", "Is", "Rin", "Cfb", , "Lprim", "Y2", "Zload".

For one-port elements:

- **a** – positive terminal,
- **b** – negative terminal.

For two-port elements except OpAmp

- **a1** – positive terminal of the 1st port,
- **a2** – negative terminal of the 1st port,
- **b1** – positive terminal of the 2nd port,
- **b2** – negative terminal of the 2nd port.

p – parameter or parameters if p is a list.

IC – initial conditions at 0-minus: Vo for capacitors, Io for inductors, [Io1,Io2] for linear inductive transformers.

Calling SALECx:

Laplace Transform s-domain

SALECx[circuitSpecification]

Phasor Transform j*omega-domain, sinusoidal steady state

SALECx[circuitSpecification, omega]

omega [radian/second] – angular frequency

See the SALECx syntax details and element catalog in **SALECx.mac** script file that accompanies this paper.

III. SALECx Symbolic Simulation Examples

Assume that SALECx has been installed in the directory "C:\SALECx\" as the mac file "SALECx.mac".

A simple capacitor circuit is shown in Fig. 1. The capacitor is initially charged and its preinitial voltage is V_0. Current of the ideal voltage source is presented to specify the reference direction.

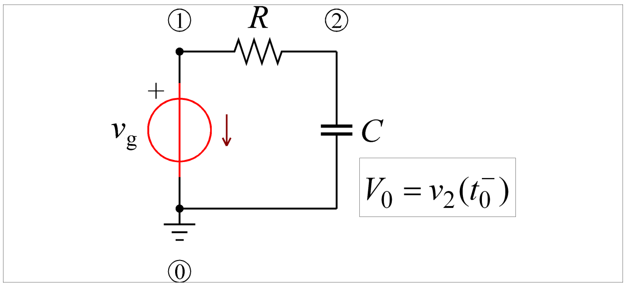


Figure 1: Simple capacitor circuit.

SALECx is loaded by the Maxima command

```
(%i1)   load("C:\\SALECx\\SALECx.mac");
(%o1)   C:\SALECx\SALECx.mac
```

The circuit is textually specified as a list (netlist) of element specifications.

```
(%i2)   VgRCVo_Shema: [
          ["V", "Vg", 1, 0, Vg],
          ["R", "R", 1, 2, R],
          ["C", "C", 2, 0, C, Vo]];
(VgRCVo_Shema) [[V,Vg,1,0,Vg],[R,R,1,2,R],[C,C,2,0,C,Vo]]
```

First, SALECx is invoked to carry out the Phasor Transform domain analysis, the steady-state sinusoidal analysis, at a frequency omega (ω) [radian/second]. All inital conditions are ignored.

```
(%i3)   VgRCVo_Response_PT: SALECx(VgRCVo_Shema,omega);
Phasor Transform at angular frequency  ω
```

(VgRCVo_Response_PT) $[V_1=Vg, V_2=\frac{Vg}{\%i\ C\ R\ \omega+1}, I_{Vg}=-\frac{\%i\ C\ Vg\ \omega}{\%i\ C\ R\ \omega+1}]$

The complex capacitor voltage is V_{20} and it is obtained from the V[2] variable.

```
(%i4)   V2PT: V[2], VgRCVo_Response_PT;
```

(V2PT) $\frac{Vg}{\%i\ C\ R\ \omega+1}$

Next, SALECx is invoked to perform the Unilateral Laplace Transform domain analysis for the complex frequency s [radian/second].

```
(%i5)   VgRCVo_Response: SALECx(VgRCVo_Shema),
         SALECxPrint: true;
```

The option "SALECxPrint: true" instructs SALECx to print some analysis details.

```
Symbolic Analysis of Linear Electric Circuits with Maxima
SALECx version 1.0, Prof. Dr. Dejan Tošić, tosic@etf.rs
Number of nodes excluding 0 node:   2
Electric circuit specification: [[V,Vg,1,0,Vg],[R,R,1,2,R],[C,C
,2,0,C,Vo]]
Supported element:  [true,true,true]
Element values:  [Vg,R,C]
Initial conditions:  [false,false,Vo]
```

MNA equations: $[\frac{V_1-V_2}{R}+I_{Vg}=0, V_2\ C\ s-C\ Vo+\frac{V_2-V_1}{R}=0, V_1=Vg]$

MNA variables: $[V_1, V_2, I_{Vg}]$

(VgRCVo_Response) $[V_1=Vg, V_2=\frac{C\ R\ Vo+Vg}{C\ R\ s+1}, I_{Vg}=-\frac{C\ Vg\ s-C\ Vo}{C\ R\ s+1}]$

The corresponding complex capacitor voltage depends on the initial condition, now. The excitation is assumed to be a step function.

```
(%i6)   V2s: V[2], VgRCVo_Response, Vg=Vstep/s;
```

(V2s) $\frac{\frac{Vstep}{s}+C\ R\ Vo}{C\ R\ s+1}$

The time-domain capacitor voltage, for $t > 0$, can be computed by the Maxima **ilt** function, which performs the Inverse Unilateral Laplace Transform.

```
(%i7) v2ilt: ilt(V2s,s,t), expand;
```

$$\text{(v2ilt)}\quad -Vstep\ \%e^{-\frac{t}{C\,R}} + Vo\ \%e^{-\frac{t}{C\,R}} + Vstep$$

The result can be rewritten for a desired form, e.g.

```
(%i8) v2t: factorout(v2ilt,Vstep,Vo);
```

$$\text{(v2t)}\quad (Vo - Vstep)\ \%e^{-\frac{t}{C\,R}} + Vstep$$

In both analyses the source current is a MNA variable because it cannot be expressed in terms of the node voltages. That is, the ideal independent voltage source is not a voltage-controlled element.

Figure 2 presents an OTA-C (**O**PERATIONAL **T**RANSCONDUCTANCE **A**MPLIFIER with **C**APACITORS) lowpass and highpass 2nd-order filter realization.

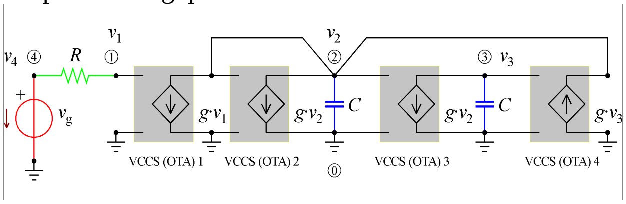


Figure 2: OTA-C filter realization.

The corresponding netlist and response generated by SALECx follow.

```
(%i2) OTA_C_shema: [
        ["V", "Vg", 4, 0, Vg],
        ["R", "R", 4, 1, R],
        ["VCCS", "OTA1", [1,0], [2,0], g],
        ["VCCS", "OTA2", [2,0], [2,0], g],
        ["VCCS", "OTA3", [2,0], [3,0], g],
        ["VCCS", "OTA4", [3,0], [0,2], g],
        ["C", "C1", 2, 0, C],
        ["C", "C2", 3, 0, C]
      ]$
(%i3) OTA_C_response: SALECx(OTA_C_shema);
```

$$\text{(OTA\_C\_response)}\quad [V_1 = Vg,\ V_2 = -\frac{C\,Vg\,g\,s}{C^2 s^2 + C\,g\,s + g^2},\ V_3 = \frac{Vg\,g^2}{C^2 s^2 + C\,g\,s + g^2},\ V_4 = Vg,\ I_{Vg} = 0]$$

```
(%i4) Hs2bandpass: V[2]/Vg, OTA_C_response;
```

$$\text{(Hs2bandpass)}\quad -\frac{C\,g\,s}{C^2 s^2 + C\,g\,s + g^2}$$

```
(%i5) Hs3lowpass: V[3]/Vg, OTA_C_response;
```

$$\text{(Hs3lowpass)}\quad \frac{g^2}{C^2 s^2 + C\,g\,s + g^2}$$

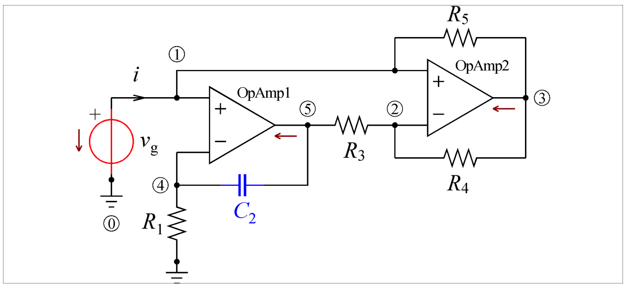


Figure 3: Riordan gyrator synthetic inductor.

Synthetic inductor, which is realized with the Riordan gyrator network, is shown in Fig. 3. The proof-of-concept symbolic analysis follows. The circuit is inductorless but, theoretically, the impedance seen by the source is purely inductive.

Wilkinson power divider, which is realized with ideal lossless transmission line sections, is shown in Fig. 4. The corresponding symbolic analysis with SALECx, performed in the Phasor Transform domain, verifies that the circuit equally splits (divides) input power to the loads $R_2 = R$ and $R_3 = R$, i.e. $V_2 = V_3$.

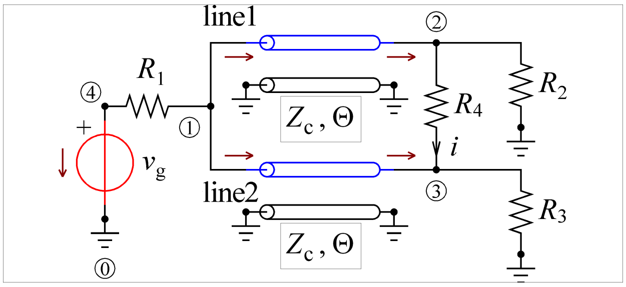


Figure 4: Wilkinson power divider.

Doubly terminated lossless transmission line section is shown in Fig. 5.

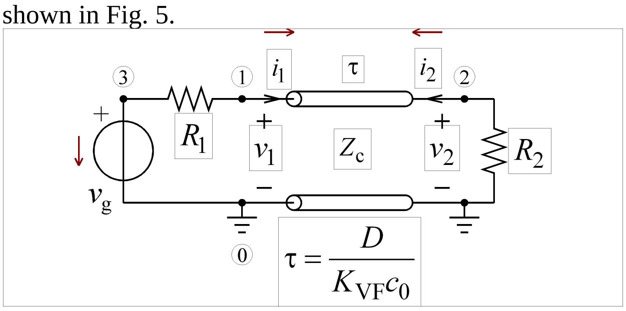
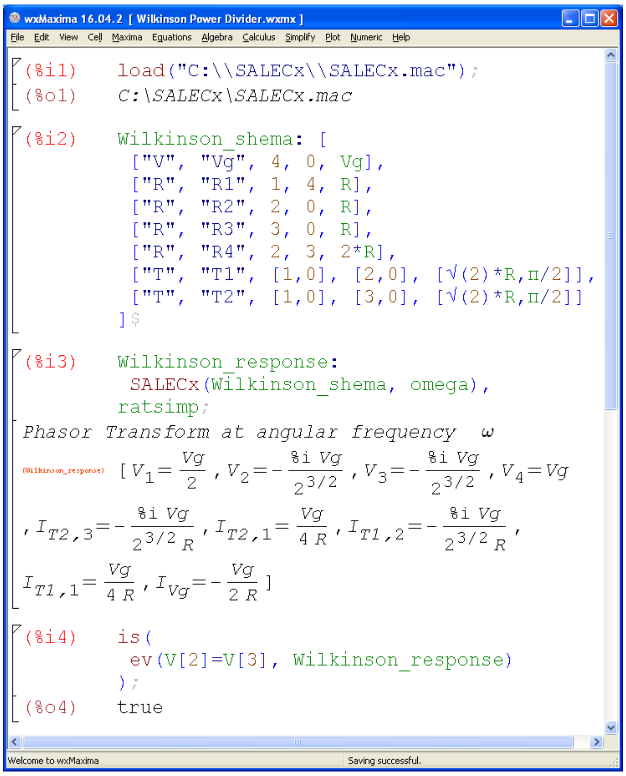
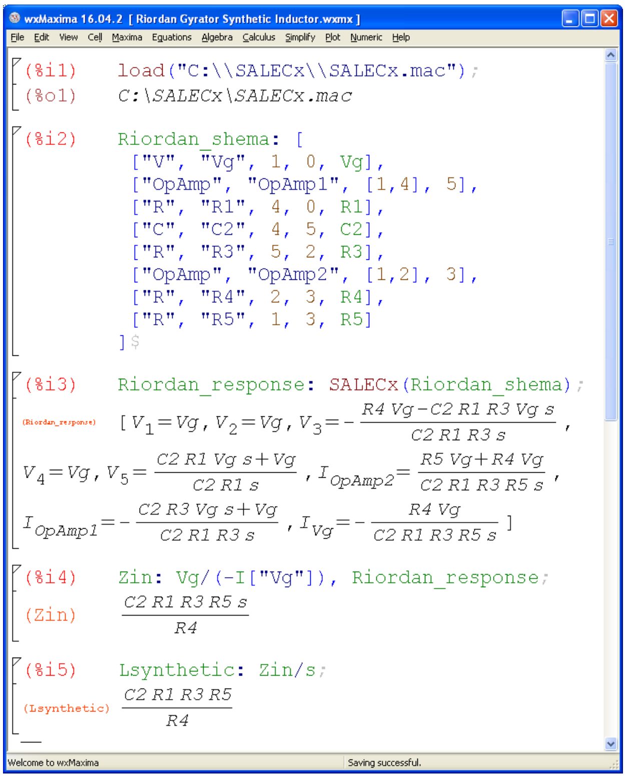
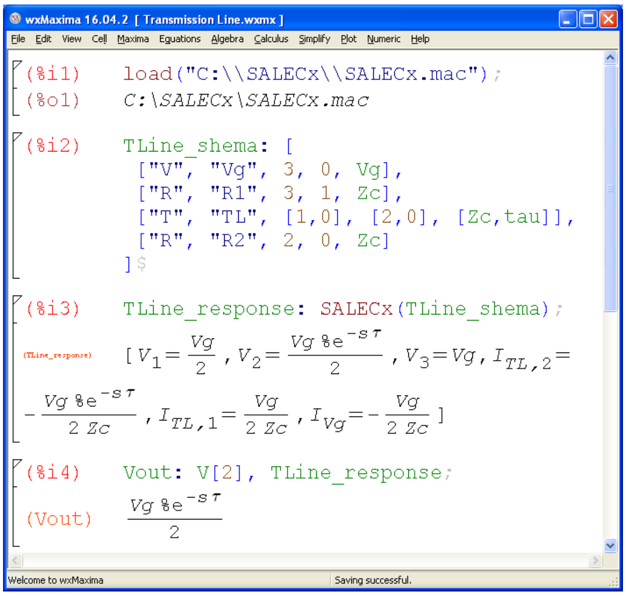


Figure 5: Transmission line circuit; the Laplace transform domain.

The corresponding symbolic analysis with SALECx, performed in the Unilateral Laplace Transform domain, verifies that the circuit acts as a delay line.







IV. Conclusion

Automated computer-aided symbolic analysis of linear time-invariant electric circuits, implemented in software SALECx, has been presented. Symbolic simulator SALECx, written in Maxima CAS, receives a textual circuit description in the form of a netlist and generates closed-form analytic expressions for the circuit response. The analysis is performed in the complex domain of the Unilateral Laplace Transform or the Phasor transform.

Engineers, educators and students can benefit from SALECx when exploring design alternatives, verifying the circuit performance, or carrying out the proof-of-concept analyses. The future directives might be an integration of SALECx with a schematic capture editor so the user can specify circuits pictorially.

Acknowledgments

We thank Prof. Dr. Predrag Pejović for permanent encouragement and valuable discussions related to the SALECx project.

This work was supported by the Ministry of Education, Science, and Technological Development of the Republic of Serbia.

References

[1] G. Gielen, P. Wambacq, and W. Sansen, “Symbolic Analysis Methods and Applications for Analog Circuits: A Tutorial Overview,” *Proceedings of the IEEE*, vol. 82, no. 2, pp. 286–304, Feb. 1994.

[2] F. Fernández, A. Rodríguez-Vázquez, J. Huertas, and G. Gielen, *Symbolic analysis techniques: Applications to analog design automation*. New York, NY: Wiley-IEEE Press, 1997.

[3] Z. Qin, S. X. D. Tan, C.-K. Cheng, *Symbolic Analysis and Reduction of VLSI Circuits*. New York, NY: Springer, 2005.

[4] M. Fakhfakh (ed.), *Design of Analog Circuits through Symbolic Analysis*. Bentham, 2012.

[5] G. Shi, S. X.-D. Tan, E. T. Cuautle, *Advanced Symbolic Analysis for VLSI Systems*. New York, NY: Springer, 2014.

[6] Maxima CAS (Compute Algebra System), free open-source, available at http://maxima.sourceforge.net [accessed] Aug. 2019.

[7] C. A. Desoer, E. S. Kuh, *Basic Circuit Theory*. New York, NY: McGraw-Hill, 1969.

[8] L. O. Chua, C. A. Desoer, and E. S. Kuh, *Linear and Nonlinear Circuits*. New York, NY: McGraw-Hill, 1987.

[9] C. K. Alexander, M. N. O. Sadiku, *Fundamentals of Electric Circuits*, 6/e. New York, NY: McGraw-Hill, 2017.

[10] J. W. Nilsson, S. A. Riedel, *Electric Circuits*, 10/e. Upper Saddle River, NJ: Prentice Hall, 2015.

[11] J. D. Irwin, R. M. Nelms, *Basic Engineering Circuit Analysis*, 11/e. Hoboken, NJ: Wiley, 2015.

[12] J. A. Svoboda, R. C. Dorf, *Introduction to Electric Circuits*, 9/e. Hoboken, NJ: Wiley, 2014.

[13] W. H. Hayt, Jr., J. E. Kemmerly, S. M. Durbin, *Engineering circuit analysis*, 8/e. New York, NY: McGraw-Hill, 2012.

[14] F. N. Najm, *Circuit Simulation*. Hoboken, NJ: Wiley, 2010.

[15] O. Wing, *Classical Circuit Theory*. New York, NY: Springer Science+Business Media, LLC, 2008.

[16] W.-K. Chen (Ed.), *Circuit Analysis and Feedback Amplifier Theory*. Boca Raton, FL: CRC Press, Taylor & Francis Group, 2006.

[17] A. L. Shenkman, *Transient Analysis of Electric Power Circuits Handbook*. Dordrecht, The Netherlands: Springer, 2005.

[18] A. L. Shenkman, *Circuit Analysis for Power Engineering Handbook*. Dordrecht, The Netherlands: Springer, 1998.

[19] P. R. Clayton, *Analysis of Multiconductor Transmission Lines*, 2/e. Hoboken, NJ: Wiley IEEE Press, 2008.

[20] D. Bindner, M. Erickson, *A Student's Guide to the Study, Practice, and Tools of Modern Mathematics*. Boca Raton, FL: CRC Press, Taylor & Francis Group, 2018.

[21] T. K. Timberlake, W. Jr. Mixon, *Classical Mechanics with Maxima*. New York, NY: Springer, 2016.

[22] G. S. Tuteja, *Practical Mathematics Using Maxima*. India: International Book House Pvt Ltd, 2018.

[23] Z. Hannan, *wxMaxima for Calculus I, II*. Solano Community College, 2015.

[24] D. V. Tošić, M. M. Potrebić, *Electric Circuit Theory Lecture Notes* (in Sebian) http://tek.etf.rs [accessed] Aug. 2019.