

Greener Journal of Physics and Natural Sciences

ISSN: 2384-6410

Submitted: 11/10/2017

Accepted: 18/10/2017

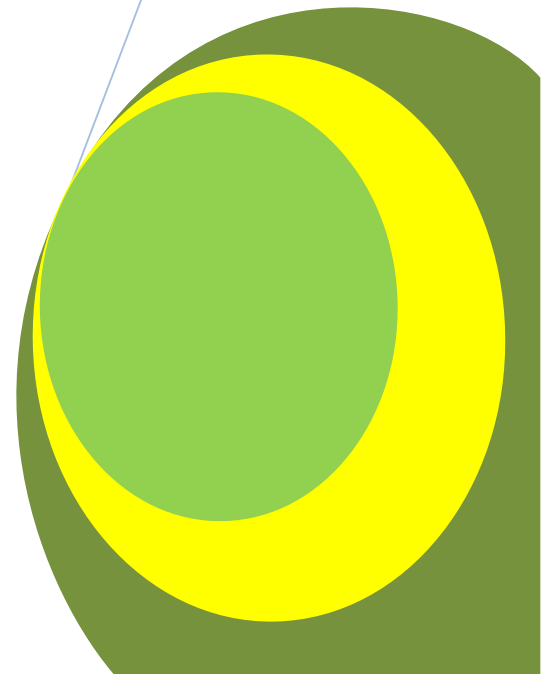
Published: 30/10/2017

DOI: <http://doi.org/10.15580/GJPNS.2017.2.101117147>

Theoretical Matrix Study of Rigid Body Pseudo Translational Motion

By

Anastas Ivanov



Research Article (DOI: <http://doi.org/10.15580/GJPNS.2017.2.101117147>)

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Anastas Ivanov

Todor Kableshkov University of Transport, Sofia, Bulgaria.

E-mail: aii2010@ abv. bg

ABSTRACT

In this paper, a *pseudo* translational motion of free asymmetrical rigid body to an absolute reference system is studied. A private kind of Theorem of change of generalized body impulse is formulated. This theorem is called Theorem of change of *pseudo* generalized body impulse. A private kind of Condensed Lagrange equations is formulated. These equations are called Condensed Lagrange equations for study of *pseudo* translational motion of free asymmetrical rigid body. Using that theorem and those equations, the *pseudo* translational motion of the rigid body is successfully studied. The paper is theoretical, but it gives a base for a number of applications. For example, these are investigations of the motion, stability and management of satellite. The other main application is free or forced small body vibrations. Moreover, the obtained formulas are appropriate for computer numerical integrations by contemporary mathematical programs.

Key words: rigid body, pseudo translational motion, generalized body impulse, condensed Lagrange equations.

1. INTRODUCTION

The presented work is a continuation of the article [1]. Here, the obtaining of differential equations, describing a rigid body *pseudo*-translational motion (RBPTM), is treated. This motion is a particular type of rigid body general motion (RBGM) when the spherical component is very small [2]. In the mentioned article [2], a linearization of the transition matrices and Cardan kinematic equations was performed.

The rigid body *pseudo*-translational motion is the basis of very important motions for engineering practice. These are the rigid body small three dimensional vibrations and multi-body small three dimensional vibrations, where the bodies are connected each others with elastic and damping elements [3, 4, 5]. The engineering study of these vibrations requires the use of modern computer programs [6, 7]. One of the most convenient programs for such study is MatLab, which works with scalars and matrices. That is why the statement in this article entirely in a matrix form is done.

The small vibrations theory of simple or complex mechanical systems is already developed. Today, using this theory, one of the most advanced methods for dynamical study of such mechanical systems is applied. This is the Finite Element Method used for dynamical study of complicated mechanical systems [8, 9, 10].

In order to obtain the system of differential equations describing RBGM, two basic theorems of Dynamics are required: Theorem of change the quantity of motion and Theorem of change the kinetic moment [11, 12, 13, 14]. In the work [1], these two theorems were united in one. This new theorem was called Theorem of change the rigid body generalized impulse (TCRBGI). In this theorem, the actual spherical component of this motion is taken into account. When a rigid body *pseudo*-translational motion is studied, the spherical component is not revealed completely. Therefore, it is interesting from a theoretical point of view how TCRBGI can use for RBPTM. In addition, Condensed Lagrange equations were defined in the work [1]. The present work shows the specific peculiarities of using these equations.

2. KINETICS CHARACTERISTICS

An asymmetric free rigid body (B) that achieves a *pseudo*-translational motion is considered. The movement of the body is counted against another body (A) conditionally assumed to be immovable. A fixed coordinate system $N \xi \eta \zeta$ is connected to it (Fig.1).

Two coordinate systems are introduced at arbitrary point O in the body (B).

The first coordinate system $OXYZ$ moves translational and the second coordinate system $Oxyz$ is steadily connected to the body (B).

The spherical component of the rigid body motion is described by Cardan angles ψ , θ and φ . They are very convenient to set the initial conditions and eventually for linearization of the angular rotations [2].

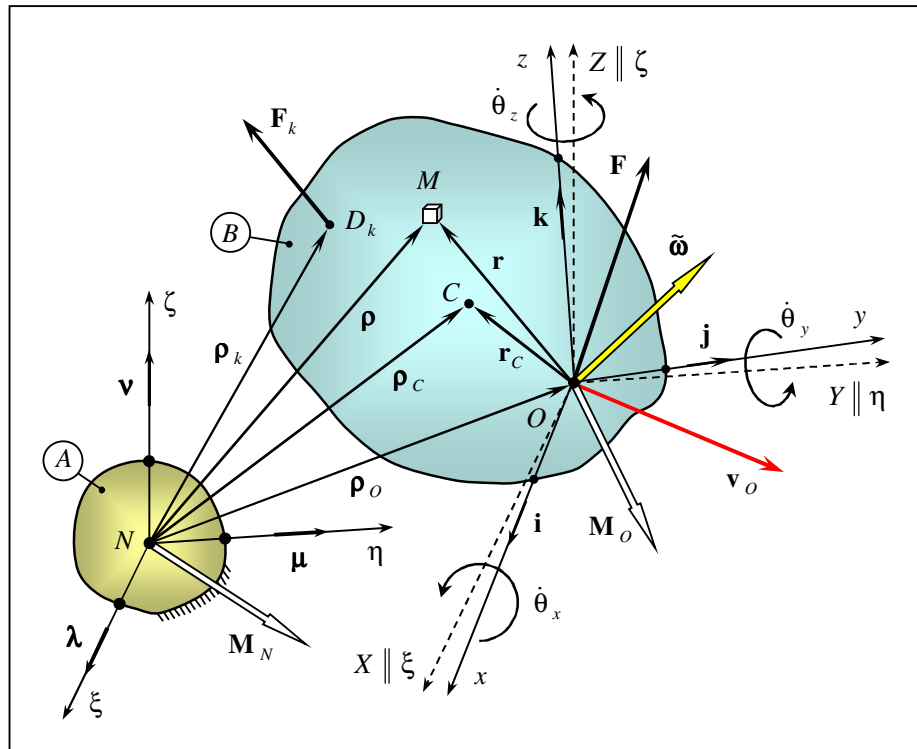


Fig.1: *Pseudo* translational motion of free rigid body

It is assumed that the two most important Kinematics characteristics are already known. These are the velocity of the pole O and the **vector-pseudo angular velocity** of the body. They are defined by the following vectors:

$$(1) \quad \mathbf{v}_o = \dot{\boldsymbol{\rho}}_o = \langle \dot{\xi}_o \quad \dot{\eta}_o \quad \dot{\zeta}_o \rangle^T,$$

$$(2) \quad \tilde{\boldsymbol{\omega}} = \langle \dot{\theta}_x \quad \dot{\theta}_y \quad \dot{\theta}_z \rangle^T \approx \langle \dot{\theta}_X \quad \dot{\theta}_Y \quad \dot{\theta}_Z \rangle^T.$$

The law of body *pseudo* translational motion is set with the **vector-pseudo generalized coordinates**:

$$(3) \quad \tilde{\mathbf{q}}_o = \langle \xi_o \quad \eta_o \quad \zeta_o \quad \theta_x \quad \theta_y \quad \theta_z \rangle^T \approx \langle \xi_o \quad \eta_o \quad \zeta_o \quad \theta_X \quad \theta_Y \quad \theta_Z \rangle^T.$$

For further presentation of the theory in this article, it is necessary to define a new vector that combines the vector velocity of the pole O and the vector-pseudo angular velocity of the body. It has also a dimension 6×1 and has the following type:

$$(4) \quad \tilde{\dot{\mathbf{q}}}_o = \begin{bmatrix} \mathbf{v}_o \\ \tilde{\boldsymbol{\omega}} \end{bmatrix} = \begin{bmatrix} \dot{\boldsymbol{\rho}}_o \\ \tilde{\boldsymbol{\omega}} \end{bmatrix},$$

$$(5) \quad \tilde{\dot{\mathbf{q}}}_o = \langle \dot{\xi}_o \quad \dot{\eta}_o \quad \dot{\zeta}_o \quad \dot{\theta}_x \quad \dot{\theta}_y \quad \dot{\theta}_z \rangle^T \approx \langle \dot{\xi}_o \quad \dot{\eta}_o \quad \dot{\zeta}_o \quad \dot{\theta}_X \quad \dot{\theta}_Y \quad \dot{\theta}_Z \rangle^T.$$

The name of the vector $\tilde{\dot{\mathbf{q}}}_o$ is **vector-pseudo generalized velocity** of the body at an arbitrary chosen pole O from it.

The body mass center C is defined by the absolute radius vector \mathbf{p}_C and the relative radius vector \mathbf{r}_C , respectively:

$$(6) \quad \mathbf{p}_C = \langle \xi_C \quad \eta_C \quad \zeta_C \rangle^T,$$

$$(7) \quad \mathbf{r}_C = \langle X_C \quad Y_C \quad Z_C \rangle^T.$$

A vector-*pseudo* generalized velocity of the body at a chosen pole O , which coincides with the mass center C , is also defined:

$$(8) \quad \tilde{\mathbf{q}}_C = \begin{bmatrix} \mathbf{v}_C \\ \tilde{\boldsymbol{\omega}} \end{bmatrix} = \begin{bmatrix} \dot{\mathbf{p}}_C \\ \tilde{\boldsymbol{\omega}} \end{bmatrix},$$

$$(9) \quad \tilde{\mathbf{q}}_C = \langle \dot{\xi}_C \quad \dot{\eta}_C \quad \dot{\zeta}_C \quad \dot{\theta}_x \quad \dot{\theta}_y \quad \dot{\theta}_z \rangle^T \approx \langle \dot{\xi}_C \quad \dot{\eta}_C \quad \dot{\zeta}_C \quad \dot{\theta}_x \quad \dot{\theta}_y \quad \dot{\theta}_z \rangle^T.$$

The free ideal rigid body that performs a *pseudo* translational motion is considered to be homogeneous. The mass properties of the body are defined by its mass m and by the diagonal mass matrix:

$$(10) \quad \mathbf{M} = \mathbf{diag} [m]_3.$$

The inertial properties of the body are defined by two tensors of inertia, which are constructed by constant elements. For the pole O these tensors have the following form:

$$(11) \quad \mathbf{J}_O = \begin{bmatrix} \bar{J}_x & -\bar{J}_{xy} & -\bar{J}_{xz} \\ -\bar{J}_{yx} & \bar{J}_y & -\bar{J}_{yz} \\ -\bar{J}_{zx} & -\bar{J}_{zy} & \bar{J}_z \end{bmatrix} \approx \begin{bmatrix} \bar{J}_X & -\bar{J}_{XY} & -\bar{J}_{XZ} \\ -\bar{J}_{YX} & \bar{J}_Y & -\bar{J}_{YZ} \\ -\bar{J}_{ZX} & -\bar{J}_{ZY} & \bar{J}_Z \end{bmatrix}.$$

These tensors for the pole O , which coincides to the mass center C , can be written as follows:

$$(12) \quad \mathbf{J}_C = \begin{bmatrix} J_x & -J_{xy} & -J_{xz} \\ -J_{yx} & J_y & -J_{yz} \\ -J_{zx} & -J_{zy} & J_z \end{bmatrix} \approx \begin{bmatrix} J_X & -J_{XY} & -J_{XZ} \\ -J_{YX} & J_Y & -J_{YZ} \\ -J_{ZX} & -J_{ZY} & J_Z \end{bmatrix}.$$

Due to the arbitrary choice of the pole O , a reverse symmetrical tensor of the mass static moments is used. It also is constructed by constant elements, namely:

$$(13) \quad \mathbf{S}_C = m \cdot \mathbf{R}_C, \quad \text{where}$$

$$(14) \quad \mathbf{R}_C = \begin{bmatrix} 0 & -z_C & y_C \\ z_C & 0 & -x_C \\ -y_C & x_C & 0 \end{bmatrix} \approx \begin{bmatrix} 0 & -Z_C & Y_C \\ Z_C & 0 & -X_C \\ -Y_C & X_C & 0 \end{bmatrix}.$$

All the forces, acting on the body, are reduced to a point O to main force \mathbf{F} and a main moment \mathbf{M}_O .

The following vector is defined:

$$(15) \quad \tilde{\mathbf{D}}_O = \begin{bmatrix} \mathbf{M} & \mathbf{S}_C^T \\ \mathbf{S}_C & \mathbf{J}_O \end{bmatrix} \cdot \begin{bmatrix} \mathbf{v}_O \\ \tilde{\boldsymbol{\omega}} \end{bmatrix}.$$

Its name is **vector-*pseudo* generalized impulse** of a rigid body for the pole O .

The matrix:

$$(16) \quad \mathbf{A}_o = \begin{bmatrix} \mathbf{M} & \mathbf{S}_C^T \\ \mathbf{S}_C & \mathbf{J}_o \end{bmatrix},$$

defines the mass and inertial properties of this asymmetric rigid body when for a pole is chosen the point O .

The formula (15) in a shortened vector-matrix form is written as follows:

$$(17) \quad \tilde{\mathbf{D}}_o = \mathbf{A}_o \cdot \dot{\tilde{\mathbf{q}}}_o.$$

A vector-*pseudo* generalized impulse of a rigid body for its mass center C is defined by the following vector-matrix expression:

$$(18) \quad \tilde{\mathbf{D}}_c = \begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{J}_c \end{bmatrix} \cdot \begin{bmatrix} \mathbf{v}_c \\ \tilde{\boldsymbol{\omega}} \end{bmatrix}.$$

The matrix:

$$(19) \quad \mathbf{A}_c = \begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{J}_c \end{bmatrix},$$

defines the mass and inertial properties of this asymmetric ideal rigid body when the pole O is chosen to coincide with the mass center C .

Formula (18) can be written in a shortened vector-matrix form as follows:

$$(20) \quad \tilde{\mathbf{D}}_c = \mathbf{A}_c \cdot \dot{\tilde{\mathbf{q}}}_c.$$

Finally, a vector-*pseudo* generalized impulse of this rigid body for the immovable pole N is introduced. That vector is defined by the formulas:

$$(21) \quad \tilde{\mathbf{D}}_N = \tilde{\mathbf{D}}_o + \bar{\mathbf{T}}_o \cdot \tilde{\mathbf{D}}_c, \quad \text{where}$$

$$(22) \quad \bar{\mathbf{T}}_o = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \tilde{\mathbf{R}}_o & \mathbf{0} \end{bmatrix},$$

$$(23) \quad \tilde{\mathbf{R}}_o = \begin{bmatrix} 0 & -\zeta_o & \eta_o \\ \zeta_o & 0 & -\xi_o \\ -\eta_o & \xi_o & 0 \end{bmatrix}.$$

Formulas (17) and (20) in equation (21) are substituted and the following equation is obtained:

$$(24) \quad \tilde{\mathbf{D}}_N = \mathbf{A}_o \cdot \dot{\tilde{\mathbf{q}}}_o + \bar{\mathbf{T}}_o \cdot \mathbf{A}_c \cdot \dot{\tilde{\mathbf{q}}}_c.$$

The ***pseudo kinetic energy*** of this asymmetrical rigid body has the following vector-matrix form:

$$(25) \quad \tilde{E}_k = \frac{1}{2} \cdot \langle \mathbf{v}_o \quad \tilde{\boldsymbol{\omega}} \rangle^T \cdot \begin{bmatrix} \mathbf{M} & \mathbf{S}_C^T \\ \mathbf{S}_C & \mathbf{J}_o \end{bmatrix} \cdot \begin{bmatrix} \mathbf{v}_o \\ \tilde{\boldsymbol{\omega}} \end{bmatrix}.$$

Formula (25) could be written in a shortened vector-matrix type as follows:

$$(26) \quad \tilde{E}_k = \frac{1}{2} \cdot \dot{\tilde{\mathbf{q}}}_O^T \cdot \mathbf{A}_O \cdot \dot{\tilde{\mathbf{q}}}_O .$$

If the pole O coincides with the mass center C , the *pseudo* kinetic energy will be determined by well known König theorem:

$$(27) \quad \tilde{E}_k = \frac{1}{2} \cdot \langle \mathbf{v}_C \quad \tilde{\boldsymbol{\omega}} \rangle^T \cdot \begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{J}_C \end{bmatrix} \cdot \begin{bmatrix} \mathbf{v}_C \\ \tilde{\boldsymbol{\omega}} \end{bmatrix} .$$

The formula (27) can be written in a shortened vector-matrix form as follows:

$$(28) \quad \tilde{E}_k = \frac{1}{2} \cdot \dot{\tilde{\mathbf{q}}}_C^T \cdot \mathbf{A}_C \cdot \dot{\tilde{\mathbf{q}}}_C .$$

A **vector-real generalized force** of this rigid body at pole O is defined:

$$(29) \quad \mathbf{Q}_O = \begin{bmatrix} \mathbf{F} \\ \mathbf{M}_O \end{bmatrix} .$$

A *vector-real* generalized force of this rigid body at pole N is also defined:

$$(30) \quad \mathbf{Q}_N = \begin{bmatrix} \mathbf{F} \\ \mathbf{M}_N \end{bmatrix} = \begin{bmatrix} \mathbf{F} \\ \mathbf{M}_O + \tilde{\mathbf{R}}_O \cdot \mathbf{F} \end{bmatrix} .$$

The relationship between these two vectors is realized by the equality:

$$(31) \quad \mathbf{Q}_N = \mathbf{Q}_O + \bar{\mathbf{T}}_O \cdot \mathbf{Q}_O .$$

The vector \mathbf{Q}_N could be defined by other way.

Let a fixed number of external forces \mathbf{F}_k act on the rigid body, applied at points D_k , ($k=1,2,\dots,h$), and they are defined by radius vectors \mathbf{p}_k , (Fig.1).

The possible power of these forces, with a possible infinitely small change of the rigid body *real* generalized velocity, is determined:

$$(32) \quad \begin{aligned} \delta P &= \sum_{k=1}^h \mathbf{F}_k^T \cdot \delta \mathbf{v}_k = \mathbf{F}^T \cdot \delta \mathbf{v}_O + \left(\mathbf{M}_O + \tilde{\mathbf{R}}_O \cdot \mathbf{F} \right)^T \cdot \delta \tilde{\boldsymbol{\omega}} = \\ &= \delta \mathbf{v}_O^T \cdot \mathbf{F} + \delta \tilde{\boldsymbol{\omega}}^T \cdot \left(\mathbf{M}_O + \tilde{\mathbf{R}}_O \cdot \mathbf{F} \right) = \delta \langle \mathbf{v}_O^T \quad \tilde{\boldsymbol{\omega}}^T \rangle \cdot \begin{bmatrix} \mathbf{F} \\ \mathbf{M}_O + \tilde{\mathbf{R}}_O \cdot \mathbf{F} \end{bmatrix} = \\ &= \delta \langle \mathbf{v}_O \quad \tilde{\boldsymbol{\omega}} \rangle^T \cdot \left(\begin{bmatrix} \mathbf{F} \\ \mathbf{M}_O \end{bmatrix} + \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \tilde{\mathbf{R}}_O & \mathbf{0} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{F} \\ \mathbf{M}_O \end{bmatrix} \right) = \\ &= \delta \dot{\tilde{\mathbf{q}}}^T \cdot \left(\mathbf{Q}_O + \bar{\mathbf{T}}_O \cdot \mathbf{Q}_O \right) . \end{aligned}$$

Then the vector-*real* generalized forces of the rigid body at pole N will be obtained:

$$(33) \quad \mathbf{Q}_N = \frac{\delta P}{\delta \ddot{\mathbf{q}}} = \frac{\delta \dot{\mathbf{q}}^T}{\delta \ddot{\mathbf{q}}} \cdot (\mathbf{Q}_o + \bar{\mathbf{T}}_o \cdot \mathbf{Q}_o) = \bar{\mathbf{E}} \cdot (\mathbf{Q}_o + \bar{\mathbf{T}}_o \cdot \mathbf{Q}_o), \quad \text{where}$$

$$(34) \quad \bar{\mathbf{E}} = \mathbf{diag} [1]_6 .$$

So, the most important Kinetics characteristics in the vector-matrix form are defined. They are necessary to introduce a private kind of the Theorem of change the rigid body generalized impulse, which is described in the next paragraph.

3. THEOREM OF CHANGE THE RIGID BODY *PSEUDO* GENERALIZED IMPULSE

The theorem states: *The first time derivative of the rigid body **pseudo** generalized impulse for a fixed pole is equal to its **real** generalized force determined for that pole.*

The mathematical record of the stated above theorem has the form:

$$(35) \quad \frac{d \tilde{\mathbf{D}}_N}{dt} = \mathbf{Q}_N .$$

Formulas (24) and (31) are substituted in equation (35) and the following expression is obtained:

$$(36) \quad \frac{d}{dt} (\mathbf{A}_o \cdot \dot{\mathbf{q}}_o + \bar{\mathbf{T}}_o \cdot \mathbf{A}_c \cdot \dot{\mathbf{q}}_c) = \mathbf{Q}_o + \bar{\mathbf{T}}_o \cdot \mathbf{Q}_o .$$

The time derivative in equation (36) is performed:

$$(37) \quad \mathbf{A}_o \cdot \ddot{\mathbf{q}}_o + \dot{\mathbf{A}}_o \cdot \dot{\mathbf{q}}_o + \dot{\bar{\mathbf{T}}}_o \cdot \mathbf{A}_c \cdot \dot{\mathbf{q}}_o + \bar{\mathbf{T}}_o \cdot \dot{\mathbf{A}}_c \cdot \dot{\mathbf{q}}_c + \bar{\mathbf{T}}_o \cdot \mathbf{A}_c \cdot \ddot{\mathbf{q}}_c = \mathbf{Q}_o + \bar{\mathbf{T}}_o \cdot \mathbf{Q}_o .$$

The following detailed calculations are done below:

$$(38) \quad \bar{\mathbf{T}}_o \cdot \dot{\mathbf{A}}_c \cdot \dot{\mathbf{q}}_c = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \tilde{\mathbf{R}}_o & \mathbf{0} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{v}_c \\ \tilde{\boldsymbol{\omega}} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix} ,$$

$$(39) \quad \bar{\mathbf{T}}_o \cdot \mathbf{A}_c \cdot \ddot{\mathbf{q}}_c = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \tilde{\mathbf{R}}_o & \mathbf{0} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{J}_c \end{bmatrix} \cdot \begin{bmatrix} \dot{\mathbf{v}}_c \\ \dot{\tilde{\boldsymbol{\omega}}} \end{bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \tilde{\mathbf{R}}_o & \mathbf{0} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{M} \cdot \dot{\mathbf{v}}_c \\ \mathbf{J}_c \cdot \dot{\tilde{\boldsymbol{\omega}}} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \tilde{\mathbf{R}}_o \cdot \mathbf{M} \cdot \dot{\mathbf{v}}_c \end{bmatrix} .$$

Taking into account the equations (38) and (39), equation (37) is simplified to the following type:

$$(40) \quad \mathbf{A}_o \cdot \ddot{\mathbf{q}}_o + \dot{\mathbf{A}}_o \cdot \dot{\mathbf{q}}_o + \dot{\bar{\mathbf{T}}}_o \cdot \mathbf{A}_c \cdot \dot{\mathbf{q}}_c + \begin{bmatrix} \mathbf{0} \\ \tilde{\mathbf{R}}_o \cdot \mathbf{M} \cdot \dot{\mathbf{v}}_c \end{bmatrix} = \mathbf{Q}_o + \begin{bmatrix} \mathbf{0} \\ \tilde{\mathbf{R}}_o \cdot \mathbf{F} \end{bmatrix} .$$

Now, the theorem of the mass center motion is used:

$$(41) \quad \mathbf{M} \cdot \dot{\mathbf{v}}_c = \mathbf{F} .$$

Through this theorem, equation (40) takes the following form:

$$(42) \quad \mathbf{A}_o \cdot \ddot{\mathbf{q}}_o + \dot{\mathbf{A}}_o \cdot \dot{\mathbf{q}}_o = \mathbf{Q}_o - \dot{\bar{\mathbf{T}}}_o \cdot \mathbf{A}_c \cdot \dot{\mathbf{q}}_c .$$

So, the equation (42) is obtained by the Theorem of change the rigid body *pseudo* generalized impulse at the fixed pole N .

Now, the same theorem, but to the moving pole O will be applied. For this purpose, a following vector is introduced:

$$(43) \quad \tilde{\mathbf{Q}}_O^* = \mathbf{Q}_O - \dot{\tilde{\mathbf{T}}}_O \cdot \mathbf{A}_C \cdot \dot{\tilde{\mathbf{q}}}_C .$$

This vector is called **vector-real kinetic pseudo generalized force**.

Through this introduced new vector, equation (42) is recorded as follows:

$$(44) \quad \mathbf{A}_O \cdot \ddot{\tilde{\mathbf{q}}}_O + \dot{\mathbf{A}}_O \cdot \dot{\tilde{\mathbf{q}}}_O = \tilde{\mathbf{Q}}_O^* .$$

The above equation can be written even shorter, namely:

$$(45) \quad \frac{d \tilde{\mathbf{D}}_O}{dt} = \tilde{\mathbf{Q}}_O^* .$$

The equation (45) is performed the Theorem of change the rigid body *pseudo* generalized impulse but applied to the moving pole O .

This variant of the theorem speaks that way: *The first time derivative of the rigid body pseudo generalized impulse for a movable pole is equal to its real kinetic pseudo generalized force determined for that pole.*

Let us assume the pole O coincides with the mass center C . Then equations (42), (43) and (44) takes the following kind:

$$(46) \quad \mathbf{A}_C \cdot \ddot{\tilde{\mathbf{q}}}_C + \dot{\mathbf{A}}_C \cdot \dot{\tilde{\mathbf{q}}}_C = \mathbf{Q}_C - \dot{\tilde{\mathbf{T}}}_C \cdot \mathbf{A}_C \cdot \dot{\tilde{\mathbf{q}}}_C .$$

$$(47) \quad \tilde{\mathbf{Q}}_C^* = \mathbf{Q}_C - \dot{\tilde{\mathbf{T}}}_C \cdot \mathbf{A}_C \cdot \dot{\tilde{\mathbf{q}}}_C ,$$

$$(48) \quad \frac{d \tilde{\mathbf{D}}_C}{dt} = \tilde{\mathbf{Q}}_C^* .$$

And now, let us develop in detail the following vector-matrix product:

$$(49) \quad \begin{aligned} \dot{\tilde{\mathbf{T}}}_C \cdot \mathbf{A}_C \cdot \dot{\tilde{\mathbf{q}}}_C &= \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \dot{\tilde{\mathbf{R}}}_C & \mathbf{0} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{J}_C \end{bmatrix} \cdot \begin{bmatrix} \dot{\tilde{\mathbf{v}}}_C \\ \dot{\tilde{\boldsymbol{\omega}}}_C \end{bmatrix} = \\ &= \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \dot{\tilde{\mathbf{R}}}_C & \mathbf{0} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{M} \cdot \dot{\tilde{\mathbf{p}}}_C \\ \mathbf{J}_C \cdot \dot{\tilde{\boldsymbol{\omega}}}_C \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \dot{\tilde{\mathbf{R}}}_C \cdot \mathbf{M} \cdot \dot{\tilde{\mathbf{p}}}_C \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M} \cdot \dot{\tilde{\mathbf{R}}}_C \cdot \dot{\tilde{\mathbf{p}}}_C \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix} . \end{aligned}$$

Therefore the equations (46), (47) and (48) take the following form:

$$(50) \quad \mathbf{A}_C \cdot \ddot{\tilde{\mathbf{q}}}_C + \dot{\mathbf{A}}_C \cdot \dot{\tilde{\mathbf{q}}}_C = \mathbf{Q}_C ,$$

$$(51) \quad \tilde{\mathbf{Q}}_C^* = \mathbf{Q}_C ,$$

$$(52) \quad \frac{d \tilde{\mathbf{D}}_C}{dt} = \mathbf{Q}_C .$$

Equation (52) performs the Theorem of change the rigid body *pseudo* generalized impulse for the pole O , which coincides with the mass center C . This variant of the theorem speaks that way: *The first time derivative of the rigid body pseudo generalized impulse for the body mass center is equal to its real generalized force determined for that center.*

It is obvious that this Theorem of change the rigid body *pseudo* generalized impulse, written by equation (35), (45) and (52), has the same structure.

4. CONDENSED LAGRANGE EQUATIONS

The following Condensed Lagrange equations are defined:

$$(53) \quad \frac{d}{dt} \left[\frac{\partial (E_k + \tilde{E})}{\partial \dot{\mathbf{q}}_o} \right] = \mathbf{Q}_o + \bar{\mathbf{T}}_o \cdot \mathbf{Q}_o .$$

The scalar quantity is constructed in the following way:

$$(54) \quad \tilde{E} = \dot{\mathbf{q}}_o^T \cdot \tilde{\mathbf{F}} ,$$

$$(55) \quad \tilde{\mathbf{F}} = \bar{\mathbf{T}}_o \cdot \tilde{\mathbf{D}}_C = \bar{\mathbf{T}}_o \cdot \mathbf{A}_C \cdot \dot{\mathbf{q}}_C .$$

Formulas (26) and (54) in equations (53) are substituted:

$$(56) \quad \frac{d}{dt} (\mathbf{A}_o \cdot \dot{\mathbf{q}}_o) + \frac{d \tilde{\mathbf{F}}}{dt} = \mathbf{Q}_o + \bar{\mathbf{T}}_o \cdot \mathbf{Q}_o ,$$

$$(57) \quad \frac{d}{dt} (\mathbf{A}_o \cdot \dot{\mathbf{q}}_o) + \frac{d}{dt} (\bar{\mathbf{T}}_o \cdot \mathbf{A}_C \cdot \dot{\mathbf{q}}_C) = \mathbf{Q}_o + \bar{\mathbf{T}}_o \cdot \mathbf{Q}_o ,$$

$$(58) \quad \mathbf{A}_o \cdot \ddot{\mathbf{q}}_o + \dot{\mathbf{A}}_o \cdot \dot{\mathbf{q}}_o + \bar{\mathbf{T}}_o \cdot \mathbf{A}_C \cdot \dot{\mathbf{q}}_C + \bar{\mathbf{T}}_o \cdot \dot{\mathbf{A}}_C \cdot \dot{\mathbf{q}}_C + \bar{\mathbf{T}}_o \cdot \mathbf{A}_C \cdot \ddot{\mathbf{q}}_C = \mathbf{Q}_o + \bar{\mathbf{T}}_o \cdot \mathbf{Q}_o .$$

The equation (58) is fully coincides with the equations (37) but it is obtained by the other way.

Now, the Condensed Lagrange equations, but with the pole O , which is coincided with the mass center C will be applied. For this purpose, the equation (53) will be written in the following type:

$$(59) \quad \frac{d}{dt} \left[\frac{\partial (E_k + \tilde{E})}{\partial \dot{\mathbf{q}}_C} \right] = \mathbf{Q}_C + \bar{\mathbf{T}}_C \cdot \mathbf{Q}_C .$$

The scalar \tilde{E} is constructed in the following way:

$$(60) \quad \tilde{E} = \dot{\mathbf{q}}_C^T \cdot \tilde{\mathbf{F}} , \quad \text{where}$$

$$(61) \quad \tilde{\mathbf{F}} = \bar{\mathbf{T}}_C \cdot \tilde{\mathbf{D}}_C = \bar{\mathbf{T}}_C \cdot \mathbf{A}_C \cdot \dot{\mathbf{q}}_C .$$

Formulas (28) and (60) in equations (59) are substituted and the following equation is obtained:

$$(62) \quad \frac{d}{dt} (\mathbf{A}_C \cdot \dot{\mathbf{q}}_C) + \frac{d \tilde{\mathbf{F}}}{dt} = \mathbf{Q}_C + \bar{\mathbf{T}}_C \cdot \mathbf{Q}_C ,$$

$$(63) \quad \frac{d}{dt} (\mathbf{A}_C \cdot \dot{\mathbf{q}}_C) + \frac{d}{dt} (\bar{\mathbf{T}}_C \cdot \mathbf{A}_C \cdot \dot{\mathbf{q}}_C) = \mathbf{Q}_C + \bar{\mathbf{T}}_C \cdot \mathbf{Q}_C ,$$

$$(64) \quad \mathbf{A}_C \cdot \ddot{\mathbf{q}}_C + \dot{\mathbf{A}}_C \cdot \dot{\mathbf{q}}_C + \bar{\mathbf{T}}_C \cdot \mathbf{A}_C \cdot \dot{\mathbf{q}}_C + \bar{\mathbf{T}}_C \cdot \dot{\mathbf{A}}_C \cdot \dot{\mathbf{q}}_C + \bar{\mathbf{T}}_C \cdot \mathbf{A}_C \cdot \ddot{\mathbf{q}}_C = \mathbf{Q}_C + \bar{\mathbf{T}}_C \cdot \mathbf{Q}_C .$$

The following detailed calculations are performed:

$$(65) \quad \bar{\mathbf{T}}_C \cdot \dot{\mathbf{A}}_C \cdot \dot{\tilde{\mathbf{q}}}_C = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \tilde{\mathbf{R}}_C & \mathbf{0} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{v}_C \\ \tilde{\boldsymbol{\omega}} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix},$$

$$(66) \quad \bar{\mathbf{T}}_C \cdot \mathbf{A}_C \cdot \ddot{\tilde{\mathbf{q}}}_C = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \tilde{\mathbf{R}}_C & \mathbf{0} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{J}_C \end{bmatrix} \cdot \begin{bmatrix} \dot{\mathbf{v}}_C \\ \dot{\tilde{\boldsymbol{\omega}}} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \tilde{\mathbf{R}}_C \cdot \mathbf{M} \cdot \dot{\mathbf{v}}_C \end{bmatrix},$$

$$(67) \quad \dot{\bar{\mathbf{T}}}_C \cdot \mathbf{A}_C \cdot \dot{\tilde{\mathbf{q}}}_C = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \dot{\tilde{\mathbf{R}}}_C & \mathbf{0} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{J}_C \end{bmatrix} \cdot \begin{bmatrix} \dot{\mathbf{v}}_C \\ \dot{\tilde{\boldsymbol{\omega}}} \end{bmatrix} = \\ = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \dot{\tilde{\mathbf{R}}}_C & \mathbf{0} \end{bmatrix} \cdot \begin{bmatrix} \mathbf{M} \cdot \dot{\boldsymbol{\rho}}_C \\ \mathbf{J}_C \cdot \dot{\tilde{\boldsymbol{\omega}}} \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \dot{\tilde{\mathbf{R}}}_C \cdot \mathbf{M} \cdot \dot{\boldsymbol{\rho}}_C \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M} \cdot \dot{\tilde{\mathbf{R}}}_C \cdot \dot{\boldsymbol{\rho}}_C \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \end{bmatrix}.$$

Taking into account the equality (65), (66) and (67), equation (64) is simplified to the following form:

$$(68) \quad \mathbf{A}_C \cdot \ddot{\tilde{\mathbf{q}}}_C + \dot{\mathbf{A}}_C \cdot \dot{\tilde{\mathbf{q}}}_C + \begin{bmatrix} \mathbf{0} \\ \tilde{\mathbf{R}}_C \cdot \mathbf{M} \cdot \dot{\mathbf{v}}_C \end{bmatrix} = \mathbf{Q}_C + \begin{bmatrix} \mathbf{0} \\ \tilde{\mathbf{R}}_C \cdot \mathbf{F} \end{bmatrix}.$$

Through the Theorem of mass center motion, equation (68) takes the following simpler form:

$$(69) \quad \mathbf{A}_C \cdot \ddot{\tilde{\mathbf{q}}}_C + \dot{\mathbf{A}}_C \cdot \dot{\tilde{\mathbf{q}}}_C = \mathbf{Q}_C.$$

Equation (69) is fully coincides with the equation (50). Moreover, equation (69) can be obtained by the following type of Condensed Lagrange equation, namely:

$$(70) \quad \frac{d}{dt} \left[\frac{\partial E_k}{\partial \dot{\tilde{\mathbf{q}}}_C} \right] = \mathbf{Q}_C.$$

Therefore the two forms of Condensed Lagrange equations from (59) and (70) lead to the same differential equation, namely equation (69). Or in other words, differential equation (69) can be obtained successfully by using of Condensed Lagrange equations from (59) or by using of Condensed Lagrange equations from (70).

5. DEVELOPMENT OF THE SYSTEM OF DIFFERENTIAL EQUATIONS

First, the variant when the pole O do not coincide with the mass center C will be developed. The time derivative of matrix \mathbf{A}_O is a zero matrix, namely:

$$(71) \quad \dot{\mathbf{A}}_O = \mathbf{0}.$$

The formula (71) in equation (42) is substituted and then the following equation is obtained:

$$(72) \quad \mathbf{A}_O \cdot \ddot{\tilde{\mathbf{q}}}_O = \mathbf{Q}_O - \dot{\bar{\mathbf{T}}}_O \cdot \mathbf{A}_C \cdot \dot{\tilde{\mathbf{q}}}_C.$$

Equation (72) is a vector-matrix record of a non-linear system of six differential equations describing the *pseudo* translational motion of a free asymmetric ideal rigid body at arbitrary chosen pole O .

Now, the following links are used:

$$(73) \quad \dot{\tilde{\mathbf{q}}}_C = \mathbf{K}_C \cdot \dot{\tilde{\mathbf{q}}}_O, \quad \text{where}$$

$$(74) \quad \mathbf{K}_C = \begin{bmatrix} \mathbf{E} & \mathbf{R}_C^T \\ \mathbf{0} & \mathbf{E} \end{bmatrix}.$$

The formula (73) is substituted in equation (72) and the following equation is obtained:

$$(75) \quad \mathbf{A}_O \cdot \ddot{\mathbf{q}}_O + \dot{\mathbf{T}}_O \cdot \mathbf{A}_C \cdot \mathbf{K}_C \cdot \dot{\mathbf{q}}_O = \mathbf{Q}_O, \quad \text{where}$$

$$(76) \quad \dot{\mathbf{T}}_O = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \dot{\mathbf{R}}_O & \mathbf{0} \end{bmatrix},$$

$$(77) \quad \dot{\mathbf{R}}_O = \begin{bmatrix} 0 & -\dot{\xi}_O & \dot{\eta}_O \\ \dot{\xi}_O & 0 & -\dot{\xi}_O \\ -\dot{\eta}_O & \dot{\xi}_O & 0 \end{bmatrix}.$$

After numerical integration of the differential equation (75) the law of absolute *pseudo* translational motion of a free asymmetrical ideal rigid body at arbitrary chosen pole O will be found.

Now, the variant when the pole O coincides with the mass center C will be developed. The time derivative of matrix \mathbf{A}_C is a zero matrix, namely:

$$(78) \quad \dot{\mathbf{A}}_C = \mathbf{0}.$$

The formula (78) is substituted in equations (50) or (69) and the following equation is obtained:

$$(79) \quad \mathbf{A}_C \cdot \ddot{\mathbf{q}}_C = \mathbf{Q}_C.$$

Equation (79) is a vector-matrix record of a non-linear system of six differential equations describing the *pseudo* translational motion of a free asymmetric ideal rigid body when the chosen pole O coincides with the mass center C .

The differential equations (75) and (79) are very convenient for numerical integration.

6. CONCLUSION

Some new kinetic characteristics for an ideal rigid body have been introduced. The main important are the **vector-pseudo angular velocity**, the **vector-pseudo generalized coordinates**, the **vector-pseudo generalized velocity**, the **vector-pseudo generalized impulse** and **pseudo kinetic energy**.

A private form of the Theorem of change the rigid body generalized impulse for the fixed pole N or for the movable pole O is defined. This new kind theorem is called **Theorem of change the rigid body pseudo generalized impulse**. It is applied to study the *pseudo* translational motion of a free asymmetric ideal rigid body. The directly defining of this new kind of theorem became possible thanks to introducing the new kinetics characteristics, using the matrix operations, and using the basis type of the Theorem of change the rigid body general motion [1, 2].

Condensed Lagrange equation for the rigid body *pseudo* translational motion is applied successfully. This matrix equation leads to the same result as the Theorem of change the rigid body *pseudo* generalized impulse.

This theory is very important for study the **small three dimensional vibrations** of a single rigid body or rigid bodies mechanical system with elastic and damping connecting elements.

The obtained system of nonlinear and linear differential equations in matrix form is convenient for a numerically integrating by the contemporary mathematical programs which is projected to use matrices and matrix calculations, for example MatLab, MathCAD, MuPAD and so on.

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Cite this Article: Ivanov A (2017). Theoretical Matrix Study of Rigid Body Pseudo Translational Motion. *Greener Journal of Physics and Natural Sciences*, 3(2): 021-031, <http://doi.org/10.15580/GJPNS.2017.2.101117147>