A liquefaction surface to describe liquefaction phenomena in unsaturated sandy soils

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ABSTRACT: The energetic interpretation of liquefaction tests indicates that the behavior of unsaturated sands can be related to the specific energy spent to liquefaction In this paper, an insight is proposed on the role played by the two components of such a specific energy: the specific volumetric energy to liquefaction ($E_{v,liq}$) and the specific deviatoric energy to liquefaction ($E_{s,liq}$). It is argued that $E_{v,liq}$ (which is nil in saturated soils) can be considered as a state variable to quantify the increase of cyclic resistance caused by a reduced degree of saturation, being thus constant along a given cyclic resistance curve. $E_{s,liq}$ is instead related to a specific value of cyclic resistance ratio or number of cycles at liquefaction via the value of the volumetric component. In fact, for saturated tests ($E_{v,liq}$=0) $E_{s,liq}$ is constant for a fixed number of cycles at liquefaction (N_{liq}). For unsaturated tests, $E_{s,liq}$ increases as $E_{v,liq}$ increases for a given value of N_{liq}. The three variables ($E_{v,liq}$, $E_{s,liq}$ and N_{liq}) define a "liquefaction surface" describing the resistance to liquefaction of unsaturated soils.

1 INTRODUCTION

Liquefaction may occur in a shallow, saturated and loose sandy soil subjected to a seismic ground motion. The increase of pore water pressure produce a decrease of effective stresses and a subsequent loss of shear strength, causing damage to structures and infrastructures during earthquakes. Several mitigation techniques were studied to reduce liquefaction potential. One of the least invasive and promising technologies against liquefaction seems to be the Induced Partial Saturation (IPS). IPS consists in injecting some amount of air/gas in the voids of soil (Eseller Bayat et al., 2012), increasing the resistance to liquefaction of the soil, as shown by several research (Chaney, 1978; Yoshimi et al., 1989; Ishihara et al., 2002; Yegian, 2007; Wang et al., 2016; Mele et al., 2018). Okamura and Soga (2006) explained that the presence of air in the voids increases the resistance to liquefaction in two ways: the first mechanism is connected to the volumetric stiffness of gases (lower than that of water). During undrained cyclic loading, the soil tends to reduce its volume and the gas phase decreases, reducing the pore pressure build-up. The second mechanism is due to the matric suction of unsaturated soils, which increases the stiffness and strength of soils (Bishop and Blight, 1963) and it is relevant for low degree of saturation (S_r). The effect of the desaturation on three different kinds of soil has been studied by Mele et al. (2018) through cyclic triaxial tests, comparing the results of saturated and unsaturated tests. The results are usually interpreted in the CSR vs. N_{cyc} plane, being CSR the *Cyclic Stress Ratio* defined as:

$$CSR = \frac{q_d}{2 \cdot \sigma'_c} \qquad (1)$$

where q_d is the cyclic deviatoric stress and σ'_c is the confining effective stress, while N_{cyc} is the applied number of constant amplitude stress cycles.

Liquefaction occurs, conventionally, when the double strain amplitude (ε_{DA}) is equal to 5% according to strain criterion or when the excess pore pressure ratio (R_u) is 0.90 (stress criterion) with R_u defined as the ratio between Δu and σ'_0 , where Δu is the excess of pore air pressure for the specimen with positive suction measurement, otherwise it is the excess of pore water pressure (Wang et al., 2016), while σ'_0 is the confining stress. The number of cycles where liquefaction is triggered is called N_{liq} (number of cycles at liquefaction). For $N_{cyc}=N_{liq}$ the applied cyclic stress ratio represents the *Cyclic Resistance Ratio* CRR. The results of cyclic tests are reported in the plan CRR-N_{liq}, where the cyclic resistance curve can be plotted for a fixed soil in known conditions. The cyclic resistance curve, in fact, depends on the state of the soil: confining stress (σ'_0), void ratio (e) and thus relative density (D_r) and degree of saturation (S_r). A new energetic approach was introduced by Mele et al. (2018) to quantify the resistance to liquefaction of unsaturated soils. In this paper new considerations about this model have been discussed to better understand the behaviour of unsaturated sandy soils in cyclic undrained conditions.

2 THE ENERGETIC MODEL IN LIQUEFACTION TESTS

2.1 *The theoretical principles of the energetic model*

Even under the unsaturated condition, specimens can lose effective stress due to cyclic loading in undrained conditions, reaching liquefaction (Unno et al., 2008). To better understand the meaning of liquefaction in unsaturated soils the definition of effective stress has to be given. In this research the expression proposed by Bishop (1963) was used:

$$\sigma'_{un} = (\sigma - u_a) + \chi \cdot (u_a - u_w) \tag{2}$$

where σ is the total stress and u_a, u_w e χ are respectively the pore air pressure, the pore water pressure and the material parameter accounting for the effect of the degree of saturation. The term (σ-u_a) is called “net stress”, while (u_a-u_w) is the “matric suction” (s). In this paper, the parameter χ is assumed equal to the degree of saturation S_r ($\leq$100%).
Based on Eq. (2), it can be understood that in unsaturated soils, liquefaction occurs when both the pore air and water pressure are equal to the initial total confining pressure (Unno et al., 2008), but Mele et al. (2018) showed that, unlike saturated tests, in unsaturated conditions the stress and strain criterion give different results in term of N_{liq}, and in particular, N_{liq} evaluated by stress criterion (R_u=0.90) is higher than that evaluated by strain criterion (ε_{DA}=5%). The latter one has been adopted by Mele et a. (2018) to plot the cyclic resistance curve of unsaturated liquefaction tests carried out on three fine sands differing by gradation and mineralogical origin. For completeness material properties of the studied sands are reported in Table 1.

Table 1. Material Properties of the Soils (Mele et al., 2018).

Material Property	Sant’Agostino Sand	Bauxite	Inagi Sand
Fines Content (<0.075 mm) (%)	20.0	40.6	29.5
Specific Gravity, G_S	2.674	2.642	2.656
D_{50} (mm)	0.200	0.200	0.115
e_{max}	1.01	-	1.645
e_{min}	0.37	-	0.907

During the undrained cyclic triaxial loading of loose unsaturated sands, volumetric strains (ε_v) increase until to reach a final value at liquefaction ($\varepsilon_{v,fin}$) which depends only on the initial state, defined for instance by the degree of saturation (S_{r0}), the void ratio (e_0) and the initial net stress (σ-u_a)$_0$. In fact, whatever the applied CSR was, $\varepsilon_{v,fin}$ is the same for fixed initial conditions.

$\varepsilon_{v,fin}$ can be easily found by simple thermodynamic considerations (isothermal condition), applying Boyle and Mariotte law and it is reported below (Okamura and Soga, 2006; Mele et al., 2018):

$$\varepsilon_{v,fin} = \frac{e_0}{1+e_0} \cdot (1 - S_{r0}) \cdot \left(1 - \frac{u_{a,0}}{\sigma_c}\right) \tag{3}$$

where σ_c is the constant confining total stress while $u_{a,0}$ is the initial air pressure.
For known initial conditions (σ'_0, e_0 and Sr_0) of a soil, average curves can be found in the ε_v:σ'_{un} plane for the experimental tests reported by Mele et al. (2018) as shown in Figure 1a. Plotting the curves in a non-dimensional plane $\sigma'_{un}/\sigma'_{un,0}$:$\varepsilon_v/\varepsilon_{v,fin}$, a unique curve (Fig. 1b) is obtained for all the experimental results, having the expression:

$$\frac{\sigma'_{un}}{\sigma'_{un,0}} = 1 - \left(\frac{\varepsilon_v}{\varepsilon_{v,fin}}\right)^{1.7} \tag{4}$$

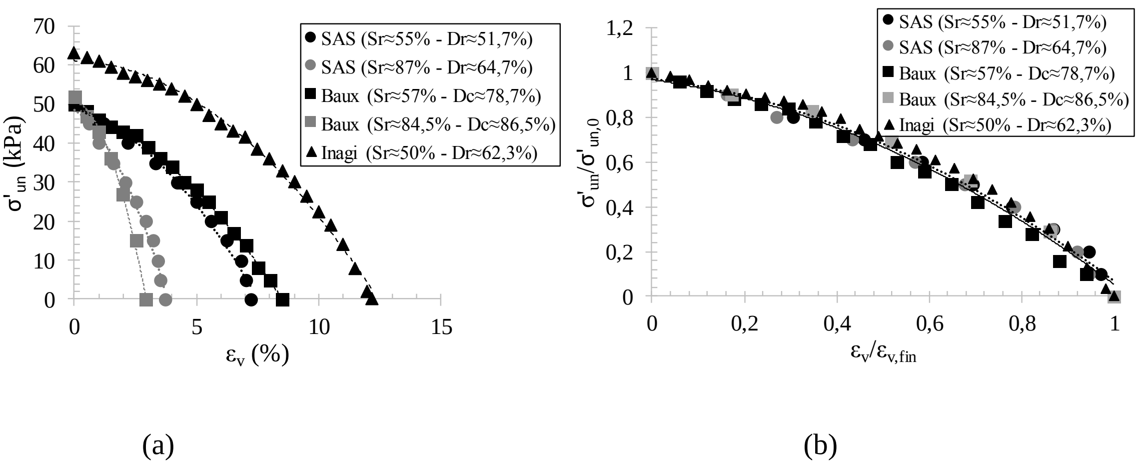

Figure 1. Results of unsaturated tests in the plane: σ'_{un} with ε_v (a) and $\sigma'_{un}/\sigma'_{un0}$ with $\varepsilon_v/\varepsilon_{v,fin}$ (Mele & Flora, 2018).

Even though this curve is obtained in similar conditions: low confining stresses and low relative densities, it can be used as a general rule in liquefaction tests because liquefaction takes place under these conditions.

2.2 *Specific energy to liquefaction*

Starting from thermodynamic considerations, where a partially saturated soil can be considered as a thermodynamically open system and under the further hypothesis of isothermal process, constant mass and air as an ideal gas, the specific energy to reach liquefaction (E_{tot}) can be introduced, defined as the sum of the specific volumetric energy at liquefaction ($E_{v,liq}$) and the specific deviatoric energy at liquefaction ($E_{s,liq}$).
$E_{v,liq}$ is the energy that the soil, under cyclic loading in undrained conditions, spends to change its volume, so it is null in undrained cyclic tests in saturated conditions (ε_v=0). $E_{v,liq}$ can be defined as the sum of three components: the volumetric specific energy to liquefaction of soil skeleton ($E_{v,sk,liq}$), the volumetric specific energy to liquefaction of water ($E_{w,liq}$), the volumetric specific energy to liquefaction of air ($E_{air,liq}$). They represent the work done by the volumetric deformation of the soil skeleton and the work caused by the flow of mass of water and air into the system of pores, respectively.
The volumetric specific energy of soil skeleton ($E_{v,sk,liq}$) can be found using the following equation:

$$E_{v,sk,liq} = \int_{0}^{\varepsilon_{v,liq}} \left[(\sigma - u_a) + sS_r \right] \cdot d\varepsilon \tag{5}$$

Where $(\sigma - u_a)$ is the net stress, s is the suction; S_r the degree of saturation, while $d\varepsilon_v$ is the increment of volumetric strain during undrained cyclic loading. $E_{v,sk,liq}$ depends just on confining stress, void ratio and degree of saturation ($E_{v,sk,liq} = f(\sigma'_0, e_0, S_{r0})$). This integral can be evaluated using the average curve of Figure 1a.
Energetic contributions of water and air are given by the following equation, respectively:

$$E_{w,liq} = -\int_{S_{r0}}^{S_{r,liq}} \frac{e(S_r)}{1+e(S_r)} s(S_r) \cdot dS_r \tag{6}$$

$$E_{air,liq} = \frac{e_0}{1+e_0}(1 - S_{r0}) u_{a,liq} d(\ln \rho_{a,liq}) \tag{7}$$

Where $u_{a,liq}$ and $\rho_{a,liq}$ are air pore pressure and mass density of air at liquefaction, respectively.
$E_{v,liq}$ is a state parameter which depends on confining stress, void ratio and degree of saturation, while it does not depend on the applied CSR. It can be considered as a syntethic parameter which summarizes the three state parameters that identify a cyclic resistance curve (e, σ' and S_r), while it is null for saturated soil in undrained condition, being null volumetric strains.
The second contribution in the total specific energy to liquefaction is given by $E_{s,liq}$. It is due only to soil skeleton and it is defined as the sum of the areas of all the cycles in the ε_s-q plane (D_{cyc} in Fig. 2 for a single cycle) up to liquefaction, where ε_s is the deviatoric strain and q is the cyclic deviatoric stress. $E_{s,liq}$ can be formally written with the following equation:

$$E_{s,sk,liq} = \sum_{N_{cyc}=1}^{N_{cyc}=N_{liq}} \iint_{D_{cyc}} dq \cdot d\varepsilon_s \tag{8}$$

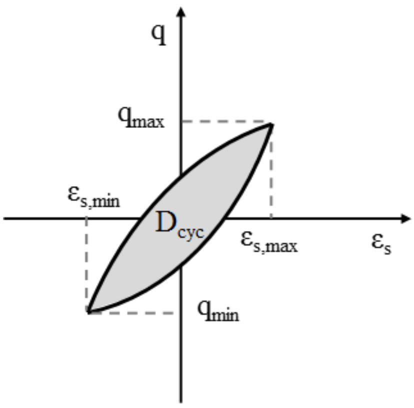

Figure 2. Cycle in q-ε_s plane (Mele et al., 2018).

Unlike $E_{v,liq}$, $E_{s,liq}$ is strongly dependent on the applied CSR, in addition to soil properties and soil state. While the volumetric part can identify the position of the cyclic resistance curve of a soil, the deviatoric part is the energetic variable that, for a given value of $E_{v,liq}$ defines the cyclic resistance CRR and thus N_{liq} (Mele & Flora, 2018).

2.3 *Main results of the energetic model*

As mentioned above E_{vliq} is a state parameter which can identify the position of a cyclic resistance curve in unsaturated conditions. For an unsaturated sandy soil with known initial conditions, a value of E_{vliq} can be associated to its cyclic resistance curve. Fixing a number of cycles at liquefaction of 15, for example, the Figure 3 (by Mele & Flora, 2018) shows as the difference between the CRR of unsaturated soil (CRR_{un}) and the CRR of a saturated soil (CRR_s) increases when E_{vliq} increases, where p_a in horizontal axis of Figure 3 is the atmospheric pressure, which is introduced to make the relationship non-dimensional.

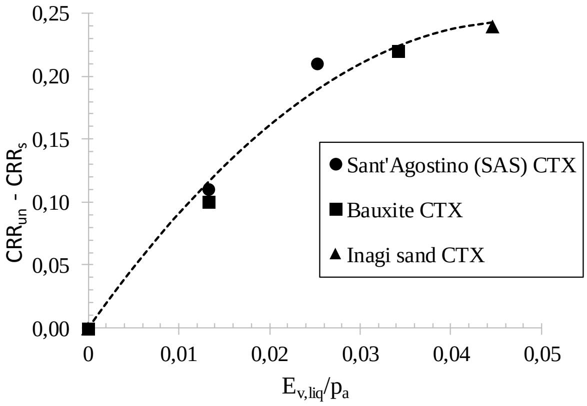

Figure 3. Relationship between CRR_{un}-CRR_s with $E_{v,liq}/p_a$ (modified by Mele & Flora, 2018).

It means that E_{vliq} can indicate the increment of resistance of an unsaturated soil compared with a saturated one at the same conditions. $E_{s,liq}$ instead, as already mentioned defines the CRR. A strong correlation between $E_{s,liq}$ and E_{vliq} has been found by (Mele & Flora, 2018) as reported in Figure 4. Also in this case, p_a is needed to make the relationship non-dimensional.

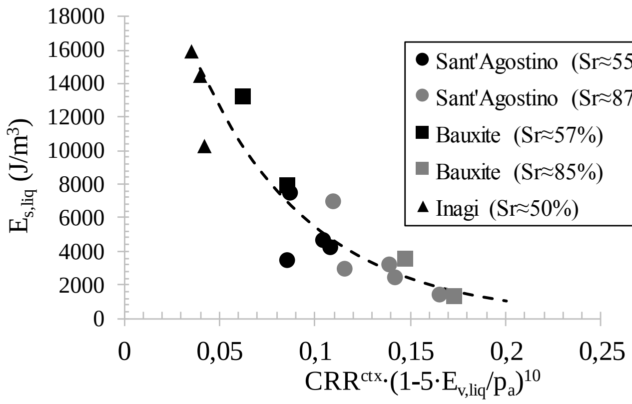

Figure 4. Cycle in q-ε_s plane (Mele & Flora 2018).

3 THE ROLE OF ENERGETIC VARIABLES IN LIQUEFACTION TESTS

3.1 *Experimental evidences*

$E_{s,liq}$ is an important variable which can be used also to identify the attainment of liquefaction.

E_s increases with number of cycles and the maximum gradient of this curve corresponds to the attainment of liquefaction (Mele & Flora, 2018). Mele et al. (2019) have shown the behavior of E_s with N_{cyc} for saturated soils, and the most important result is that, regardless of the position of cyclic resistance curve, $E_{s,liq}$ reaches the same value for equal N_{liq}.

In order to verify this result in unsaturated conditions, and thus with $E_{v,liq}$ not null (and variable), three tests, carried out by Mele et al. (2018) have been compared in term of E_s – N_{cyc} (Fig. 5). The tests have been chosen among the unsaturated tests of Mele et al. (2018) where liquefaction occurs at a very similar N_{liq} (Table 2).

Table 2. State properties and energetic components of tests (modified by Mele & Flora, 2018).

Test	Material	σ'$_{un}$ (kPa)	e_0	S_{r0} (%)	CRR	N_{liq}*	$E_{v,liq,av}$ (J/m^3)	$E_{s,liq}$ (J/m^3)
U_SA8	SAS	48.8	0.59	87.6	0.258	9.6	1192	2486
U_BA5	Bauxite	48.4	0.75	85.0	0.279	8.3	1200	3622
U_IN3	Inagi	62.3	1.14	52.0	0.404	8.6	4035	10306

*evaluated for ε_{DA}=5%

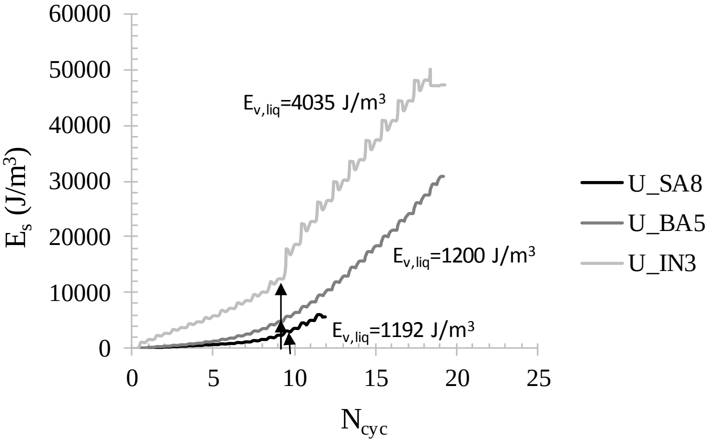

Figure 5. E_s with N_{cyc} for different $E_{v,liq}$.

These three tests seem to confirm that the maximum gradient of the curve E_s-N_{cyc} is reached in correspondence of N_{liq} evaluated with strain criterion (ε_{DA}=5%) and indicated by arrows in Figure 5. Even though N_{liq} is not exactly the same in these tests, it can be noted that, unlike saturated tests, $E_{s,liq}$ is not constant for a fixed value of N_{liq}. In particular, $E_{s,liq}$ is much higher as $E_{v,liq}$ increases. This has a clear physical meaning: by reducing the degree of saturation, E_v increases and so thus the deviatoric energy needed to liquefy.

$E_{s,liq}$ of all experimental tests of Mele et al. (2018) can be plotted with N_{liq} for several values of $E_{v,liq}$ (Fig. 6). $E_{v,liq}$=0 corresponds to saturated tests; in this case, regardless of the kind of sand, $E_{s,liq}$ is the same for a fixed value of N_{liq}. The straight line, in this case, seems to be horizontal because of the small range of $E_{s,liq}$ (500-900 J/m^3). For higher values of $E_{v,liq}$ the slope of the curve is much more important and when $E_{v,liq}$ increases $E_{s,liq}$ increases at a fixed number of cycles. Considering a N_{liq} equal to 15, it can be noted that the gradient of the relationship $E_{s,liq}$ - $E_{v,liq}$ increases with $E_{v,liq}$, as shown in Figure 7.

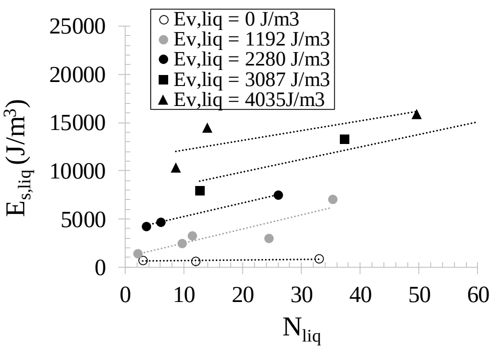

Figure 6. $E_{s,liq}$ with N_{liq} for fixed values of $E_{v,liq}$.

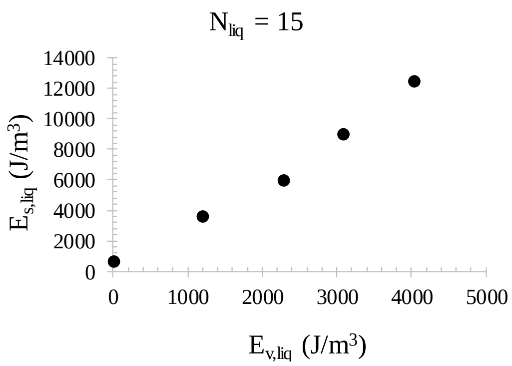

Figure 7. $E_{s,liq}$ with $E_{v,liq}$ for N_{liq} = 15.

3.2 *Theoretical interpretation*

These experimental results show that $E_{s,liq}$ is strongly dependent on the state of soil, represented by $E_{v,liq}$, which seems to be able to summarize three important state variables in liquefaction tests, such as σ'_0, e_0 and S_{r0}. If $E_{v,liq}$ can express the state of the soil, N_{liq} can be considered as the variable which describes the energy of the seismic action, and is typically related to the earthquake magnitude (M) (e.g. Seed & Idriss, 1982). $E_{s,liq}$, instead, is the energetic variable which describes the response of the soil in particular state conditions ($E_{v,liq}$) to a fixed seismic event (N_{liq}), allowing to quantify the resistance to liquefaction because of its dependence on CRR (Fig. 4). It means that the liquefaction behaviour of unsaturated soils can be described by three variables: $E_{v,liq}$, $E_{s,liq}$ and N_{liq}. The results of the experimental tests may be reported in the space $E_{v,liq}$-N_{liq}-$E_{s,liq}$. Fig. 8 shows a surface in such a plane connecting the experimental points reported in Fig. 6. Then, Figs. 6 and 7 are just cross sections of this surface.

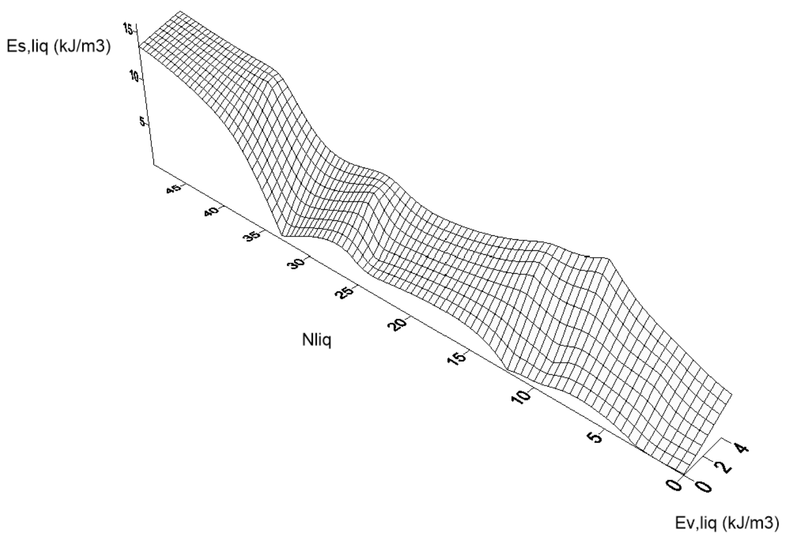

Figure 8. Liquefaction surface $E_{v,liq}$-N_{liq}-$E_{s,liq}$.

The surface reported in Fig. 8 is not regular as it connects a limited number of experimental points. Assuming to have a more regular and known surface, interpolating a larger number of data, it would describe the behaviour of a sandy soil in particular conditions (described by $E_{v,liq}$) subjected to a seismic event (N_{liq}) in terms of the deviatoric energy $E_{s,liq}$ needed to liquefy. The latter is connected to CRR_{un} (Fig. 4).

4 CONCLUSIONS

In order to better understand the role played by the specific volumetric and deviatoric energy at liquefaction, a further insight has been carried out analysing the results of experimental data already published by the authors.. $E_{v,liq}$ is a synthetic variable to identify the state of the soil in unsaturated conditions. Plotting E_s with N_{cyc} for unsaturated tests, for similar values of N_{liq}, it can be noted that $E_{s,liq}$ is different in these tests, unlike saturated tests, where regardless of the soil and the applied CSR, $E_{s,liq}$ is the same for fixed N_{liq}. This experimental evidence proves that $E_{s,liq}$, is dependent on $E_{v,liq}$ (and thus has to be constant for saturated soils, for which $E_{v,liq}$ is constant and in particular nil), increasing as $E_{v,liq}$ increases. For a given number of cycles to liquefaction (in particular N_{liq}=15, which is often considered in literature as a reference value) a correlation has been found in the paper between the two variables. A more general representation of the results can be obtained by looking at Fig. 8, in which the liquefaction behavior of loose sands is fully represented by a surface in the $E_{v,liq}$-N_{liq}-$E_{s,liq}$ plane. $E_{v,liq}$ summarizes the state of soil, N_{liq} represents earthquake magnitude, while $E_{s,liq}$ may express the response of the soil subjected to a seismic event. A correlation between $E_{s,liq}$ and CRR_{un}, allows to quantify the resistance of an unsaturated soil. Further tests are needed to confirm this result in different conditions in term of relative density and confining stress.

5 ACKNOWLEDGEMENTS

This work was carried out as part of the European project Horizon 2020 – Assessment and Mitigation of liquefaction potential across Europe: A holistic approach to protect structures infrastructures for improved resilience to earthquake – induced liquefaction disasters – "LIQUEFACT" (grant agreement No. 700748).

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