Inflation, curvature and kinetic dominance

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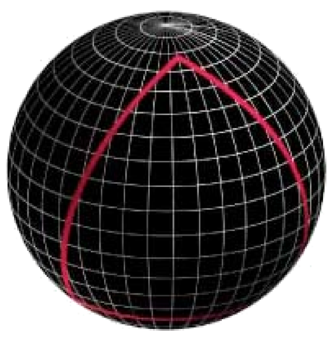
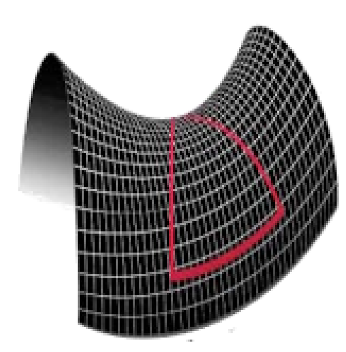
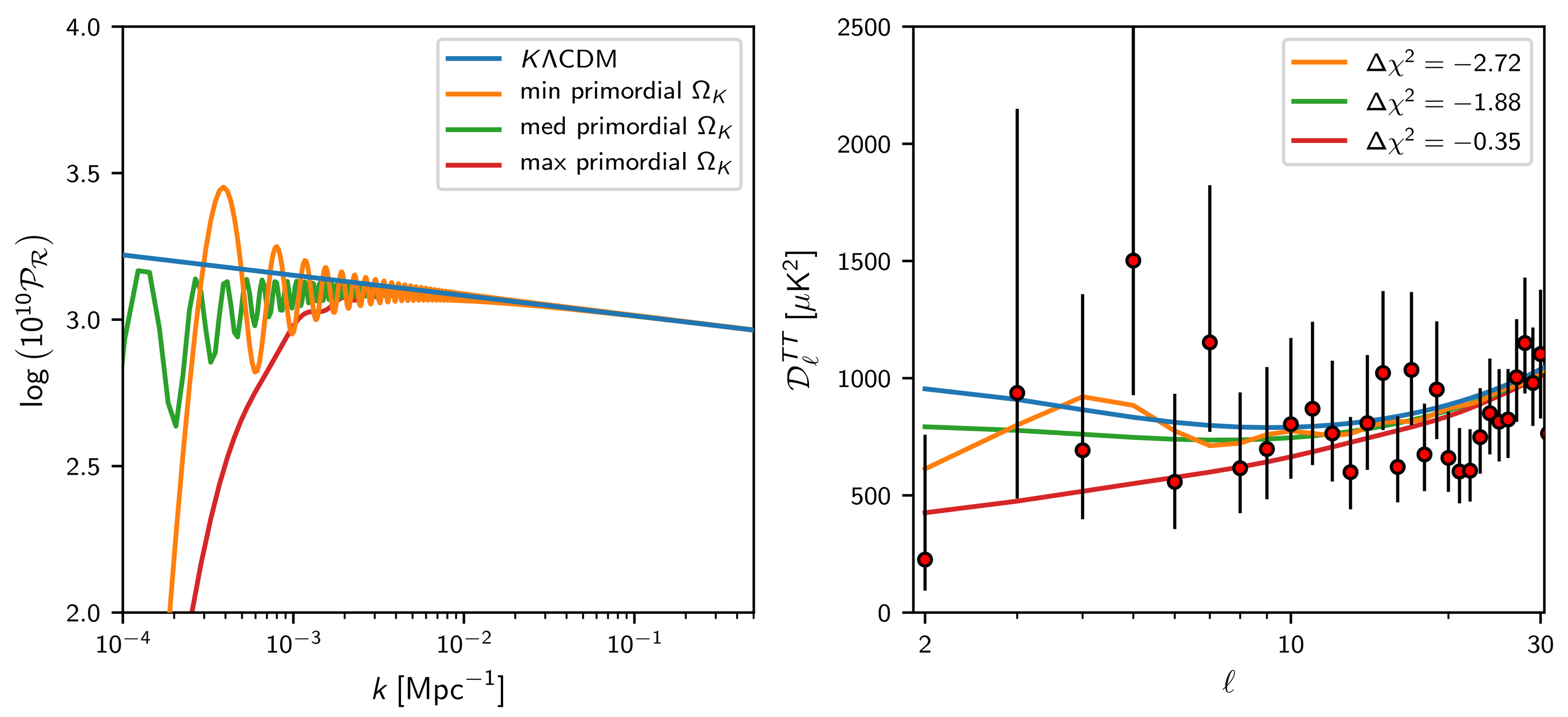


Despite the long history of cosmological models which include spatial curvature, there is a strong research community bias toward a flat universe. This is partly on theoretical grounds, but mostly derived from historical observations. Today, the Planck 2018 cosmic microwave background data suggest a model that is closed at $\Omega_K \sim -4.5\% \pm 1.5\%$, with betting odds of 50 : 1 against a flat universe. Other datasets such as CMB lensing or baryon acoustic oscillations which strongly suggest a flat universe are suspiciously inconsistent at 2.5 to 3σ with CMB data alone, and should not be confidently combined until this tension is released. If this is not a systematic effect in the `plik` or late-time likelihoods, then the inclusion of primordial curvature in models of inflation may explain further unresolved features in the cosmic microwave background, with implications for the Hubble tension.



Curved primordial power spectra



Primordial and CMB power spectra computed from the curved Mukhanov-Sazaki equation at the Planck best-fit parameter values [1]. In all cases, there is a generic suppression of power at large angular scales and oscillatory features. The jagged edges of the curves at low-k arise from the discrete wavevectors for closed universes. The improvement in $\Delta\chi^2$ relative to a spectrum without features is shown in the right-hand figure legend, with negative values indicating a closer fit to the data.

Setting initial conditions for the mode evolution in the curved regime is non-trivial as for small to medium modes the background is not in a de-Sitter state. Renormalised stress-energy initial conditions [2] provide a better fit in comparison with Bunch-Davies vacua.

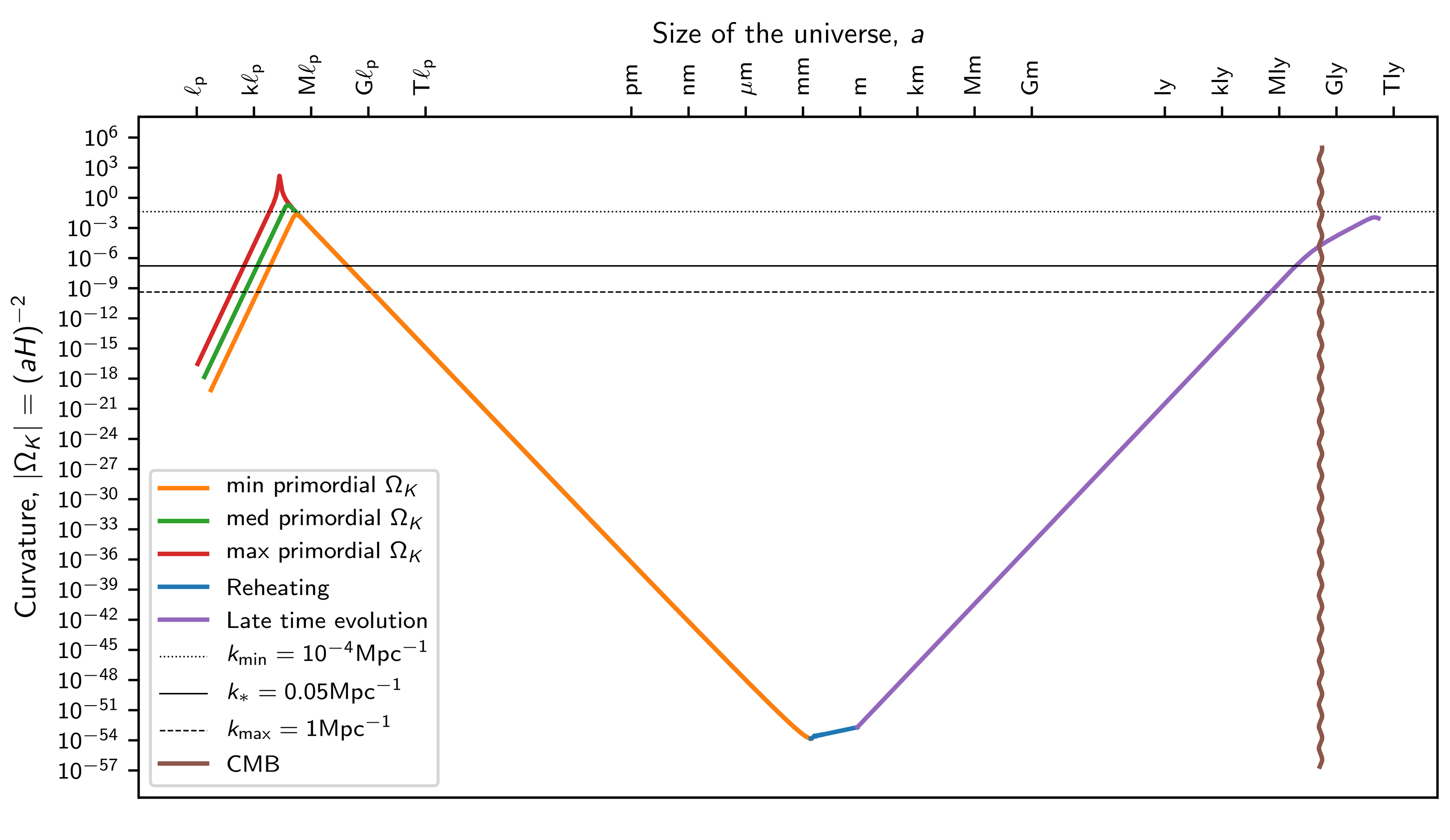
The Mukhanov-Sazaki equation

The comoving curvature perturbation for curved universes evolves according to [1]

$$0=\ddot{\mathcal{R}}+\frac{\left(H+\frac{2\dot z}{z}\right)\mathcal{D}^2-\frac{3KHz^2}{2a^2}}{\mathcal{D}^2-\frac{Kz^2}{2a^2}}\dot{\mathcal{R}}+\frac{K\left(1+\frac{z^2}{2a^2}-\frac{2}{H}\frac{\dot z}{z}\right)\mathcal{D}^2+\frac{K^2z^2}{2a^2}-\mathcal{D}^4}{a^2\left(\mathcal{D}^2-\frac{Kz^2}{2a^2}\right)}\mathcal{R},\qquad \mathcal{D}^2=\nabla^2+3K.$$

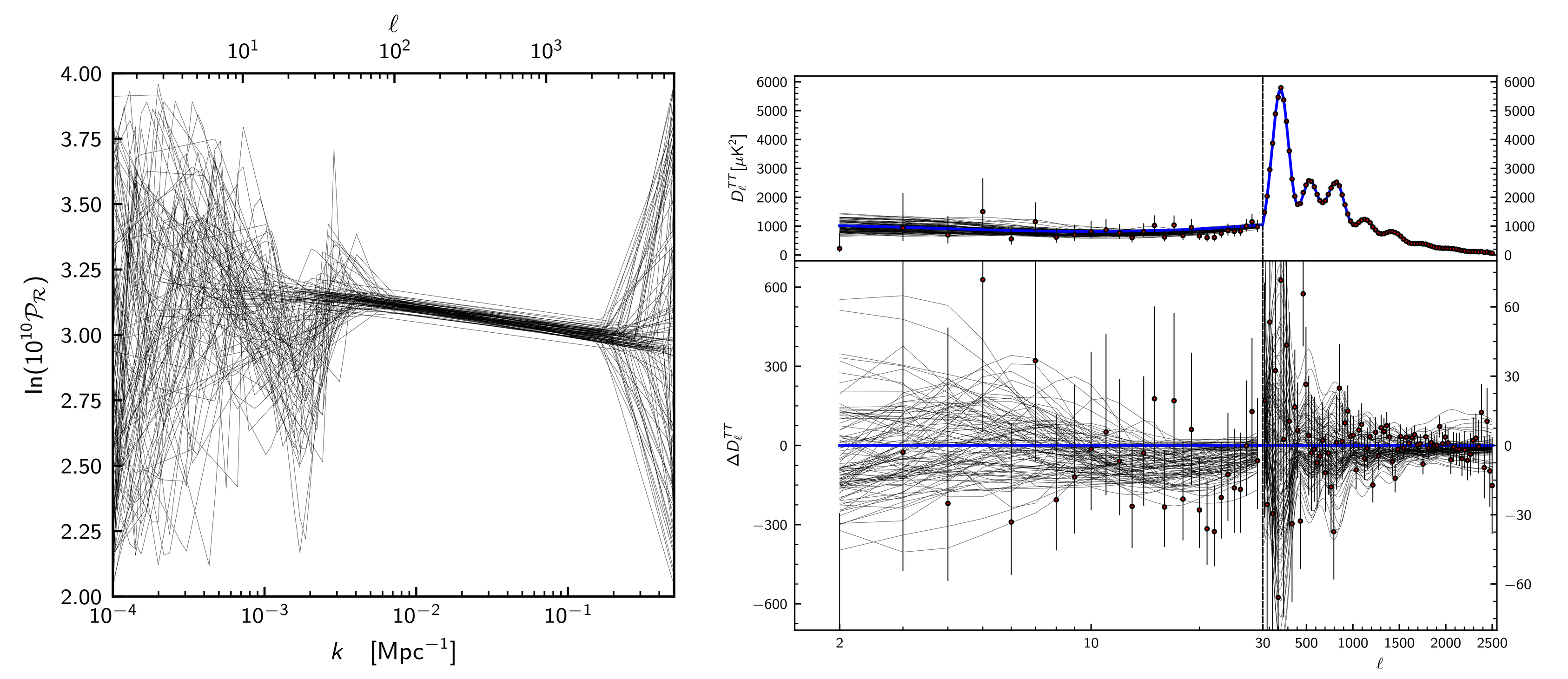
This equation is considerably more complicated than for the flat case, resulting in the non-trivial behaviour for the primordial power spectrum at low k seen above. The equation reduces to the flat equivalent if $K = 0$ or $k \gg 1$. Solving these mode equations rapidly through the non slow-roll regime requires novel techniques for solving ODEs [3–6].

Horizon history for curved universes



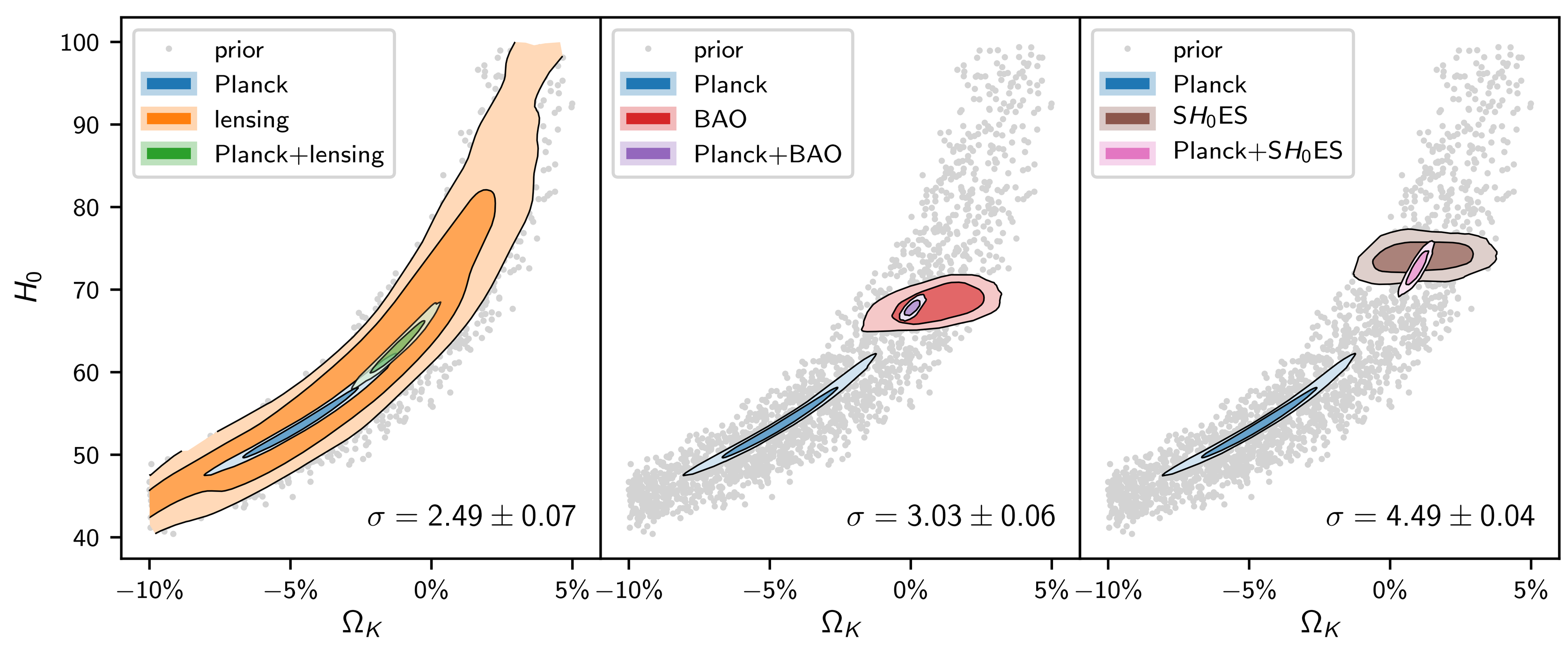
Curvature history for a $K\Lambda$CDM universe with best-fit baseline Planck 2018 parameter values and a $\phi^{4/3}$ inflationary potential. One can see the transition through various evolutionary phases: kinetic dominance [7–10], fast-roll inflation, slow-roll inflation, reheating, radiation, matter, CMB crossing and dark energy domination. The horizontal lines linking the primordial and late time universe indicate the CMB observational window. The presence of any detectable late-time curvature provides a strong justification for "just enough inflation" theories, as curvature breaks slow-roll inflation at early times. Interestingly, for Planck best-fit cosmological parameters, ϕ^2 and Starobinsky inflation potentials are incapable of producing a consistent horizon history, producing too much inflation to match onto late time evolution without superluminal reheating.

Reconstructed primordial power spectra



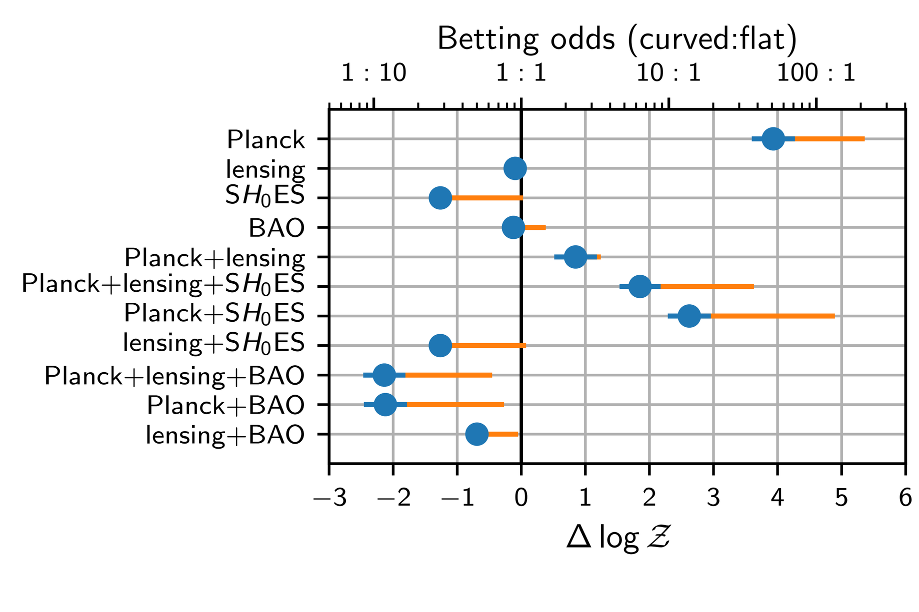
Free-form reconstructions of features in the primordial power spectrum [11], produced using `PolyChord` [12] and plotted using `fgivenx` [13] for a linear spline reconstruction with 8 movable knots. Although these reconstructions were applied to flat universes, the features recovered are similar to the generic predictions for curved primordial and CMB power spectra.

Curvature tension



Curvature tension [14] plotted using `anesthetic` [15]. The first panel shows that when combined with Planck, CMB lensing draws the posterior significantly toward flatness at a tension of 2.5σ measured using the suspiciousness statistic [16,17]. The second panel shows BAO's (BOSS+RSD) preference for a flat universe. The BAO posterior is disconnected from the Planck posterior, at a tension of 3σ. Finally, in the third panel the Planck-SH_0ES inconsistency in the curved case is shown to be 4.5σ. In light of these tensions, Planck and CMB lensing should not be combined when constraining curved models.

Evidence for a closed universe



Model comparison [12] between curved and flat cosmologies [14]. Positive Bayesian log-evidence indicates favouring of curved universes. The Occam regularisation penalty associated with the additional Ω_K parameter is shown in orange, estimated via the difference in KL divergence between the two models. Planck CMB data (`plik`) alone prefer closed universes with odds of 50 : 1.

References

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