Response of Beam with Viscously Damped Axial Force on Elastic Foundation

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Abstract:
This paper examined the response of beam with viscously damped axial force on elastic foundation. The effects of the moving loads on beam with respect to the moving force and the moving mass were examined. The fourth order partial differential equation which is the governing equation was solved in form of series solution to obtain two distinct equations for both the moving force and the moving mass. These were finally solved numerically using maple software to determine the behaviour of the system under consideration. The results of the findings revealed that as damping coefficient (ω) increases the deflection decreases for both the moving force and the moving mass. In the same way, it was also noted that the deflection decreases as axial force (S) increases for both the two cases. It was also discovered that as elastic foundation (K) increases the deflection increases for both the moving force and the moving mass. Lastly, it was showed that the displacement for the moving mass is greater than that of the moving force.

***Key words**: Beam, Axial force, Viscously damped, Elastic foundation and Moving loads.*

1. Introduction

This paper combined axial force, elastic foundation and viscous damping with moving force and moving mass. The damping is divided into three types which are; viscous damping, column damping and structural damping but for the purpose of this study, viscous damping is considered.

Adetunde and Baba (2008) worked on dynamic response of loads on viscously damped axial force Rayleigh beam. The theory was based on orthogonal functions and finite central difference method was employed to solve the governing differential equation numerically. The results showed that the deflection of a viscously damped axial force Rayleigh beam for both the moving force and moving mass increase with increasing mass of load and that the deflection due to moving mass is greater than that of moving force. Akour (2010 and 2012) examined dynamics of nonlinear beam on elastic foundation and parametric study of nonlinear beam vibration resting on linear elastic foundation respectively. The governing equations were derived using Hamilton's principle. The study examined both linear and nonlinear cases and they were analyzed numerically using Runge-Kutta technique. Damping coefficient, natural frequency and coefficient of the nonlinearity were the three main parameters that were investigated and comparison were made between the linear and the nonlinear. It was discovered from the study that as long as the damping coefficient is positive and non zero the system is stable and controllable. Furthermore, it was showed that as the nonlinearity increases, more damping is required to prevent the system from moving towards chaos and also that the strength of the nonlinearity is inversely proportional to the square of the radius of gyration. Are, Idowu and Gbadeyan (2013) examined vibration of damped simply supported orthotropic rectangular plates on elastic winkler foundation, subjected to moving loads. The effects of some physical phenomena were investigated and the results revealed that the elastic foundation made the system to stabilize and reduced the possibilities of resonance greatly. Borş and Milchiş (2013) evaluated dynamic response of beams on elastic foundation with axial load. In the study, the vibrations of beams on elastic foundations with respect to axial load and dynamic external forces were examined considering Winkler`s model. It was revealed that when the axial load is increasing due to the rigidity loss the values of natural frequencies decreases. Idowu et al (2013) worked on dynamic effects of viscous damping with respect to moving loads. Fourth order partial differential equation was examined where the dynamic effect of viscous damping was investigated. The method employed in this study is the central difference scheme of the finite deference and it was found that the presence of elastic foundation stabilizes the system and reduces the possibility of resonance. Kumari, Sahoo and Sawant (2012) enumerated dynamic response of railway track using two parameter models. An infinite Euler–Bernoulli beam of constant cross-section resting on an elastic foundation was considered. The beam in this case is subjected to a constant point load moving with a constant speed. The results of the study showed that the absolute value of the deflection of the beam at subcritical speed increases with increase of the load velocity. Again, it was discovered that when the load travels at the critical speed, Peak maximum deflection appears. Sangoniyi and Akinpelu (2017) considered dynamics response of beam on elastic foundation with axial force to partially distributed moving loads. The fourth order partial differential equation was first reduced to ordinary second order differential equation by assumed a solution in term of series solution and Maple software was finally used to solve the equation numerically. The results of the findings showed that as the axial force increases, the displacement decreases under the action of the moving force while in the case of the moving mass, as the axial force increases the displacement is equally increases. More so, it was noted that displacement decreases as the elastic foundation (k) increases for both the moving force and the moving mass and deflection due to moving mass is greater than that of the moving force when comparison was made. Senalp et al (2010) evaluated Dynamic response of a finite length Euler-Bernoulli beam on linear and nonlinear viscoelastic foundations to a concentrated moving force. Galerkin method was used to solve the governing equations of motion and it was deduced from the results that increase in the damping ratio (ξ) resulted into decrease in the dynamic deflections for both linear and nonlinear cases. Also, the dynamic response of the beam is always greater for the nonlinear foundation model, when compared to the linear foundation model. Finally, the distribution of deflection is symmetrical for small values of the damping ratio and this symmetry is distorted with increasing damping. In this paper the response of beam with viscously damped axial force on elastic foundation is considered. Wang et al (2011) examined transient responses of beam with elastic foundation supports under moving wave load excitation. Effects of the stiffness of the elastic foundation base, the traveling speed of load and viscous damping were examined. It was showed that an increase in velocity parameter of the moving loads resulted in the increase in dynamic deflections of the beam within the range of values considered.

This paper considered the combination of the axial force, elastic foundation and viscously damped beam unlike the existing ones including the above papers which have been reviewed that treated the three stated terms separately.

2. The governing Equation of the problem

The governing equation of the response of beam with viscously damped axial force on elastic foundation, is given as:

$$\frac{\partial^2}{\partial x^2}\left(EI(x)\left(\frac{\partial^2 u}{\partial x^2}+a_1\frac{\partial^3 u}{\partial x^2\partial t}\right)\pm sy\right)+m(x)\frac{\partial^2 u}{\partial t^2}+C(x)\frac{\partial u}{\partial t}+kY=F(x,t) \tag{1}$$

Subjected to the following boundary and initial conditions:

$$y(0,t)=y''(0,t)=0 \tag{2}$$

$$y(L,t)=y''(L,t)=0 \tag{3}$$

$$y(x,0)=y'(x,0)=0 \tag{4}$$

The applied force per unit length $F(x,t)$ is the uniform partially distributed moving load defined as

$$F(x,t)=\left(Mg-M\left(\frac{\partial^2 u}{\partial t^2}+2v\frac{\partial^2 u}{\partial x\partial t}+v^2\frac{\partial^2 u}{\partial x^2}\right)\right)\left(H\left(x-\xi+\frac{\varepsilon}{2}\right)-H\left(x-\xi-\frac{\varepsilon}{2}\right)\right) \tag{5}$$

Where EI is the flexural rigidity, a_1 is the stiffness proportionality factor called damping complex or radius of gyration, y is the deflection/transverse displacement, x is the Coordinate, t is the time, c (x) is the external damping force per unit length, M is the mass of the load or beam, m is the mass per unit length, g is the acceleration due to gravity, H is the Heaviside unit function, Ɛ is the length of the beam, $\xi = vt + \frac{\varepsilon}{2}$ is the fixed length of the load, V is the constant velocity of the load, s is the axial force, and F(x, t) is the applied force.

3. Method of Solution

Combining equations (1) and (5) to have

$$\frac{\partial^2}{\partial x^2}\left(EI(x)\left(\frac{\partial^2 u}{\partial x^2}+a_1\frac{\partial^3 u}{\partial x^2\partial t}\right)\pm sy\right)+m(x)\frac{\partial^2 u}{\partial t^2}+c(x)\frac{\partial u}{\partial t}+kY=\left(Mg-M\left(\frac{\partial^2 u}{\partial t^2}+2v\frac{\partial^2 u}{\partial x\partial t}+v^2\frac{\partial^2 u}{\partial x^2}\right)\right)\left(H\left(x-\xi+\frac{\varepsilon}{2}\right)-H\left(x-\xi-\frac{\varepsilon}{2}\right)\right) \quad (6)$$

To solve (6) a solution is assumed in the form of;

$$U(x,t)=\sum_{i=1}^{n}Y_i(x)T_i(t) \quad . \quad (7)$$

Substitute (7) into (6) resulted in;

$$EI(x)\sum_{i=1}^{n}y_i^{iv}(x)T_i(t)+EI(x)a_1\sum_{i=1}^{n}y_i^{iv}(x)\dot{T}_i(t)\pm s\sum^{n}y_i^{''}(x)T_i(t)+m(x)\sum_{i=1}^{n}y_i(x)\ddot{T}_i(t)+c(x)\sum_{i=1}^{n}y_i(x)\dot{T}_i(t)+k\sum_{i=1}^{n}y_i(x)T_i(t)=$$
$$\left(Mg-M\sum_{i=1}^{n}Y_i(x)\ddot{T}_i(t)+2MVY_i^{'}(x)\dot{T}_i(t)-MV^2Y_i^{''}(x)T_i(t)\right)\left(H\left(x-\xi+\frac{\varepsilon}{2}\right)-H\left(x-\xi-\frac{\varepsilon}{2}\right)\right) \quad (8)$$

Multiplying equation (8) by y_j (x) and integrate along the length of the beam yield

$$\int_0^L\left(EI(x)\sum_{i=1}^{n}y_i^{iv}(x)T_i(t)+EI(x)a_1\sum_{i=1}^{n}y_i^{iv}(x)\dot{T}_i(t)\pm s\sum^{n}y_i^{''}(x)T_i(t)+m(x)\sum_{i=1}^{n}y_i(x)\ddot{T}_i(t)+c(x)\sum_{i=1}^{n}y_i(x)\dot{T}_i(t)+k\sum_{i=1}^{n}y_i(x)T_i(t)\right)y_j(x)dx=\text{Mg}$$
$$\int_0^L y_i(x)\left(H\left(x-\xi+\frac{\varepsilon}{2}\right)-H\left(x-\xi-\frac{\varepsilon}{2}\right)\right)dx \quad -M\sum_{i=1}^{n}\ddot{T}_i(t)\int_0^L y_i(x)y_j(x)\left(H\left(x-\xi+\frac{\varepsilon}{2}\right)-H\left(x-\xi-\frac{\varepsilon}{2}\right)\right)dx$$
$$-2MV\sum_{i=1}^{n}\dot{T}_i(t)\int_0^L y_i^{'}(x)y_j(x)\left(H\left(x-\xi+\frac{\varepsilon}{2}\right)-H\left(x-\xi-\frac{\varepsilon}{2}\right)\right)dx$$
$$-MV^2\sum_{i=1}^{n}T_i(t)\int_0^L y_i^{''}(x)y_j(x)\left(H\left(x-\xi+\frac{\varepsilon}{2}\right)-H\left(x-\xi-\frac{\varepsilon}{2}\right)\right)dx \quad (9)$$

Using the orthogonality condition $\int_{x_1}^{x_2}x_i(x)\delta(x-x_i)dx=x-x_i=x_j(x)$ such that $x_0<x_1<x_2$

Thus, (9) can then be expressed as

$$EIT_i(t)+EIa_1\dot{T}_i(t)\pm sT_i(t)+m(x)\ddot{T}_i+C(x)\dot{T}_i(t)+kT_i(t)=Mg\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}}y_j(x)dx$$
$$-M\sum_{i=1}^{n}\ddot{T}_i(t)\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}}y_i(x)y_jdx-2MV\sum_{i=1}^{n}\dot{T}_i(t)\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}}y_i^{'}(x)y_jdx-MV^2\sum_{i=1}^{n}T_i(t)\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}}y_i^{''}(x)y_jdx \quad (10)$$

For free vibration, $EIy^{iv}=m\omega^2y$ so equation (10) gives

$$m\omega^2 T_i(t) + m\omega^2 a_i \dot{T}_i(t) \pm sT_i(t) + m(x)\ddot{T}_i(t) + c(x)\dot{T}_i(t) + kT_i(t)$$
$$= Mg\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}} y_j(x)dx - M\sum_{i=1}^{n}\ddot{T}_i(t)\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}} y_i(x)y_j dx - 2MV\sum_{i=1}^{n}\dot{T}_i(t)\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}} y_i'(x)y_j dx - MV^2\sum_{i=1}^{n}T_i(t)\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}} y_i''(x)y_j dx \quad (11)$$

For simply supported axial displacement;

$$Y_i(x) = \sqrt{\frac{2}{L}}\sin\left(\left(1+\frac{s}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIi^4\pi^4}\right)^{1/4}\frac{i\pi}{L}x\right) \quad (12)$$

Differentiate (12) and substitute in (11) accordingly resulted into

$$m\ddot{T}_i(t) + \left(m\omega^2 a_i + c(x)\right)\dot{T}_i(t) + \left(m\omega^2 + s + k\right)T_i(t) = Mg\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}}\sqrt{\frac{2}{L}}\sin\left(\left(1+\frac{s}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right)^{1/4}\frac{j\pi}{L}x\right)dx$$
$$-M\sum_{i=1}^{n}\ddot{T}_i(t)\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}}\sqrt{\frac{2}{L}}\sin\left(\left(1+\frac{s}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIi^4\pi^4}\right)^{1/4}\frac{i\pi}{L}x\right)\left(\sqrt{\frac{2}{L}}\sin\left(\left(1+\frac{s}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right)^{1/4}\frac{j\pi}{L}x\right)\right)dx$$
$$-2MV\sum_{i=1}^{n}\dot{T}_i(t)\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}}\sqrt{\frac{2}{L}}\left(\left(1+\frac{s}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIi^4\pi^4}\right)^{1/4}\frac{i\pi}{L}\right)\cos\left(\left(1+\frac{s}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIi^4\pi^4}\right)^{1/4}\frac{i\pi}{L}x\right)\left(\sqrt{\frac{2}{L}}\sin\left(\left(1+\frac{s}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right)^{1/4}\frac{j\pi}{L}x\right)\right)dx$$
$$+MV^2\sum_{i=1}^{n}T_i(t)\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}}\sqrt{\frac{2}{L}}\left(\left(\left(1+\frac{s}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIi^4\pi^4}\right)^{1/4}\frac{i\pi}{L}\right)\right)^2\sin\left(\left(1+\frac{s}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIi^4\pi^4}\right)^{1/4}\frac{i\pi}{L}x\right)\left(\sqrt{\frac{2}{L}}\sin\left(\left(1+\frac{s}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right)^{1/4}\frac{j\pi}{L}x\right)\right)dx$$

(13) is written as

$$m\ddot{T}_i(t) + \left(m\omega^2 a_i + c(x)\right)\dot{T}_i(t) + \left(m\omega^2 \pm s + k\right)T_i(t) = Mg\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}}\sqrt{\frac{2}{L}}\sin\left(B\frac{j\pi}{L}x\right)dx$$
$$-M\sum_{i=1}^{n}\ddot{T}_i(t)\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}}\left(\sqrt{\frac{2}{L}}\sin\left(A\frac{i\pi}{L}x\right)\left(\sqrt{\frac{2}{L}}\sin\left(B\frac{j\pi}{L}x\right)\right)\right)dx$$

$$-2Mv\sum_{i=1}^{n}\dot{T}_i(t)\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}}\left(\sqrt{\frac{2}{L}}\,A\frac{i\pi}{L}\right)\cos\left(A\frac{i\pi}{L}x\right)\left(\sqrt{\frac{2}{L}}\sin\left(B\frac{j\pi}{L}x\right)\right)dx$$

$$+Mv^2\sum_{i=1}^{n}T_i(t)\int_{\xi-\frac{\varepsilon}{2}}^{\xi+\frac{\varepsilon}{2}}\left(\sqrt{\frac{2}{L}}\,A\frac{i\pi}{L}\right)^2\sin\left(A\frac{i\pi}{L}x\right)\left(\sqrt{\frac{2}{L}}\sin\left(B\frac{j\pi}{L}x\right)\right)dx \qquad (14)$$

Where;

$$A = \left(1+\frac{s}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIi^4\pi^4}\right)^{1/4} \text{ and } B = \left(1+\frac{s}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right)^{1/4}$$

Applying trigonometry identity and further simplification of (14) gives;

$$\ddot{T}_i(t)+\left(\omega^2 a_i+\frac{c}{m}(x)\right)\dot{T}_i(t)+\left(\omega^2+\frac{s}{m}+\frac{k}{m}\right)T_i(t)=\frac{2Mg\sqrt{2L}}{Bmj\pi}\left(\sin Bj\frac{\pi}{L}\xi\sin Bj\pi\frac{\varepsilon}{2L}\right)$$

$$+\frac{M}{Am\pi}\sum_{i=1}^{n}\ddot{T}i(t)\left[\frac{1}{(i+j)}\left(2\cos\frac{A\pi}{L}(i+j)\xi\sin\frac{A\pi}{2L}(i+j)\varepsilon\right)-\frac{1}{(i-j)}\left(2\cos B\frac{\pi}{L}(i-j)\xi\sin B\frac{\pi}{2L}(i-j)\varepsilon\right)\right]$$

$$+\frac{2MVA}{mL}\sum_{i=1}^{n}\dot{T}_i(t)\left[\left(\frac{1}{(i+j)}\left(-2\sin\frac{A\pi}{L}(i+j)\xi\sin\frac{A\pi}{2L}(i+j)\varepsilon\right)\right)-\left(\frac{1}{(i-j)}\left(-2\sin\frac{B\pi}{L}(i-j)\xi\sin\frac{B\pi}{2L}(i-j)\varepsilon\right)\right)\right]$$

$$-\frac{MV^2A^2}{mL^2}\sum_{i=1}^{n}T_i(t)\left[\left(\frac{1}{(i+j)}\left(2\cos\frac{A\pi}{L}(i+j)\xi\sin\frac{A\pi}{2L}(i+j)\varepsilon\right)\right)-\left(\frac{1}{(i-j)}\left(2\cos\frac{B\pi}{L}(i-j)\xi\sin\frac{B\pi}{2L}(i-j)\varepsilon\right)\right)\right] \qquad (15)$$

Substitute for A and B into equation (15) and rearranged to have;

$$-\frac{2M}{m\left(1+\frac{s}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIi^4\pi^4}\right)^{1/4}\pi}\sum_{i=1}^{n}\ddot{T}_i(t)\left(\begin{array}{l}\frac{1}{(i-j)}\cos\left(1+\frac{s}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right)^{1/4}\frac{\pi}{L}(i-j)\xi\sin\left(1+\frac{s}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right)^{1/4}\frac{\pi}{2L}(i-j)\varepsilon\\ -\frac{1}{(i+j)}\cos\left(1+\frac{s}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIi^4\pi^4}\right)^{1/4}\frac{\pi}{L}(i+j)\xi\sin\left(1+\frac{s}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIi^4\pi^4}\right)^{1/4}\frac{\pi}{2L}(i+j)\varepsilon\end{array}\right)$$

$$+\frac{4MV}{mL}\left(\left(1+\frac{s}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right)^{1/4}\frac{i\pi}{L}\right)\sum_{i=1}^{n}\dot{T}_i(t)\left(\begin{array}{l}\frac{1}{(i-j)}\sin\left(1+\frac{s}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right)^{1/4}\frac{\pi}{L}(i-j)\xi\sin\left(1+\frac{s}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right)^{1/4}\frac{\pi}{2L}(i-j)\varepsilon\\ -\frac{1}{(i+j)}\sin\left(1+\frac{s}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIi^4\pi^4}\right)^{1/4}\frac{\pi}{L}(i+j)\xi\sin\left(1+\frac{s}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIi^4\pi^4}\right)^{1/4}\frac{\pi}{2L}(i+j)\varepsilon\end{array}\right)$$

$$+\frac{2MV^2}{mL^2}\left(\left(1+\frac{s}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIi^4\pi^4}\right)^{1/4} i\pi\right)^2 \sum_{i=1}^{n} T_i(t)\left(\begin{array}{l}\frac{1}{(i-j)}\cos\left(1+\frac{s}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right)^{1/4}\frac{\pi}{L}(i-j)\xi\sin\left(1+\frac{s}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right)^{1/4}\frac{\pi}{2L}(i-j)\varepsilon \\ -\frac{1}{(i+j)}\cos\left(1+\frac{s}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIi^4\pi^4}\right)^{1/4}\frac{\pi}{L}(i+j)\xi\sin\left(1+\frac{s}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIi^4\pi^4}\right)^{1/4}\frac{\pi}{2L}(i+j)\varepsilon\end{array}\right) \quad (16)$$

4. Discussion of Results

In this paper, two cases were considered; the moving force and the moving mass.

4.1 The moving force

In this case, the inertia effect of the moving load is neglected from equation (16) and only the first term on the right hand side was put into consideration resulted to the case of moving force, thus;

$$\ddot{T}_i(t)+2\omega_b\dot{T}_i(t)+\left(\omega^2+\frac{s}{m}+\frac{k}{m}\right)T_i(t)=\frac{\sqrt{8L}}{m\left(1+\frac{s}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right)^{1/4}}Mg\sin\left(1+\frac{s}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right)^{1/4} j\frac{\pi}{L}\xi\sin\left(1+\frac{s}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right)^{1/4}\frac{j\pi}{2L}\varepsilon \quad (17)$$

Where;

$2\omega_b$ is assumed to be $\omega^2 a_i+\frac{c}{m}(x)$

4.2 The moving mass

In this case, both the inertia and force effects of the moving load were put into consideration in (16) to have;

$$\left[1+\frac{2M}{m\left(1\pm\frac{s}{EI}\left(\frac{L}{i\pi}\right)^2\right)^{1/4}\pi}\sum_{i=1}^{n}\left(\begin{array}{l}\frac{1}{(i-j)}\cos\left(1\pm\frac{s}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right)^{1/4}\frac{\pi}{L}(i-j)\xi\sin\left(1\pm\frac{s}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIi^4\pi^4}\right)^{1/4}\frac{\pi}{2L}(i-j)\varepsilon \\ -\frac{1}{(i+j)}\cos\left(1\pm\frac{s}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIi^4\pi^4}\right)^{1/4}\frac{\pi}{L}(i+j)\xi\sin\left(1\pm\frac{s}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIi^4\pi^4}\right)^{1/4}\frac{\pi}{2L}(i+j)\varepsilon\end{array}\right)\right]\ddot{T}_i(t)$$

$$\left(\left(\omega^2 a_i+\frac{c}{m}(x)\right)-\frac{4MV}{mL}\left(\left(1+\frac{s}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIi^4\pi^4}\right)^{1/4} i\pi\right)\sum_{i=1}^{n}\left(\begin{array}{l}\frac{1}{(i-j)}\sin\left(1\pm\frac{s}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right)^{1/4}\frac{\pi}{L}(i-j)\xi\sin\left(1\pm\frac{s}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right)^{1/4}\frac{\pi}{2L}(i-j)\varepsilon \\ -\frac{1}{(i+j)}\sin\left(1\pm\frac{s}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIi^4\pi^4}\right)^{1/4}\frac{\pi}{L}(i+j)\xi\sin\left(1\pm\frac{s}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIi^4\pi^4}\right)^{1/4}\frac{\pi}{2L}(i+j)\varepsilon\end{array}\right)\right)\dot{T}_i(t)$$

$$\left(\left(\omega^2\pm\frac{s}{m}+\frac{k}{m}\right)-\frac{2MV^2\left(\left(1+\frac{s}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIi^4\pi^4}\right)^{1/4} i\pi\right)^2}{mL^2}\sum_{i=1}^{n}\frac{1}{(i-j)}\left(\begin{array}{l}\cos\left(1\pm\frac{s}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right)^{1/4}\frac{\pi}{L}(i-j)\xi\sin\left(1\pm\frac{s}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right)^{1/4}\frac{\pi}{2L}(i-j)\varepsilon \\ -\frac{1}{(i+j)}\cos\left(1\pm\frac{s}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right)^{1/4}\frac{\pi}{L}(i+j)\xi\sin\left(1\pm\frac{s}{EI}\left(\frac{L}{i\pi}\right)^2+\frac{kL^4}{EIi^4\pi^4}\right)^{1/4}\frac{\pi}{2L}(i+j)\varepsilon\end{array}\right)\right)T_i(t)=$$

$$\frac{\sqrt{8L}}{m\left(1\pm\frac{s}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right)^{1/4}}Mg\sin\left(1\pm\frac{s}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right)^{1/4}j\frac{\pi}{L}\xi\sin\left(1\pm\frac{s}{EI}\left(\frac{L}{j\pi}\right)^2+\frac{kL^4}{EIj^4\pi^4}\right)^{1/4}\frac{j\pi}{L}\varepsilon \quad (18)$$

(17) and (18) were solved numerically by the help of maple software and the adoption of the following numerical values: $\lambda = 0.1$, E = 2.07x10^{11}, R:= 55.6x10^{6}, a = 1.0 ε = 0.1, L = 10, α = 10, θ = 0.1, m = 70, V=3.3, g = 9.8, π =3.142, c = 0.1 and $\omega = \sqrt{\frac{\lambda^4 EI}{m}}$ used by Sangoniyi and Akinpelu (2017).

Figures 1 and 2 are the graphs of displacement against time for moving force and moving mass respectively for various values of damping coefficient (ω). It was observed that as ω increases the deflection decreases for both the moving force and the moving mass.

Figures 3 and 4 are the graphs of displacement against time for moving force and moving mass respectively for various values of axial force (s). It was showed that the deflection decreases for both the two cases as axial force (s) increases.

Figures 5 and 6 are the graphs of displacement against time for moving force and moving mass respectively for various values of elastic foundation (k). It was discovered that as elastic foundation (k) increases the deflection decreases for both the moving force and the moving mass.

Figure 7 showed that the deflection under the action of moving mass is greater than that under the action of the moving force.

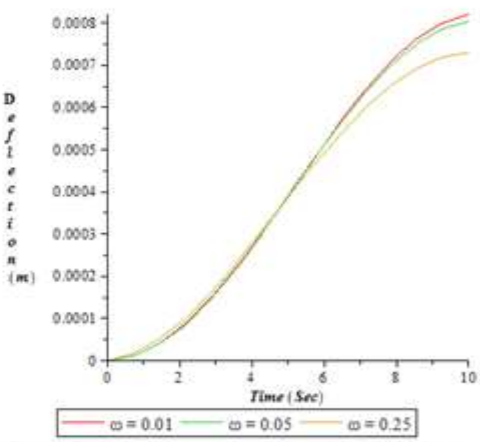

Figure 1: Displacement against time under the action of moving force for various values of damping coefficient

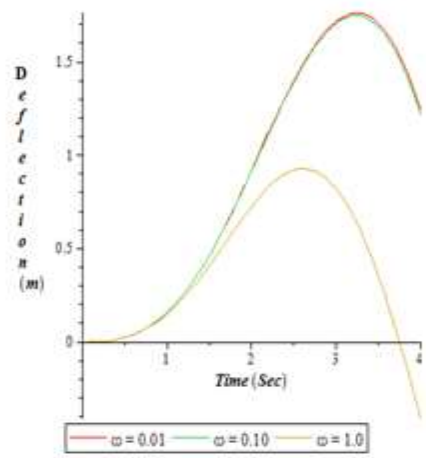

Figure 4: Displacement against time under the actio of moving mass for various values of axial force (S)

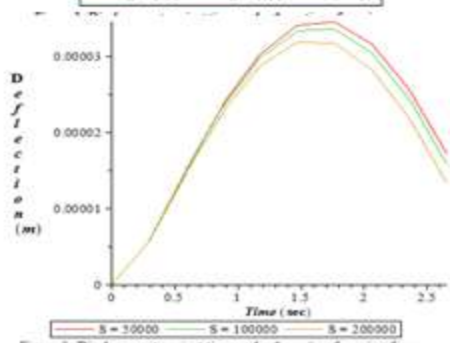

Figure 3: Displacement against time under the actio of moving force for various values of axial force (S)

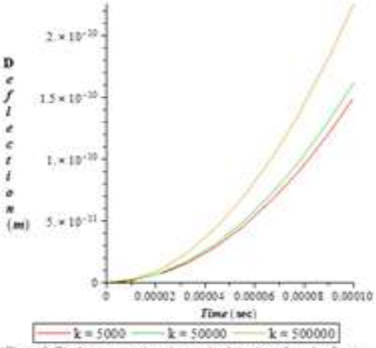

Figure 5: Displacement against time under the action of moving force for various values of elastic foundation (k)

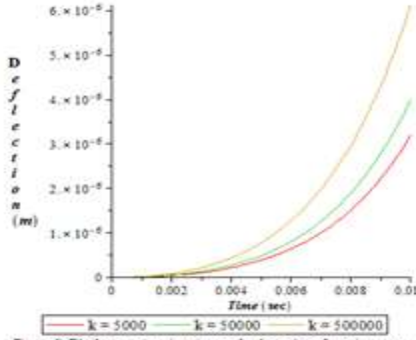

Figure 6: Displacement against time under the action of moving mass for various values of elastic foundation (k)

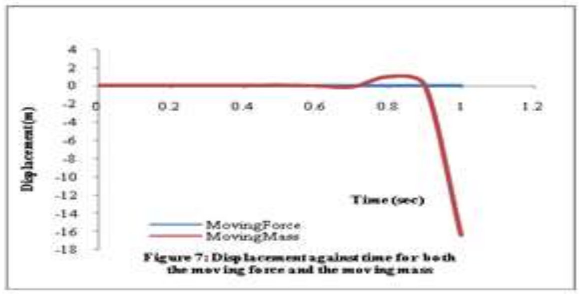

Figure 7: Displacement against time for both the moving force and the moving mass

5. Conclusion

Dynamics response of beam with viscously damped axial force on elastic foundation was examined. The fourth order partial differential equation (the governing equation) was solved in form of series solution to obtain two distinct equations which are for both the moving force and the moving mass and were finally solved numerically by maple software. The results of the study revealed that as damping coefficient (ω) increases the deflection decreases for both the moving force and the moving mass. It was also showed that the deflection decreases as axial force (S) increases for both the two cases. It was discovered that as elastic foundation (K) increases the deflection increases for both the moving force and the moving mass. Lastly, it was showed that the displacement for the moving mass is greater than that of the moving force.

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