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Electromagnetic Wave Propagation Equations in 2D by Finite Difference Method

N. Fusun Oyman Serteller

Abstract**—**In this paper, the techniques to solve time dependent electromagnetic wave propagation equations based on the Finite Difference Method (FDM) are proposed by comparing the results with Finite Element Method (FEM) in 2D while discussing some special simulation examples. Here, 2D dynamical wave equations for lossy media, even with a constant source, are discussed for establishing symbolic manipulation of wave propagation problems. The main objective of this contribution is to introduce a comparative study of two suitable numerical methods and to show that both methods can be applied effectively and efficiently to all types of wave propagation problems, both linear and nonlinear cases, by using symbolic computation. However, the results show that the FDM is more appropriate for solving the nonlinear cases in the symbolic solution. Furthermore, some specific complex domain examples of the comparison of electromagnetic waves equations are considered. Calculations are performed through Mathematica software by making some useful contribution to the programme and leveraging symbolic evaluations of FEM and FDM.

Keywords**—**Finite difference method, finite element method, linear-nonlinear PDEs, symbolic computation, wave propagation equations.

I. Introduction

MODELLING, control and design of many electrical systems involve calculating a linear or nonlinear partial differential equation (PDE). Common analytical techniques, based on the assumption that nonlinearities are relatively insignificant, sometimes strongly affect the solution with respect to the real physics of the phenomenon. Therefore, seeking exact and approximate solutions is still a significant problem, and it becomes increasingly important to be familiar with all traditional and recently developed methods for finding exact and approximate solutions and the implementations of these methods [1]-[3], [8], [17].

Computer algebra systems provide the possibility to conduct both numerical and symbolic computations [5], [6], [16]. Therefore, many traditional algorithms can be improved, sometimes considerably via embedding symbolic parts into the numerical algorithms. These hybrid techniques involve numeric and symbolic manipulations to provide arbitrary precision in defeating instability problems and reduce the number of iterations in general [5], [6], [9], [11].

In the literature, 1D and 2D dynamic and non-dynamic wave propagation equations and their approximate solving techniques have been much studied recently [5]-[16]. In this paper, the 2D time-dependent wave propagation equation (PDEs) solved by FEM and FDM permits the cooperation of symbolic parameters penetrating into the solution using Mathematica toolbox. By means of the Mathematica program and solving techniques, some linear and nonlinear wave equations or PDEs are handled by examples in general forms. The computer algebra system helps the equation solving process to be easily implemented and completed in a short time while providing different ideas for the phenomena and medium [4]-[6], [14]. The problems are solved with the discretization method using FEM and FDM to examine the compression of the methods [7], [9], [11], [12]. In particular, a 2D lossy medium alone and a lossy medium with a nonlinear source, which are the examples of problems that have difficulties being resolved via equations because of the structure of nonlinearity, inaccurate approximation, difficulties in meshing and non-rapidly convergent calculations [10], [15], are being considered and visualized. The main objective of this contribution is to introduce a comparative study for the usability of the methods and to show that both methods can be applied to the wave problems in either linear or nonlinear cases. The solution methods applied using a computer are easy and more convenient to implement while searching the problems' features and diversity with some special initial and boundary conditions. Although for most cases in the literature, FEM appears to be more appropriate to obtain the results of the problems [2], here, after many comparing studies and plotting results, the FDM solution and results are considered to be more appropriate for some nonlinear cases such as lossy medium with or without source [9], [13]. In addition, when the constants value in the equation is needed to increase, the FEM solution accuracy stays behind the FDM solution. However, in some cases, the advantages of two methods can be combined, i.e. efficiency of FDM and convenience and flexibility for free boundary of FEM [11]. Calculations are performed through Mathematica, which provides some useful contributions in terms of programming and leveraging the visualization of the symbolic evaluations of FEM and FDM via its simulation visualization. The results show that these methods are efficient and convenient and can be applied to a large class of wave propagation problems. However, after the first figure, since the FDM solution is apparently more convenient to use in solving such problems, FDM has been used to solve some specific following problem equations.

II. Methods

In this study, the structure of the FEM and FDE equations is presented. The principal algorithm for the FEM and FDM for the electromagnetic wave propagation equation, including all

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possible wave propagation distributions on the domain (Ω) with initial and boundary conditions, involves a nonlinear or linear PDE written in the general form which are shown as:

$$\begin{cases} \frac{\partial^2 u(t,x,y)}{\partial x^2} + \frac{\partial^2 u(t,x,y)}{\partial y^2} + u(t,x,y)\frac{1}{v^2}\frac{\partial^2 u(t,x,y)}{\partial t^2} = u(t,x,y) \\ \forall\ x,y\ \partial\Omega, \forall t \end{cases} \quad (1)$$

If the initial conditions are applied in general as following:

$$u(0,x,y)=0$$
$$\frac{\partial u(t,x,y)}{\partial t}=0,\ t=0. \quad (2)$$

The boundary conditions are applied like:

$$u(t,-L,y)=u(t,L,y)=u(t,x,-L)=u(t,x,L)=S\text{in}(wt-x)$$
$$x\in[-L,L];\ \ y\in[-L,L]. \quad (3)$$

In Cartesian coordinates, system $u(t,x,y)$ satisfies the 2D scalar wave equation in the x direction from the boundaries which are pulsed with time-dependence of the fields, where w is the angular frequency, L denotes the length of domain in x and y directions and v denotes the wave speed in medium. The discussion of the optimization and convergence of the medium and source analysis and the method of building of the wave propagation are not the subjects of this paper; for more details, refer to the work of [4], [10].

Fig. 1 shows the 2D scalar wave equation using a time equation schematic representation on the domain. All the materials in the wave propagation equation examples are completely open and customizable which fulfills one's needs or simply creates the functions to use Mathematica's built-in tools as building functions [5], [6], [16]. Here, PDEs are defined, modified and improved using Mathematica tools. Further to this study, the dynamic analysis of the subject can be studied to see the detail of the wave behavior on domain using animation or dynamic tool, see appendix.

III. Numerical Results and Discussion

In this section, the feasibility and efficiency of the FEM and FDM are simulated via the cases below in the sense that their analytical answers are not known in advance. Initial and boundary conditions are taken equal for different PDE examples to show the equations' variation on the domain.

A. Lossless Media with Free Source

The wave propagation equations are best described in spatial and temporal lossless media and source free conditions as linear case in 2D as:

$$\begin{cases} \frac{\partial^2 u(t,x,y)}{\partial x^2} + \frac{\partial^2 u(t,x,y)}{\partial y^2} - \frac{1}{v^2}\frac{\partial^2 u(t,x,y)}{\partial t^2} = 0 \\ -L \le x \le L,\ -L \le y \le L \\ \text{under initial conditions, } u(t,x,y)=0,\ t=0 \text{ and,} \\ \text{(Nuemann condition) } \frac{\partial u(t,x,y)}{\partial t}=0,\ t=0 \\ \text{Boundary condition, } u(t,-L,y)=u(t,L,y)=u(t,x,-L)=u(t,x,-L)=S\text{in}(wt-x) \end{cases} \quad (4)$$

FEM results at a given time for the equations are given below.

In Figs. 2 (a)-(d), simulation of the FEM solution with the triangle element on the domain is solved for time dependent. Since the drawing the triangular mesh for finite elements darkens the figures, no meshing is shown in Figs. 1 (b)-(d). The wave propagation equation is realized with some difficulty in Mathematica's own solving system [16]. Especially, the solution techniques and the computational process, which includes the constant value of v, directly affect the solution stability and inconsistency of the results. A stable increment in the value of the constants and stability of the initial conditions should be an appropriate process; otherwise, the PDE's solution cannot be obtained via the FEM solution [6], [11]. The FDM solutions are given below.

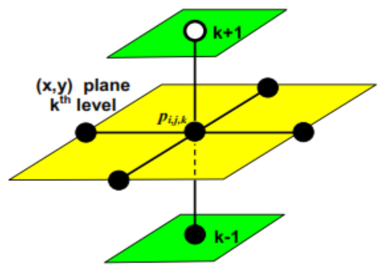
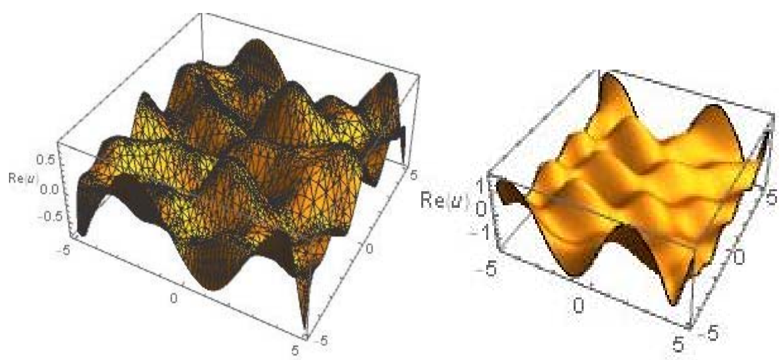


Fig. 1 Schematic representation of the 2D scalar wave equation with the (t,x,y) operator [1]



(a) (b)

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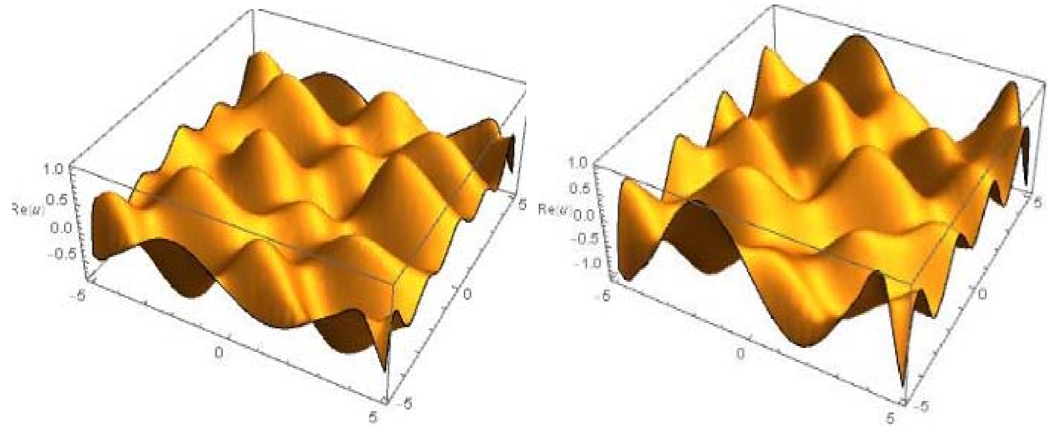
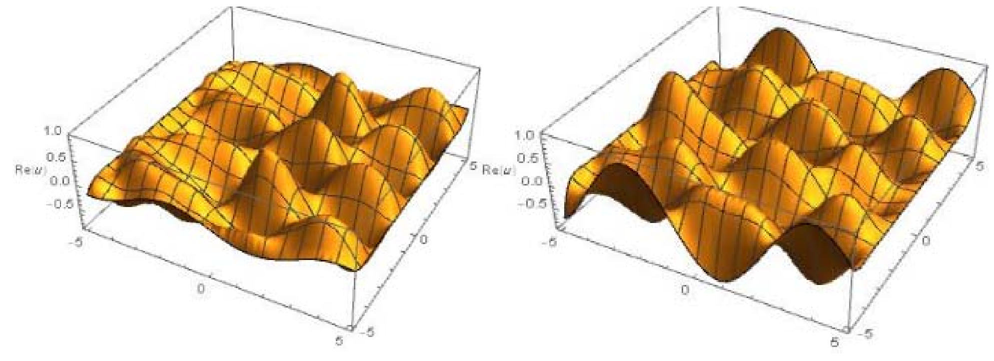
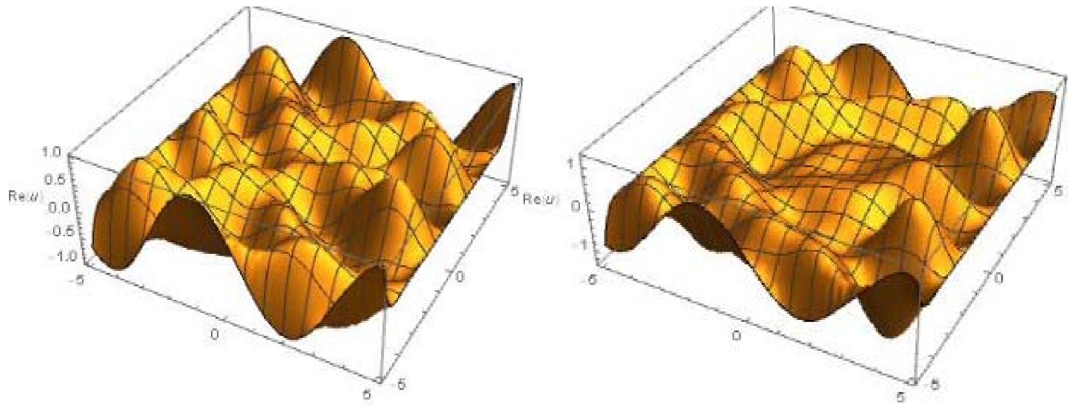


(c) (d)

Fig. 2 FEM solution for (4) (alphabetical order defines t=1 s t=2 s, t=3s, t=4 s, respectively).



(a) (b)



(c) (d)

Fig. 3 FDM Results for (4) t=1 s, t=2 s, t=3s,t=4 respectively

The differences between the two methods with time variation are visualized in Figs. 2 and 3. When the constant values are increased in equations, FDM solution gives more details and more accurate value (u) on the domains, depending on time, in the linear case. Because the linear equation case is previously realized in more details, the following examples are only solved for FDM. Moreover, FDM solving system has a short CPU time (in Fig. 7 (b)) and few acceptable warnings results which are like in FEM in differential equations solving [11], [12]. The visualized solutions for (4) without the Neumann condition are below.

The appearance of the solution exhibits the classical wave propagation behavior over time with less wave peaks. The values of the solution of wave propagation are the nearly same with those of the previously solved examples (Figs. 2 and 3, $\mathrm{Re}(u)$ values), and each figure in Fig. 4 changes in a different manner over time, as shown in Figs. 4 (a)-(d). It has the shortest CPU time which is shown in Figs. 7 (b).

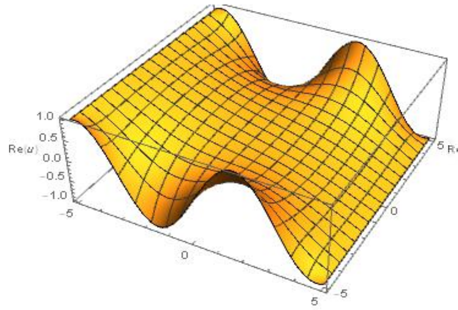
A. Lossy Media without a Source

In a lossy or inhomogeneous, nonlinear medium, the nonlinear equation with the same initial and boundary conditions is given in (5) [13], [14]. The wave propagation mathematical equations are written as:

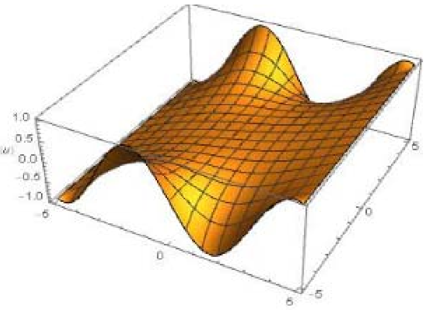
$$\begin{cases} -\dfrac{\partial^2 u(t,x,y)}{\partial x^2} - e^{-x}\dfrac{\partial^2 u(t,x,y)}{\partial y^2} + \dfrac{1}{v^2}\dfrac{\partial^2 u(t,x,y)}{\partial t^2} = 0 \\ -L \le x \le L,\ -L \le y \le L\ , \\ \text{initial condition};\ u(0,x,y)=0, \quad \text{and} \quad \dfrac{\partial u(t,x,y)}{\partial t} = 0, t = 0, \\ \text{Boundary condition};\ u(t,-L,y) = u(t,x,-L) = u(t,-L,y) = u(t,L,y) = S\text{in}(wt - x) \end{cases} \tag{5}$$

To demonstrate the nonlinear case of wave lossy propagation in a standard setting, an arbitrary spatially dependent coefficient e^{-x} multiplication with a time fractional derivative is also included into (5).

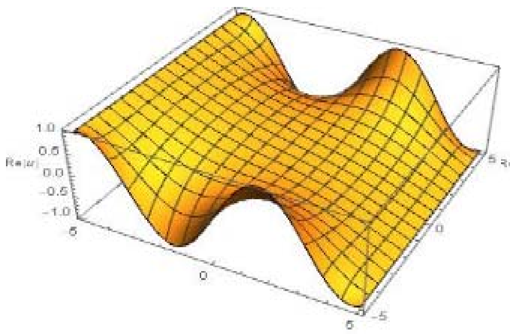
The visualization of (5) is displayed in Figs. 5 (a)-(d). The theoretical explanation [15], completely corresponds to the visualization of the solution of (5) with the decreasing effect of the lossy medium. Although the lossy effect is seen by the time, in detail from Figs. 5 (c) and (d), the constant values play important role in the solving equation. Additionally, an increment in the constant's value makes the lossy equation solution unpredictable in both FEM and FDE solution. However, FDM solution gives more stable and more accurate results which are supported with the theory in [1], [15].



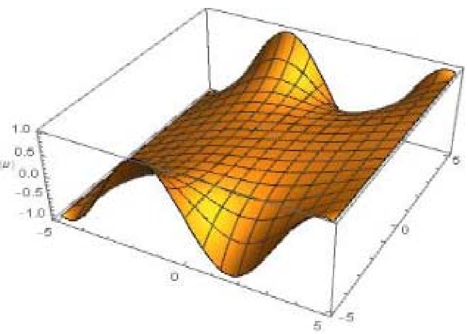
(a)



(b)

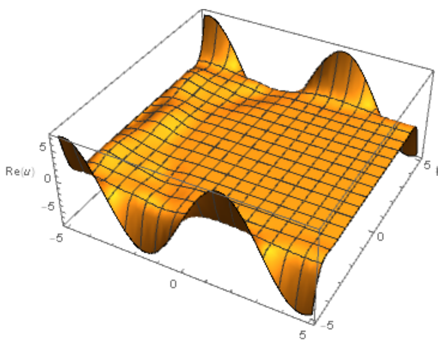


(c)

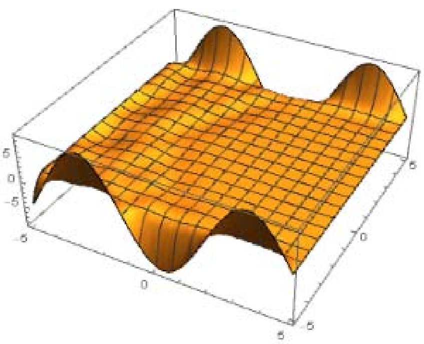


(d)

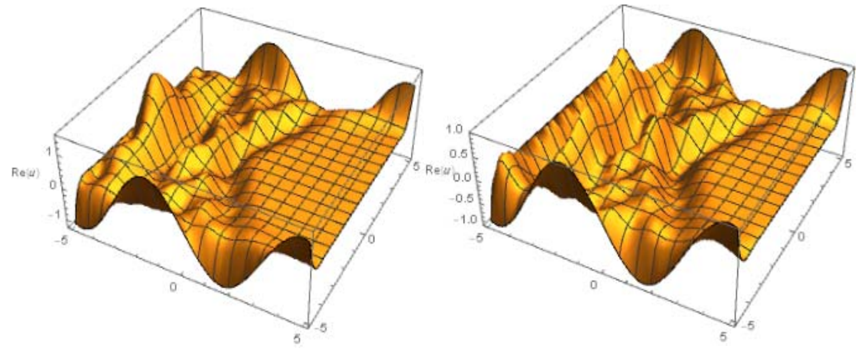
Fig. 4 FDM Results for (4)



(a)

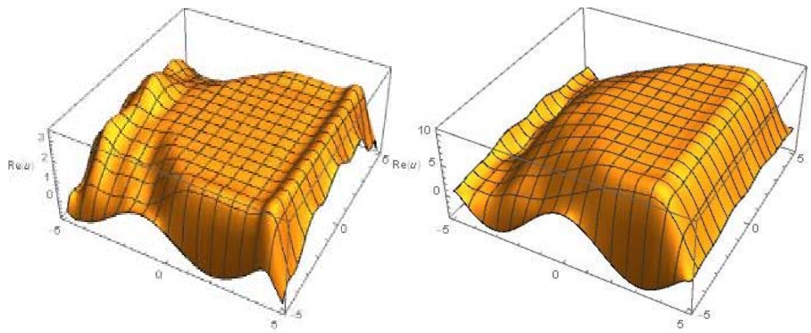


(b)

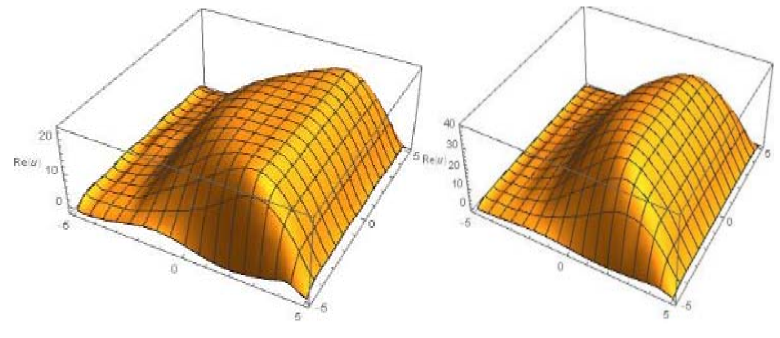


(c) (d)

Fig. 5 FDM analysis result for (5)



(a) (b)



(c) (d)

Fig. 6 FDM equation for (6)

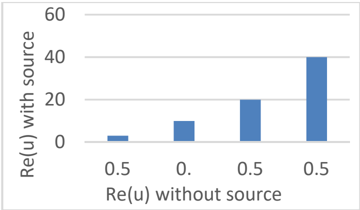
A. Lossy Media with a Source

The wave equations in a lossy medium with a source element are written as:

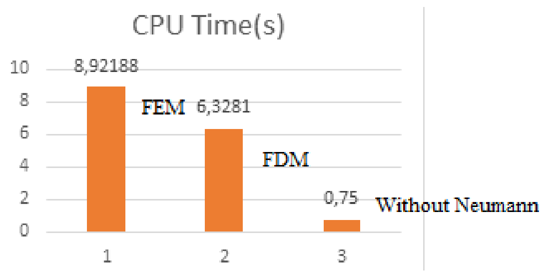
$$\begin{cases} -\dfrac{\partial^2 u(t,x,y)}{\partial x^2} - \dfrac{\partial^2 u(t,x,y)}{\partial y^2} + e^{-x}\dfrac{1}{v^2}\dfrac{\partial^2 u(t,x,y)}{\partial t^2} = K \\ K \geq 0, \\ -L \leq x \leq L, -L \leq y \leq L \ , \\ \text{initial condition, } u(0,x,y)=0,\ \dfrac{\partial u(t,x,y)}{\partial t}=0, t=0, \\ \text{Boundary condition; } u(t,-L,y)=u(t,L,y)=u(t,x,-L)=u(t,x,L)=S\text{in}(wt-x) \end{cases} \quad (6)$$

The source constant element is denoted as K . If the source element is considered depending on the dimensions and time in (6), then Fig. 6 plotting is nearly similar to Fig. 2 [9], [12]. The FDM solution in a lossy medium with a source element makes the nonlinear cases the more difficult to solve. However, this type of equation is the most encountered in real wave equations [12]-[14].

In Figs. 6 (a)-(d), the increasing effect of the source element on the domain is clearly seen orderly. Over the time, the impact of the source element's appearance becomes tapered, as shown in Fig. 6 (d) [15]. In Fig. 7 (a), dominant source effect is seen clearly.



(a)



(b)

Fig. 7 (a) Source effect (b)CPU time

IV. Conclusion

Preliminary studies on examples of the linear or nonlinear PDEs, and even inhomogeneous PDEs, in 2D wave propagation equations are presented. Some of the complex and most common problems encountered are solved by using Mathematica tools, such as FEM and FDM. The overall results indicate that FEM and FDM are reliable and efficient in numerically solving all types of PDEs in electromagnetic wave propagation equations. Preliminary studies on examples of the linear or nonlinear PDEs, and even inhomogeneous PDEs, in 2D wave propagation equations are presented. Some of the complex and most common problems encountered are solved by using Mathematica tools, such as the methods of FEM and FDM. Also, it has been experienced that although FEM has some advantages in lossless medium such as the more clear visualization, more suitable constant value penetration, the FDM solution has more appropriate and has more accurate results in lossy medium with source or without source equations.

The result of the comparison of the cases indicates some important data:

- The effect of a lossy medium is observed clearly at the boundaries, which plays a decreasing role in values.
- The constant source propagation affects all the regions as time increases.
- If the initial Neumann conditions are not used, then the solution of the equations does not permit radical changes in appearance.

Mathematica solving system enables some modifications and certain extensions in studying wave propagation equations that overcome the equations' disadvantages and nonlinearity and satisfy the requirements of experimental comparison. However, further and developed computational research is required on large and complex regions.

Appendix

$$\text{wavesol} = \text{NDSolve}[\{D[u[t,x,y],t,t] = \\ = v^2 D[u[t,x,y],x,x] + v^2 D[u[t,x,y],y,y], u[0,x,y] = \\ = 0, u[t,-5,y] == u[t,5,y] == 0, u[t,x,-5] == u[t,x,5] = \\ = \text{Sin}[\omega t - x], u^{(1,0,0)}[0,x,y] = \\ = 0\}, u, \{t,0,4\}, \{x,-5,5\}, \{y,-5,5\}, \text{PrecisionGoal} \to 1, \text{Method} \\ \to \{\text{"DAEInitialization"} \to \{\text{"Collocation"}, \text{"CollocationDirection"} \\ \to \text{"Forward"}\}\}]]$$

$$\text{Animate}[\text{Plot3D}[\text{wavesol}[t,x,y], \{x,-5,5\}, \{y,-5,5\}, \text{PlotRange}- \\ > \text{All}, \text{PlotPoints} \to 15, \text{Method} \\ \to \text{"FiniteElement"}, \text{MaxRecursion} \to 3, \text{Mesh} \\ \to \text{All}], \{t,1,4\}]$$

For CPU time;

$$\text{Timing}[\text{Table}[\text{Plot3D}[\text{wavesol}[t,x,y], \{x,-5,5\}, \{y,-5,5\}, \text{PlotRange} \\ \to \text{Full}, \text{Method} \to \text{"FiniteElement"}, \text{PlotPoints} \\ \to 20, \text{MaxRecursion} \to 2(*, \text{Mesh} \to \text{None}*)], \{t,1,4\}]]$$

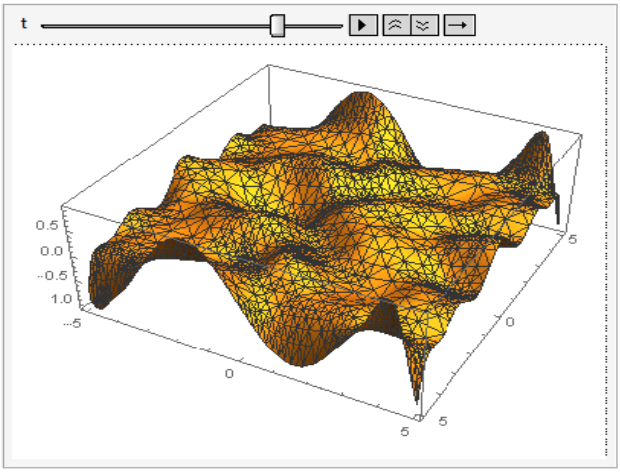


Fig. 8 Animation for 2D time dependent wave propagation equation

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